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Optical and Photoelectric Analog of the Eye

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A photoelectric analog of the visual system is constructed in conformance with anatomical data. The analog has the form of a color television camera chain feeding electrical signals to a "computer" (the brain). Evaluation of characteristics is limited to elements preceding the computer, and particularly to the "luminance channel" of the color system.

The primary photoelectric transfer characteristics $\bar{n}_1 = f(E_r)$ of the receptors (rods and cones) are computed as a function of retinal illumination (E_r) from threshold signal-to-noise ratios in the effective image area of point sources, disks, and other test objects. The effective image area, which is the convolution of the object area with the sampling area of the visual system, is determined from its Fourier spectrum. The constants of the transfer functions are established from the optical constants of the eye, its storage-time function, and the maximum transfer ratio of statistical units of the rod and cone systems. There is little room for variation of constants, if they are to remain in agreement with observed values.

The incomplete dc restoration in the system (differentiation of edges) is taken into account as a negative image component caused by feedback. System design principles are used as a guide in calculating the signal integration by retinal elements and the relative photoconductor gain characteristics of the receptors which are part of a system of interdependent functions including the primary characteristic, the over-all transfer characteristic to the optic nerve lines, and the four spatial integration characteristics represented by the equivalent passbands of lens and retina for the rod and cone systems.

The final solution is perhaps not completely unique for all functions, but does not violate or disagree with fundamental principles or observations as demonstrated by a comparison of the operating characteristic of the analog with the Munsell lightness scale, its noise level with the perception of external noise; and its statistical transfer ratio, relative gain, gamma, and feedback with observed data. The acuity, contrast sensitivity, and threshold visibility of point sources of the analog are, of course, in inherent agreement with corresponding properties of the eye.

A. DESCRIPTION OF SYSTEM

A VISUAL system performs five major functions: (1) formation of an optical image on a photosensitive surface (retina); (2) conversion of light energy into electrical signals (photoreceptors); (3) control of signal levels by storage time, spatial integration, negative feedback, and encoding (bipolar and ganglion cells); (4) matrixing of primary color signals (ganglion cells) to form a set of color-ratio signals depending on illumination and suitable for economic transmission over a limited number of "lines" (optic nerve fibers) to a computer (the brain); and (5) correction, interpretation, and correlation of these signals in the computer by comparison with stored information. A simplified schematic of the retina performing the functions (2) to (4) is shown in Fig. 1. Block diagrams of an analogous sequential system are shown in Figs. 2 and 3.

The analog is patterned after a television system. It does not duplicate the performance of the visual system in detail, because it is a sequential system and because many of the functions of the visual system are either not yet known or understood as pointed out in an excellent paper by Talbot.[1] Servo and feedback systems (shown in broken lines) indicate some of the automatic control functions performed in the "camera" or by a "computer" which has enormous memory capacity and which also interprets coded image signals in correlation with other senses. Evaluation of system constants is limited by available data to the "luminance-channel" (preceding the computer), which can simulate quite accurately principal properties of the process of black-and-white vision.

The liquid-immersed optical system of the eye is replaced by an automatically focused lens having a focal length in air of 17 mm and an iris controlled by the

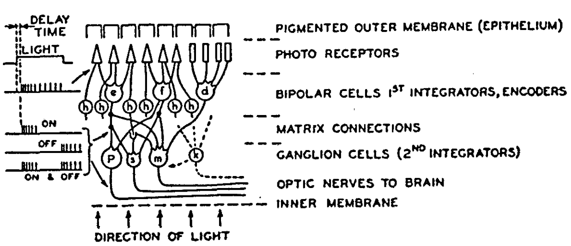

FIG. 1. Simplified drawing of retinal elements.

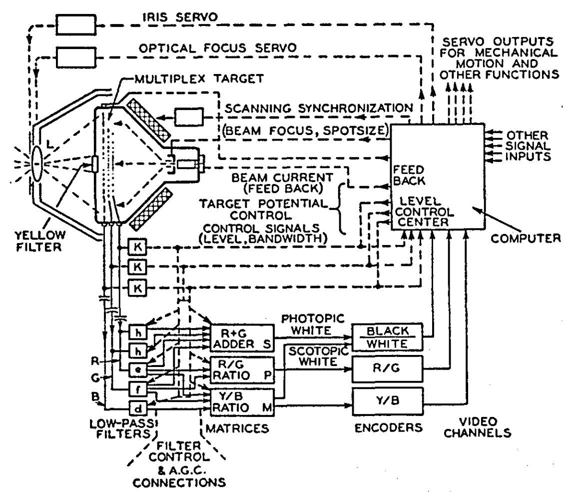

FIG. 2. Block diagram of analog system representing the eye.

[1] S. A. Talbot, J. Opt. Soc. Am. 41, 895–941 (1951).

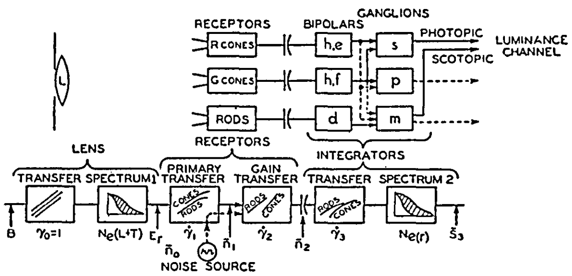

FIG. 3. Block diagram of luminance channel and its characteristic functions.

average field luminance $\bar{B}$.[2] The retinal illumination E_r (see Table I) is expressed in trolands, defined by

$$E_r = 2.69\delta^2 B, \tag{1}$$

where B = field luminance in foot-lamberts, and δ = iris diameter in mm.

The 126 million photoreceptors of the retina (5% cones and 95% rods)[3] which are connected by a maze of direct and diffuse connections through bipolar and ganglion cells to a much smaller number (1 million approximately) of optic nerve fibers or "lines" to the brain (see Fig. 1), are replaced by the photoconductive target of a storage-type color camera tube containing a mosaic of photoconductive cells. The target area is shared by two types of cells having different photoconductor characteristics: the blue-green sensitive "rods" (sparse in the center) and the "cones" (sparse in the periphery) of which there are two kinds, characterized by broad overlapping spectral response characteristics peaked in the red or green, respectively. The stored signals from these receptors are released and transmitted in sequence, by action of a noise-free scanning beam, into three video and three control channels.

The bipolar cells, ganglion cells, and the retinal potential mechanism,[1,3,4] which at high illumination levels gather and relay signals from a few receptors and at low levels integrate signals from thousands of receptors (particularly rods) for transmission over a single nerve fiber, are replaced by correspondingly labeled filters (h,e,f,d) having automatic band-width control, three color mixers (s,p,m) which establish the photopic luminosity response ($R+G$) of the "luminance" channel (S), and the R/G and Y/B color-ratio signals (p,m) (if one accepts validity of the Adams-Muller color system[1]), and also the scotopic bluish "white" response.

Special control lines and filters (K) are provided for automatic and localized control of gain, band width, color matrixing, and dc restoration, stabilized by feedback from the computer. Pulse encoders are located after these elements. (In the eye, the encoding occurs early in bipolar and ganglion cells.)

It is important to keep in mind that the eye is a system transmitting signals simultaneously over one million separate channels having a small band width (in the order of 500 cps), which in the analog, is simulated by a single-channel television system. Spatial integration of signals in a simultaneous system causes a change of signal levels and gamma but not of band width in an individual line; temporal integration by filters in a sequential system causes a change of band width only. It is therefore necessary to introduce a special transfer characteristic into the sequential system (analog) to duplicate the effect of retinal integration on signal intensities, while the multiplicity of branch lines and integrating elements in the retina can be replaced by much fewer electrical filters in the analog system as indicated in Fig. 3.

The present analysis deals particularly with the dual luminance channel and that portion of the retina which the eye chooses automatically for best vision. The basic characteristics of this specific part of the visual system are represented by the functional diagram in Fig. 3.

The low-gamma over-all transfer characteristic $\bar{n}_3 = f(B)$, which enables the eye to function over a range of 10^{-5} to 10^4 foot-lamberts is subdivided into four characteristics: (1) The transfer characteristic $E_r = f(B)$ of the optical system which is substantially linear at any one lens stop; (2) the dual transfer characteristics $\bar{n}_1 = f(E_r)$ of the primary photoelectric processes in the receptors (rods and cones); (3) the dual transfer characteristics $\bar{n}_2 = f(\bar{n}_1)$ describing the relative receptor gain functions; and (4) the dual transfer characteristics $\bar{n}_3 = f(\bar{n}_2)$, describing the level change caused by integration in bipolar and ganglion cells.

The spatial integration in the system is described in the frequency domain by the Fourier spectra (1) of the optical system and the Fourier spectra (2) of the retinal integration area. The source of random fluctuations in the primary process converting photons to electrons is indicated by a noise generator.

