Variable-Resolution Displays: A Theoretical, Practical, and Behavioral Evaluation

Derrick J. Parkhurst and **Ernst Niebur,** The Johns Hopkins University, Baltimore, Maryland

Variable-resolution display techniques present visual information in a display using more than one resolution. For example, gaze-contingent variable-resolution displays allocate computational resources for image generation preferentially to the area around the center of gaze, where visual sensitivity to detail is the greatest. Using such displays reduces the amount of computational resources required as compared with traditional uniform-resolution displays. The theoretical benefits, implementational issues, and behavioral consequences of variable-resolution displays are reviewed. A mathematical analysis of computational efficiency for a two-region variable-resolution display is conducted. The results are discussed in relation to applications that are limited by computational resources, such as virtual reality, and applications that are limited by bandwidth, such as internet image transmission. The potential for variable-resolution display techniques as a viable future technology is discussed.

INTRODUCTION

One of the factors determining the quality of a visual display is the presented level of detail. The detail that can be rendered in real time for most applications is limited by the available processing power (e.g., in virtual reality applications) or the available communication bandwidth (e.g., in image transmission applications). In light of these restrictions, it is important to allocate resources efficiently. Presenting a uniform level of detail across a display wastes resources because the human visual system does not process visual detail equally over the whole visual field. Rather, it focuses processing resources near the center of gaze. This property of the visual system can be exploited to minimize resource requirements by rendering a high degree of visual detail only in a portion of the display. Variable-resolution displays do this by presenting visual information using more than one level of detail.

A number of variable-resolution display techniques have been developed. First, gaze-contingent variable-resolution displays allocate resources for image generation preferentially to the area around the center of gaze. It is known that the visual sensitivity of the human visual system is at least an order of magnitude greater at the center of gaze than in the peripheral visual field (Adler, 1965). If this is to be exploited, the viewer's point of gaze must be tracked and the visual display updated in real-time so as to maintain a high level of detail at that location.

The potential resource savings (compared with presentation in uniform resolution) are substantial given that with this technique, a high level of detail need be maintained only in a small area around the center of gaze. This is especially true for head-mounted or projection-based displays that subtend a large portion of the visual field; however as will be shown later, substantial savings can also be obtained for smaller displays such as computer monitors. (Note that for the purposes of this paper, "smaller" and "larger" displays should always be understood in terms of visual angle.)

Gaze-contingent variable-resolution displays have been used primarily in virtual reality

Address correspondence Ernst Niebur, Zanvyl Krieger Mind/Brain Institute, Johns Hopkins University, Baltimore, MD, 21218; niebur@jhu.edu. **HUMAN FACTORS,** Vol. 44, No. 4, Winter 2002, pp. 611–629.

applications (Danforth, Duchowski, Geist, & McAliley, 2000; Duchowski, 1998a; Levoy & Whitaker, 1990; Murphy & Duchowski, 2001; Ohshima, Yamamoto, & Tamura, 1996). Users are presented with a display that subtends most of the visual field by way of a head-mounted display or a large projection screen. The benefit of using these large displays is that they increase the feeling of reality or immersion in the virtual environment. Given that a computer must generate the visual display in real time and that the available computational resources are usually insufficient to display a high level of visual detail across the whole display, a lower level of detail is sometimes used. Alternatively, a trade-off can be made whereby temporal resolution (i.e., the frame update rate) is sacrificed to obtain a higher spatial resolution.

Unfortunately, low update rates impair users' ability to interact naturally with virtual environments and may lead to simulator sickness. Simulator sickness can include a number of symptoms, including disorientation, nausea, and headaches, and is thought to be caused by inconsistent visual, proprioceptive, and vestibular inputs (McCauley & Sharkey, 1992). In fact, simulator sickness has been a major factor limiting virtual reality applications (Pausch, Crea, & Conway, 1992). Gaze-contingent variable-resolution display techniques can reduce the likelihood of simulator sickness and improve the quality of virtual reality displays by shifting resources normally dedicated to generating detail in the periphery to increasing frame update rates while maintaining a high level of detail only at the point of gaze where it is most needed.

Although we focus on the application of gaze-contingent variable-resolution displays in virtual reality, another application worth mentioning involves low-vision enhancement. Patients with retinal pathologies such as macular degeneration can experience reduced vision in the fovea because of local degeneration of the photoreceptors. In this instance, gaze-contingent variable-resolution displays can be used to remap the visual input projected to the damaged fovea to other nearby functioning retinal locations. Analogously, patients with peripheral scotomas caused by, for example, a stroke can benefit from a similar remapping. For an excellent review of the issues surrounding such low-vision enhancement research, see Dagnelie and Massof (1996).

A second type of variable-resolution display technique allocates resources for image generation preferentially to the most salient scene locations (Duchowski & McCormick, 1995a, 1996, 1998), to locations where viewers are likely to fixate (Duchowski, 1998b; Duchowski & McCormick, 1998; Stelmach, Tam, & Hearty, 1991), or to locations specified by user input (Chang, Yap, & Yen, 1997; Frajka, Sherwood, & Zeger, 1997; Geisler & Perry, 1998, 1999). The primary advantage of this technique is that it does not require eye tracking equipment, nor does it require invasive procedures sometimes associated with eye tracking (e.g., the restriction of head movements). The major disadvantage of this technique is that the viewer's exact fixation locations are unknown, and therefore it is possible that locations displayed in low detail may be fixated. To reduce the likelihood of this happening, multiple regions of high detail can be maintained and centered on salient scene locations that are determined a priori (Basu and Wiebe, 1998; Duchowski, 1998b; Duchowski & McCormick, 1995b, 1995a). This technique has lower computational efficiency than gaze-contingent techniques because the overall resolution of the display increases with the number of regions of high detail.

This second type of variable-resolution display has been used primarily in internet image transmission applications such as teleconferencing, talemedicine, and video compression. In such applications the limiting factor is not computational power but the bandwidth or latency of the communication channel. Using a variable-resolution technique minimizes the total amount of data that must be transmitted when channel bandwidth is limited, or it prioritizes the most important information to be sent first when channel latency is high.

For example, in teleconferencing applications, Internet bandwidths are typically insufficient to transmit high-resolution video at standard refresh rates. By selectively transmitting particular spatial regions in high resolution (e.g., the face of a speaking person), one can substantially increase frame update rates without a noticeable amount of perceptual degradation occurring. In talemedicine or data visualization applications, when retrieving static high-resolution images (e.g., MRI scans) over a limited communication channel is desired, it is advantageous to transmit the most relevant information first. In video compression applications, regions of interest are usually specified ahead of time on the basis of human input or by heuristic algorithms; in data visualization applications, regions of interest are usually selected on line in parallel with data transmission. Although this is less common, gaze-contingent variable-resolution displays have also been applied to internet transmission applications (Kortum & Geisler, 1996).

In the remainder of this article, we review the theoretical benefits, practical constraints, and behavioral consequences associated with both types of variable-resolution displays. First, we conduct an analysis of the potential computational savings that can be achieved using variable-resolution display techniques and examine the results of this analysis in the context of different applications. Then we discuss the practical constraints in implementing variable-resolution displays. Finally, we review the behavioral consequences of using variable-resolution displays in terms of perceptual quality, task performance, and eye movement measures. We conclude with a brief discussion of future directions for variable-resolution display research.

POTENTIAL COMPUTATIONAL BENEFIT

Given that the central part of the visual field is an order of magnitude more sensitive and many orders of magnitude smaller than the rest of the visual field, it has been pointed out that variable-resolution displays have the potential of being much more computationally efficient compared with uniform-resolution displays. Although this logic is frequently cited, a detailed analysis of the potential computational benefit is lacking. In the theoretical analysis that follows, we quantitatively examine the efficiency of a two-region variable-resolution display (see Figure 1). The two-region display consists of a central high-resolution region (centered on the viewer's point of gaze in gaze-contingent applications) surrounded by a low-resolution region. The two-region display is the most common variable-resolution display in use and is usually preferred over other approaches (e.g., more than two regions, or a continuous mapping) because substantial computational resources may be required to generate more complex variable-resolution displays in real time (but see Duchowski, 2000, and Lee, Pattichis, & Bovik, 2001, for progress toward efficient display methods).

