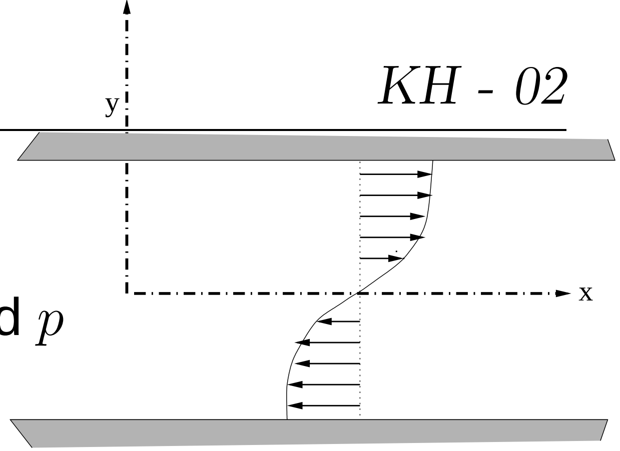
Nonlinear evolution of the Kelvin-Helmholtz instability

keppens@rijnh.nl

- the Kelvin-Helmholtz instability in compressible HD
- Linear (2D) results

  $\Rightarrow$ HD versus MHD: parallel and anti-parallel $\mathbf{B}$ configurations
- Non-linear (2D) behaviour
- 3D simulations: A case study

Kelvin-Helmholtz instability

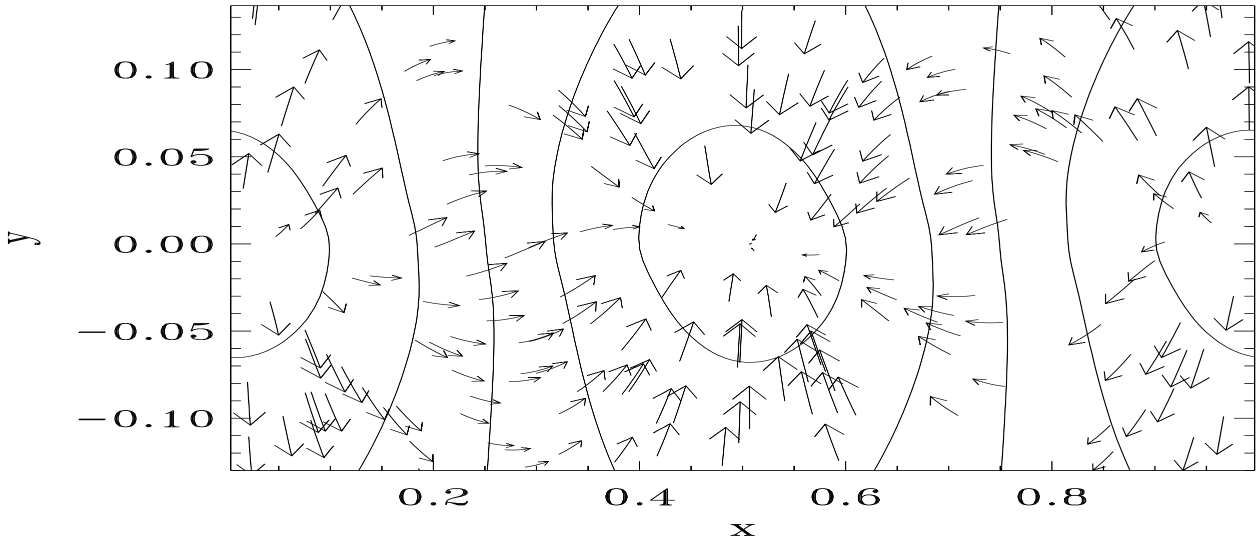


- 2D (x, y)-plane, compressible HD, constant ρ and p
- shear $\tanh(y/a)$ horizontal v_x flow profile, width $2a$
- perturb with vertical flow $\delta v_y \sim \sin(k_x x)$

  $\Rightarrow$ within shear layer: $-\rho v_x' \delta v_y$ horizontal force

  $\Rightarrow$ density ($\propto p$) redistributed in compression-depression pattern

  $\Rightarrow$ pressure perturbation $\rightarrow$ pressure gradient force field

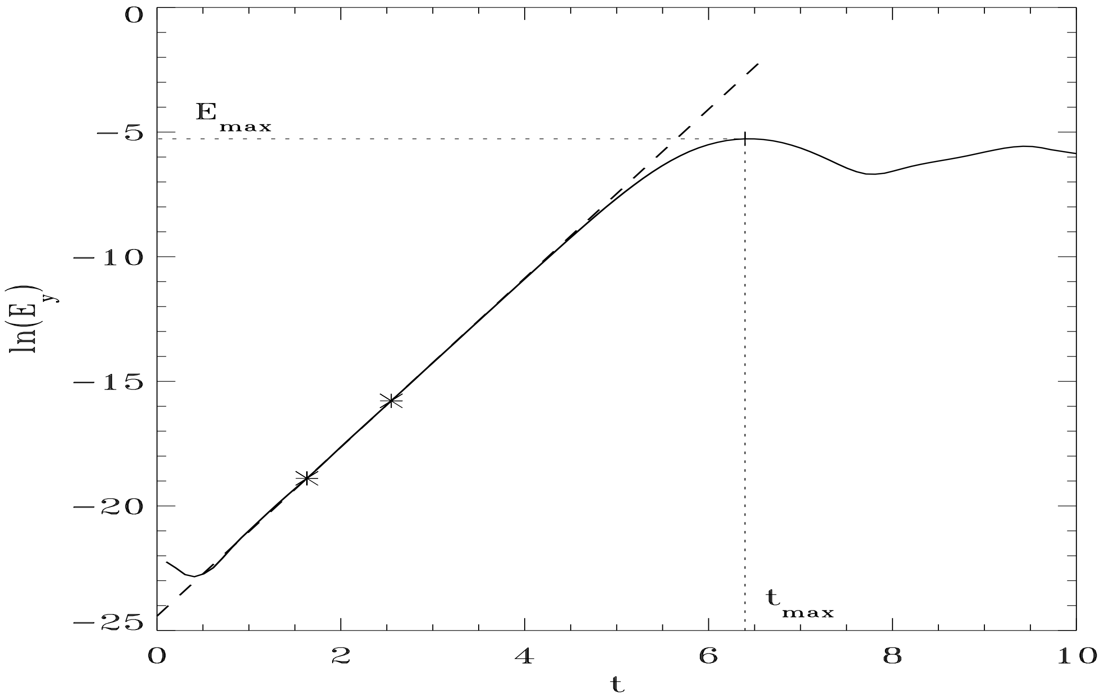


$\Rightarrow$ amplifies the vertical velocity perturbation: unstable!