B. CHARACTERISTICS AND CONSTANTS

Data on visual performance are obtained to a large extent by observation of thresholds: incremental brightness steps, contrast thresholds for small areas, acuity, visibility of point sources, etc. The purposeful organization of the visual system revealed by its anatomy, and the electronic nature of signal transmission suggest strongly that thresholds of perception are determined by statistical fluctuations ("noise") in the transduced quantized energy, which is limited at low illumination by the physical size of the collector (lens) and at higher illumination by the maximum energy level which can be transduced by the system. The latter is undoubtedly

[2] S. G. DeGrott, J. Opt. Soc. Am. 42, 492 (1952).
[3] M. H. Pirenne and E. J. Denton, Nature 170, 1039 (1952).
[4] S. Polyak, *The Retina* (University of Chicago Press, Chicago, 1941).

in agreement with the energy supply and rate of visual information which can be processed by the computer.

The average photon density ($\bar{n}_0$) in the optical energy input can be computed from data on lens constants, spectral response, and transmittance factors of the eye, and limits for the maximum transfer factor $G_1=\bar{n}_1/E_r$ of the primary characteristic $\bar{n}_1=f(E_r)$ can be established from estimates of the number of photons producing on the average, one independent signal unit.

The quantity counted statistically as an independent event in a quantized energy state of flux will be defined as a *statistical unit*, (abbreviated s.u.). One s.u. may consist of a single photon, electron, or atom, or it may consist of aggregates containing any number of such "quanta." The absorption of a single photon (one s.u.) by a photoconductor, for example, may excite one or a number of electrons into a carrier state (depending on relative energy). During the lifetime of these carriers (which is a function of several parameters), electrons from an external energy source can drift through the photoconductor, with the result that the number of output quanta can range from zero to many powers of ten. The output quantum-aggregate appears or disappears as a unit with the input unit, and must therefore be counted likewise as a single event (one s.u.) regardless of the energy amplification or growth of the unit. It follows that the number of s.u.'s in the transduced signal cannot exceed the number of s.u.'s of the input signal. *The statistical transfer ratio** is therefore

$$\mu_s=\frac{\text{number of output s.u.}}{\text{number of input s.u.}}\leqq 1. \qquad (2)$$

The standard deviation and the electrical signal-to-noise ratio [R] of a system is determined in principle by a count of statistical units (s.u.) in a specified sampling area $\bar{a}$, and is therefore unaffected by linear amplification. The primary transfer characteristic $\bar{n}_1=f(E_r)$ which relates the average density $\bar{n}_1$ of electronic signal units (in s.u.) to the retinal illumination can therefore be computed from threshold signal-to-noise ratios. Over-all transfer characteristics $\bar{s}_3=f(B)$ or $\bar{s}_3=f(E_r)$ can be obtained by integration of threshold intensity steps, and the relative "amplifier" transfer characteristics $\bar{n}_2=f(\bar{n}_1)$, $\bar{n}_3=f(\bar{n}_2)$, and $\bar{n}_3=f(\bar{n}_1)$ can be established subsequently from the above characteristics and the sampling areas of the eye. The average electron densities $\bar{n}_1$, $\bar{n}_2$, and $\bar{n}_3$ are therefore obtained in s.u.'s, and no information can be given on the growth and number of electrons in the unit except by direct measurement[7] of photoreceptor and nerve signal currents. A quantitative evaluation of the spatial integrating properties of the visual system in terms of equivalent sampling apertures $\bar{a}$ or equivalent passbands N_e is essential for these computations.

* It is apparent that this ratio does not express an efficiency, although it is often designated as the quantum efficiency of the process.

TABLE I. Relation of field luminance and retinal illumination.

B (foot-lambert)	E_r (trolands)
10^{-6}	1.35×10^{-4}
4×10^{-6}	$5.\ 4\times10^{-4}$
10^{-5}	1.32×10^{-3}
4×10^{-5}	$5.\ 1\times10^{-3}$
10^{-4}	1.21×10^{-2}
4×10^{-4}	$4.\ 5\times10^{-2}$
10^{-3}	1.08×10^{-1}
4×10^{-3}	$3.\ 8\times10^{-1}$
10^{-2}	$8.\ 7\times10^{-1}$
4×10^{-2}	3.05
10^{-1}	7.0
4×10^{-1}	23.5
1	51.
4	167.
10	350.
40	1000.
100	2050.
400	5900.
1000	12 000.

1. Fourier Spectra and Equivalent Passbands $[N_e=f(E_r)]$ of the Visual System

The integrating properties of an imaging system can be specified by either the intensity functions or the Fourier spectra of its point images. The spectrum representation is more suitable for evaluation of systems containing a number of elements. The sine-wave amplitude coefficients $r_{\tilde{\psi}}$ of the spectrum are determined as a function of line number N, $r_{\tilde{\psi}}=F(N)$. In television terminology, the line number is the number of half-cycles in a unit length $N=2f$. Integration furnishes the noise-equivalent passband[5,6]

$$N_e=\int r_{\tilde{\psi}}^2dN, \qquad (3)$$

which is related to the effective sampling area by

$$\bar{a}=N_e^{-2}. \qquad (4)$$

It is known from electroretinographic studies,[7] that a continuous optical signal evokes continuous electrical impulse signals at the receptor output, and that counter polarization (inhibition or negative feedback) very rapidly suppresses the steady signal term, resulting in a short burst of impulses from the ganglion cells at the start or end (or both) of the optical signal (see Fig. 1).

The fact that the main signal-transmitting system of the eye does not transmit dc signals[8] is indicated in Fig. 2 by blocking capacitors in the main signal leads. The simultaneous system of the eye also contains these "blocking capacitors" and therefore provides "spot

[5] O. H. Schade, Natl. Bur. Standards Circ. 526, 1954, pp. 231–258.
[6] O. H. Schade, J. Soc. Motion Picture Television Engrs. **56**, 137–171 (1951); **58**, 181–222 (1952); **61**, 97–164 (1953); **64**, 593–617 (1955).
[7] H. K. Hartline, J. Opt. Soc. Am. **30**, 239–247 (1940).
[8] Riggs, Ratliff, Cornsweet, and Cornsweet, J. Opt. Soc. Am. **43**, 495–501 (1953).

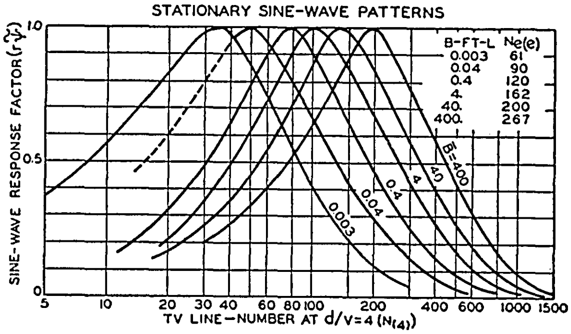

FIG. 4. Sine-wave response of eye to stationary test patterns (relative units for 4 to 1 viewing distance).

wobble" (ocular tremor) to generate continuous ac signals from small objects and contours, which otherwise can be seen only in a transient state.[8] For restoration of continuous signals from larger areas the computer receives auxiliary level signals from a relatively small number of special dc-responsive photoreceptors, but it does not need a perfect dc restoration because it has access to a vast memory of objects and shapes for interpretation. The over-all sine-wave response of such a system, therefore, can be expected to decrease, perhaps even to zero, at very low line numbers, although its square-wave response will remain high (observed value ~0.8), because the "differentiated" edges of long square waves are transmitted owing to ocular tremor, while their levels are at least partially restored by the action of the dc restorer and memory in the computer. This theory is confirmed by measurement of over-all sine-wave response characteristics (Fig. 4) obtained from a "standard observer" selected because of his normal visual characteristics.† Each curve in Fig. 4 represents from 200 to 600 observations (see Appendix). A portion of the observed amplitude loss at low line numbers must be attributed to the fact that very long sine waves (N approaching 0) cannot be reproduced by an imaging process because of practical field limitations. These limitations, however, are counteracted by motion of the eyeball. A second set of observations was made, therefore, using sine-wave bar patterns drifting through the field of vision at a very low rate (in the order of one cycle per second). The response characteristics obtained by this procedure (Fig. 5) represent more closely the effective response at low line numbers (see further down). From these observed characteristics a third family of characteristics (Fig. 6) with $r_{\tilde{\psi}}=1$ at $N=0$ can be extrapolated to represent the normal photoelectric response without the "blocking capacitors" in Figs. 2 or 3. The line-number spectrum in the Figs. 4, 5, and 6 is given in television lines in the vertical frame dimension V (N = number of half-cycles per unit length) at a viewing distance $d=4V$. The line number per millimeter at the retina is $N/mm=N_{(4)}/4.25$. The sine-wave response of the eye in these units is shown in Fig. 7.