The approach we take to quantitatively determine the potential efficiency of variable-resolution display techniques is to compare the computational effort required to generate a uniform resolution display with that required for an equivalent two-region variable-resolution display. Both displays are equivalent in the sense that they must enable viewers to perform a task that requires high spatial resolution at the center of gaze (e.g., object identification). For the

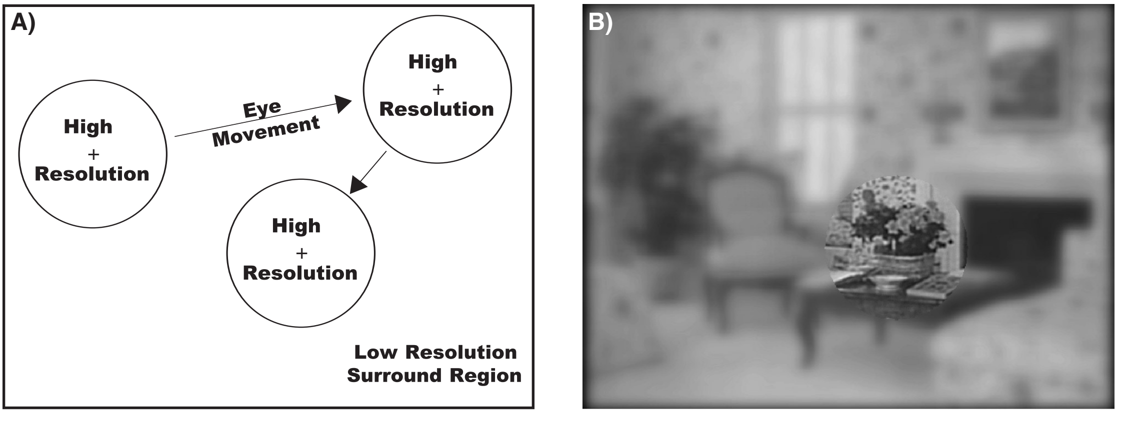

Figure 1. (A) A schematic representation of a gaze-contingent, two-region, variable-resolution display. The high-resolution central region tracks the viewer's center of gaze. (B) A variable-resolution display used in Parkhurst et al. (2000) study, with the high-resolution central region centered on the flowers.

uniform-resolution display, this is achieved by maintaining high resolution across the entire display. For the variable-resolution display, high resolution is maintained only in the central region. The resolution in the surrounding region is constrained to be only as high as that dictated by visual system sensitivity. Given that these simple constraints still leave the size of the central region undetermined, the size of the central region will be arbitrarily selected in order to maximize display efficiency.

Calculating Optimal Central Region Size and Computational Gain

We start from the premise that a fixed number of computational resources are available and that these resources are divided into two parts. One part is devoted to generate the central region, and the remaining part is used to generate the surrounding region in the periphery. A simple measure of the computational cost C is the number of elementary features to be painted in every frame, computed as the area S of the displayed region multiplied by the square of its linear resolution R:

$$C = S \cdot R^2. \tag{1}$$

The linear resolution is defined in terms of the highest spatial frequency displayed. This is a simple but realistic measure of the computational cost for bitmapped raster operations and has the advantage of being independent of hardware details. For three-dimensional graphic engines used in virtual reality applications, replacing R^2 with a measure in terms of polygons/area would yield a closer approximation of the actual computational cost. A discussion at the end of this section describes such an approach.

Let us divide the total visual field area S into two regions, a central region of size s around the center of gaze and the remainder in the surround, $S - s$. Let us further assume that ν_c is the linear resolution required in the central region for a particular task and ν_s is the lower resolution in the surround.

In the case of a uniform resolution display, high resolution ν_c is maintained everywhere. The computational effort required for this display is therefore $C_0 = S\nu_c^2$. In contrast, for the variable-resolution display, high resolution is maintained only in the central region, and the surrounding region is presented in a lower resolution. The computational effort required for the variable-resolution display C is the sum of that required to display the central region ($s\nu_c^2$) and that required for the surrounding region [$(S - s)\nu_c^2$]. The computational gain I is defined as the ratio of computational costs C_0 and C,

$$I = \frac{C_0}{C} = \frac{S\nu_c^2}{s\nu_c^2 + (S - s)\nu_s^2}. \tag{2}$$

The gain indicates how many times more efficient the variable-resolution technique is relative to the uniform-resolution technique. Considering the case in which one has a circular center of radius ρ inside a rectangular display, $s = \pi\rho^2$, Equation 2 becomes

$$I = \frac{S\nu_c^2}{\pi\rho^2\nu_c^2 + (S - \pi\rho^2)\nu_s^2}. \tag{3}$$

To determine the resolution of the surround ν_s, psychophysical measures of human visual system sensitivity can be used to minimize the perception of blur in the periphery. Presenting a very low resolution in the periphery will result in the perception of blur, whereas presenting a very high resolution will waste computational resources. The resolution of the surrounding region is therefore set to the highest resolution detectable in that region. It is known that the spatial frequency sensitivity (i.e., visual acuity) $\nu(x)$ of the human visual system decreases with eccentricity x and can be approximated as

$$\nu(x) = \alpha / x, \tag{4}$$

where α is a constant of proportionality. This approximation is used because psychophysical and anatomical evidence indicates that the relationship between the reciprocal of spatial frequency sensitivity (i.e., the minimum angle of resolution) and eccentricity is approximately linear. This relationship only slightly deviates from linearity in the fovea and extreme periphery (Anstis, 1974; Van Essen & Anderson, 1990). When the central region is centered on the point of gaze, the most sensitive part of the surrounding region is near the high-to-low resolution border at radius ρ, and therefore the resolution in the surround becomes $\nu_s = \alpha/\rho$. Using this in Equation 3, we obtain

$$I = \frac{S\nu_c^2}{\pi\rho^2\nu_c^2 + S(\alpha/\rho)^2 - \pi\alpha^2}. \tag{5}$$

Is there an optimal value to be chosen for ρ that maximizes computational gain? If we choose it very small (say, $\rho = 1°$; it cannot be zero), then the surround will be represented with the relatively high resolution of the visual system at 1° and the computational gain will be minimal. On the other hand, if ρ is chosen very large, a lower resolution is used for the periphery but the presentation of the large, high-resolution central region is very costly and the gain is again minimal. In between these two values is an optimal ρ that compromises between the cost of displaying the center and the cost of displaying the surround.

We can compute an optimal ρ by minimizing the cost of the variable-resolution display compared with the uniform resolution display in Equation 5. This is done by differentiating I with respect to ρ and setting the result to zero. It is shown in the Appendix that the result is

$$\rho = \sqrt[4]{\frac{S\alpha^2}{\pi\nu_c^2}}. \tag{6}$$

Both the size of the display S and the central region resolution ν_c are determined by the particular application, whereas α is a constant and can be determined from the well-studied spatial frequency transfer function of the visual system. Virsu and Rovamo (1979) measured the threshold at which sinusoidal gratings extending 5° could be detected 50% of the time for a range of eccentricities and spatial frequencies. A least-squares estimate of $\alpha = 15$ cycles was obtained from a fit of Equation 4 to the optimal spatial frequency as a function of eccentricity. The implications of using this particular measure of visual sensitivity are discussed later.

Application of Analysis

The results of our analysis can be applied to any arbitrary display device if the display area and the display resolution are both known. To obtain the optimal central region size, which maximizes the computational gain, the display area S (in degrees squared) and the linear resolution of the display ν_c (in cycles per degree) are used in Equation 6. The values of π and α are given, and therefore the optimal central region size ρ can be directly computed. To determine the computational gain, the optimal central region size ρ, the display area S, and the resolution of the display ν_c are used in Equation 5. Again the values of π and α are given, and the computational gain I can be directly computed. This procedure is followed below for two example displays, one relevant to virtual reality applications and one relevant to image transmission applications.

Given a large, high-resolution display, such as that desirable in inversive virtual reality systems, a substantial computational savings can be obtained by using variable-resolution techniques. To determine the potential savings I for such an application, the area of the display S is set to the size of the human visual field and the central region resolution ν_c is set to the maximum resolution of the visual system at the center of gaze. We take a conservative estimate of the visual field size to be 180° horizontal × 100° vertical (some estimates of the useful visual field size are as large as 200° × 135°; Wandell, 1995), and we estimate the maximal linear resolution of the visual system as 60 cycles/°, which agrees well with that indicated by foveal cone spacing and psychophysical measures (Wendell, 1995). Using Equation 6 to calculate the optimal radius ρ gives 4.4°, which, if used in Equation 5 to obtain a savings factor, results in a gain of $I = 150$. This result indicates that the uniform resolution display requires 150 times more resources than the equivalent variable-resolution display.

With image transmission applications, the potential savings are likely to be less substantial primarily because computer monitors typically subtend only a portion of the visual field. To determine the potential savings I for this application, the area of a typical monitor S can be approximated to be 900°^{2} (30° horizontal × 30° vertical). A typical monitor resolution of approximately 15 cycles° can be used for the central region resolution ν_c (derived from a standard resolution of 1024 × 768 pixels). These values assume a typical viewing distance of 70 cm with a 21-inch monitor. Again, using Equation 6 to calculate the optimal radius ρ gives 4.1°, which, if used in Equation 5 to obtain a savings factor, results in a gain of $I = 8.7$. This result indicates that the uniform resolution display requires 8.7 times more resources than the equivalent variable-resolution display.

