Zadanie 3.

200 wektorów losowych $\sim N(0, I_{100\times100})$

$X^1 = (X_1^1 \ldots X_{100}^1)$ $\quad$ zbinol$(X^1 \ldots X^{200})$ $\Longrightarrow$

$\vdots$

$X^{200} = (X_1^{200} \ldots X_{100}^{200})$

$$\Longrightarrow X = \begin{pmatrix} X^1 \\ \vdots \\ X^{200} \end{pmatrix} = \begin{pmatrix} X_1^1 & \ldots & X_{100}^1 \\ \vdots & & \vdots \\ X_1^{200} & \ldots & X_{100}^{200} \end{pmatrix}$$

$X = X_{200\times100}$

$$? \quad \tilde{X} = \begin{pmatrix} \tilde{X}^1 \\ \vdots \\ \tilde{X}^{200} \end{pmatrix} = \begin{pmatrix} \tilde{X}_1^1 & \ldots & \tilde{X}_{100}^1 \\ \vdots & & \vdots \\ \tilde{X}_1^{200} & \ldots & \tilde{X}_{100}^{200} \end{pmatrix}; \quad \tilde{X}_{200\times100}$$

$\tilde{X}$ - 200 w. losowych $\sim N(0, \Sigma)$

$$\Sigma = \Sigma_{100\times100} = \begin{pmatrix} 1 & 0{,}9 & \ldots & 0{,}9 \\ 0{,}9 & \ddots & & \vdots \\ \vdots & & \ddots & \\ 0{,}9 & \ldots & & 1 \end{pmatrix}$$

$\underset{200\times100}{\tilde{X}} = \underset{200\times100}{X A} \quad \Rightarrow \quad A = A_{100\times100}$

$\tilde{X}^1 = X^1 A \qquad \underset{1\times100}{\overline{\quad \tilde{X}^1 \quad}} = \underset{1\times100}{\overline{\quad X^1 \quad}} \underset{100\times100}{\begin{pmatrix} \; A \; \end{pmatrix}}$

$\Rightarrow (\tilde{X}^1)^T = A^T \underbrace{(X^1)^T}_{\sim N(0, I)}$

$$\underset{100\times1}{(\tilde{X}^1)^T} = \underset{100\times100}{A^T} \; \underset{100\times1}{(X^1)^T}$$

$\Rightarrow (\tilde{X}^1)^T \sim N(0, A^T A)$

$\tilde{X}^1 \sim N(0, A^T A)$

A^T - dolnotrójkątna $\Rightarrow$ A - górnotrójkątna

A^T $\qquad\qquad\qquad$ A

Rozkład Choleskiego $\qquad$ $A^T \Rightarrow A$

$$\tilde{X} = XA = \begin{array}{c} \begin{pmatrix} \tilde{x}_1^1 & \cdots & \tilde{x}_{100}^1 \\ \vdots & & \vdots \\ \tilde{x}_1^{200} & \cdots & \tilde{x}_{100}^{200} \end{pmatrix} \\ \tilde{X}_1 \quad \cdots \quad \tilde{X}_{100} \end{array} \begin{array}{c} \tilde{x}^1 \\ \vdots \\ \tilde{x}^{200} \end{array} \sim N(0, \Sigma)$$

próba losowa

/Hogg ta inny, str 228/

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} \Rightarrow \bar{y} = \frac{1}{m} \sum_{j=1}^{m} y_j$$

mean( )

$$S_y^2 = \frac{1}{m-1} \sum_{j=1}^{m} (y_j - \bar{y})^2$$

var( )

$$z = \begin{pmatrix} z_1 \\ \vdots \\ z_m \end{pmatrix} \qquad S_{yz}^2 = \frac{1}{m-1} \sum_{j=1}^{m} (y_j - \bar{y})(z_j - \bar{z})$$

cov( )

średnia:

$$\bar{X} = \frac{1}{200} \sum_{j=1}^{200} \tilde{x}^j = (\text{mean}(\tilde{X}_1), \ldots, \text{mean}(\tilde{X}_2))$$

$$S^2 = S_{100}^2 = \begin{pmatrix} S_{1,1}^2 & \cdots & S_{1,100}^2 \\ \vdots & \ddots & \vdots \\ S_{1,1000}^2 & \cdots & S_{100,100}^2 \end{pmatrix}$$

$$S_{ii}^2 = \text{var}(\tilde{X}_i), \qquad S_{ij}^2 = \text{cov}(\tilde{X}_i, \tilde{X}_j)$$