The response of the perfect f:7.25 lens ($\bar{B}=400$ foot-lamberts) for light having the relative energy distribution of the luminosity curve is shown for comparison. The characteristics, Fig. 7, represent the product of the sine-wave spectra of lens and retina. They do not include the loss of amplitude response at low line numbers by incomplete restoration which must be taken into consideration to obtain the dynamic operating response of the eye to complex wave forms *as interpreted by the brain.* Memory and experience contribute much to the interpretation of levels, and low line-number square waves, for example, appear to contain low-order harmonic components larger than are indicated by mathematical synthesis of sine-wave components from Fig. 7. The suppression of the low-frequency portion of the spectrum by the blocking capacitors and the subsequent partial restoration can be represented separately by the spectrum of a high-pass filter in cascade with the normal spectra of Fig. 7, or it can be regarded as a short low-pass spectrum with *negative* amplitude coefficients which is *added* to the normal positive low-pass spectra as indicated in Fig. 8(a). In the space domain these representations correspond to the addition of a large-diameter point image of *negative* intensity to the normal positive point image [see Fig. 8(b)] and provide a realistic analog of the

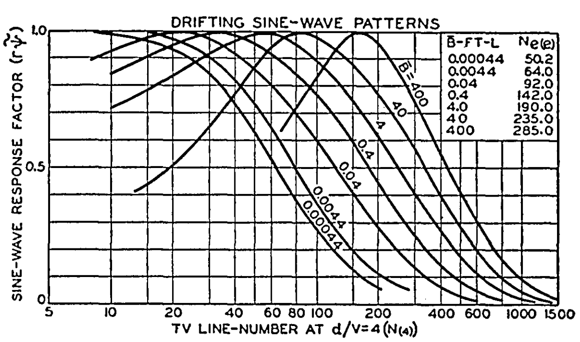

FIG. 5. Sine-wave response of eye to drifting test patterns (relative units for 4 to 1 viewing distance).

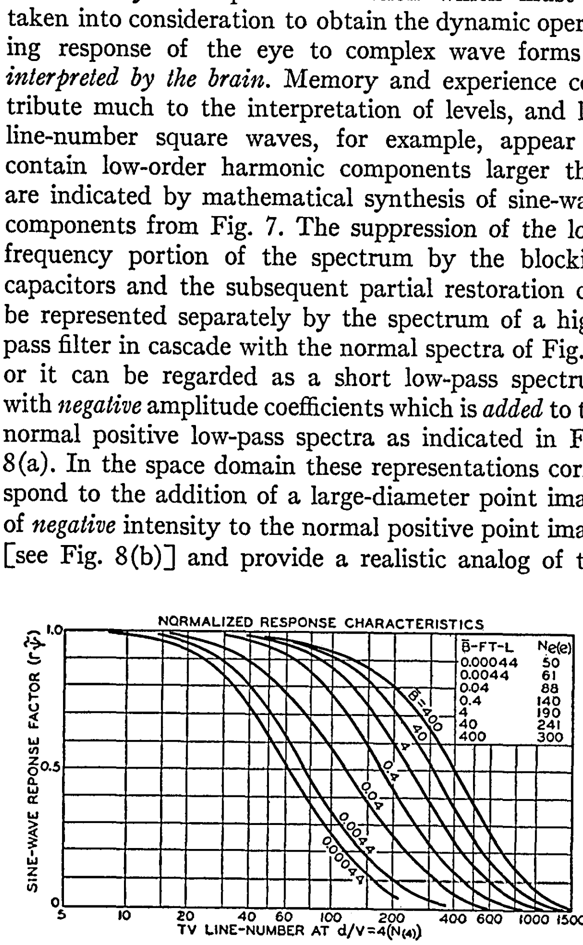

FIG. 6. Normalized sine-wave spectra of optical system (relative units for 4 to 1 viewing distance).

† Various observers of different age had similar response characteristics.

masking process caused by counter polarization in the retina.

2. Subdivision of Over-All Spectrum and Specification by Equivalent Passbands N_e

The over-all spectrum of the visual system is the sum of a long positive and a short "negative" low-pass spectrum. The positive spectrum of the system is the product of the normal low-pass spectra‡ of its components (lens, ocular tremor, and retinal elements). The "negative" spectrum of the dc-blocking and level restoring mechanism (encoding and decoding of signals) is also a normal low-pass spectrum except for its *negative amplitude coefficients*. Each component spectrum can be represented by a noise-equivalent passband N_e.

The equivalent passbands of either positive or of negative normal spectra can be combined separately with good accuracy[6] by the simple rule

$$N_{e(\text{total})}{}^{-2}=N_{e(1)}{}^{-2}+N_{e(2)}{}^{-2}\cdots+N_{e(n)}{}^{-2}. \quad (5)$$

The final combination of positive and negative over-all passbands $N_{e(\text{total}+)}$ and $N_{e(\text{total}-)}$, however, must be made by addition of their spectra. For the purposes of this paper it is sufficiently accurate to approximate the actual positive and negative over-all spectra by error curve spectra computed from their equivalent passbands, with the amplitude response

$$r^{\sim}{}_{(\text{equiv})}=\exp-(0.2\pi N/N_e)^2. \quad (6)$$

The over-all passband is then computed from their sum by Eq. (2). A series of auxiliary curves [Fig. 8(c)] showing the ratio $N_{e(\text{total})}/N_{e(+)}$ as a function of the passband ratio $N_{e(-)}/N_{e(+)}$ were computed for various values of the energy ratio $(W-/W)$ in the negative and positive point images to facilitate rapid evaluation of image passbands.

The equivalent passbands $N_{e(e+)}=f(E_r)$ of the observed main spectra (Fig. 7) are shown in Fig. 9. The numerical subdivision according to Eq. (5) listed in

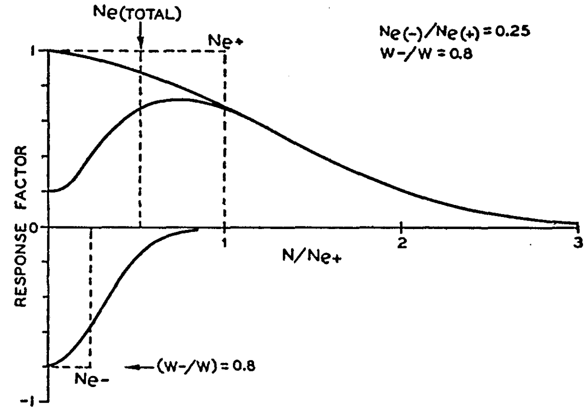

FIG. 8(a). Spectrum with deficient low-frequency response represented as the sum of two low-pass spectra with positive and negative amplitude coefficients respectively.

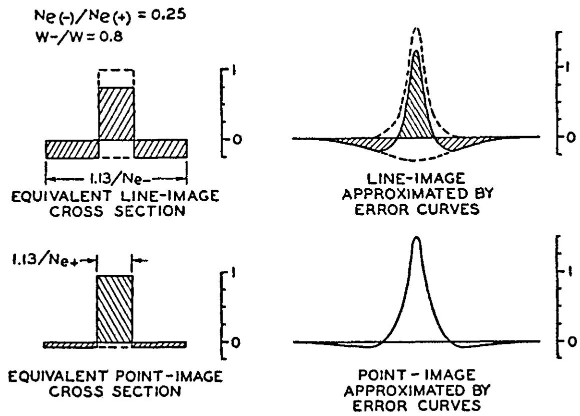

FIG. 8(b). Line image and point image of spectrum in Fig. 8(a).

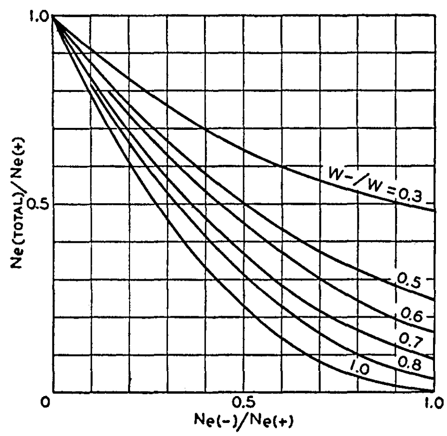

FIG. 8(c). Equivalent passband of composite spectrum as function of its components.

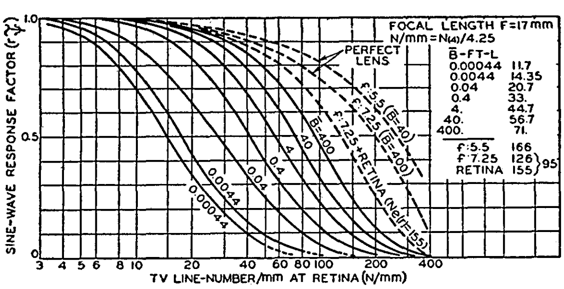

FIG. 7. Normalized sine-wave spectra of optical system in retinal units.

Table II, is initially an estimate governed by the retinal anatomy, influenced by the observation that the lens performance can be improved noticeably by corrective glasses even at low illumination levels. The estimate is strengthened by requirements for good system "design." The substantially parallel course of

‡ A spectrum is "normal" when its amplitudes are steadily decreasing as a function of line-number, similar to an error curve. Spectra of abnormal shape can often be replaced by sums of several "normal" spectra.

