人. 记 Σ 特征值分解 $\Sigma = P\Lambda P^T = \sum_k \lambda_k a_k a_k^T$

$$\sum_k \lambda_k a_{ik}^2 = \sum_k \lambda_k e_i^T a_k a_k^T e_i$$

$$= e_i^T \left[\sum_k \lambda_k a_k a_k^T \right] e_i$$

$$= e_i^T \Sigma e_i = \sigma_{ii}$$

故 $\sum_k \rho^2 (Y_k, X_i) = \sum_k \lambda_k a_{ik}^2 / \sigma_{ii} = 1$

2. $n=6$, $\bar{X} = \sum_i \frac{1}{n} X_i = [5,4]^T$

$$S = \frac{1}{n-1} \sum_i (X_i - \bar{X})(X_i - \bar{X})^T = \begin{bmatrix} 4 & \frac{17}{5} \\ \frac{17}{5} & 3.2 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & \frac{17}{40}\sqrt{5} \\ \frac{17}{40}\sqrt{5} & 1 \end{bmatrix}$$

$|\lambda I - R| = 0 \Rightarrow \lambda_1 = 1 + \frac{17}{40}\sqrt{5}$, $\lambda_2 = 1 - \frac{17}{40}\sqrt{5}$

对应单位特征向量为 $\alpha_1 = [\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]^T$, $\alpha_2 = [\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}]^T$

$\cancel{(\tilde{X})_{ij}}$ $(\tilde{X})_{ij} = \dfrac{x_{ij} - \bar{x}_j}{\sqrt{\sigma_{jj}}} \Rightarrow \tilde{X} = \begin{bmatrix} -\frac{3}{2} & -\frac{1}{2} & 0 & 0 & \frac{1}{2} & \frac{3}{2} \\ -\frac{\sqrt{5}}{2} & -\frac{\sqrt{5}}{4} & -\frac{\sqrt{5}}{4} & 0 & \frac{\sqrt{5}}{2} & \frac{3\sqrt{5}}{4} \end{bmatrix}^T$

得分 $Y_1 = \tilde{X}\alpha_1$, $Y_2 = \tilde{X}\alpha_2$

(第一、二主成分) $Y_1 = \frac{1}{2\sqrt{2}} [-3-\sqrt{5}, -1-\frac{\sqrt{5}}{2}, -\frac{\sqrt{5}}{2}, 0, 1+\sqrt{5}, 3+\frac{3}{2}\sqrt{5}]^T$

$Y_2 = \frac{1}{2\sqrt{2}} [-3+\sqrt{5}, -1+\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}, 0, 1-\sqrt{5}, 3-\frac{3}{2}\sqrt{5}]^T$