- 2D studies: HD and MHD uniform horizontal ($\|$ flow) and reversed $\mathbf{B}$

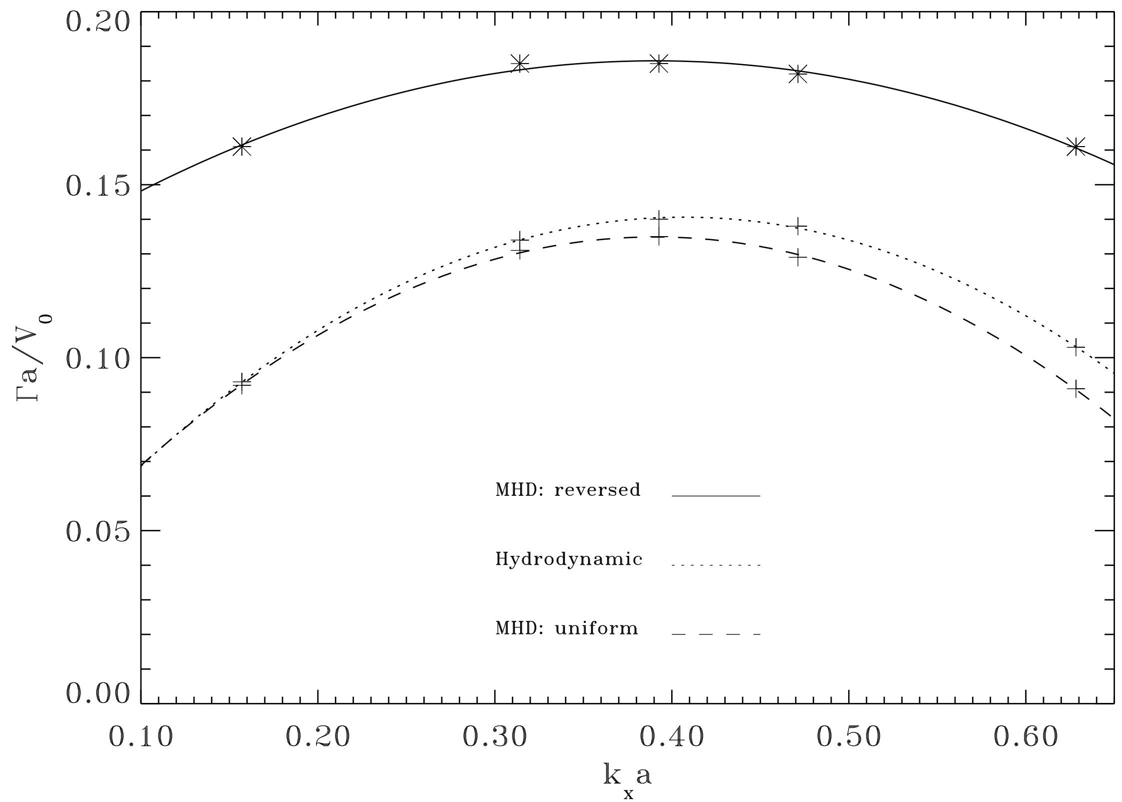
  $\Rightarrow$ reversed field: uniform strength, direction flips at $y = 0$

- linear and non-linear behaviour: growth & saturation
- monitor vertical kinetic energy: deduce growth rate and E_{max}



Linear (2D) results

- reference case with $V_0 = 0.645$, $a = 0.05$, $k_x = 2\pi$, $B_0 = 0.129$
  - $\Rightarrow$ KH unstable for HD, initially weak $\mathbf{B}$ field in MHD
  - $\Rightarrow$ dimensionless $k_x a = 0.314$, $M_s = 0.5$, $M_a = 5$, $\beta = 120$
  - $\Rightarrow$ growth rates agree with LEDAFLOW results!
- dependence of growth Γ on wavelength

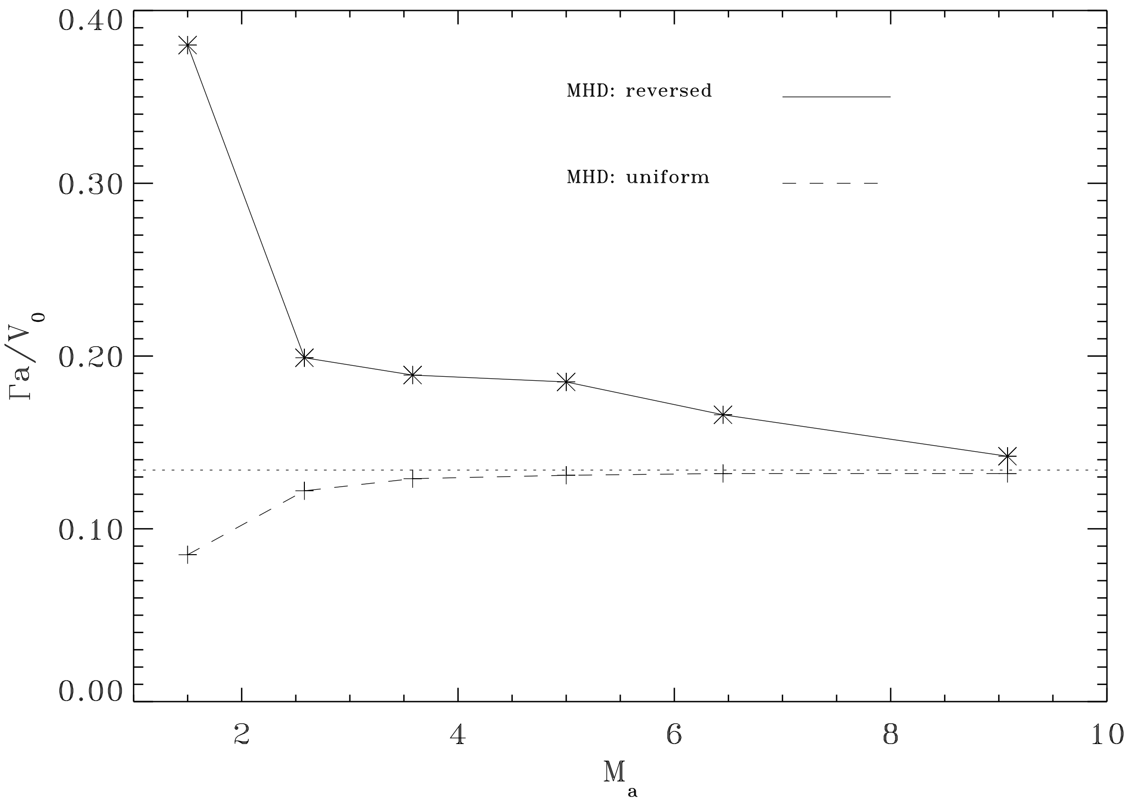


- stabilizing influence of uniform $\mathbf{B}$

  $\Rightarrow$ tension in field lines

- destabilizing influence of reversed $\mathbf{B}$ stronger for $\mathbf{B}$ $\nearrow$

  $\Rightarrow$ growth rate as function of Alfvén Mach M_a



Nonlinear results

- KH instability for reference parameter values

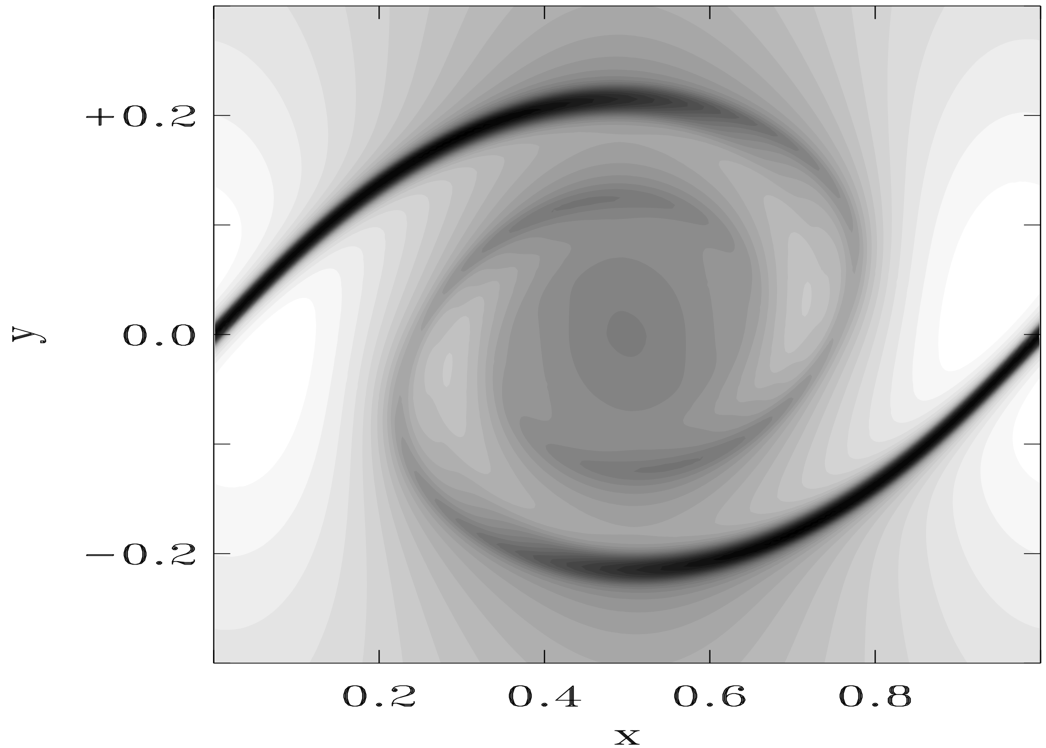
  $\Rightarrow$ weak initial $\mathbf{B}$ ($\beta = 120$): vortical flow pattern sets in

