MRE Reconstruction:
Inverting the wave equation

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Outline of This Talk

1. Why we need MRE
2. Data Reconstruction and current Problems
3. First Experiments
4. What comes next

Why do we need MRE?

E Barnhill et al. / Medical Image Analysis 35 (2017) 133–145

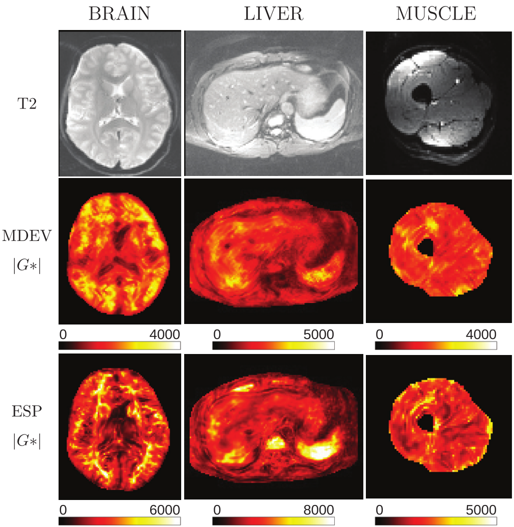

Fig. 9. Comparison of $|G^*|$ and φ maps using the MDEV and ESP pipelines. All values are in Pascals.

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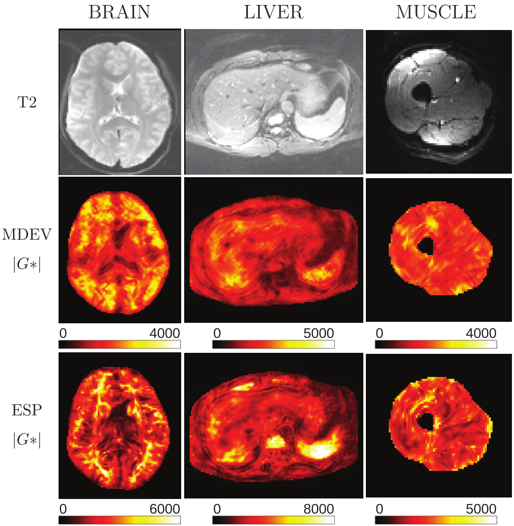

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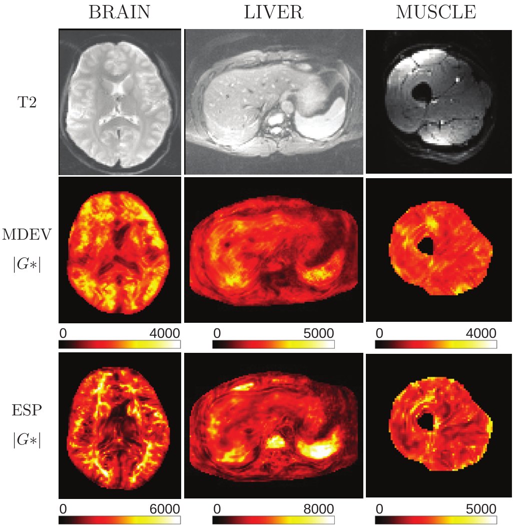

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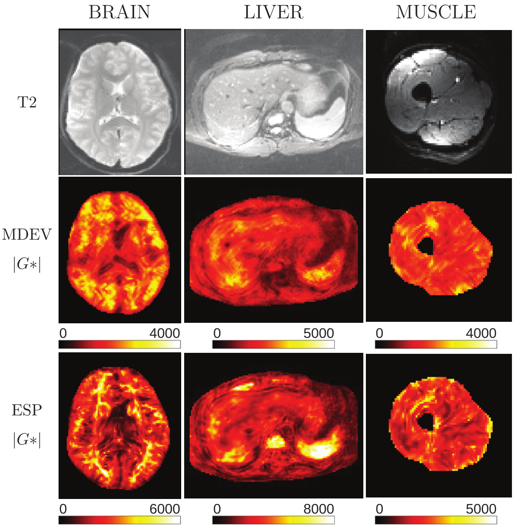

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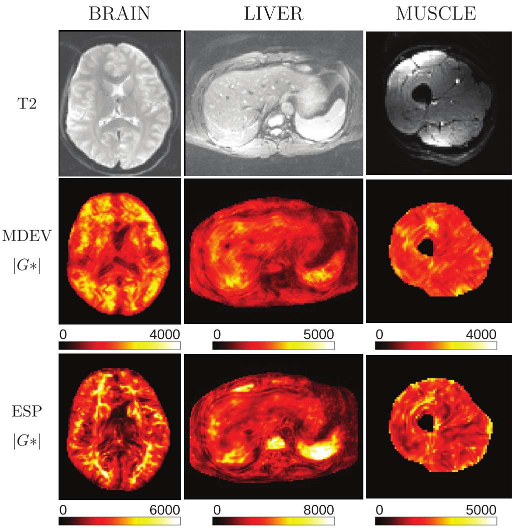

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Why do we need MRE?