Interestingly, Kortum and Geisler (1996) found similar gains in efficiency with an applied rather than theoretical evaluation of a gaze-contingent variable-resolution display in an image transmission application.

Analysis Implications

To summarize these results, Figure 2 shows a plot of the computational gain I for the optimal central region radius ρ determined for a range of central region resolutions (ν_c) and display areas (S). As can be seen from the figure, the greatest gain occurs with a combination of high resolution and large display size ideal for virtual reality applications. Yet, in agreement with the rough estimates from the previous sections, significant savings on the order of a factor of 10 can still be obtained for smaller, low-resolution displays typical of image transmission applications. Note also that central region size and display area have a multiplicative effect in that substantial increases in gain are observed only when both display factors increase simultaneously. An additional result of practical importance is that gain increases linearly with central region resolution, whereas gain tends to asymptote off as display area increases. Accordingly, it is advantageous when designing a variable-resolution display to select a higher resolution monitor rather than a monitor of larger size. All other things being equal (e.g., cost), trading display size for resolution will enable computational resources to be more efficiently used.

The analysis presented in this section applies to the most common variable-resolution display, the two-region display. This display takes into account the drop in visual sensitivity in the periphery and presents two resolutions rather than one resolution, as is done in standard uniform-resolution displays. Using such a display can result in a significant savings, ranging from a factor of 10 to 150, depending on the display device. What is the gain if more than two regions are used? Our analysis does not directly address this, but it is clear that using more regions to obtain a better approximation to the drop in visual sensitivity will certainly increase the potential savings, making variable-resolution approaches all the more valuable.

Better approximations than the two-region case are typically avoided because the additional computational cost of generating a multiregion display in real time can be prohibitive, to the

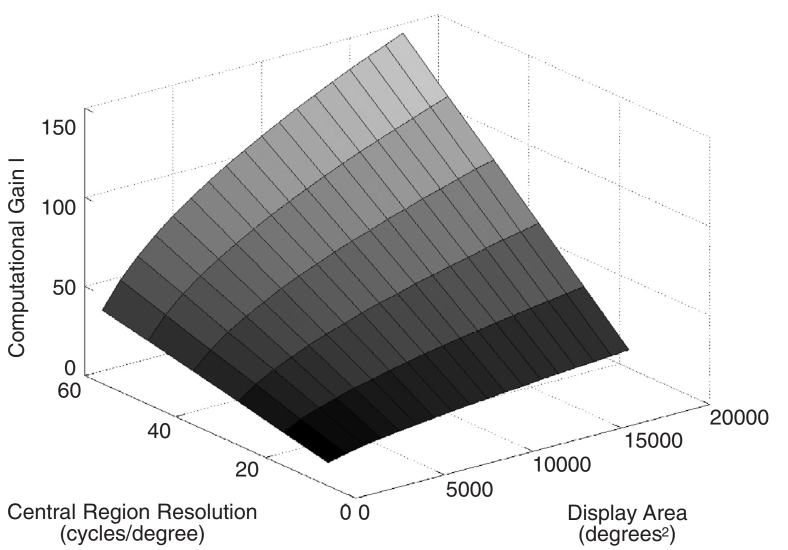

Figure 2. Computational gain I as a function of central region resolution ν_c (cycles/degree) and display area S (degrees squared). Gain increases with both increasing central region resolution and increasing display size.

point that more resources are used to generate the display than are saved. The optimal approximation will be dependent on the implementational details of the particular approximation used and therefore must be evaluated on a case-by-case basis.

Generalizing to Virtual Reality

Throughout our analysis we have equated the computational cost to render a display with the area of that display times its linear resolution squared (see Equation 1). For the majority of applications that utilize two-dimensional bitmap representations, this is a realistic measure of the computational cost. On the other hand, this relationship is not as direct for three-dimensional object representations (e.g., meshes or volumes) used in virtual reality applications.

The primary difficulty comes from the fact that unlike for bitmap graphics, the complexity of object representations does not map directly, one-to-one, onto displayed resolution. Rather, there is a nontrivial relationship between displayed resolution and three-dimensional object geometry. Taking, for example, a polygonal object representation, the exact mapping between number of polygons that are used to render a model and the degree to which those polygons are resolvable at a given resolution will depend on the chosen rendering methods (e.g., lighting and shading techniques). Research on the relationship between model complexity and image quality is in its infancy (but see Lindstrom & Turk, 2000). For these reasons, we do not attempt to quantify this relationship in our analysis.

Does this fact alter the conclusions of our analysis for virtual reality applications? If we assume that the relationship between the amount of visual detail (i.e., resolution) available in a rendered image of a model increases, on average, with the complexity of the model, then our conclusions go unaltered. In fact this is a reasonable assumption. For example, adding more polygons to an object mesh should, in general, increase the amount of rendered detail. Clearly, the addition of particular polygons can lead to a temporary reduction in detail because of occlusion, but as more polygons are added, rendered detail will tend to increase.

Furthermore, in practice, this relationship holds. A large literature on model simplification bears this out (for a recent review, see Luebke, 2001). In addition, such gaze-contingent variable-resolution displays have been implemented in virtual reality applications and shown to provide substantial computational savings (e.g., Danforth et al., 2000; Luebke, Hallen, Newlield, & Watson, 2000, Murphy & Duchowski, 2001).

It is straightforward to apply our analysis to a particular variable-resolution display in a virtual reality application. All that needs be done is to replace the resolution term R^2 in Equation 1 with a measure of model complexity that is applicable to the rendering system. For example, with a polygonal mesh representation, resolution should be replaced with polygons per unit area. Once this is done, the results and conclusions of our analysis apply.

PRACTICAL ISSUES

Although variable-resolution display techniques are clearly efficient, their application has been limited. These techniques have had the most success in the internet video transmission domain, where low bandwidths heavily restrict real-time multimedia content. Gaze-contingent variable-resolution displays have seen a much more limited application and are found primarily in high-end virtual reality systems used for flight or combat simulations (e.g., see Fernie, 1995). This section discusses the practical issues that have constrained the use of variable-resolution display techniques in applied settings.

Display Quality

There are two applied approaches to using variable-resolution displays. First are displays that maintain visual detail at any given point in the display, at or above the level determined by the human visual system. The goal of this approach is to minimize or eliminate the user's perception that a variable-resolution display is being used but still obtain an increase in display efficiency. This approach has been applied most often to virtual reality applications in which high resolution was maintained in a relatively large window at the point of gaze surrounded by low resolution in the periphery. Although maintaining such a large high-resolution window is resource inefficient, it is still much more efficient than maintaining high resolution across the *entire* display.

Second, there are displays that maintain visual detail at a lower level than that determined by the human visual system. Typically this approach is taken so that no resources are wasted by presenting more detail than can be perceived or to improve the gain in display efficiency at the cost of introducing perceptual artifacts (e.g., blur in the periphery). This approach is most often applied in bandwidth-limited image transmission applications in which the potential gain is reduced by using the small, low-resolution monitors found with desktop computers. For example, peripheral resolution can be sacrificed to increase frame update rates in video streams (Reeves & Robinson, 1996). The perceptual benefits of a greater temporal resolution outweigh the consequences of the visible reduction in peripheral resolution because the increased sense of motion tends to keep the viewer's attention centered on the regions of high resolution. The approach adopted by a particular application will depend on the relative importance of display efficiency compared with perceptual and behavioral consequences. The consequences of this choice must then be evaluated experimentally, as will be discussed later.

Gaze-Contingent Displays

Gaze-contingent variable-resolution techniques select the region of interest by actively tracking the viewer's eyes and maintaining a high level of detail at the point of gaze. Widespread application of such displays has been hampered by a number of difficulties. First, the technique requires fast and continuous tracking of the center of gaze. Although eye-tracking technology has been available for many years using a variety of methods (e.g., Purkinje reflection-based methods, contact lens-based eye coil systems, and electro-ocularography; see Young & Sheena, 1975, for a survey of classical eye-tracking technology), these techniques have been either too expensive for widespread application, too imprecise, or too invasive for routine use with humans.

It is only with the relatively recent advent of video-based eye tracking that simple and minimally invasive eye-tracking facilities are becoming widely available. Currently video sampling rates of 60 Hz are standard, but hardware capable of 250 Hz is now available and rapidly becoming common. The accuracy of video-based eye trackers is much improved compared with previous methods and is often better than 0.5°. Furthermore, the need for restrictive head restraint has been eliminated with the use of head-mounted eye-tracking methods and active head-tracking techniques.

