##w203: Statistics for Data Science; Thursdays, Section 6: Samantha Williams

Section 2: Proof Practice

For this question please do the work (probably with pen and paper, although you should feel free to write a `.Rmd` solution if you're interested in the practice with), and then enter your solutions into the Gradescope assignment slot that is called "Unit 1 – Proof Practice."

Math proofs have a bad reputation: many of the instructors have negative memories of rote tasks about proving the Pythagorean Theorem, or some other such task. Perhaps you do too?

However, in this course and throughout many parts of data science the work that you will do will involve arguing for why the approach that you have taken is the *best* of the available options. A math proof is just one of these forms of argument.

For the question that follows, a compelling argument – a *proof* – should show the following two facts:

a. That the value you arrive at is possible (for example, for the maximum possible value, you might state that the maximum possible value is 1); but, importantly,
b. That there are no ways to arrive at a value that is higher (or lower as the case may be) that the value you arrive at.

Is part (b) that really makes this question a proof.

3. (6 points total) On the Overlap of Two Events

Suppose for events A and B, $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{4}$, but we have no more information about the events.

1. (3 points) What are the maximum and minimum possible values for $P(A \cap B)$?

Given that $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{4}$ the maximum value of $P(A \cap B)$ is the smallest of the two probabilities.

Maximum value of $P(A \cap B) = \frac{1}{2}$

Given that any probability cannot exceed 1 and based on the maximum value of $P(A \cap B)$ the $P(A \cap B)^C = \frac{1}{2}$ we can assume that $P(A \cup B) = 1$.

Using the addition rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ we can find the minimum value of $P(A \cap B)$.

$$1 = \frac{1}{2} + \frac{3}{4} - P(A \cap B) = \frac{5}{4} - P(A \cap B)$$

$$P(A \cap B) = \frac{5}{4} - 1 = \frac{1}{4}$$

2. (3 points) What are the maximum and minimum possible values for $P(A|B)$?

Given the limited information about $P(A)$ and the $P(B)$, we can find the maximum possible value for $P(A|B)$ using the conditional formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$ and plug in the maximum possible value of the $P(A \cap B) = 1/2$ or the $P(A)$.

$$P(A|B) = \frac{P(A)}{P(B)} = \frac{\frac{1}{2}}{\frac{3}{4}} = 2/3$$

Using the conditional formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$ and plugging in the minimum possible value of the $P(A \cap B) = 1/4$ we can determine the minimum possible value for $P(A|B)$ as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = 1/3$$