  $\Rightarrow$ magnetic field dragged into spiral configuration

  $\Rightarrow$ eventually gets amplified in thin spiral sheetlike structure

  $\Rightarrow$ $\mathbf{B}$ locally dynamically important: halts further winding

  $\Rightarrow$ where $\mathbf{B}$ is amplified in sheets: cospatial low density lanes

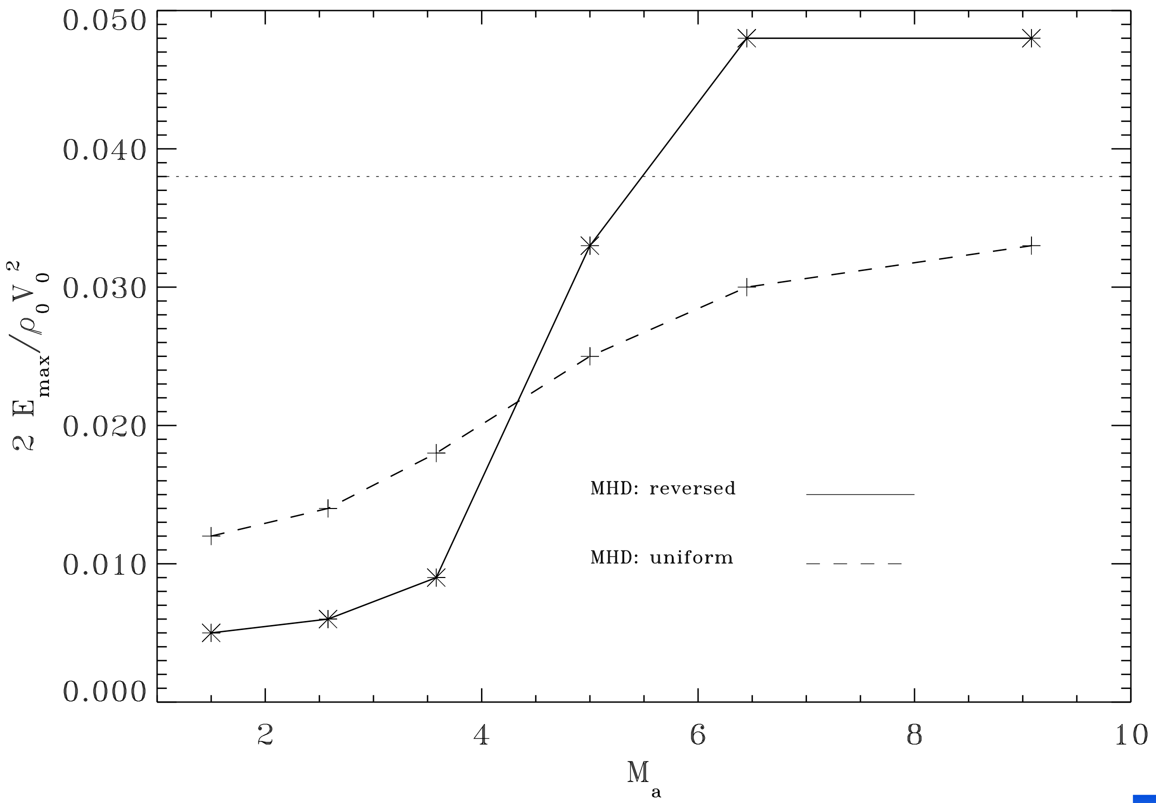


- study of saturation energy level E_{max} versus $k_x a$, M_s, and M_a

  $\Rightarrow$ indication of non-linear saturation

- dependence on Alfvén Mach number (initial field strength)

  $\Rightarrow$ reversed case may saturate above/intermediate/below HD value



- reversed case: initial $\mathbf{J} = \nabla\times\mathbf{B}$ infinitely thin sheet

  $\Rightarrow$ limit $b \to 0$ with $B_x = \tanh(y/b)$ [then $p(y)$]

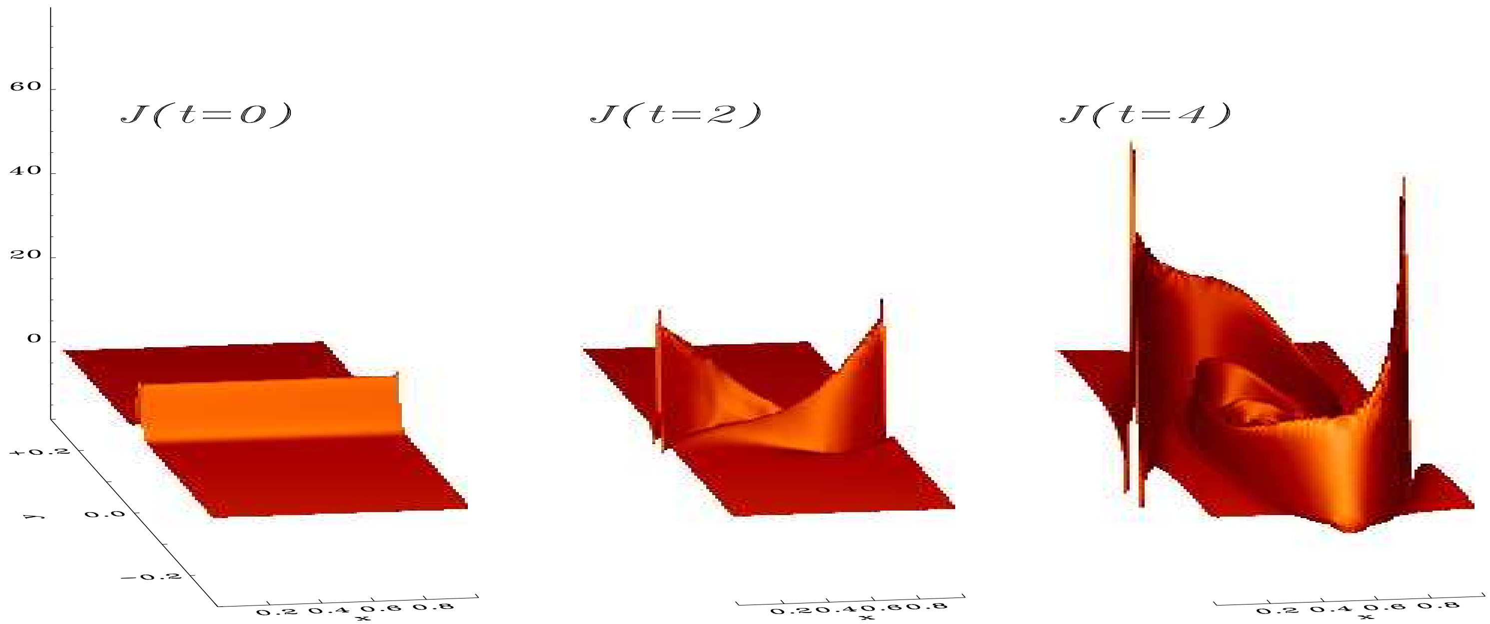
  $\Rightarrow$ additional pinching mode (with reconnection) accessible