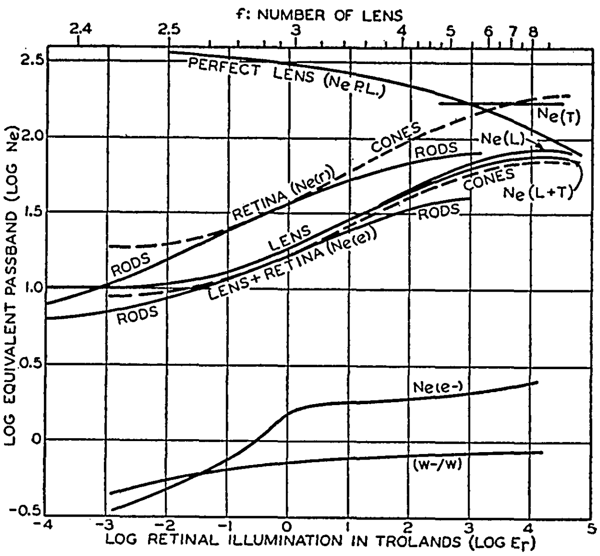

FIG. 9. Equivalent passbands N_e of optical system and components as a function of retinal illumination.

the characteristics $N_{e(r)}$ and $N_{e(L)}$ in Fig. 9 provides optimum conditions for the detection of small signals in the presence of random fluctuations (noise), which require that the passband $N_{e(r)}$ following the noise source has a value at which it just begins to affect the signal. This condition occurs for a ratio $N_{e(r)}/N_{e(L)}$ in the order of two. The subdivision into rod and cone branches, and further refinement of this estimate is dictated by system calculations as explained in subsequent sections.

It must be kept in mind that the equivalent passband N_e does not necessarily have a fixed relation to the resolving power or response limit of a spectrum.

At large apertures ($f/2.5$) the sine-wave spectrum of the simple aspherical lens of the eye departs considerably from that of a perfect lens which establishes a theoretical limit. At small lens stops ($f/7$) the lens of the eye (like many simple lenses) approaches theoretical values for red and green light (cone branch) but it is poorer for greenish blue light (rods) because of chromatic aberration. For constant-energy white light the performance of the lens is reduced further because of chromatic aberration. At high light levels in the eye, this defect is counteracted by a yellow-dye filter over the central color-sensitive field, which drastically reduces the blue sensitivity, resulting in a much improved sine-wave response over the visible green-peaked spectral range (luminosity curve) interpreted by the brain as photopic "white" light.

The structure of tightly packed color-sensitive elements in the central area (fovea) is sufficiently fine (cone diameter 2.2 microns[4,9]) to permit integration of the cell structure over an area approximately 6 microns in diameter by the ocular tremor[10,11] (size of scanning spot in Fig. 1). The passbands $N_{e(r)}$ and $N_{e(T)}$ are wide enough to have little effect on the lens performance.

The equivalent passband $N_{e(e-)}$ of the negative spectrum representing the dc blocking and restoring mechanism, and particularly its relative energy ratio $(W-/W)$ to the positive spectrum is probably multivalued, depending on intensity distribution, duration, and size of the image. The function $N_{e(e-)}=f(E_r)$ shown in Fig. 9 has been computed from Blackwell's data for circular areas up to 360′ in size (see further down). The curve $(W-/W)$ gives the relative energy in the negative point image (or relative amplitude coefficient at $N=0$) with respect to the energy in the positive point-image for steady vision.

3. Limits for the Primary Transfer Ratio μ_s§

The light flux at the retina is computed as for a conventional optical system. For the focal length $F=17$ millimeters, the luminous retinal flux ψ_r is given by

$$\psi_r/\text{mm}^2=9.3B\tau\delta^2\times10^{-9}\ \text{lumen}, \tag{7}$$

where

B = external field luminance in foot-lamberts,
τ = transmittance of lens and color filters,
δ = diameter of lens stop in mm.

There are advantages to be gained by assigning a uniform transfer ratio μ_{s1} (independent of wavelength) to the primary photoelectric process, and include the variation in spectral response of the eye in the transmittance function of color filters and ocular media preceding the receptor surface. Because of the definition of the lumen unit, (which represents the energy passed by a luminosity color filter), the number of photons that must be emitted per second by a blackbody at 5400°K to yield one lumen of white light in the range $\Delta\lambda=0.4\ \mu$ to $0.73\ \mu$ is 1.3×10^{16}.‖ It follows from Eq. (7) that the photon density at the retinal surface (number of photons per mm² per sec) is expressed by

$$\bar{n}_0=1.21B\tau\delta^2\times10^8\ \text{s.u./mm}^2/\text{sec}. \tag{8a}$$

Substituting Eq. (1) results in

$$\bar{n}_0=4.5\tau E_r\times10^7\ \text{s.u./mm}^2/\text{sec}. \tag{8b}$$

The primary photoelectron density $\bar{n}_1$ caused by the photon absorption $\bar{n}_0$ is determined by the statistical transfer ratio $\mu_{s1}=\bar{n}_1/\bar{n}_0$ [Eq. (2)] and is expressed by

$$\bar{n}_1=4.5\tau\mu_{s1}E_r\times10^7\ \text{s.u./mm}^2/\text{sec}. \tag{9}$$

[9] Brian O'Brien, J. Opt. Soc. Am. 41, 882–894 (1951).

[10] Lorrin A. Riggs, J. Opt. Soc. Am. 42, 287 (1952).

[11] G. C. Higgins and K. F. Stults, J. Opt. Soc. Am. 42, 872 (1952).

§ See Eq. (2).

‖ This number is roughly 3 times higher than the amount of luminous energy passed by the luminosity filter.

The transfer ratio μ_{s1} may vary as a function of illumination as described by the transfer characteristic, $\bar{n}_1=f(E_r)$, but cannot exceed unity at any point of the actual characteristic $\bar{n}_1=f(E_r)$ to be computed later.

The total relative energy transmittance τ to the rods or cones is computed as the integral of the products of relative light energy and absolute transmittance of the "color filter" and ocular media over the spectral range $\Delta\lambda=0.4$ to $0.73\,\mu$. The white light energy is reduced by the relative spectral transmittance of the broadband color filters and tinted ocular media to approximately 28% for the rods (scotopic luminosity curve) and to 32% each for the two cone types. The absolute transmittance τ_r to the rods is reduced by a 50% absorption in the yellow-tinted ocular media,[12] to $\tau_r\simeq 0.14$. In the foveal region the yellow "filter" drastically attenuates blue light, but it does not reduce the energy to the spectrally only slightly different "red" and "green" sensitive cones[13] by more than 40%, resulting in a higher over-all transmission factor of $\tau_c\simeq 0.19$ for the cones.

The maximum transfer ratio μ_{s1} of both rods and cones is assumed to have a similar value, because a high-acuity color system is as much or even more in need of a good statistical transfer ratio to attain adequate signal-to-noise ratios at medium light levels, as is a monochrome system which is called upon to transduce image signals at extremely low illumination. The transfer ratio μ_{s1} of the rods has been investigated frequently and is reported to lie between 0.5 and unity. The primary photoelectron density obtained with Eq. (9) and the values $\tau_r=0.14$, and $\tau_c=0.19$, in the most efficient section of the function $\bar{n}_1=f(E_r)$ should hence remain within these limits:

$$\left.\begin{array}{l}\text{for rods: } 6.3E_r\times 10^6>\bar{n}_1>3.15E_r\times 10^6\\ \text{for cones: } 8.55E_r\times 10^6>\bar{n}_1\end{array}\right\}. \quad (10)$$

4. Calculation of Transfer Characteristics from Threshold Data

Some Fundamental Relations

Equivalent areas and passbands.—The optical image (a_1) of a circular test area (a_0) of uniform intensity can be computed by convolution (scanning) of the test area with the point-image of the lens. The electrical images (a_2 to a_3, see Fig. 3), containing the integration by additional system elements, are formed by progressive convolutions with the point images of the added system elements. These images have a nonuniform intensity distribution and must be converted for "noise" calculations into equivalent images or sampling areas $\bar{a}$ of uniform intensity. It is convenient to replace the spatial description of object and point-images by their transforms or Fourier spectra, and express each spectrum by its equivalent passband [Eq. (3)] which is directly related to the equivalent sampling area by Eq. (4). For the condition that all spectra are normal low-pass spectra, the total equivalent image area and passband is obtained with good accuracy by simple addition of the equivalent areas or squared reciprocal passbands of the components [compare Eq. (5)]. For example,

$$\bar{a}_1\simeq\bar{a}_0+\bar{a}_L+\bar{a}_T,$$

corresponding to

$$\bar{a}_1=N_{e(1)}^{-2}\simeq N_{e(0)}^{-2}+N_{e(L)}^{-2}+N_{e(T)}^{-2}. \quad (11)$$

The equivalent area and passband of circular test objects of uniform intensity are obtained as follows: The object area $a_0=\delta^2\pi/4$ is usually specified by the subtended angle α' (in minutes). For a focal length $F=17$ mm, the angle α' corresponds to the diameter $\delta=4.95\alpha'$ microns. The equivalent passband is then given in television lines by

$$\left.\begin{array}{l}N_{e(0)}=a_0^{-\frac{1}{2}}=\bar{a}_0^{-\frac{1}{2}}=2/\delta\sqrt{\pi}\\ \text{or}\quad N_{e(0)}=228/\alpha'(TV\text{ lines/mm})\end{array}\right\}. \quad (12)$$

Contrast and Signal Transfer

The object contrast is defined by the ratio $C_0=\Delta B/B=\Delta E_r/E_r$. The contrast C_i of the retinal image is, therefore,

$$\left.\begin{array}{l}C_i=C_0\bar{a}_0/\bar{a}_1\\ \text{or}\quad C_i=C_0N_{e(1)}^2/N_{e(0)}^2\end{array}\right\}. \quad (13)$$

