

Inference rules in natural deduction.

	<i>Name</i>	<i>rule</i>	<i>coq forward</i>	<i>coq backward</i>
1	$\wedge I$	$\frac{H_1 : A \quad H_2 : B}{A \wedge B}$	<code>pose proof(conj H1 H2).</code>	<code>split.</code>
2	$\wedge E_1$	$\frac{H_1 : A \wedge B}{A}$	<code>pose proof(proj1 H1).</code>	<code>—</code>
3	$\wedge E_2$	$\frac{H_1 : A \wedge B}{B}$	<code>pose proof(proj2 H1).</code>	<code>—</code>
4	$\vee I_1$	$\frac{H_1 : C}{C \vee X}$	<code>pose proof(or_introl (B:=X) H1).</code> or <code>pose proof(@or_introl _ X H1).</code>	<code>left.</code>
5	$\vee I_2$	$\frac{H_1 : C}{X \vee C}$	<code>pose proof(or_intror (A:=X) H1).</code> or <code>pose proof(@or_intror X _ H1).</code>	<code>right.</code>
6	$\vee E$	$\frac{H_1 : A \vee B \quad H_2 : A \Rightarrow C \quad H_3 : B \Rightarrow C}{C}$	<code>pose proof(or_ind H2 H3 H1).</code> or <code>destruct H1.</code> (<i>H₂</i> and <i>H₃</i> are not needed)	<code>—</code>
7	$\Rightarrow I$	assumption/discharge	<code>assert(A->B). intros Hn.</code>	<code>intros Hn.</code>
8	$\Rightarrow E$	$\frac{H_1 : A \Rightarrow B \quad H_2 : A}{B}$	<code>pose proof(H1 H2).</code>	<code>apply H1.</code>
9	$\neg I$	$\frac{H_1 : A \Rightarrow \perp}{\neg A}$	<code>pose proof (H1:~A).</code>	<code>—</code>
10	$\neg E$	$\frac{H_1 : \neg A}{A \Rightarrow \perp}$	<code>unfold not in *.</code>	<code>unfold not.</code>
11	$\Leftrightarrow I$	$\frac{H_1 : A \Rightarrow B \quad H_2 : B \Rightarrow A}{A \Leftrightarrow B}$	<code>pose proof(conj H1 H2:A<->B).</code>	<code>split.</code>
12	$\Leftrightarrow E$	$\frac{H_1 : A \Leftrightarrow B \quad H_2 : (\dots A \dots)}{(\dots B \dots)}$	<code>rewrite H1 in H2.</code> or <code>rewrite <- H1 in H2.</code>	<code>rewrite H1.</code> or <code>rewrite <- H1.</code>
13	$\perp E$	$\frac{H_1 : \perp}{X}$	<code>destruct H1.</code>	<code>exfalso.</code>
14	$\top I$	$\frac{}{\top}$	<code>pose proof(I).</code>	<code>—</code>
15	$\forall I$	assumption/discharge	<code>—</code>	<code>intros c</code>
16	$\forall E$	$\frac{H_1 : \forall x, Px \quad c \text{ is a constant}}{Pc}$	<code>pose proof(H1 c).</code>	<code>—</code>
17	$\exists I$	$\frac{c \text{ is a constant}}{\frac{H_1 : Pc}{\exists x, Px}}$	<code>pose proof(ex_intro P c H1).</code>	<code>exists c.</code>
18	$\exists E$	$\frac{H_1 : \exists x, Px \quad c \text{ is a constant (fresh)}}{Pc}$	<code>destruct H1.</code>	<code>—</code>
19	$= I$	$\frac{c \text{ is a constant}}{c = c}$	<code>pose proof(eq_refl c).</code>	<code>reflexivity</code>
20	$= E$	$\frac{c \text{ is a constant} \quad d \text{ is a constant} \quad H_1 : c = d \quad H_2 : Pc}{Pd}$	<code>rewrite H1 in H2.</code> or <code>rewrite <- H1 in H2.</code>	<code>rewrite H1.</code> or <code>rewrite <- H1.</code>
21	LEM	$\frac{}{X \vee \neg X}$	<code>pose proof (classic X).</code>	<code>—</code>