- current sheet gets amplified by vortex flow

  $\Rightarrow$ strong current sheet $\Rightarrow$ resistive dissipation important

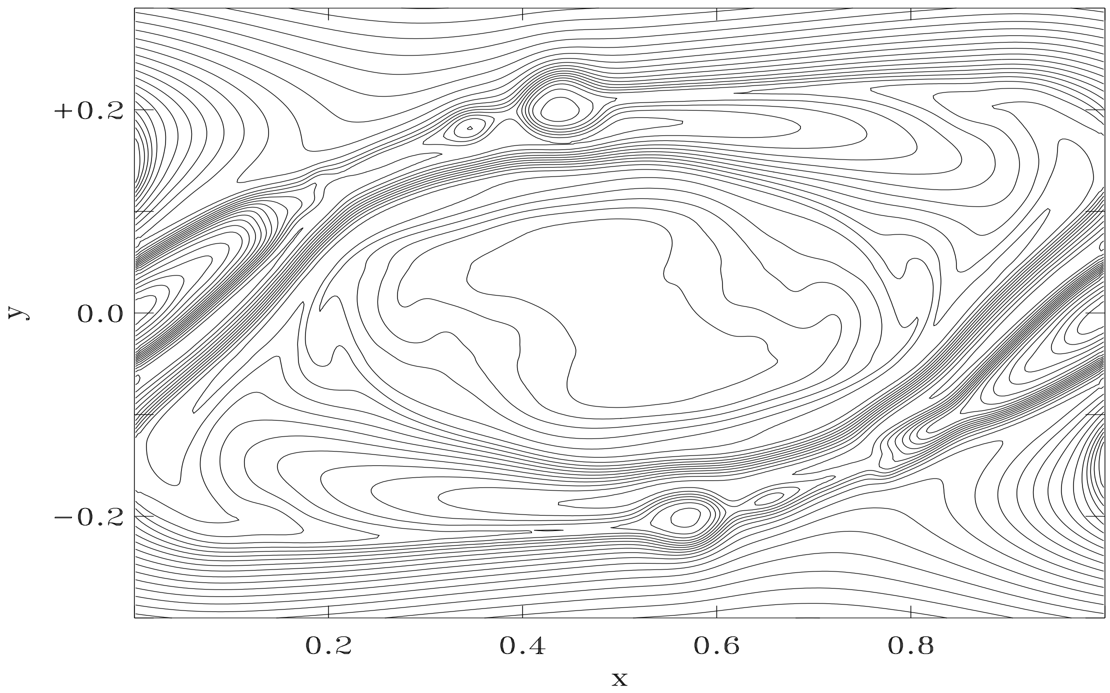
  $\Rightarrow$ systematic study in resistive MHD for $\eta \to 0$ (finer grid)

  $\Rightarrow$ extra magnetic energy tapped



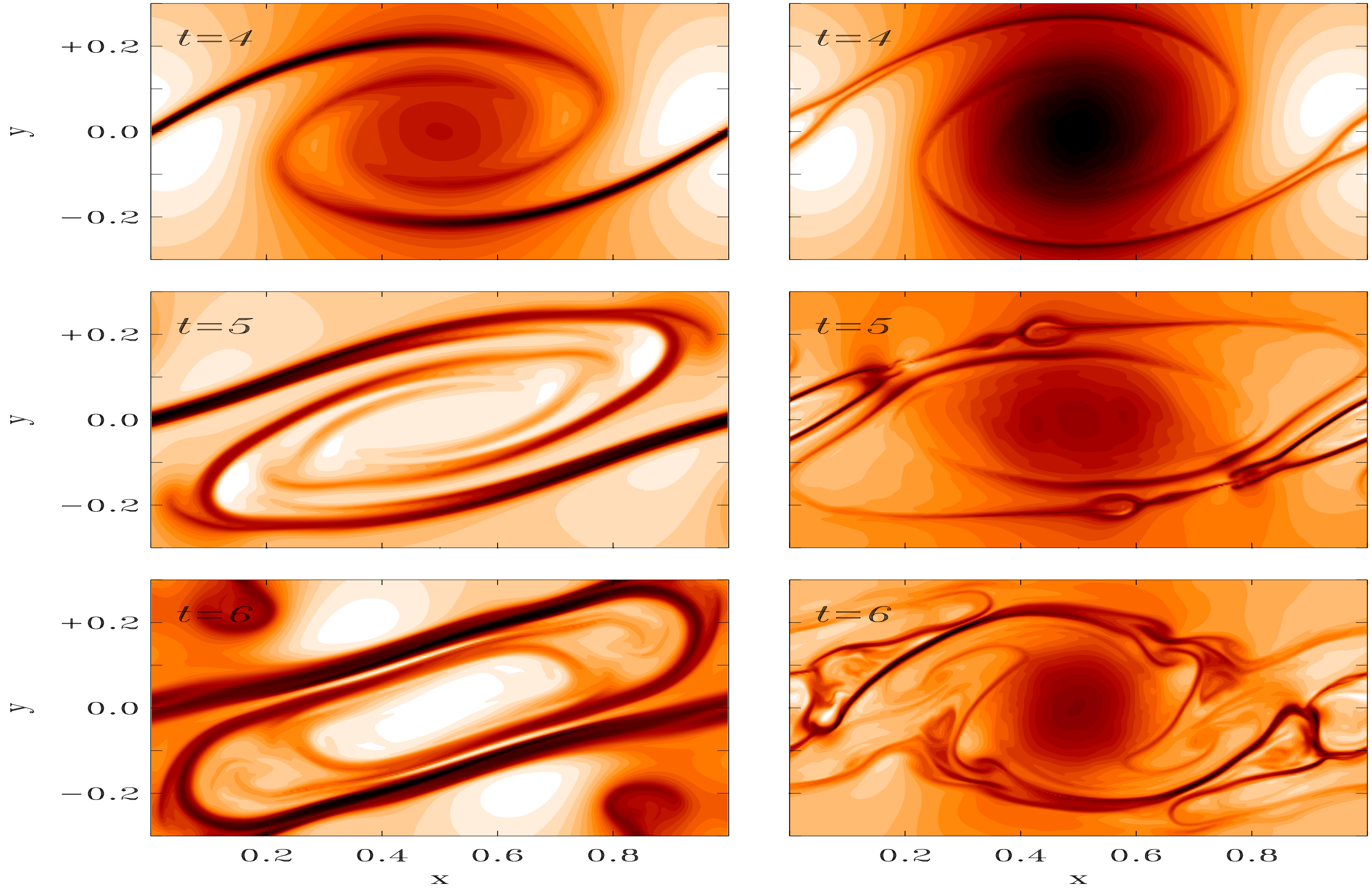
- anti-parallel field lines pushed together

  $\Rightarrow$ magnetic islands form (tearing unstable)



- reconnection plays role sooner than for uniform case
- turbulent state sets in fast, complicating saturation behaviour
- compressibility: density deviations up to 40 %