A second limitation stems from the need for real-time updates of the visual display synchronized to follow the viewer's center of gaze. In older implementations, special video-swapping boards or multiple video boards were used to achieve reasonable update rates (van Diepen, 1997; van Diepen, De Graef, & Van Renabergen, 1994). Fortunately, the recent market drive for inexpensive video boards optimized for 3-D gaming environments has minimized the cost and increased the availability of suitable video hardware, especially for applications in virtual reality.

Although the price of current gaze-contingent variable-resolution displays is substantially less than that of their predecessors, available basic eye-tracking systems cost on the order of \$10 000 US. Although this price may be too high for consumer-level visual displays, it is well within the range of even low-end virtual reality systems. Given that one can theoretically increase the display efficiency by approximately 150 times using these techniques, this cost is much less than that required to increase the computational power of such a system by the same factor. Certainly for high-end virtual reality systems, in which obtaining faster computational hardware becomes exponentially more expensive and sometimes impossible, the obtainable increase in efficiency is well worth the price.

Currently, the primary factor limiting widespread application of gaze-contingent displays is not technological but, rather, economic. Eye-tracking technology has advanced to the point that noninvasive video-based techniques are sufficiently fast and accurate to be effectively used in gaze-contingent applications. Similarly, video hardware capable of rapidly updating the visual display is already available. The viability and widespread use of gaze-contingent applications now depends primarily on the development of low-cost eye-tracking systems.

Region of Interest Displays

The purchase of eye-tracking systems can be avoided by constructing variable-resolution displays in which the locations of high resolution are determined by other means. This selection can be made off line ahead of time, or on line in an interactive fashion using types of user input that are easier to obtain than eye movements.

Off-line selection of regions of interest is primarily used in internet image transmission applications. For example, streaming digital video content can be processed ahead of time in variable resolution to significantly reduce bandwidth requirements (Osberger, Maeder, & Bergmann, 1998).

Off-line selection can be achieved in a variety of ways. First, manual selection can be made using heuristic rules (e.g., "always select faces"), but this method is somewhat arbitrary and time consuming in applications other than teleconferencing (Basu & Wiebe, 1998). Second, regions of interest can be determined beforehand using the eye movements of a representative set of participants (Duchowski & McCormick, 1998; Stelmach et al., 1991). For example, regions that are frequently fixated by participants during viewing can be preferentially selected for presentation in a high resolution. The advantage of this method is that it results in regions of interest that are accurately placed for a typical viewer. The disadvantage of this method is that eye-tracking equipment must be available at some point, although not during image transmission, as is required of gaze-contingent displays. Finally, computational models of visual selective attention based on the human visual system can be used to predict regions of interest (Itti, Niebur, & Koch, 1998; Niebur & Koch, 1996; Osberger & Maeder, 1998). The primary advantage of this method is the high level of automaticity in the selection and subsequent encoding that can be achieved. Implementation and validation of such models are in progress (Parkhurst, Law, & Niebur, 2002).

Regions of interest can also be selected on line during image transmission via user input. This method is particularly useful in telemedicine or data visualization applications, in which a significant amount of information must be transmitted across a limited information channel. Typically users select a particular spatial region for enhancement based on an initial low-detail representation. Interactive region of interest selection is superior to predetermined region of interest selection for certain applications, in that viewers can select regions of interest in a dynamic fashion and in a way that may not necessarily be linked to visual salience.

For example, a physician may wish to inspect a relatively homogeneous region in a magnetic resonance imaging scan for signs of cancerous growth. Automated salience-based encoding schemes may inappropriately classify this region as one of low importance and represent it in low detail. User interaction allows the observer to select this region of low visual salience and assign it high priority for transmission. The cost of interactive inspection is that dedicated hardware must be in place to downgrade the high-detail, server-side image information, matching it to the level of detail requested by the user. Depending on the representation of the information, significant server-side computational resources are required to accomplish this (see Frajka et al., 1997, for a discussion on this topic).

General Limitations

In general, widespread use of variable-resolution techniques has also been hampered by the quality of available video displays.

Variable-resolution techniques can be applied to at least three major types of displays. First and most commonly, desktop displays used in Internet image transmission applications or virtual gaming environments can take advantage of these techniques. Desktop displays commonly subtend 20 to 30° of visual angle at normal viewing distances and can support the display of just over one million pixels. Although desktop displays are clearly limited in available resolution and display size, application of variable-resolution techniques can still potentially provide an order-of-magnitude savings.

Second, head-mounted displays in virtual reality applications represent a case in which a much more substantial gain might be obtained. Unfortunately, most commonly available head-mounted displays are similarly limited in quality, with displays typically spanning 30 to 60° of visual angle and supporting the display of fewer than one million pixels. The potential gain for head-mounted displays is somewhat larger than for desktop displays because of the greater field of view.

Finally, displays that utilize multiple projectors or video screens are available and stand to benefit substantially from variable-resolution display technology. These displays can subtend a very large field of view, often occupying a whole wall, and can display as many as 18 million pixels (e.g., see Funkhouser & Kai, 2000; Schikoreet al., 2000). The potential gains of variable-resolution techniques are limited by the cost and availability of large, high-resolution displays, but as display technology advances, so will the appeal of these techniques.

It is also worth noting that as display technology advances, computational hardware will also advance. Indeed, at some point in the distant future, the available computational resources will be capable of presenting very high resolution over the entire visual field. This does not negate the usefulness of variable-resolution techniques because it will still be advantageous to use these techniques to shift resources to other computationally intensive tasks – for example, predicting future eye movements.

BEHAVIORAL CONSEQUENCES

Given the complexity of the human visual system and the difficulty of predicting the behavioral and perceptual consequences of using variable-resolution display techniques, it is important that these techniques be experimentally examined. Primarily because of the aforementioned technical difficulties, only a handful of researchers have attempted to conduct experimental evaluations of these displays.

Most research has centered on using these techniques (through foveal or peripheral degradation) to better understand the process of reading (McConkie & Rayner, 1975) and scene perception (van Diepen, Wampers, & d'Ydewalle, 1998). For a review of these approaches, see Rayner (1998). A few studies have attempted to evaluate the effects of using gaze-contingent, variable-resolution display techniques. Unfortunately, most of these studies examined only perceptual or performance measures.

A complete evaluation of variable-resolution displays requires not only examining measures of task performance (e.g., reaction time and accuracy) and perceptual reports but also eye movement measures (e.g., fixation duration and saccade length). Although perceptual reports and task performance are directly relevant in applied settings, eye movement measures can provide a more sensitive measure of potential problems.

Gaze-Contingent Displays

The use of gaze-contingent displays introduces potential technical complications that must be experimentally examined. For example, the visual update rate of the display must be sufficiently fast and synchronized to follow eye movements. The total time to make a gaze-contingent display change is a function of the eye tracker sampling rate, the saccade detection delay, the software drawing routine speed, and the refresh rate of the display.

To determine the update rate that is required to avoid perception of a display change, Loschky and McConkie (2000) presented participants with a gaze-contingent display showing photographic images. A high-resolution image was presented during fixations and a low-resolution image during eye movements. The delay in replacing the low-resolution image with the high-resolution image and eye movement was varied. Participants were unable to detect the lower resolution if the display was updated within 12 ms of the end of the preceding eye movement.

In a related study, Inhoff, Starr, Liu, and Wang (1998) evaluated a gaze-dependent display in a reading paradigm and found that neither reading performance nor eye movement measures were correlated with update delays in the range of 5 to 16 ms. Such an extended reduction in sensitivity may be supported by saccadic suppression of visual information, which has been shown to be effective prior to, during, and just after a saccade (Burr, Morrone, & Ross, 1994; Hubel, 1988; Volkman, Riggs, White, & Moore, 1978).

It is important to note that although participants are at above-chance levels in detecting lower resolution if the display is not rapidly updated, this does not imply that longer delays are visually disturbing or harm task performance. To address this question, Loschky and McConkie (2000) asked participants to search for an object in a photographic image using a gaze-contingent, two-region, variable-resolution display with display updates that were delayed for as long as 45 ms from the beginning of each fixation. Task performance was not significantly affected in any condition. On the other hand, average fixation duration was prolonged by approximately 20 ms compared with a full-resolution condition. This result is important because it indicates that long update delays do not necessarily affect task performance; furthermore, given that eye movements were affected, task performance effects and eye movement effects are not necessarily associated.

It is also important to note that even though participants are unable to accurately detect low resolution when updated rapidly, this does not imply that the lower resolution had no effect. In fact, O'Regan (1990) argued that using gaze-contingent displays can disrupt normal eye movements because of the visible flicker and reduced contrast associated with updating a display at the point of fixation.