The transfer characteristic $\bar{n}_1=f(E_r)$ of the primary process relates the electron density $\bar{n}_1$ (number of s.u./mm²/sec) to the retinal illumination E_r. This transfer characteristic cannot be assumed *a priori* to be linear in all of its sections. The transfer of large "level-signals" measured from the absolute zero ($E_r=0$, $\bar{n}_1=0$) is expressed by the transfer factor

$$G_1=\bar{n}_1/E_r, \quad (14)$$

while the transfer of small signal increments ($\pm\Delta E_r<0.2E_r$) is determined by the transfer gradient

$$g_1=d\bar{n}_1/dE_r. \quad (15)$$

The transfer ratio

$$g_1/G_1=(d\bar{n}_1/\bar{n}_1)/(dE_r/E_r)=d(\log\bar{n}_1)/d(\log E_r)$$

is defined as the *point gamma* $\dot{\gamma}$ of the transfer process

$$\dot{\gamma}_1=g_1/G_1. \quad (16)$$

The image contrast ratio $C_i=\Delta E_r/E_r$ is, therefore, transduced into the electrical signal contrast ratio

$$\Delta\bar{n}_1/\bar{n}_1=\dot{\gamma}_1C_i,\ C_i<0.2. \quad (17)$$

[12] E. Ludwigh and E. F. McCarthy, Arch. Ophthalmol. (Chicago) 20, 37 (1938), G. Wald, Science 101, 653 (1945).
[13] H. L. DeVries, J. Opt. Soc. Am. 36, 121–127 (1946).

TABLE II. Equivalent passbands (N_e) of visual system components (see Fig. 9).

$\log E_r$	Rods $N_{e(e+)}$	$N_{e(r)}$	Cones $N_{e(e+)}$	$N_{e(r)}$	$N_{e(e-)}$	$(W-/W)$	$f/$	$N_{e(p.L.)}$	Rods $N_{e(L)}$	Cones $N_{e(L)}$
−4.0	6.0	7.4							10	
−3.4	6.7	8.9					2.4	381	10	
−3.0	7.2	10.5	8.9	19.1	0.33	0.43			10	
−2.4	8.0	13.1	9.1	19.1	0.4	0.5			10.2	
−2.0	8.9	15.9	9.35	19.5	0.48	0.55	2.5	366	10.8	
−1.4	10.2	20.4	10.75	22	0.61	0.60			11.7	
−1.0	11.4	23.5	11.8	24.6	0.74	0.645	2.66	344	13.0	
−0.4	14.5	31	14.5	31	1.07	0.69			16.4	
0.	16.6	35.6	16.6	37.2	1.5	0.71	2.95	310	18.7	18.7
0.6	21.4	46.	21.4	48.	1.7	0.74			24.2	24.2
1.0	24	52.5	25.7	59	1.78	0.76	3.22	284		29.
1.6	30	62.5	34.7	79.5	1.86	0.78				39.6
2.0	34	67.6	40.7	98	1.9	0.80	4.1	223		46.4
2.6	38	77.5	50.	123.5	2.0	0.815				57.8
3.0	40	79.5	56.5	141.5	2.1	0.83	5.5	166.5		66.0
3.6		79.5	66	166.	2.3	0.83				79.0
4.0			71	182.	2.46	0.85	8.	114.		86.0
4.6			71	191.						85.5
Column #	1	2	3	4	5	6	7	8	9	10

1, 3 Equivalent passband of eye (normal positive component)
2, 4 Equivalent passband of retinal elements (normal positive component)
5 Equivalent passband of feedback image (negative image component)
6 Relative energy of negative to positive point-image (see Fig. 8)
7 $f/$ number of eye
8 Equivalent passband of perfect lens for white light (luminosity weighting) (see reference 6)
9, 10 Equivalent passband of lens (eye)

Calculation of Primary Photoelectric Transfer Characteristics $\bar{n}_1 = f(E_r)$

The detection of an area a_0 in the presence of random fluctuations ("noisy" background) requires that the signal-to-noise ratio be equal to or larger than a certain threshold value K. This principle was expressed by Rose[14] who estimated a value $K \simeq 5$. ¶It was recognized that the constant relates to a probability. The value K is therefore a function of various conditions such as: storage time, time of observation, presence of interfering structures, the degree of information on existence, location, size, shape, color, steadiness of the object. In short, if a trained observer knows exactly "what to look for" and is given enough time, the presence or absence of a single "grain" at a particular location may be sufficient for detection of an object in a single image frame by a complicated process of mental correlation. For a fixed set of conditions, K remains constant, but it may have a value considerably smaller than five.

The method of computing $\bar{n}_1 = f(E_r)$ consists in deriving an expression for the threshold signal-to-noise ratio at the point of observation in the visual system, and solving for $\bar{n}_1$ by letting the signal-to-noise ratio equal an arbitrary constant K'. The functions $c\bar{n}_1 = f(E_r)$ obtained from various sets of data are made to coincide by adjustment of respective K'-values. The average characteristic $c\bar{n}_1 = f(E_r)$ is finally adjusted to agree with the probable transfer ratio μ_{s1} and storage time of the system which eliminates the factor c and yields probable values for $\bar{n}_1$ and K.

The threshold data (size α' and contrast C_0) for areas a_0 of uniform intensity, available from observations of contrast thresholds,[15] visibility of points sources,[16] and measurements of acuity,[3,17] can be processed in the same manner.

Threshold observations deal by nature with incremental signals in the order of the random deviation. The image contrast C_i [Eq. (13)] is quite small because of integration by the point image of the lens, even though the object contrast C_0 may have values far greater than unity, as, for example, in point source observations.

The incremental electrical signal ΔS expressed by the number of signal units (electrons) in the equivalent image area $\bar{a}_1$ is

$$\Delta S = \bar{n}_1 \bar{a}_1 T,$$

where $\bar{n}_1$ = average electron density in s.u., T = time of integration (storage time).

The "noise" is the rms value of random deviations from the average number of signal units in the main level-signal in areas $\bar{a}_1$. It is equal to the square root of the main signal

$$|N| = S^{\frac{1}{2}} = (\bar{n}_1 \bar{a}_1 T)^{\frac{1}{2}}.$$

The threshold signal-to-noise ratio for the test image

¶ The author observed a value $K \simeq 3$ at that time on television images.

[14] A. Rose, J. Opt. Soc. Am. 38, 196–208 (1948).

[15] Blackwell, J. Opt. Soc. Am. 36, 624–643 (1946).

[16] Knoll, Tousey, and Hulburt, J. Opt. Soc. Am. 36, 480–482 (1946).

[17] S. Hecht, J. Gen. Physiol. 11, 225 (1928).

TABLE III. Evaluation of tentative values $\dot{\gamma}_1^2\bar{n}_1$ for $T=0.1$ sec.

$\log E_r$	$N_{e(0)}$	Acuity $N_{e(e)}$	$N_{e(3)}$	$\dot{\gamma}_1^2\bar{n}_1$	Rods $E_r/\Delta E_r$	Cones $E_r/\Delta E_r$	Rods $\dot{\gamma}_1^2\bar{n}_1$	Cones $\gamma_1^2\bar{n}_1$
−4.0	0.95	6.0	0.94	46.2	1.3		122.	
−3.75	1.83	6.3	1.75	185.				
−3.4	3.5	6.7	3.1	790.	3.5		1.11×10^3	
−3.0	5.9	7.2	4.57	2.92×10^3	6.0	1.0	3.76×10^3	
−2.4	10.5	8.0	6.35	1.51×10^4	11.	3.9	1.56×10^4	
−2.0	14.5	8.9	7.6	3.85×10^4	15.	7.6	3.6×10^4	
−1.4	22.5	10.2	9.3	1.48×10^5	23.6	18	1.18×10^5	
−1.0	30.	11.4	10.7	3.56×10^5	31.	31	2.54×10^5	2.54×10^5
−0.4	51.	14.5	13.9	1.77×10^6	44.	59	8.2×10^5	1.48×10^6
0.	79.5	16.6	16.3	7.55×10^6	50.	95	1.39×10^6	5×10^6
0.6	141.5	21.4	21.2	4.5×410^7	58.	159	3.1×10^6	2.3×10^7
1.0	200	25.7	25.2	1.28×210^8	62.	210	4.5×10^6	5.9×10^7
1.6	300	34.7	34.6	3.41×10^8	65.	282	7.7×10^6	1.94×10^8
2.0	356	40.7	40.5	4.9×10^8	65.	316	0.9×10^6	3.35×10^8
2.6	428	50.	49.7	6.9×10^8		355		6.4×10^8
3.0	450	56.5	56.5	6.5×10^8		363		8.5×10^8
3.6	490	66.	65.5	6.8×10^8		375		1.11×10^9
4.0	500	71.	69.5	6.55×10^8		372		1.41×10^9
						370		1.4×10^9

$\dot{\gamma}_1^2\bar{n}_1$ computed with Eq. (20)
$K=2.25$, $T-0.1$, $C_0=1$

$\dot{\gamma}_1^2\bar{n}_1$ computed with Eq. (22)
$K=0.45c^{\frac{1}{2}}$, $T=0.1$, $N_{e(e)}$ from Table II

is hence

$$(\Delta S/|N|)_1=\Delta\bar{n}_1(\bar{a}_1T/\bar{n}_1)^{\frac{1}{2}}. \qquad (18)$$