- density evolution for case without and with **J** -sheet



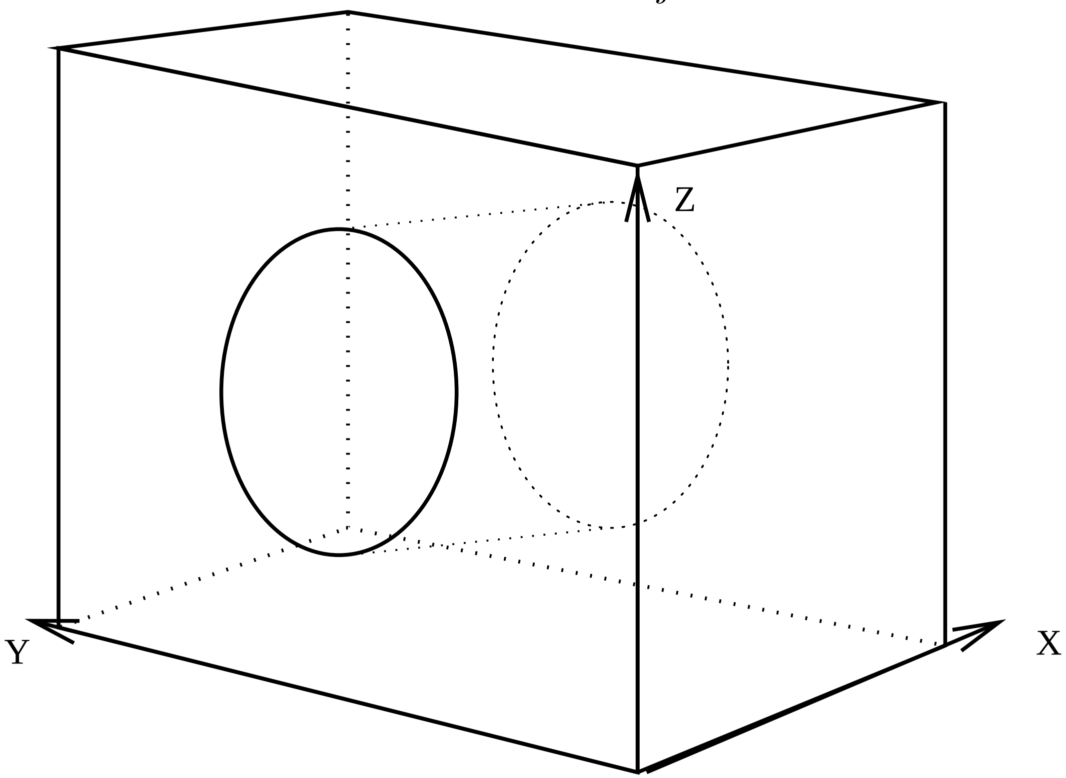
⇒ induced island formation (‘tearing’) when **J** -sheet

⇒ 2D current-vortex transits to magnetically modified turbulence

⇒ KH unstable shear flow triggers small-scale reconnection events

3D KH case study

- Model [astrophysical] jet of radius R_{jet}



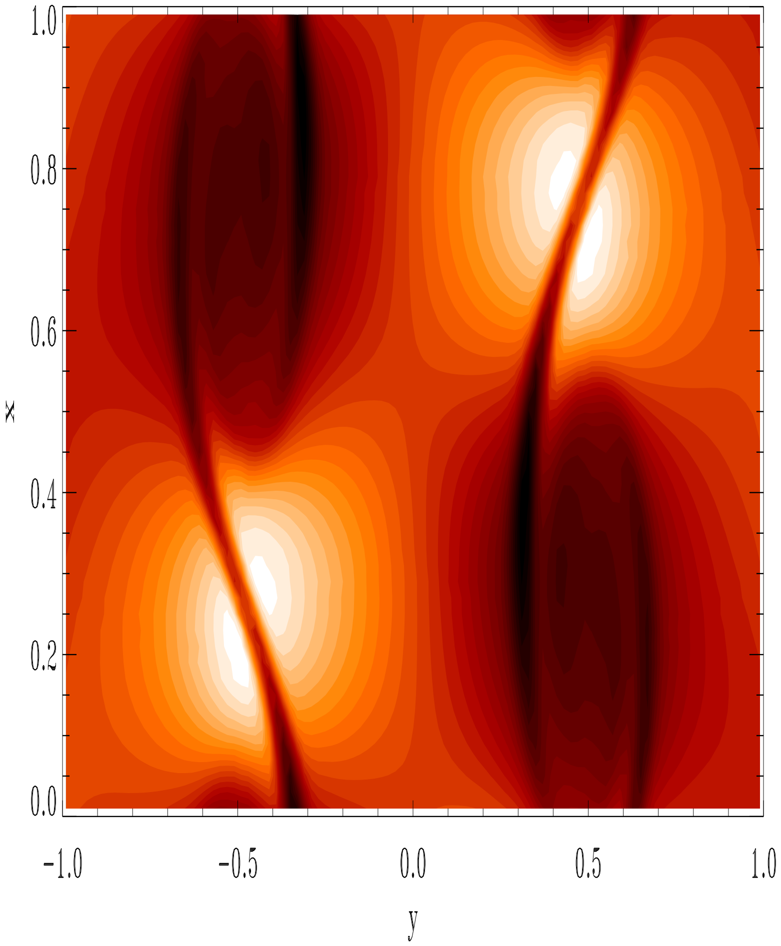
$\Rightarrow$ shear flow (width $2a$) across its circumference

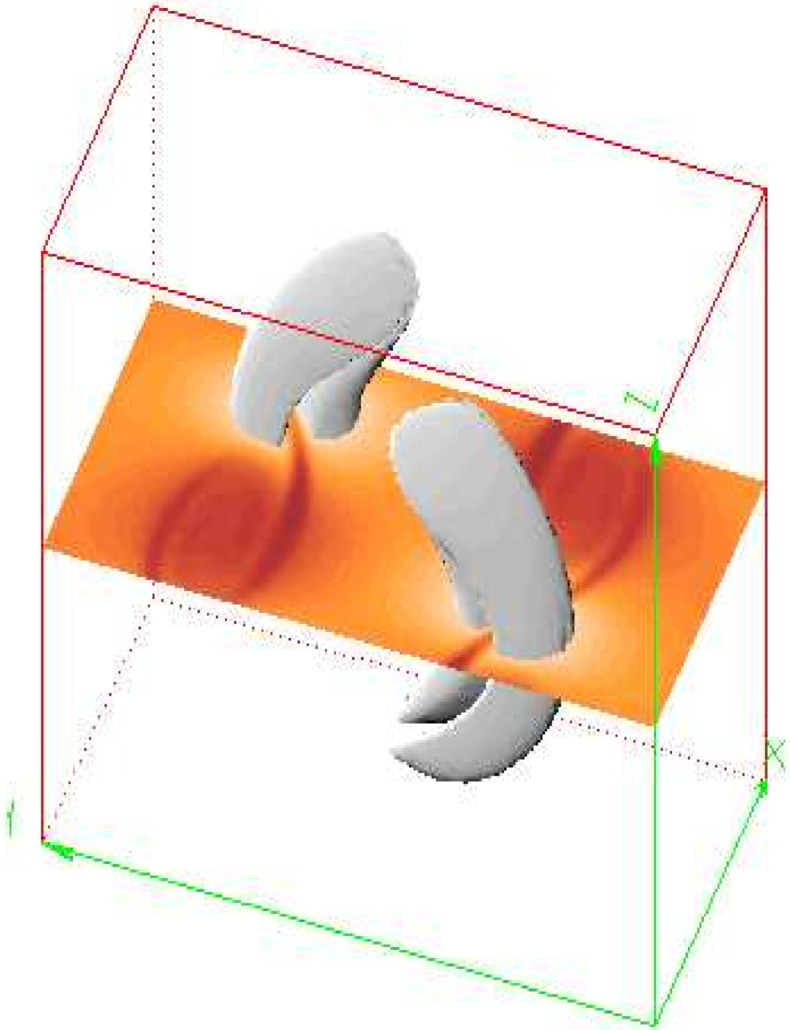
$\Rightarrow t = 0$ parallel uniform $\mathbf{B}$, reference $V_0 = 0.645$, $a = 0.05$, $B_0 = 0.129$