Shioiri (1993) explored this question by presenting photographic images to participants for a later recognition memory test. During the study phase the images were either blurred or blanked for delay periods as short as 25 ms and as long as 200 ms following the beginning of each fixation. Across all delays, fixation durations were prolonged by approximately the same amount of time as the full resolution images were delayed. This again indicates that eye movements are susceptible to display update delay. It is unclear whether this proportional delay holds for shorter, imperceptible delays, as might be predicted from these results.

The studies of gaze-contingent displays cited in this section provide significant, insight but further research is needed. It is clear that at some point, as update delay increases, not only will perceptual and eye movement effects occur, but task performance should also be affected. What is unclear is the point at which this occurs and whether this point is dependent on the task. To help answer this question, a study examining the interrelationships among task performance, perceptual report, and eye movement measures as a function of update delay is needed. Additionally, research examining the consequences of display update delay needs to be conducted across a wide range of tasks and conditions that are likely to be differentially affected by update delays.

Variable Resolution: Central Region Size and Peripheral Resolution

Experimental studies employing variable-resolution techniques have primarily used two-region displays to examine the importance of available resolution in vision. Two variables that are manipulated in these studies are the size of the central region and the resolution in the periphery. Each variable controls the amount of available resolution, but in different ways. Increasing central region size provides more resolution near the center of gaze, where it is most likely to be useful in identification tasks, whereas increasing peripheral resolution provides more information in the periphery, where it is most likely to be useful in target localization tasks. The following paragraphs describe experimental studies that examined the effects of manipulating these two variables.

Saida and Ikeda (1979) examined picture memory using a gaze-contingent variable-resolution display. Participants were required to memorize black-and-white drawings of common scenes for a later recognition memory test. During the study phase participants viewed the drawings using a gaze-contingent variable-resolution display in which a square window with side length ranging in size from 3 to 11° followed the point of gaze. Outside the display the picture was blanked. Each drawing was presented only once and was displayed for a duration ranging from 0.5 to 20 s. These researchers found that the amount of time required to reach 70% correct recognition in the test phase decreased with increasing window size. The 11° window was found to be the optimal window size because it resulted in performance indistinguishable from the unrestricted baseline viewing condition.

In a second experiment Saida and Ikeda (1979) reduced the image size by approximately 60% and found similar reduction in the optimal size of the window following the center of gaze, to 8°. They concluded that optimal window size is a function of the density of visual information in an area rather than the physical size of that area. Interestingly, average saccade size was found to be a function of window size; shorter saccades were observed for smaller windows. Saccade sizes were unaffected by the absolution image size.

Shioiri and Ikeda (1989) further examined picture memory using the same experimental task and a similar gaze-contingent variable-resolution display. During the study phase participants viewed drawings using a gaze-contingent variable-resolution display in which a square window with side length ranging in size from 3 to 10° tracked the point of gaze. Outside the window a portion (ranging from 60% to 100%) of the image pixels were randomly picked and blanked to white. This manipulation had a similar visual effect to that obtained by reducing spatial resolution in the periphery. These researchers found that the amount of time required to reach 75% correct recognition increased with decreasing peripheral resolution and decreasing window size. From these data they estimated an optimal window size, one in which increase in window size would no longer produce an improvement in performance. When no peripheral information was present (i.e., 100% blanked), performance increases were seen for windows up to the maximum size of 10°. When only 60% of the pixels were blanked, the optimal window was smaller, at 4°. In addition, saccade length was again found to be a function of window size, whereby smaller windows resulted in shorter saccades, by as much as a degree. Interestingly, this effect was strongest in the fully blanked condition and weakest in the 60% blanked condition. This suggests that the dependence of saccade length on window size is not attributable to the discrete high-to-low resolution border but, rather, is related to the general level of reduction in peripheral resolution.

Loschky and McConkie (2000) examined visual search performance for objects in photographic images of natural scenes. A two-region, gaze-contingent variable-resolution display was used with a circular central region with a radius that ranged from 1.6 to 4.1°. Three levels of peripheral resolution reduction, determined by the number of coefficients in a wavelet transform, were tested in the periphery. They found that detection of the peripheral resolution reduction tended to increase with decreasing central region size and decreasing peripheral resolution. Search time showed a similar pattern; slowing was observed for smaller central region sizes as well as greater peripheral resolution reduction. Saccade size effects were similar to those observed by Shioiri and Ikeda (1989); both decreasing peripheral resolution and decreasing central region size led to shorter saccade sizes. Fixation durations were not affected by manipulations in peripheral resolution except for when the central region was very small (1.6°). These researchers concluded that a gaze-contingent display with a central region radius of 4.1° supports approximately normal viewing performance and eye movements.

Using a variation on gaze-contingent variable-resolution paradigms, van Diepen and Wampers (1998) employed a two-region display in which the central region was an oval subtending 3.5 × 2.6° and the peripheral information was either low-passed, band-passed, or high-passed, but only during the first 150 ms of each fixation. After 150 ms, the periphery reverted back to full resolution. Participants were presented with computer-rendered scenes and required to search for a nonsense object. Although no difference was observed between the different filtering conditions, fixation durations were prolonged, saccades were shorter, and search time was longer than in the full-resolution baseline condition.

In a related, gaze-contingent masking paradigm, van Diepen, Ruelens, and d'Ydewalle (1999) required participants to search line drawings for nonsense objects. A mask was presented foveally for a duration of 83 ms following a delay ranging from 15 to 85 ms after each fixation. The masked covering the point of gaze was of an oval filled with random-dot noise subtending 2.5 × 1.9°. Peripheral information was always presented in full resolution. The effect of this manipulation was to increase search time and prolong fixation durations. The effect on fixation duration was related to that found by Shioiri (1993); fixation durations were prolonged the most for early mask onsets and the least with delayed mask onsets. Interestingly, saccade size was not affected, further suggesting that saccade parameters are determined by peripheral rather than central information. An identical pattern of results was obtained by Bertera and Rayner (2000) in a similar gaze-contingent foveal masking paradigm that involved searching for a letter among a randomly scattered array of distracter letters.

Taken together, these studies indicate a clear pattern of results. Small central region sizes consistently lead to slowing in search tasks and lower accuracy in memorization tasks as well as longer fixation durations and shorter saccades. Reduced resolution in the periphery also leads to slowing in search tasks and lower accuracy in memorization tasks. On the other hand, only saccade size is consistently affected by manipulations of peripheral resolution. Fixation durations are for the most part unaffected by peripheral resolution and are affected only by changes in peripheral resolution when the central region size is very small. These results suggest that peripheral information primarily affects mechanisms of saccade target selection, whereas central information primarily affects mechanisms of object identification. However, we note that central region size and peripheral resolution are interrelated in variable-resolution displays, and therefore their effects cannot be strictly dissociated.

Evaluation of Applied Variable-Resolution Displays

Although much research has gone into implementing and testing the technical aspects of variable-resolution displays in a variety of applications, only a few of these applied studies have examined the resulting behavioral and perceptual consequences. Two such applied studies of note include Kortum and Geisler (1996) and Parkhurst, Culurciello, and Niebur (2000). Kortum and Geisler (1996) evaluated a gaze-contingent variable-resolution display in which resolution continuously declined as the distance from the point of gaze increased. This was accomplished by displaying so-called SuperPixels, which increased in size in the periphery, rather than the pixels available in the full-resolution images. The average luminance across all the pixels in the full-resolution image subtended by a given SuperPixel determined that SuperPixel's luminance. Using this technique reduces the resolution of the surround more efficiently than applying traditional image-filtering techniques.

To evaluate the perceptual consequences of this display, participants viewed various photographic images and were required to make perceptual quality judgments. Participants reported that perceived quality was highest during fixation and that reduced contrast and apparent motion artifacts were visible in the periphery during eye movements. The latter effect is presumably caused by the remaining high spatial frequency content at the SuperPixel edges. Participants also reported that peripheral reduction in resolution was visible but that this fact had a minimal effect on perceptual quality measures.

Parkhurst et al. (2000) conducted an applied evaluation of a gaze-contingent, two-region, variable-resolution display. They presented participants with images of home interiors and required participants to search for specific objects. All stimuli were of approximately the same size and type (e.g., medium-sized bowls in living rooms or bedrooms). All were smaller than the smallest high-resolution window studied and could be identified with each resolution used in the central region of the variable-resolution display. These researchers argued that although variable-resolution techniques are expected to yield better performance for fine-grained detection and discrimination tasks in the central visual field (because resources are preferentially allocated to the center), performance could be degraded in tasks, like visual search, that make use of peripheral information. To find the optimal two-region display where search performance most resembled that obtained with a full-resolution display, the size of the central region radius was varied from 1 to 15°.