- diseased tissue changes mechanical
- low tech: palpation
- higher tech: ultrasound
- highest tech: MRE
- for deep tissue and brains, but non-invasive

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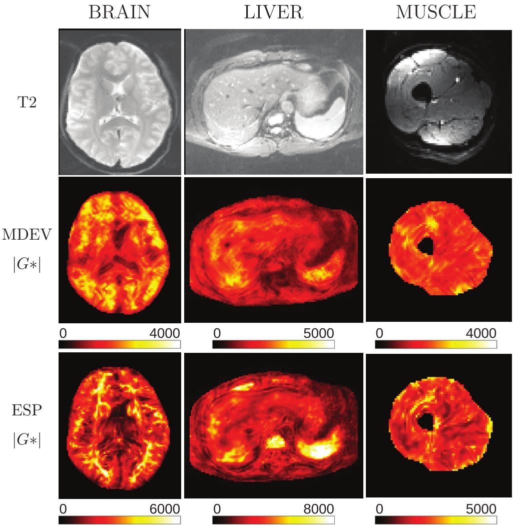

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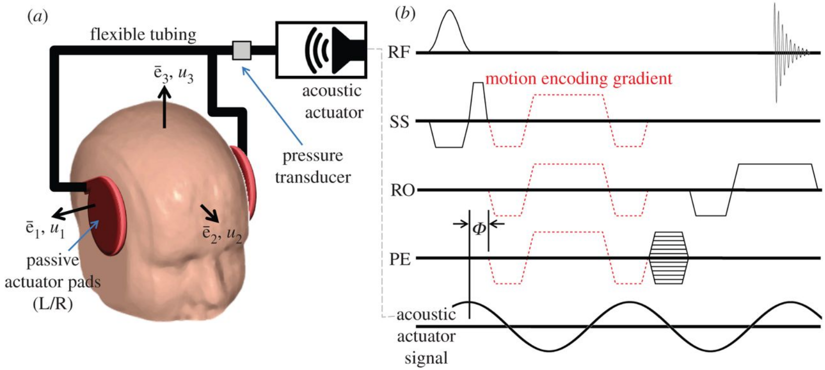

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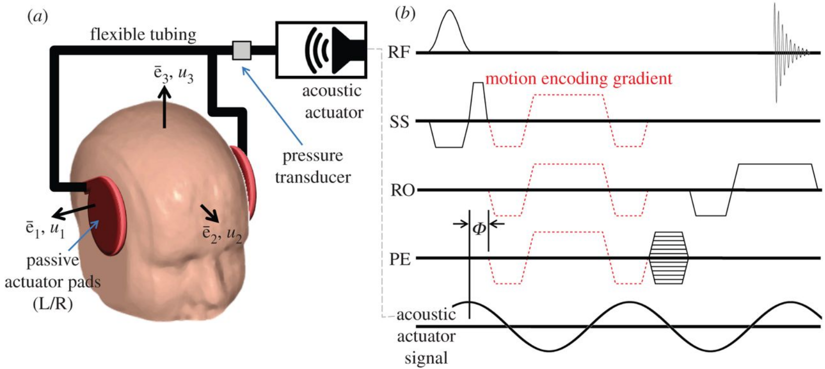

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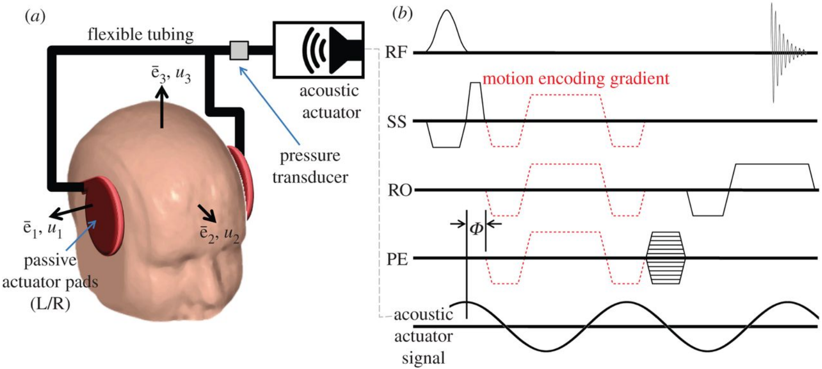

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- 72 times longer per pixel than MRI

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- Navier-Lamé equation:

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$$\left[\nabla \cdot \epsilon + \epsilon \nabla\right] \boldsymbol{\mu} = -\rho\omega^2 \mathbf{u}$$

- matrix inversion relies on correct knowledge of $\mathbf{u}'$, and $\mathbf{u}''$
  $\rightarrow$ noise can completely screw up the derivatives

Noisy displacement field

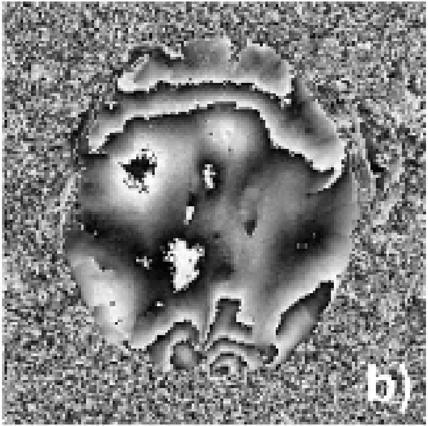

Figure: b) The human brain

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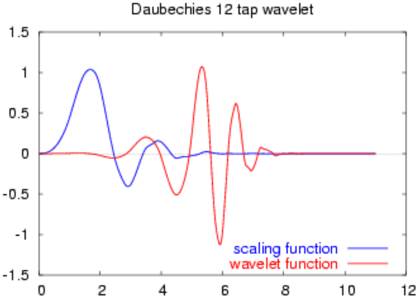

Wavelet Denoising

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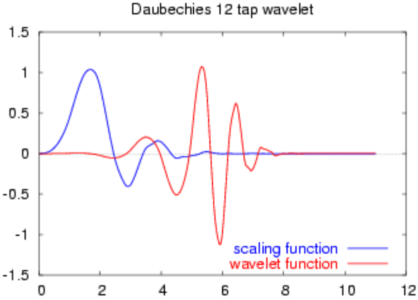
- in 2d: anisotropic structure $\rightarrow$ shearlets



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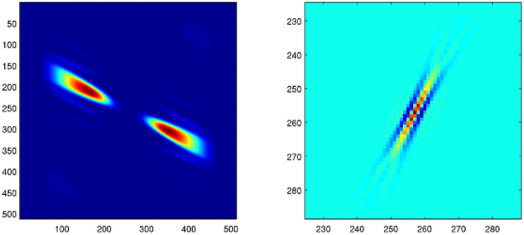

$$
\begin{aligned}
u^* &= \underset{u}{\operatorname{argmin}} \; \|u_{\text{meas}} - u\|_2^2 + \lambda \|Wu\|_0 \\
&= W^\dagger H(Wu_{\text{meas}}; \lambda)
\end{aligned}
$$

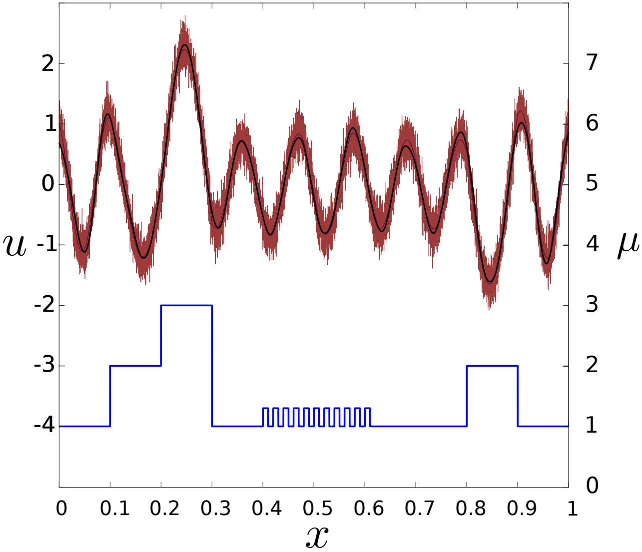

Figure: Shear parameter, displacement and noise in 1d

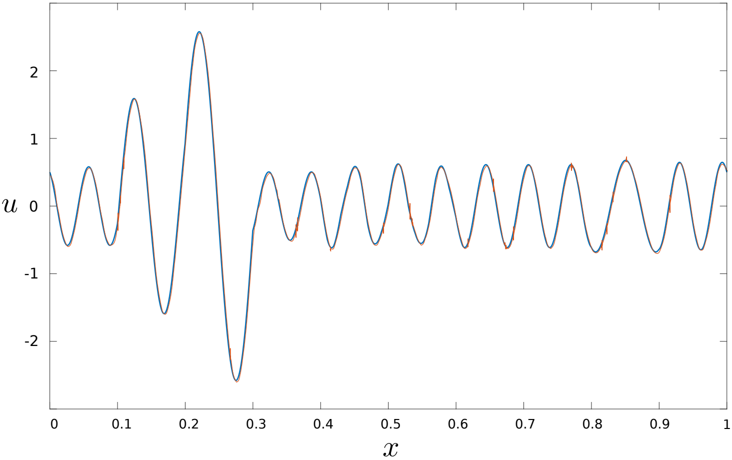

Figure: Reconstructed with Daubeshies10-wavelets and threshold of $0.01 * \max_i(u_i)$