- 3D perturbation wavenumber n along jet, m about jet axis

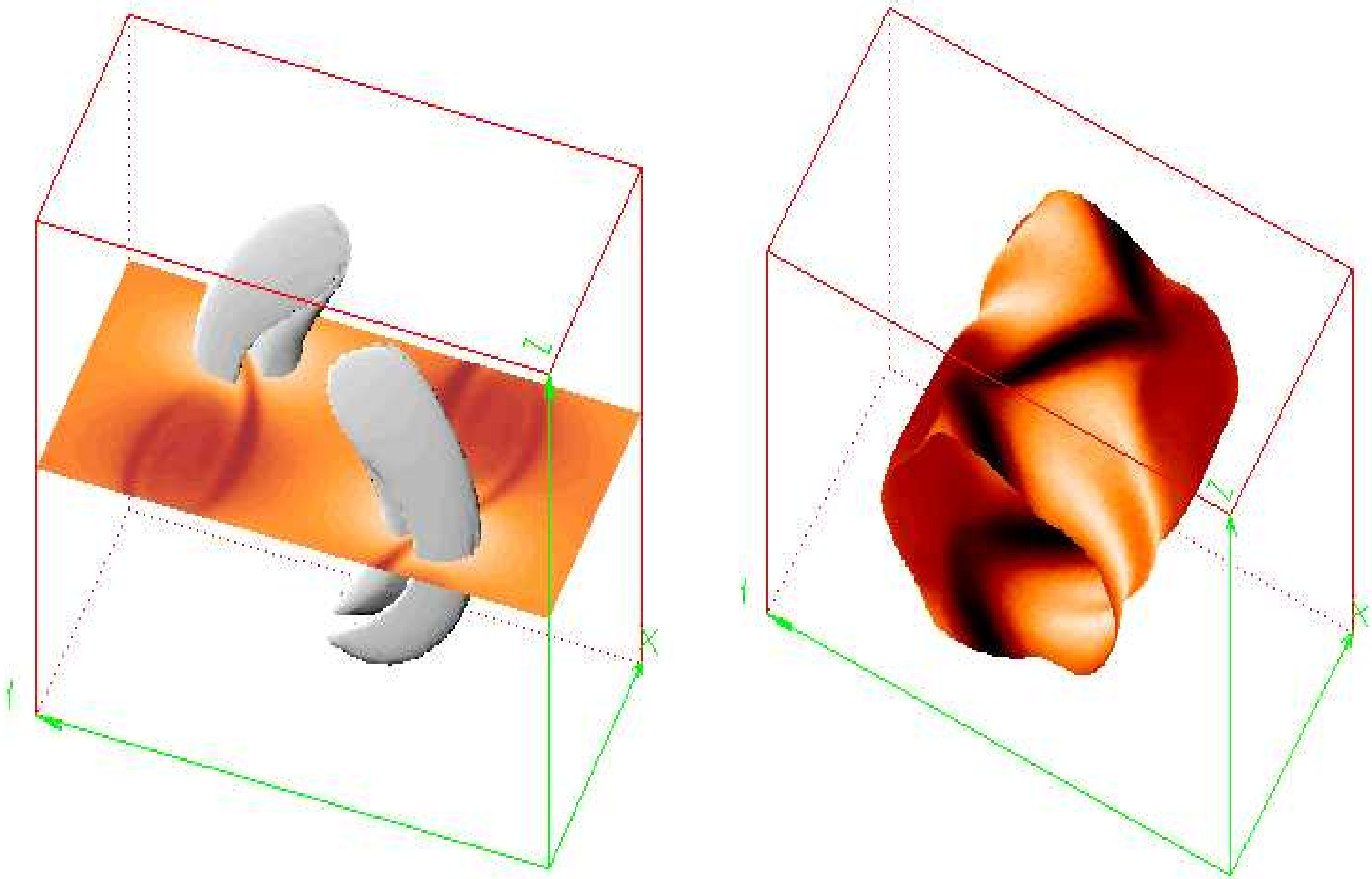
$\Rightarrow$ sideways ‘kink’ perturbation $m = 1 = n$