The applied approach Parkhurst et al. (2000) took to evaluate these displays was guided by two principles. First, no computational resources were to be wasted presenting more detail than could be processed at a particular eccentricity. Presenting more detail, as is done with traditional uniform resolution displays, wastes resources. To accomplish this, they set the resolution of the central region to be no greater than the level determined by visual sensitivity at the outer border of the central region. The effect of this constraint was that as the central region size grew, the resolution in that region dropped. This drop of resolution in the central region was visible to participants, but assured that no resources were being wasted.

Second, each experimental condition was required to use the same total amount of computational resources to generate the display. Intuitively, having a large central region of high resolution should always be advantageous as opposed to a smaller region of high resolution. Comparing the behavioral consequences observed in these two conditions would be unfair given that the display with the larger central region would require more computational resources to generate. Equating computational resources makes the comparison fair and allows generalization of the results to applied settings in which computational resources are usually constant (i.e., determined by given hardware). One effect of this constraint was that when displaying a large central region, few resources remained to generate the surrounding region, and therefore it was displayed in a low resolution. On the other hand, with a small central region more resources were available and the surround could be displayed in a higher resolution.

Together, these constraints allowed the comparison of variable-resolution displays of equivalent computational demand, none of which wasted resources by presenting more detail than could be seen by the participant. The consequence of these constraints was that the overall resolution of the display dropped as central region size grew. The resolution in the central region dropped to accommodate the first constraint (visual sensitivity), and the resolution in the peripheral region dropped to accommodate the second constraint (equivalent costs). This fact is important in interpreting the following task performance results.

The task performance results indicated a speed/accuracy trade-off. When the central region radius was small (less than 5°), participants more carefully inspected the image, tending to take more time and be more accurate than in the full-resolution baseline condition. On the other hand, when central region radius was large (greater than 5°), participants inspected the image less carefully, tending to respond rapidly and less accurately. Given that resolution of the central region was always sufficient to identify a target once fixated, the differences in accuracy and reaction time were not attributable to differences in target discriminability but, rather, to a shift in the way participants choose to perform the task.

It is well known that participants can be influenced to perform a task either accurately or rapidly, and in this paradigm Parkhurst et al. (2000) attributed this shift observed across experimental conditions to the changing amount of available resolution. When the overall resolution was high (i.e., when the central region was small), participants performed the task carefully, but when the available resolution was low (i.e., when the central region was large), participants performed the task rapidly. Interestingly, when the central region radius was 5°, both reaction time and accuracy were indistinguishable from those in the full-resolution baseline condition, indicating that participants performed the task in a similar way in both conditions. Parkhurst et al. (2000) concluded that use of a 5° central region radius for two-region variable-resolution display is optimal for visual search of natural stimuli in their applied system.

These two applied studies represent exemplar empirical evaluations of variable-resolution displays. The major contributions of applied studies such as these include validating conceptual research, fine tuning of display parameters within the applied constraints of a given application, and, most important, the discovery of unexpected behavioral consequences (e.g., speed-accuracy trade-offs) associated with use of these displays. It is only with such applied behavioral evaluations that variable-resolution displays can eventually become commonplace. Although it is important to understand these benefits, it is also important to understand the limitations of applied studies. Given that they are uniquely tailored to a particular application, it is difficult to generalize the results of such studies to other situations. Clearly, applied work must be buttressed with more controlled basic science such as that reviewed in the previous sections.

Difficulties in Applying Measures of Sensitivity

The human visual system is dynamic and

adaptive. Therefore, manipulations made on the visual input – however minor – can have behavioral and perceptual consequences. As noted earlier, using a variable-resolution display can produce significant alterations in task performance, perceived display quality, and eye movements. As a heuristic, many variable-resolution studies have proposed to use or have actually used measures of visual sensitivity to guide the choice of display parameters (e.g., resolution and central region size). Although this approach is preferable to an arbitrary selection of display parameters, there are a number of associated difficulties.

First, which measure of sensitivity is to be used? Visual sensitivity can be estimated from both biological and behavioral measures. Biological measures of visual sensitivity can be derived from neuroanatomical measures (e.g., retinal photoreceptor densities or cortical magnification factors) or from neurophysiological measures (e.g., receptive field sizes). If we take retinal photoreceptor densities, which limits the resolution of visual input, this will ensure enough visual detail for most tasks. Unfortunately, it does not necessarily provide sufficient detail for all tasks. For example, hyperacuity (i.e., sensitivity to detail smaller than the size of a single photoreceptor) is obtainable when the visual system takes advantage of the optical point spread function of the eye's lens in combination with a retinal population code.

Alternatively, a behavioral measure of sensitivity can be used. The contrast sensitivity function (Virsu & Rovamo, 1979) is frequently used for this purpose. This function is a measure of sensitivity to the contrast of sinusoidal gratings across a range of spatial frequencies and eccentricities. Unfortunately, it is only one of many behaviorally obtainable measures that describe the sensitivity to visual detail. Other measures include Vernier acuity, Landolt ring acuity, and discrimination acuity using Snellen or Sloan letters. All these measures provide quantitatively different estimates of sensitivity. Even with one measure, a variety of results may be obtained under different experimental testing conditions, including such basic parameters as ambient light levels.

Both biological and behavioral measures have limited generality outside a relatively circumscribed domain, none of which is likely to generalize to visual sensitivity while viewing natural stimuli. For example, Peli and Geri (1999) found that discrimination performance of variable-resolution images generated using contrast sensitivity functions obtained from orientation and detection tasks using either sinusoidal gratings or Gabor patches varied greatly from each other.

To compound this problem, it is well known that attention (Egeth & Yantis, 1997; Posner, 1980) can dynamically modulate the portion of the visual field that receives detailed processing; attentional allocation can be focused or diffuse. For example, focal attention acts to enhance spatial sensitivity at the location of attention (Goto, Toriu, & Tanahashi, 2001; Yeshurun & Carrasco, 2000). Furthermore, attention can be volitionally allocated to parts of the visual field independent of the point of gaze. Therefore, displaying high resolution at the point of gaze may be inappropriate when the center of gaze and the location of attention are dissociated.

It is also known that the visual system is sensitive to different spatial scales depending on task demands (Schyns & Olivia, 1997). In some tasks (e.g., object identification), visual detail may be needed in a limited spatial region, whereas in other tasks (e.g., visual search), detail may be more useful in the periphery. For example, tasks such as visual search require high spatial frequency content in the periphery for efficient guidance, whereas identification of global scene properties (e.g., scene gist) requires only low spatial frequency content in the periphery (van Diepen et al., 1998).

Until the operation and dynamics of visual selective attention are better understood in the context of a variety of tasks appropriate to natural stimuli, variable-resolution display techniques will not be able to take full advantage of the potential reduction in resolution and gain in resources that can be obtained. At this time no simple rule of thumb, measure of sensitivity, or single behavioral task can be used to authoritatively predict the perceptual and behavioral consequences associated with variable-resolution displays. This fact argues strongly for a task-specific experimental evaluation of any variable-resolution display.

CONCLUSIONS AND FUTURE DIRECTIONS

The focus of this review is the evaluation of variable-resolution displays from three perspectives. First, a theoretical evaluation of variable-resolution displays indicated that these displays provide a substantial gain in computational efficiency for virtual reality and image transmission applications. Second, the practical issues associated with variable-resolution displays were discussed in the context of a variety of applications. Although technical and economical barriers have stood in the way of widespread use of variable-resolution displays in the past, improvements in eye tracking, graphics-rendering hardware, and visual displays continue to make variable-resolution techniques more practical and advantageous. Third, the behavioral consequences associated with using variable-resolution displays were reviewed in both theoretical and applied contexts. Detailed behavioral evaluations have begun only recently, and current research indicates that variable-resolution display techniques can be used effectively to improve display efficiency without introducing significant behavioral consequences if display parameters are experimentally determined. Careful experimental studies of task performance, perceptual quality, and eye movement measures will be required to tap the whole potential of these techniques.

As discussed earlier, much research on variable-resolution display techniques has been carried out in the applied domain, where the focus has been on the development of efficient variable-resolution displays across a wide range of applications. There is a need for studies of the behavioral consequences of using variable-resolution display techniques. Important questions remain to be examined in both theoretical and applied settings.

For example, what are the effects of reducing peripheral resolution, and how do the effects vary as a function of experimental task? It is clear that reducing peripheral resolution introduces behavioral and perceptual consequences at some level, but it is also clear that the type and amount of visual information required in the peripheral visual field depends on the task and the attentional state of the observer. A detailed examination of this relationship is needed to provide better guidelines for variable-resolution display applications.

A related question concerns the relationship between perception of the variable-resolution display and its effect on task performance and eye movements. Some of the evidence reviewed suggests a dissociation between perception and behavior. Participants may indeed be able to perceive reductions in the peripheral resolution, but their performance may not be significantly affected by it. This relationship needs to be examined further.