The object contrast C_0 is introduced by substitution of Eqs. (17) and (12) for $\Delta\bar{n}_1$, yielding

$$(\Delta S/|N|)=\dot{\gamma}_1(\bar{n}_1T)^{\frac{1}{2}}C_0Ne_{(1)}/N_{e(0)}^2. \qquad (19)$$

Because the signal ΔS as well as the noise $|N|$ are small signals, their ratio is not affected by the gamma of the succeeding transfer processes $\bar{n}_3=f(\bar{n}_1)$ (see Fig. 3). The additional integration (by spectrum 2 of Fig. 3) in these processes, however, decreases the equivalent image passband from $N_{e(1)}$ to $N_{e(3)}$ in the optic nerve line where signal and noise are observed in sensation units $\bar{s}_3$. Substitution of $N_{e(3)}$ for $N_{e(1)}$ in Eq. (19) gives the threshold signal-to-noise ratio at that point:

$$(\Delta S/|N|)_S=\dot{\gamma}_1(\bar{n}_1T)^{\frac{1}{2}}C_0N_{e(3)}/N_{e(0)}^2=K.$$

Rearranging terms yields the characteristic

$$\dot{\gamma}_1^2\bar{n}_1=[KN_{e(0)}^2/C_0N_{e(3)}]^2/T=f(E_r), \qquad (20)$$

which can be solved for $\dot{\gamma}_1$ and the transfer characteristic $\bar{n}_1=f(E_r)$.

For numerical evaluation of Eq. (20), the equivalent passbend $N_{e(0)}$ is determined from the object size [Eq. (12)]; the image passband $N_{e(3)}$ is computed from $N_{e(3)}^{-2}=N_{e(0)}^{-2}+N_{e(e)}^{-2}$ [Eq. (5)], and the equivalent passband $N_{e(e)}$ of the eye listed in Table II; the contrast C_0 is given in the original threshold data as a function of illumination (B or E_r); and the storage time of the visual process is estimated tentatively at $T=0.1$ sec. Typical sets of data are given in Table III.

A graphic solution for $\dot{\gamma}_1$ and for the transfer characteristic $\bar{n}_1=f(E_r)$ illustrated in Fig. 10 which shows a portion of the tentative average function $\log\dot{\gamma}_1^2\bar{n}_1=f(\log E_r)$ obtained from four different sets of data. The function $\log\bar{n}_1=f(\log E_r)$ must be continuous and the logarithm of its gradient must equal the ordinate difference $\log\dot{\gamma}_1^2=\log\dot{\gamma}_1^2\bar{n}_1-\log\bar{n}_1$. It can therefore be constructed by connecting appropriate slope values to form a continuous function as shown in Fig. 10, starting at the point where both functions have the slope $\dot{\gamma}_1=1$. The solution is then checked numerically. It is obvious from the illustration that the solution is single valued.

The photopic branch of the computed function $\dot{\gamma}_1^2\bar{n}_1$ shown in Fig. 11 is sublinear ($\dot{\gamma}_1<1$) in the highlight range while the scotopic branch ($E_r<10^{-1}$) exhibits superlinearity towards the absolute threshold of vision. Superlinearity, however, is not found in primary characteristics, although possible in gain characteristics of photoconductors. The steep rise of the scotopic branch must therefore be attributed to an additive constant, the "dark current" ($\bar{n}_d$), and not to a high gamma of the primary transfer characteristic which, as shown in Fig. 11, continues on linearly and has a gamma of unity for $E_r<10^{-2}$. The electron density of the dark current can be computed from two expressions for the signal-to-noise ratio $[R]_u$ per unit area and time.

For the observed signal curve

$$[R]_u=(\bar{n}_1\dot{\gamma}_1^2)^{\frac{1}{2}}=(\bar{n}_1^*)^{\frac{1}{2}},$$

and for the actual signal with dark current

$$[R]_u=\bar{n}_1/(\bar{n}_1+\bar{n}_d)^{\frac{1}{2}},$$

yielding $\bar{n}_d=(\bar{n}_1^2/\bar{n}_1^*)-\bar{n}_1$. (21)

The characteristics shown in Fig. 11 are tentative characteristics computed for a fixed storage time $T=0.1$

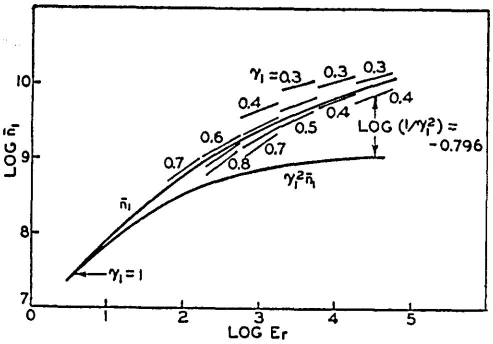

FIG. 10. Graphic solution for $\dot{\gamma}_1$ and $\bar{n}_1$ (see text).

second. Adjustment of constants to satisfy the "guide"-characteristics [Eq. (10)] furnishes relative values K' for the threshold constants. Before making corrections for the variable storage time of the visual process, a discussion of the negative image component caused by feedback (inhibition) is in order.

The Negative Image Component

It became apparent in the course of calculations that the equivalent passband $N_{e(3)}$ in Eq. (20) must contain the effect of the "blocking capacitors" and dc restoring process of the system (see Fig. 3) to avoid serious discrepancies for large test areas.

Blackwell's Table VIII contains threshold contrast data for seven different disk sizes ($\alpha=0.6'$ to $360'$) for each value B of field luminance. Calculation of $\dot{\gamma}_1^2\bar{n}_1$ with Eq. (20) based on the positive image passband $N_{e(3+)}$ only, results in systematic errors increasing with object size. The largest test object causes a decrease of

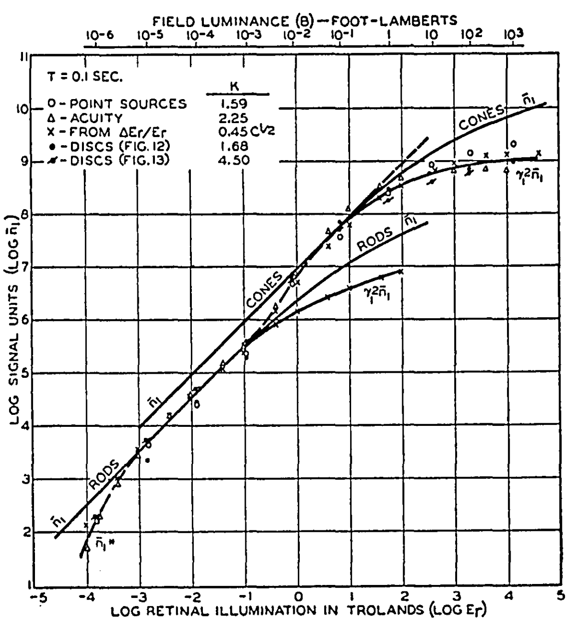

FIG. 11. Data and average tentative function $\dot{\gamma}_1^2\bar{n}_1$ for $T=0.1$ sec.

signals by a factor as large as 10^3 at high illumination as illustrated in Fig. 12, while objects reproduced as point images cause no error. This result indicates that the dc and large area signal restoration in the system is incomplete, leaving a negative image component $N_{e(3-)}$ which cannot be neglected. The seven sets of constants permit a solution for the negative passband $N_{e(e-)}$ and its relative energy $(W-/W)$, because correct constants yield a single function $\dot{\gamma}_1^2\bar{n}_1$ for *all* object sizes as shown by the closely grouped points in Fig. 12. A perfect agreement can hardly be expected in a system where the receptor population and retinal integration are far from uniform, thus making a more precise method of computation of dubious value.

The threshold constant for the known object position and relatively long observation time of 15 seconds of Blackwell's Table VIII has a low value $K'=1.68$. It is not surprising that the constant for the conditions of his Table III is considerably higher ($K'=4.5$) because the object position in this case was variable and the short exposure time of 6 seconds required rapid scanning of the test field by the eye to detect the object. The actual time of seeing the object was therefore quite short and explains the smaller amplitude of the negative image component (see Fig. 13) which does not have time to build up to its full steady value. Note that only the relative energy $(W-/W)$ of the negative point-image changes, but not its area.

A Relation Between Contrast Sensitivity $B/\Delta B$ and Primary Characteristic $\bar{n}_1=f(E_r)$

Because of the differentiation occurring in the system, the intensity difference ΔB between two large adjacent fields is transmitted only near the boundary, and the sampling area is a "*line image*" $\bar{a}_l=c/N_{e(e)}{}^2$ of the visual process, where $c/N_{e(e)}$ is the length of the line image.