- in horizontal cross-cut: doubled 2D result: ρ at $t = 4$

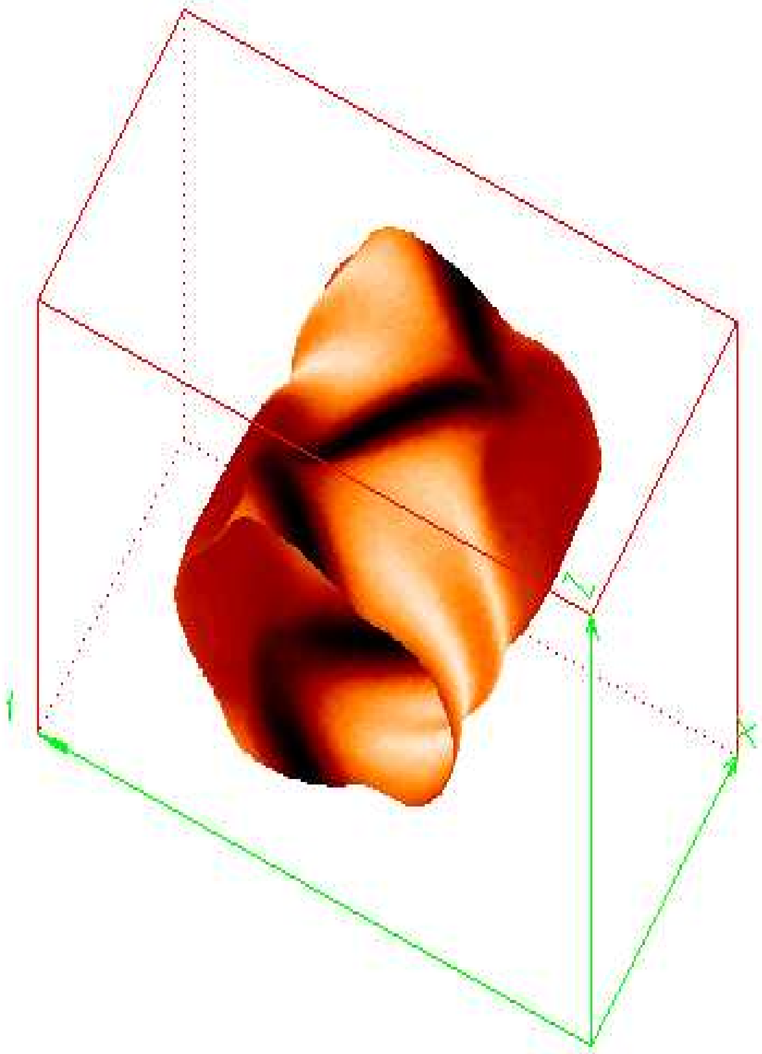


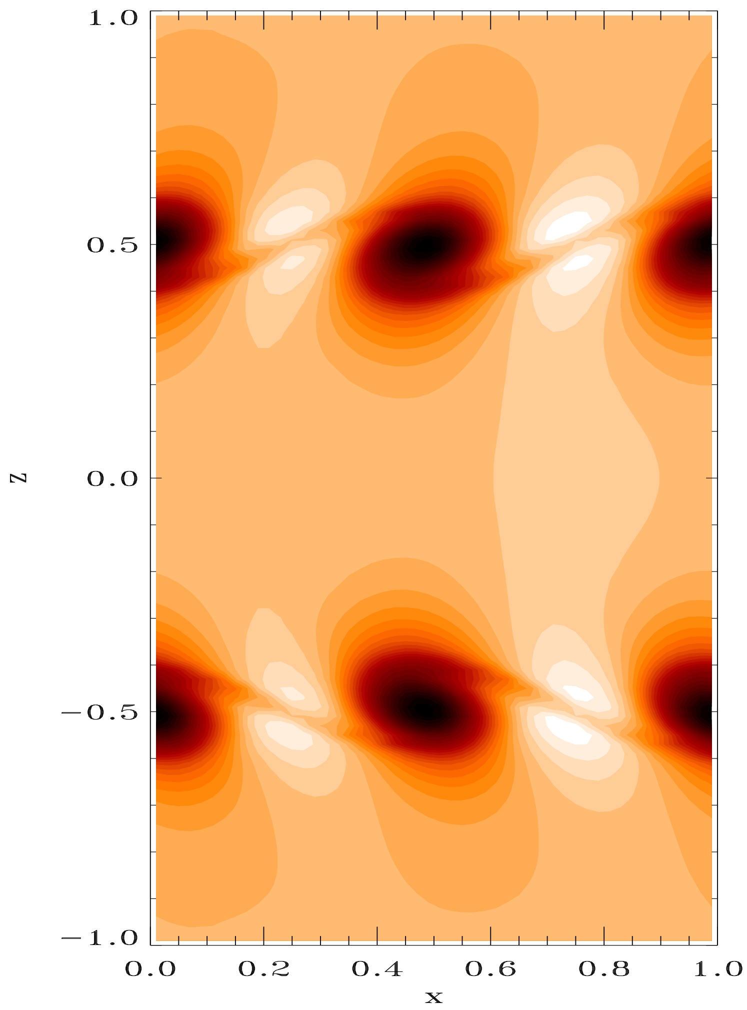


- high ρ isosurface and $v_x = 0$ jet surface colored by $p_{\rm th}$

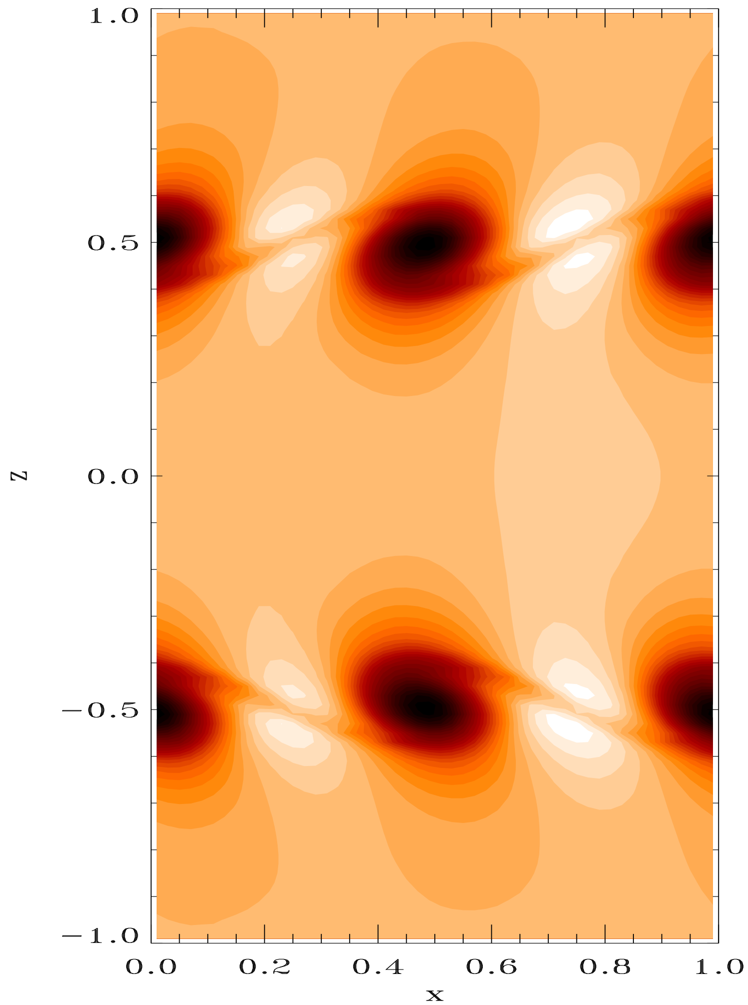


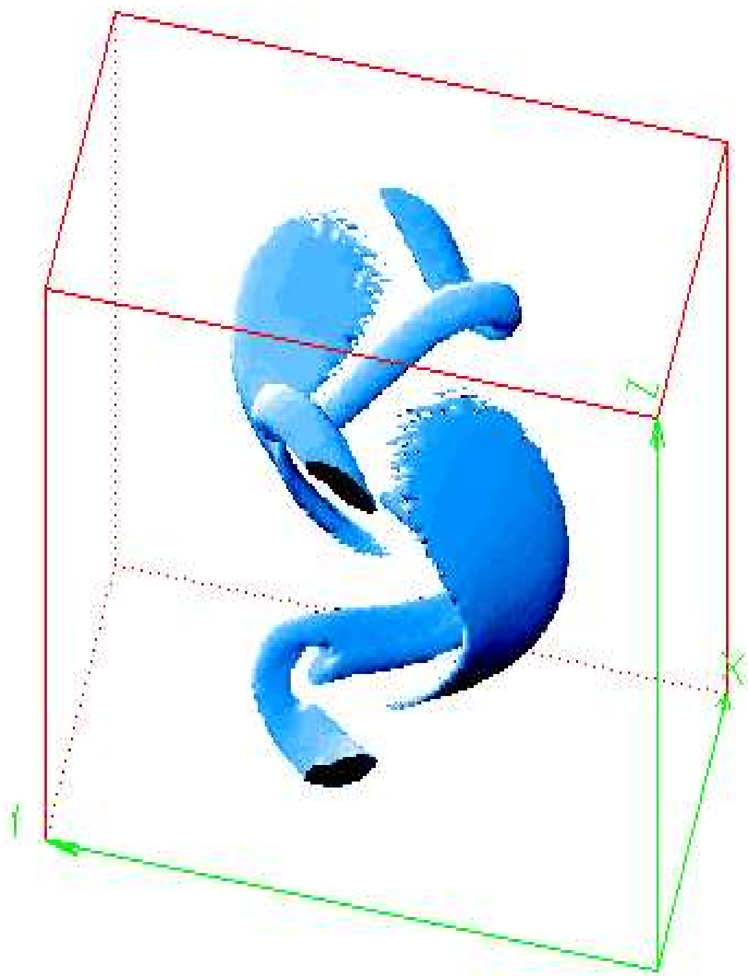
- p_{th} gradient induces wavenumber doubling on top/bottom