Even more interesting, given the fact that attention modulates visual sensitivity to detail, can predictive strategies based on models of attention be employed in variable-resolution displays so that rather than simply following the viewer's center of gaze, predictions of future fixation locations are made?

Potential directions for research are many, but all center on a better understanding of visual information processing. Overall, the logic of adapting level of detail in variable-resolution displays to the level usable by a human observer is sound, but to do this well is a much more complicated task than previously thought.

APPENDIX: DERIVATION OF EQUATION 6

It is convenient to introduce the abbreviation

$$N := Sv_c^2 \tag{7}$$

for the numerator of Equation 5 and

$$A := \pi v_c^2 \tag{8}$$

for the first term in its denominator. With these definitions, Equation 5 becomes

$$I = \frac{N}{A\rho^2 + S(\alpha/\rho)^2 - \pi\alpha^2}. \tag{9}$$

Multiplication by ρ^2 in numerator and denominator yields

$$I = \frac{N\rho^2}{A\rho^2 + S\alpha^2 - \pi\alpha^2\rho^2}. \tag{10}$$

From the discussion following Equation 5, we know that the optimal value (that is, maximal computational gain) is to be found not at the extremes of possible values of ρ, which would be 0 and ∝, but, instead, at a finite value. Therefore, the global maximum will be a local maximum (i.e., attained for finite ρ). The local maxima (and minima) of the gain function can be found by taking those values where the gain is momentarily unchanging, which is when the first derivative of the gain is zero. We can therefore find the maximal computational gain by differentiating *I* with respect to ρ, setting the result to 0 and solving for ρ.

From the chain rule, we find for all differentiable functions

$$f(\boldsymbol{x}) := \frac{u(\boldsymbol{x})}{v(\boldsymbol{x})}$$

that

$$f' = \frac{u'v - v'u}{v^2}$$

where the prime indicates the first derivative with respect to $\boldsymbol{x}$. We find

$$\frac{dI}{d\rho} = \frac{2N\rho(A\rho^4 - \pi\alpha^2\rho^2 + S\alpha^2) - N\rho^2(4A\rho^3 - 2\pi\alpha^2\rho)}{(A\rho^4 + S\alpha^2 - \pi\alpha^2\rho^2)^2} = 0. \tag{11}$$

We divide Equation 11 by ρ (which is allowed given that $\rho \neq 0$) and multiply it by its denominator to obtain

$$0 = 2NA\rho^4 - 2N\pi\alpha^2\rho^2 + 2NS\alpha^2 - 4NA\rho^4 + 2N\pi\alpha^2\rho^2. \tag{12}$$

Collecting terms, only

$$0 = -NA\rho^4 + NS\alpha^2 \tag{13}$$

remains. Given that both $\nu_c \neq 0$ and $S \neq 0$, it follows that $N \neq 0$ and $A \neq 0$, and we may therefore divide by *NA,* yielding

$$\rho^4 = \frac{NS\alpha^2}{NA} = \frac{S\alpha^2}{A}. \tag{14}$$

We now replace *A* by its definition (see Equation 8), and take the fourth root of Equation 14. Given that ρ is a radius, we have to take the positive root, yielding

$$\rho = \sqrt[4]{\frac{S\alpha^2}{\pi\nu_c^2}}. \tag{15}$$

This is Equation 6.

ACKNOWLEDGMENTS

This research was supported by National Science Foundation CAREER Grant IBN9876271 and an Alfred P. Sloan Fellowship to the second author as well as a National Institutes of Health-National Eye Institute visual neuroscience training fellowship to the first author.

REFERENCES

Adler, F. H. (1965). *Physiology of the eye* (4th ed.). Saint Louis, MO: Mosby.

Anstis, S. (1974). A chart demonstrating variations in acuity with retinal position. *Vision Research, 14,* 589–592.

Basu, A., & Wiebe, K. J. (1998). Enhancing videoconferencing using spatially varying sensing. *IEEE Transactions on Systems, Man, and Cybernetics, 28,* 137–148.

Bertera, J. H., & Rayner, K. (2000). Eye movements and the span of the effective stimulus in visual search. *Perception and Psychophysics, 63,* 576–585.

Burr, D., Morrone, C., & Ross, J. (1994). Selective suppression of the magnocellular visual pathway during saccadic eye movement. *Nature, 371,* 511–513.

Chang, E. C., Yap, C., & Yen, T.-J. (1997). Realtime visualization of large images over a thinwire. In *IEEE visualization* (pp. 45–48). New York: Institute of Electrical and Electronics Engineers.

Dagnelie, G., & Massof, R. W. (1996). Toward an artificial eye. *IEEE Spectrum, 33*(5), 20–29.

Danforth, R., Duchowski, A. T., Geist, R., & McAliley, E. (2000). A platform for gaze-contingent virtual environments. In *Proceedings of the American Association for Artificial Intelligence, Smart Graphics Symposium* (Vol. 4, pp. 66–70). Menlo Park, CA: AAAI.

Duchowski, A. T. (1998a). Incorporating the viewer's point of regard (POR) in gaze-contingent virtual environments. In M. T. Bolas, S. S. Fisher, & J. O. Merritt (Eds.), *Proceedings of SPIE: Vol. 3295. Stereoscopic displays and virtual reality systems V* (pp. 332–343). Bellingham, WA: SPIE – International Society for Optical Engineering.

Duchowski, A. T. (1998b). Representing multiple regions of interest with waveless. In S. A. Rajala & M. Rabbani (Eds.), *Proceedings of SPIE: Vol. 3309. Visual communications and image processing* (pp. 975–986). Bellingham, WA: SPIE – International Society for Optical Engineering.

Duchowski, A. T. (2000). Acuity-matching resolution degradation through wavelet coefficient scaling. *IEEE Transactions on Image Processing, 9,* 1437–1439.

Duchowski, A. T., & McCormick, B. H. (1995a). Preattentive considerations for gaze-contingent image processing. In B. E. Rogowitz & J. P. Allebach (Eds.), *Proceedings of SPIE: Vol. 2411. Human vision, visual processing, and digital display VI* (pp. 128–139). Bellingham, WA: SPIE – International Society for Optical Engineering.

Duchowski, A. T., & McCormick, B. H. (1995b). Simple multiresolution approach for representing multiple regions of interest (ROIs). In L. T. Wu (Ed.), *Proceedings of SPIE: Vol. 2501. Visual communication and image processing* (pp. 175–186). Bellingham, WA: SPIE – International Society for Optical Engineering.

Duchowski, A. T., & McCormick, B. H. (1996). Modeling visual attention for gaze-contingent video processing. In *9th Image and Mutidimensional Digital Signal Processing Workshop* (pp. 130–131). New York: Institute of Electrical and Electronics Engineers.

Duchowski, A. T., & McCormick, B. H. (1998). Gaze-contingent video resolution degradation. In B. E. Rogowitz & T. N. Pappas (eds.), *Proceedings of SPIE: Vol. 3299. Human vision and electronic imaging III* (pp. 318–329). Bellingham, WA: SPIE – International Society for Optical Engineering.

Egeth, H. E., & Yantis, S. (1997). Visual attention: Control, representation, and time course. *Annual Review of Psychology, 48,* 269–297.

Fernie, A. (1995). Head-mounted display with dual resolution. *SID Applications Digest, 3/4,* 37–40.

Frajka, T., Sherwood, P. G., & Zeger, K. (1997). Progressive image coding with spatially variable resolution. In *Proceedings of the International Conference on Image Processing* (Vol. 1, pp. 53–56). New York: Institute of Electrical and Electronics Engineers.

Funkhouser, T., & Li, K. (2000). Large-format displays. *IEEE Computer Graphics and Applications, 20*(4), 20–21.

Geisler, W. S., & Perry, J. S. (1998). A real-time foveated multiresolution system for low-bandwidth video communication. In *Proceedings of SPIE: Vol. 3299. Human vision and electronic imaging* (pp. 294–305). Bellingham, WA: SPIE – International Society for Optical Engineering.

Geisler, W. S., & Perry, J. (1999). Variable-resolution displays for visual communication and simulation. In *Proceedings of the Society for Information Display* (Vol. 30, pp. 420–423). San Jose, CA: Society for Information Display.

Goto, M., Toriu, T., & Tanahashi, J. (2001). Effect of size of attended area on contrast sensitivity function. *Vision Research, 41,* 1483–1487.

Hubel, D. H. (1988). *Eye, brain, and vision.* New York: Scientific American Library.

Inhoff, A. W., Starr, M., Liu, W., & Wang, J. (1998). Eye-movement-contingent display changes are not compromised by flicker and phosphor persistence. *Psychonomic Bulletin and Review, 5,* 101–106.