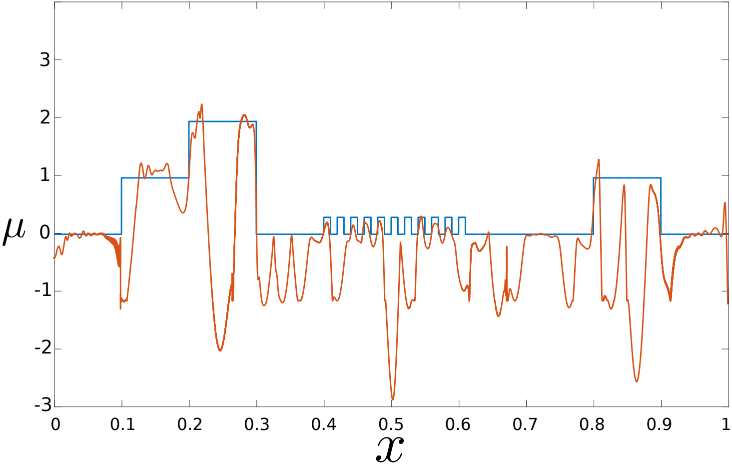

Figure: Original and reconstructed shear parameter μ.

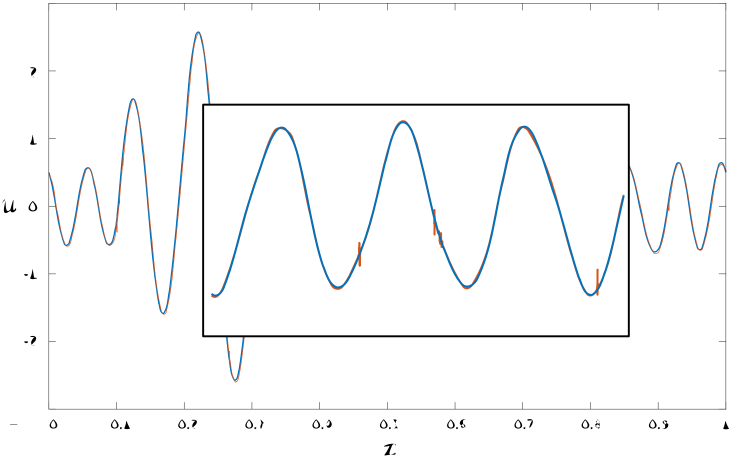

Figure: First derivative of original and denoised displacement u. Close-up reveals noise residuals on a fine scale.

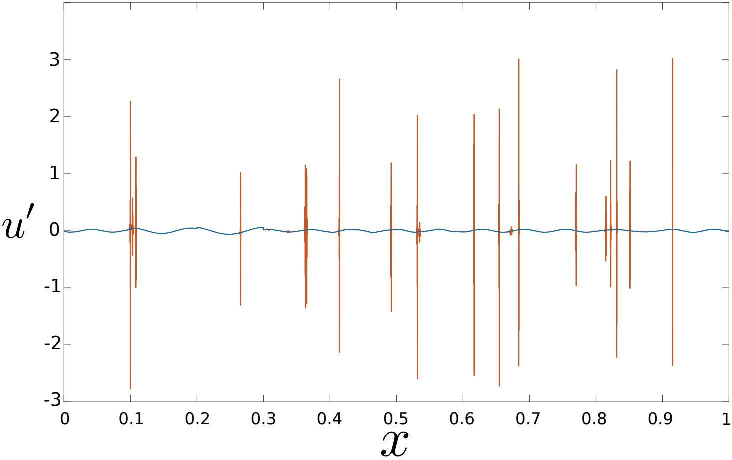

Figure: First derivative of original and denoised displacement u. Fine scale noise dominates the first (and subsequent) derivatives.

Our plan of work

- Do simuations in 1d: wavelets
- Do simulations in 2d: wavelets, shearlets
-
- Problem: Need to reconstruct the derivatives
- slight noise can lead to totally wrong derivatives –¿ inversion is useless
- MRI measurement in 3 spatial directions and 8 time steps –¿

What would be nice results

- Have better resolution of the stiffness map
- Have clinically useful values, at the moment to varying
- Have shorter acquisition times per pixel

- Problem: Need to reconstruct the derivatives
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- MRI measurement in 3 spatial directions and 8 time steps –¿