Method for motion correction

When a subject moves during the MRE acquisition, this leads to obtaining encoding maps in different anatomical reference frames, it is then necessary to spatially normalize all maps. Spatial normalization process involves 3D rotations, which furthermore mix up the displacement field components encoded onto the MR signal phase. A second correction is needed to correct this mixture by a renormalization of the phase or the displacement fields.

a. Spatial normalization

The spatial normalization is performed by using Statistical Parametric Mapping (SPM8) (UCL Institute of Neurology, London, UK) (Ref spm) implemented in Matlab software (Mathworks Inc., Natick, USA). This correction uses a rigid body transformation that consists at normalizing on a reference image (first acquired phase offset volume of the MRE data set) a source image (the other acquired phase offset volumes of the three encoding directions) with only translations and rotations. It is realized in two steps. The first step is applied on the magnitude images of the MRE data and involves the estimation of the six parameters describing the spatial transformation that enable the best match between the two images by minimizing the differences between them. The six inferred parameters are the three translations (q_1, q_2, q_3) and the three rotations (q_4, q_5, q_6) constituting the rigid body transformation. The second step is the image transformation, applied on both magnitude and phase images, from the six parameters estimated during the first step. The spatial transformation of the source images is completed by the process of resampling: the aim is to determine, for each voxel of the transformed image, the intensity corresponding to the original image. To achieve this step, an interpolation is needed, and during this work the interpolation used was a generalized interpolation. Thus, the generalized interpolation method models an image as a linear combination of basis functions, here *B-spline* functions of degree 7.

b. Displacement field renormalization

For an acquired phase offset volume along one encoding direction *i* (*i=M for the measure encoding direction, P for the phase encoding direction* or *S for the slice encoding direction*), the rigid body transformation matrix is constructed from the parameters, translations and rotations (q_1-q_6), estimated and given in step of spatial normalization. This matrix is composed of a translation matrix, $\mathrm{T_i}$, and a rotation matrix, $\mathrm{R_i}$, such as:

$$\mathrm{T_i} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}, \qquad [1]$$

$$\mathrm{R_i} = \begin{bmatrix} R_{i,1} & R_{i,2} & R_{i,3} \\ R_{i,4} & R_{i,5} & R_{i,6} \\ R_{i,7} & R_{i,8} & R_{i,9} \end{bmatrix},$$

$$\mathrm{R_i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_M & -\sin\theta_M \\ 0 & \sin\theta_M & \cos\theta_M \end{bmatrix} \begin{bmatrix} \cos\theta_P & 0 & \sin\theta_P \\ 0 & 1 & 0 \\ -\sin\theta_P & 0 & \cos\theta_P \end{bmatrix} \begin{bmatrix} \cos\theta_S & -\sin\theta_S & 0 \\ \sin\theta_S & \cos\theta_S & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad [2]$$

with $\theta_M = q_4$, $\theta_P = q_5$ and $\theta_S = q_6$.

During the acquisition, the subject motions induce a change of anatomical reference frame between each encoding of the displacement field components. After spatial normalization, which consists in the correction of the anatomical reference frame, each displacement map undergoes different rotation and translation, equivalent to a change of reference frame for a vector field. It is necessary to renormalize the displacement field in an orthonormal basis. The displacement field renormalization aims at changing the reference frame of a vector field that remains translational invariant but is affected by rotation. Those are the changes induced by spatial normalization of the subject rotation that must be corrected.

Considering the displacement field $\boldsymbol{u}$ defined by the components u_M, u_P and u_S along the encoding directions $(\boldsymbol{e_M}, \boldsymbol{e_P}, \boldsymbol{e_S})$ which form an orthonormal basis such as $\boldsymbol{u} = \begin{pmatrix} u_M \\ u_P \\ u_S \end{pmatrix}$, thus for each time offset, renormalization of the displacement field maps should allow to recover the displacement field $\boldsymbol{u}$. This is achieved by the following inverse transform:

$$\begin{pmatrix} u_M \\ u_P \\ u_S \end{pmatrix} = \mathrm{M}_{\mathrm{RBT}}^{-1} \begin{pmatrix} u_{M'} \\ u_{P'} \\ u_{S'} \end{pmatrix}, \qquad [3]$$

with $\boldsymbol{u}' = \begin{pmatrix} u_M \\ u_P \\ u_S \end{pmatrix}$, the displacement field calculated from acquired and spatially normalized displacement field maps, defined in the non-orthonormal basis $(\boldsymbol{e_{M'}}, \boldsymbol{e_{P'}}, \boldsymbol{e_{S'}})$. $\mathrm{M}_{\mathrm{RBT}}$ is the rotation matrix calculated from the three rotation matrices of each encoding direction $\mathrm{R_i}$, such as $\mathrm{M}_{\mathrm{RBT}} = \begin{bmatrix} R_{M,1} & R_{M,2} & R_{M,3} \\ R_{P,4} & R_{P,5} & R_{P,6} \\ R_{S,7} & R_{S,8} & R_{S,9} \end{bmatrix}$.