Itti, L., Niebur, E., & Koch, C. (1998). A model of saliency-based fast visual attention for rapid scene analysis. *IEEE Transactions on Pattern Analysis and Machine Intelligence, 20,* 1254–1259.

Kortum, P. T., & Geisler, W. S. (1996). Implementation of a foveated image-coding system for bandwidth reduction. In *Proceedings of SPIE: Vol. 2657. Human vision and electronic imaging* (pp. 350–360). Bellingham, WA: SPIE – International Society for Optical Engineering.

Lee, S., Pattichis, M. S., & Bovik, A. C. (2001). Foveated video compression with optimal rate control. *IEEE Transactions on Image Processing, 10,* 977–992.

Levoy, M., & Whitaker, R. (1990). Gaze-directed volume rendering. In *Proceedings of the 1990 Symposium on Interactive 3D Graphics* (pp. 217–223). New York: Association for Computing Machinery.

Lindstrom, P., & Turk, G. (2000). Image-driven simplification. *ACM Transactions on Graphics, 19,* 204–241.

Loschky, L. C., & McConkie, G. W. (2000). User performance with gaze contingent multresolutional displays. In *Proceedings of the Eye Tracking Research and Applications Symposium* (Vol. 1, pp. 97–103). New York: Association for Computing Machinery, Special Interest Group on Computer Graphics and Interactive Techniques.

Luebke, D. P. (2001). A developer's survey of polygonal simplification algorithms. *IEEE Computer Graphics and Applications, 21*(3), 24–35.

Luebke, D. P., Hallen, B., Newfield, D., & Watson, B. (2000). *Perceptually driven simplification using gaze-directed rendering* (University of Virginia Tech. Report CS-2000-04). Charlottesville: University of Virginia.

McCauley, M. E., & Sharkey, T. J. (1992). Cybersickness: Perception of self-motion in virtual environments. *Presence, 1,* 311–318.

McConkie, G. W., & Rayner, K. (1975). The span of the effective stimulus during a fixation in reading. *Perception and Psychophysics, 17,* 578–586.

Murphy, H., & Duchowski, A. T. (2001). Gaze-contingent level of detail rendering. In *EuroGraphics 2001* (short presentations, pp. 1–10). Aire-la-Ville, Switzerland: EuroGraphics.

Niebur, E., & Koch, C. (1996). Control of selective visual attention: Modeling the "where" pathway. In D. S. Touretzky, M. C. Mozer, & M. E. Hasselmo (Eds.), *Advances in neural information processing* (Vol. 8, pp. 802–808). Cambridge, MA: MIT Press.

Ohshima, T., Yamamoto, H., & Tamura, H. (1996). Gaze-directed adaptive rendering for interacting with virtual space. In *Proceedings of the IEEE Virtual Reality Annual International Symposium* (pp. 103–110). New York: Institute of Electrical and Electronics Engineers.

O'Regan, J. K. (1990). Eye movements and reading. In E. Kowler (Ed.), *Eye movements and their role in visual and cognitive processes* (pp. 395–453). Amsterdam: Elsevier.

Osberger, W., & Maeder, A. J. (1998). Automatic identification of perceptually important regions in an image. In *Proceedings of the 14th International Conference on Pattern Recognition* (pp. 701–704). Washington, DC: IEEE Computer Society.

Osberger, W., Maeder, A. J., & Bergmann, N. (1998). A perceptually based quantization technique for MPEG encoding. In B. E. Rogowitz & T. N. Pappas (Eds.), *Proceedings of SPIE: Vol. 3299. Human vision and electronic imaging III* (pp. 148–159). Bellingham, WA: SPIE – International Society for Optical Engineering.

Parkhurst, D., Culurciello, E., & Niebur, E. (2000). Evaluating variable resolution displays with visual search: Task performance and eye movements. In *Proceedings of the ACM Eye Tracking Research and Applications Symposium* (pp. 105–109). New York: Association for Computing Machinery.

Parkhurst, D. J., Law, K., & Niebur, E. (2002). Modeling the role of salience in the allocation of overt visual selective attention. *Vision Research, 42*(1), 107–123.

Pausch, R., Crea, T., & Conway, M. (1992). A literature survey for virtual environments: Military flight simulator visual systems and simulator sickness. *Presence, 1,* 344–363.

Peli, E., & Geri, G. A. (1999). Testing a simulation of peripheral vision using an image discrimination task. In *Proceedings of the Society for Information Display* (Vol. 30, pp. 424–427). San Jose, CA: Society for Information Display.

Posner, M. I. (1980). Orienting of attention. *Quarterly Journal of Experimental Psychology, 32,* 3–25.

Rayner, K. (1998). Eye movements in readings and information processing: 20 years of research. *Psychonomic Bulletin, 124,* 372–422.

Reeves, T. H., & Robinson, J. A. (1996). Adaptive foveation of MPEG video. In *Proceedings of the 4th ACM International Conference on Multimedia* (pp. 231–241). New York: Association for Computing Machinery.

Saida, S., & Ikeda, M. (1979). Useful visual field size for pattern perception. *Perception and Psychophysics, 25,* 119–125.

Schikore, D., Fischer, R., Frank, R., Gaunt, R., Hobson, J., & Whitlock, B. (2000). High-resolution multiprojector display walls. *IEEE Computer Graphics and Applications, 20,* 38–44.

Schyns, P. G., & Olivia, A. (1997). Flexible, diagnosticity-driven, rather than fixed, perceptually determined scale selection in scene and face recognition. *Perception, 26,* 1027–1038.

Shioiri, S. (1993). Postsaccadic processing of the retinal image during picture scanning. *Perception and Psychophysics, 53,* 305–314.

Shioiri, S., & Ikeda, M. (1989). Useful resolution for picture perception as a function of eccentricity. *Perception, 18,* 347–361.

Stelmach, L. B., Tam, W. J., & Hearty, P. J. (1991). Static and dynamic spatial resolution in image coding: An investigation of eye movements. In B. E. Rogowitz, M. H. Brill, & J. P. Allebach (Eds.), *Proceedings of SPIE: Vol. 1453. Human vision, visual processing, and digital display II* (pp. 147–152). Bellingham, WA: SPIE – International Society for Optical Engineering.

van Diepen, P. M. J. (1997). A pixel-resolution video switcher for eye-contingent display changes. *Spatial Vision, 10,* 335–344.

van Diepen, P. M. J., De Graef, P., & van Rensbergen, J. (1994). On-line control of moving masks and windows on a complex background using the ATVista Videographics Adapter. *Behavior Research Methods, 26,* 454–460.

van Diepen, P. M. J., Ruelens, L., and d'Ydewalle, G. (1999). Brief foveal masking during scene perception. *Acta Psychologica, 101,* 91–103.

van Diepen, P. M. J., & Wampers, M. (1998). Scene exploration with Fourier-filtered peripheral information. *Perception, 27,* 1141–1151.

van Diepen, P. M. J., Wampers, M., & d'Ydewalle, G. (1998). Functional division of the visual field: Moving masks and moving windows. In G. Underwood (Ed.), *Eye guidance in reading and scene perception* (pp. 337–355). Oxford, UK: Elsevier.

Van Essen, D., & Anderson, C. (1990). Information processing strategies and pathways in the primate retina and visual cortex. In S. F. Zornetzer, J. L. Davis, & C. Lau (Eds.), *An introduction to neural and electronic networks* (pp. 43–72). San Diego, CA: Academic.

Virsu, V., & Rovamo, J. (1979). Visual resolution, contrast sensitivity, and the cortical magnification factor. *Experimental Brain Research, 37,* 475–494.

Volkman, F. C., Riggs, L. A., White, K. D., & Moore, R. K. (1978). Contrast sensitivity during saccadic eye movement. *Vision Research, 18,* 1193–1200.

Wandell, B. A. (1995). *Foundations of vision.* Sunderland, MA: Sinauer.

Yeshurun, Y., & Carrasco, M. (2000). The locus of attentional effects in texture segmentation. *Nature Neuroscience, 3,* 622–627.

Young, L. R., & Sheena, D. (1975). Survey of eye movement recording techniques. *Behavior Research Methods, Instruments and Computers, 7,* 397–429.

Derrick J. Parkhurst received a Ph.D. in psychology in 2002 from Johns Hopkins University, where he is a postdoctoral fellow at the Zanvyl Krieger Mind/Brain Institute.

Ernst Niebur is an assistant professor in the Department of Neuroscience and the Zanvyl Krieger Mind/Brain Institute at Johns Hopkins University. He received his Dr. ès sciences in physics from the Université de Lausanne, Switzerland, in 1988.

Date received: February 15, 2001
Date accepted: January 28, 2002