The image contrast of large areas is equal to the object contrast

$$C_i=C_0=(\Delta E_r/E_r).$$

The incremental signal is hence $\Delta\bar{n}_1/\bar{n}_1=\dot{\gamma}_1(\Delta E_r/E_r)$, and the threshold signal-to-noise ratio becomes

$$(\Delta S/|N|)=\dot{\gamma}_1(\Delta E_r/E_r)\bar{n}_1c\bar{a}_lT/(\bar{n}_1c\bar{a}_lT)^{\frac{1}{2}}=K;$$

with the substitution $\bar{a}_l=c/N_{e(e)}{}^2$ one obtains

$$\dot{\gamma}_1^2\bar{n}_1=(K^2/c)[(E_r/\Delta E_r)N_{e(e)}]^2/T. \qquad (22)$$

Taking the threshold contrast for the largest disk size from Blackwell's Table VIII ($\alpha=360'$), the computed function (see Fig. 11 and Table III) is found to agree well in form with the other data, and furnishes $K'c^{-\frac{1}{2}}=0.45$. Having computed the value $K'=1.68$ from the same data on an area basis, the line image is found to have the length

$$c/N_{e(e)}=14/N_{e(e)}. \tag{23}$$

The sampling aperture of the eye for lines or edges is its line image, limited in length to fourteen equivalent point-image diameters. At a normal illumination $E_r=400(B\simeq 10$ ft-L), for example, the effective length of the eye's sampling aperture at the foveal center is $14/50=0.028$ mm, corresponding to a visual angle $\alpha\simeq 48'$. This result agrees with the observation that line objects gain at first in contrast when their length is increased. It should be mentioned again that the nonuniformity of receptor population and integration of the eye can cause considerable variation of these values at different retinal locations.

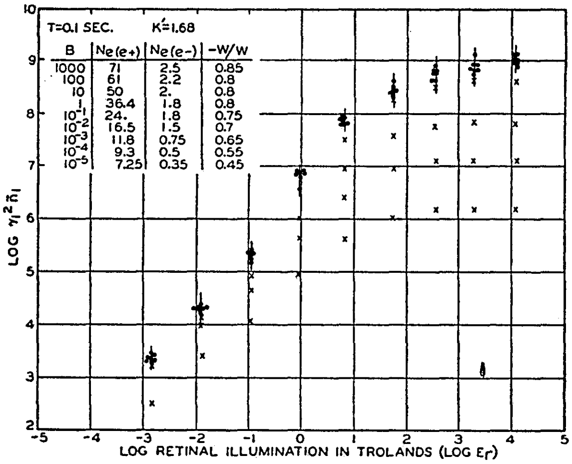

FIG. 12. The function $\dot{\gamma}_1{}^2\bar{n}_1$ computed from Blackwell's Table VIII[15] neglecting negative feedback (crosses) and corrected for negative feedback (points).

Storage and Frame Time T

The temporal integration of signal units in a visual system is in principle a compromise between frame rate and signal-to-noise ratio in one image frame. Rapid motion requires transmission of a large number of independent image frames per second, while low noise levels require a long storage time per frame. An upper limit of frame rate is set by the reaction time of the nervous system and the brain. Longer integration periods are profitable at low light values and do take place in retinal elements. To avoid energy loss, the storage and discharge mechanism must be nondissipative. The nature of neural signal transmission and the constancy of the time-intensity product (Bunsen Roscoe law)[18] within a definite time interval and its sudden departure upon exceeding this interval, support the concept that accumulation and storage of signal units in retinal elements is terminated by a sudden breakdown of a barrier, triggering the discharge of stored energy when a potential has increased to a critical value. The triggering may be controlled by the rising signal potential itself, or by a feedback potential, or by a combination of both, and may be likened to the triggering of a counter circuit or multivibrator.

It is evident that such mechanisms can cause "resonance" effects (Talbot effect) upon synchronous intermittent excitation, and that one must be cautious in interpreting flicker observations by linear circuit theory. Qualitative information on the shape of the function $T=f(E_r)$ can be obtained from observations of the critical flicker frequency (C.F.F.) *versus* illumination.[19–21] Departure of the function C.F.F.$=f(E_r)$ from a constant ratio to the contrast sensitivity function $B/\Delta B=f(E_r)$ should indicate departure from a constant storage time, provided both functions refer to identical fields of view and adapting conditions. There seem to be few data meeting this latter requirement. The detailed function $T=f(E_r)$ applying to retinal areas preferred by the eye in normal vision and corresponding to previously used data was estimated as shown in Fig. 14. The initial estimate was corrected by subsequent evaluation of various other characteristics which force corrections in certain directions because all functions form a system of interdependent equations narrowing the limits for probable values. The storage time decreases quickly from $T\simeq 0.2$ sec at the threshold of vision to $T=0.1$ sec, remaining fairly constant for the rod system. For the cone system it decreases below this value towards a plateau of approximately 0.05 sec at high illumination.

The tentative function $\dot{\gamma}_1{}^2\bar{n}_1=(E_r)$ shown in Fig. 11 and the threshold constants K' based on a constant frame time of 0.1 sec can now be corrected. The final values are given in Table IV and Fig. 14.

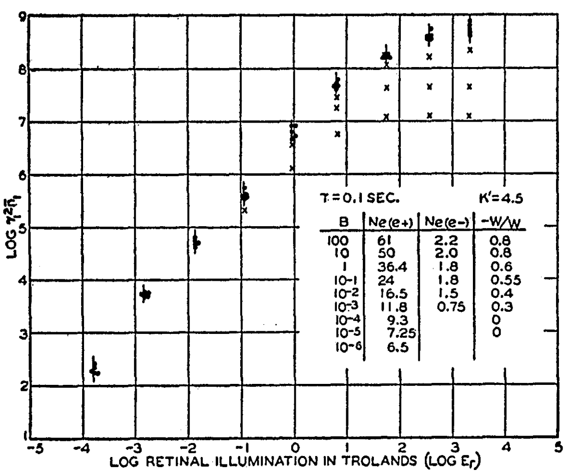

FIG. 13. The function $\dot{\gamma}_1{}^2\bar{n}_1$ computed from Blackwell's Table III neglecting negative feedback (crosses) and corrected for negative feedback (points).

[18] G. E. Long, J. Opt. Soc. Am. 41, 743–747 (1951).
[19] S. Hecht and E. L. Smith, J. Gen. Physiol. 19, 979 (1936).
[20] R. C. Brooke, J. Opt. Soc. Am. 41, 1010–1016 (1951).
[21] H. DeLange Dzn, J. Opt. Soc. Am. 44, 380–389 (1954).

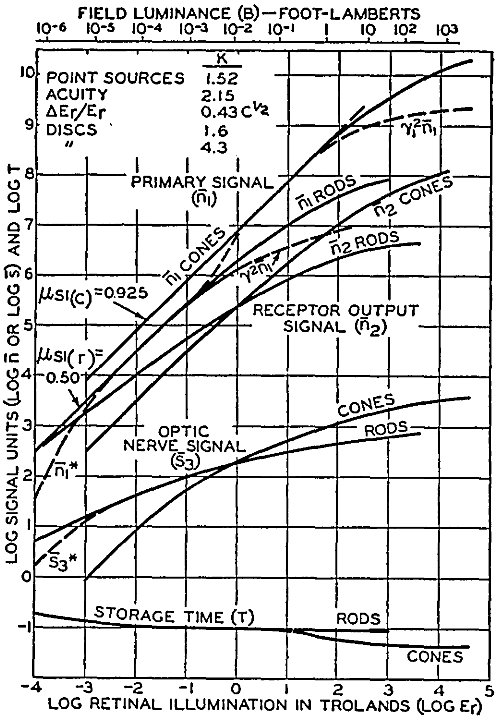

FIG. 14. Over-all and photoelectric transfer characteristics of visual process.

C. CONSTANTS AND CHARACTERISTICS OF THE VISUAL SYSTEM

1. Statistical Transfer Ratios (μ_{s1}) and Threshold Constants K

Correction of the primary characteristics for the variable frame time T furnishes the final characteristics $\bar{n}_1=f(E_r)$ for rods and cones shown in Fig. 14 (see also Table IV), with the unexpected result that the transfer ratio μ_{s1} of the cones is 86% higher than that of the rods which must therefore be set at the minimum estimate $\mu_{s1(r)}=0.5$, so that the transfer ratio of the cones $\mu_{s1(c)}=0.93$ does not exceed the possible maximum.

A decrease of rod storage time, increase of cone storage time, or a higher energy-transmission factor for the cones can change the relative values and increase $\mu_{s1(r)}$, but such adjustments appear limited to small corrections. The threshold constant has values varying from $K=1.52$ for a long observation time to $K=4.3$ for a short observation time as listed in Fig. 14.

Including light absorption in "filters" and ocular media, the *over-all conversion ratio* in s.u. from photons entering the eye to photoelectrons is thus found to be 7% for the rods and 17.6% for the cones, for the white light energy range $\Delta\lambda=0.4$ to 0.73 micron. For comparison it can be stated that a 7% conversion ratio requires that a television camera tube have a primary photocathode sensitivity of 140 μa per lumen, complete charge storage, and a noise free discharge mechanism and signal multiplication. This performance is in the order obtained recently with the fastest commercial film types.

