Proof Practice

(3 points total) Maximizing Correlation

(3 points) Show that if $Y = aX + b$ where X and Y are random variables and $a \neq 0$, $corr(X, Y) = -1$ or $+1$.

$Cov[X, Y] = E\Big[(X - E[X])(Y - E[Y])\Big]$
$E[Y] = aE[X] + b - y = aX + b$
$Y - E[Y] = a(X - E[X])$
$E[Y - E[Y]]^2 = aE[(X - E[X])(Y - E[Y])]$
$Cov[X, Y] = \frac{1}{a}V[Y]$

(Up to 3 bonus points total) Optional Advanced Exercise: Heavy Tails

One reason to study the mathematical foundation of statistics is to recognize situations where common intuition can break down. An unusual class of distributions are those we call **heavy-tailed**. The exact definition varies, but we'll say that a heavy-tailed distribution is one for which not all moments are finite.

Consider a random variable M with the following pmf:

$$p_M(x) = \begin{cases} \frac{c}{x^3}, & x \in 1, 2, 3, \dots \\ 0, & otherwise, \end{cases},$$

where c is a constant (you can calculate its value if you like, but it's not important).

1. Is $E(M)$ finite?
2. Is $V(M)$ finite?

   Heavy-tailed distributions may seem odd, but they're not as rare as you might suspect.

   Researchers argue that the distribution of wealth is heavy-tailed; so is the distribution of computer file sizes, insurance payouts, and area burned by forest fires. These random variables are problematic in that a lot of common statistical techniques don't work on them.

   In this class, we won't cover heavy tailed distributions in depth, but we want you to become alert to their possibility.

Note: Maximum score on any homework is 100%