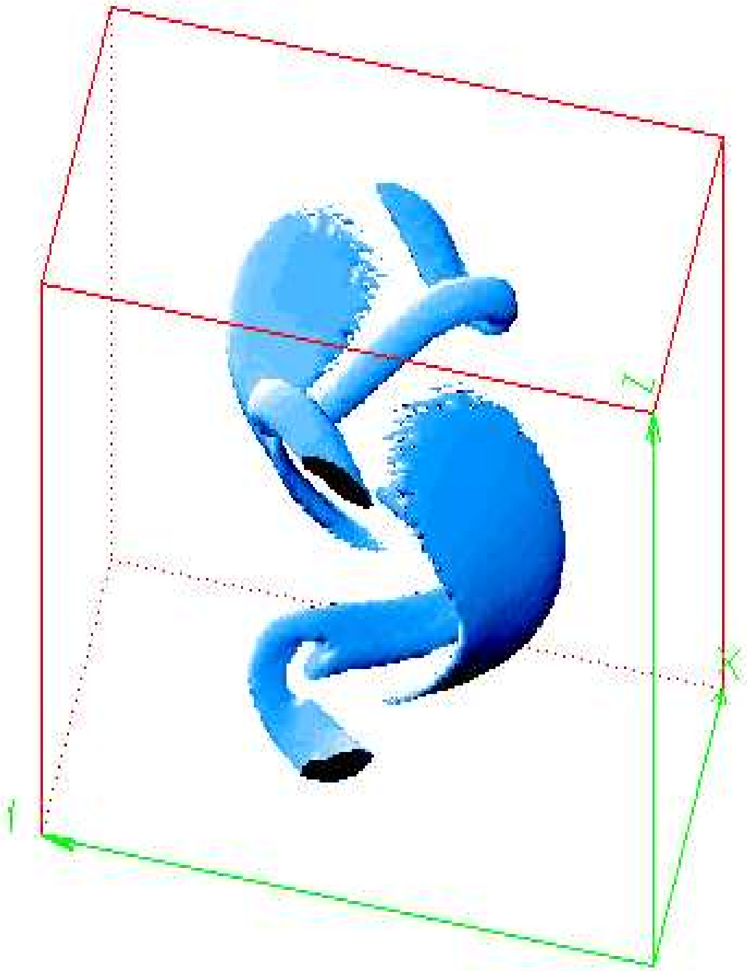


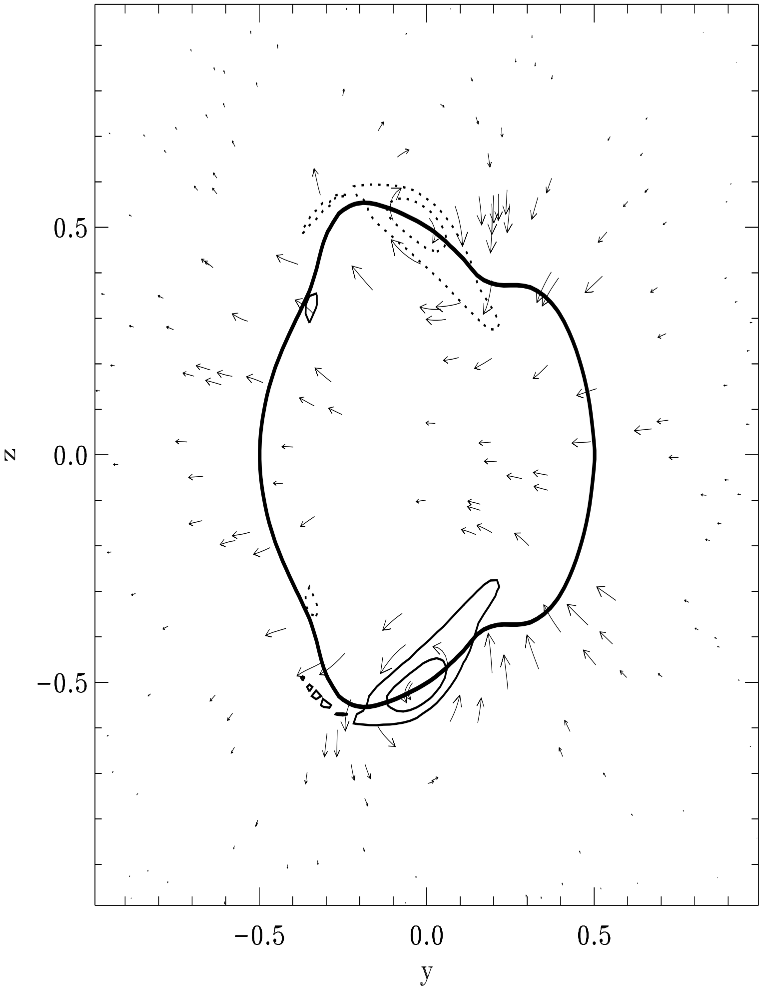
- low ρ lanes: 3D fibril and sheet structures: cospatial with high B_{pol}



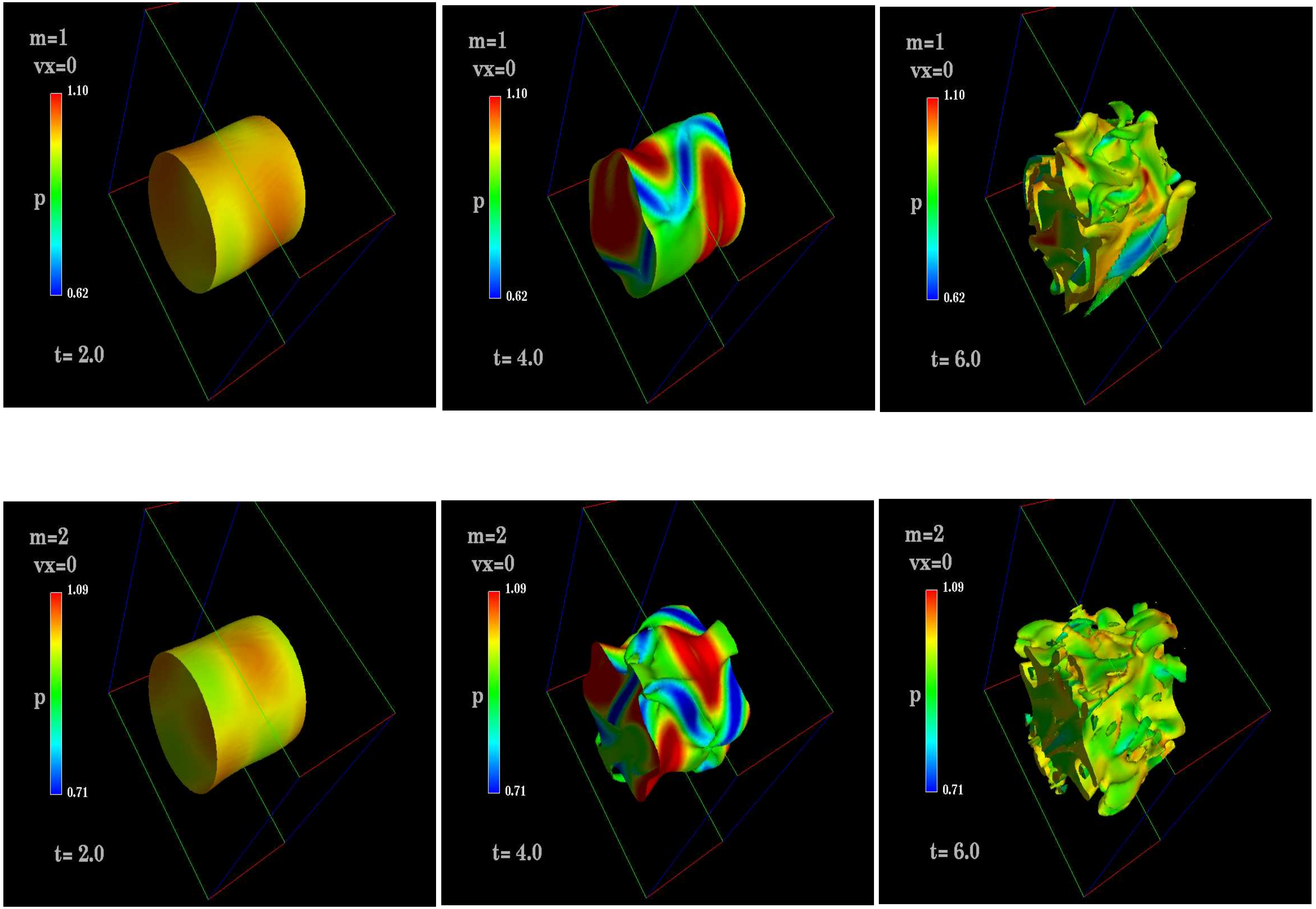


- localized 3D high $B_{\rm pol}$ regions control jet deformation





Kelvin–Helmholtz unstable jet : m=1 vrs. m=2 breakup.



References

- R. Keppens et al., *J. Plasma Physics* **61**, 1 (1999)
- R. Keppens, G. Tóth, *Physics of Plasmas* **6**(5), 1461 (1999)