2. The Primary Photoelectric Transfer Characteristics of the Receptors

The preceding calculations furnish sections of the two distinctly separate primary characteristics $\bar{n}_1=f(E_r)$ for the rods and cones shown in Fig. 14. The observed rod characteristic $\bar{n}_1^*$ (broken lines) bends down from the linear primary characteristic $\bar{n}_1$ because of the dark current which is $\bar{n}_d=2530$ s.u./mm²/sec [Eq. (21)]. The rod characteristic continues on separately beyond $E_e\simeq 0.1$. The "cone" characteristic $\bar{n}_1$ in turn does not cross the rod characteristic, because super linearity ($\gamma_1>1$) is not a property of primary characteristics. Failure to see in this portion of the cone characteristic is caused not by noise limitations [Eq. (20) or (22) do not apply] but by insufficient signal integration and gain in the high-acuity color system. The transition from one to the other curve (broken lines) is real and multivalued, because it is an averaging process by the "computer." The primary characteristic $\bar{n}_1$ computed from signal-to-noise ratios are internal receptor characteristics which do not include the receptor gain. The solution for the signal output $\bar{n}_2$ at the receptor terminals requires first an evaluation of the over-all transfer function $\bar{n}_3\cong f(E_r)$.

3. Over-All Transfer Functions and Relative Receptor Gain Characteristics

The sensation unit $\Delta\bar{s}=1$ is a subjective signal output unit in the computer. It is observable as the slope value $\Delta\bar{s}/\Delta E_r$ of the over-all transfer function $\bar{s}=f(E_r)$. A 50% probability of seeing the increment ΔE_r can be interpreted as perception of one signal unit $\Delta\bar{s}=1$, received once in two frame periods ($2T$) from the sampling area $\bar{a}_e=1/N_{e(e)}^2$ of the visual system. Because the retinal sampling area $\bar{a}_r=1/N_{e(r)}^2$ is smaller than $\bar{a}_e$, a single optic nerve fiber carries only a fraction $\Delta\bar{s}_3$ of one sensation unit:

$$\Delta\bar{s}_3=\Delta\bar{s}(\bar{a}_r/\bar{a}_e)=\Delta\bar{s}(N_{e(e)}/N_{e(r)})^2. \qquad (24)$$

The number of sensation units per second in one optic-nerve line can hence be computed by the integration

$$\bar{s}_3=f(E_r)=\tfrac{1}{2}\int(N_{e(e)}/N_{e(r)})^2/T\Delta E_r)dE_r \qquad (25)$$

(ΔE_r is taken from Blackwell's data for 360′ disks, see Table III). The sensation unit $\Delta s_3=1$ is a large encoded signal unit, (a burst of pulses) consisting of many electron units. In the simultaneous system of the eye the function $\bar{s}_3=f(E_r)$ is related to the primary signal intensity $\bar{n}_1=f(E_r)$ and the receptor output $\bar{n}_2=f(E_r)$ as follows. Division of $\bar{s}_3$ by the ratio of the retinal integration area $\bar{a}_r$ to the area $\bar{a}_{rc}$ of one photoreceptor

TABLE IV. Storage time and transfer functions of visual system (log units, except $\dot{\gamma}$).

	Rods							Cones						
log E_r	log T	log $\dot{\gamma}_1{}^2\bar{n}_1$	log $\bar{n}_1$	$\dot{\gamma}_1$	log s	log $\bar{s}_3$	log $\bar{n}_2$	log T	log $\dot{\gamma}_1{}^2n_1$	log $\bar{n}_1$	$\dot{\gamma}_1$	log s	log $\bar{s}_3$	log $\bar{n}_2$
−4.0	−0.72	1.54	2.5	1.	0.0	0.20* 0.72	2.46							
−3.4	−0.80	2.75	3.1	1.	0.56	0.795* 1.0	2.94							
−3.0	−0.86	3.33	3.5	1.	0.87	1.11* 1.2	3.25	−0.89		3.9	1.0	0.0	−0.07	2.49
−2.4	−0.92	4.04	4.1	1.	1.22	1.44* 1.47	3.71	−0.92		4.5	1.0	0.59	0.53	3.09
−2.0	−0.96	4.48	4.5	1.	1.43	1.64	4.04	−0.96		4.9	1.0	0.93	0.93	3.51
−1.4	−0.98	5.05	5.09	0.95	1.70	1.86	4.48	−0.98		5.5	1.0	1.40	1.44	4.12
−1.0	−0.99	5.41	5.48	0.90	1.85	2.0	4.74	−0.99		5.9	1.0	1.66	1.72	4.50
−0.4	−1.00	5.87	6.0	0.85	2.05	2.17	5.15	−1.0	6.17	6.5	1.0	2.04	2.06	5.04
0.0	−1.00	6.12	6.3	0.80	2.17	2.29	5.39	−1.0	6.82	6.9	1.0	2.25	2.27	5.14
0.6	−1.01	6.41	6.75	0.68	2.33	2.42	5.75	−1.03	7.5	7.5	1.0	2.50	2.55	5.91
1.0	−1.02	6.59	7.01	0.60	2.41	2.51	5.95	−1.06	7.9	7.9	1.0	2.67	2.71	6.25
1.6	−1.03	6.80	7.37	0.52	2.49	2.61	6.20	−1.13	8.42	8.5	0.95	2.875	2.93	6.73
2.0	−1.04	6.91	7.59	0.46	2.55	2.68	6.35	−1.20	8.72	8.83	0.88	3.00	3.06	7.04
2.6	−1.04		7.85		2.60	2.77	6.55	−1.27	9.01	9.31	0.71	3.156	3.25	7.43
3.0	−1.04		7.95		2.65	2.81	6.61	−1.30	9.13	9.57	0.60	3.245	3.35	7.65
3.6						2.87	6.67	−1.32	9.26	9.91	0.47	3.335	3.46	7.90
4.0								−1.33	9.31	10.11	0.40	3.392	3.53	8.05
4.6								−1.33	9.34	10.3	0.33	3.45	3.59	
	1	2	3	4	5	6	7	1	2	3	4	5	6	7

Column	
1	Storage time (see Fig. 14)
2	Computed with Eq. (20) or (22)
3	Primary photoelectric process
4	Gamma of 3
5	Sensation units (s^* including effect of dark current) per picture frame
6	Signal units per second in optic nerve [Eq. (25)], values* contain dark current effect
7	Receptor output signals [Eq. (27)]

furnishes the number of statistical signal units (s.u.) contributed by one receptor

$$\bar{n}_{(\text{receptor})}=f(E_r)=\bar{s}_3\bar{a}_{rc}/\bar{a}_r. \tag{26}$$

The number of s.u. from one square millimeter of receptor area, i.e., the receptor output signal $\bar{n}_2$, is obtained by multiplying Eq. (26) by the receptor density $(1/\bar{a}_{rc})$:

$$\bar{n}_2=f(E_r)=\bar{s}_3/\bar{a}_r=\bar{s}_3N_{e(r)}{}^2. \tag{27}$$

In the sequential analog system, the receptor output function $\bar{n}_2=f(E_r)$ bears the same relation to the primary function $\bar{n}_1=f(E_r)$ as in the simultaneous system [Eq. (25) and (27)]. The over-all transfer function, however, should be expressed by an electron density $\bar{n}_3$, requiring multiplication of $\bar{s}_3$ by the total number of nerve lines serving a unit area:

$$\bar{n}_3=\bar{s}_3N_{e(r)\,\max}{}^2\simeq 3.73\bar{s}_3\times 10^4. \tag{28}$$

The relation has no numerical significance in the visual system and because the constant is of little importance in the analog, the optic nerve signal function $\bar{s}_3=f(E_r)$ can be retained to describe the over-all transfer characteristic of the analog.

Inspection of the relations (25) and (27) reveals that the relative receptor gain function $(\bar{n}_2)$** is controlled essentially by observed values ($N_{e(r)}$ cancels out in effect), while calculation of the nerve signal $\bar{s}_3$ depends on the squared value of the estimated function $N_{e(r)}=f(E_r)$ and its subdivision into rod and cone branches. It was pointed out earlier that the maximum of $N_{e(r)}$ is determined with good accuracy by the diffraction limit of the lens and the ocular tremor, and that the general course of the functions $N_{e(r)}=f(E_r)$ is likely to follow sound "design" principles and run parallel to $N_{e(e)}$ (see Fig. 9). It is further known that the system of permanent connections branching out from each optic nerve terminates on a certain number of receptors (see Fig. 1) which determines the maximum integration for that nerve line. The maximum integration for cone lines is lower than for rod lines, but the minimum integration in the cone system is reduced by high illumination to very few cones, while the rod-lines cannot be "blocked" off from a much larger number of receptors. This "blocking" of connections (inhibition) seems to be controlled directly or indirectly by the electron density in the retina, and the location of the network in the path of light is perhaps not a biological accident, because absorbed light energy can initiate and promote various photoelectric and photochemical regulating processes.

The branch functions $N_{e(r)}=f(E_r)$ must logically be continuous functions and have a sigmoid shape being displaced against each other as shown in Fig. 9, with the cone integration beginning to change just below the cone-vision threshold. Their probable course

** It is pointed out that the statistical unit for $\bar{n}_2$ is likely to contain a larger number of electrons than the unit for $\bar{n}_1$, in order to provide sufficient energy for nerve triggering and to avoid transmission line noise.

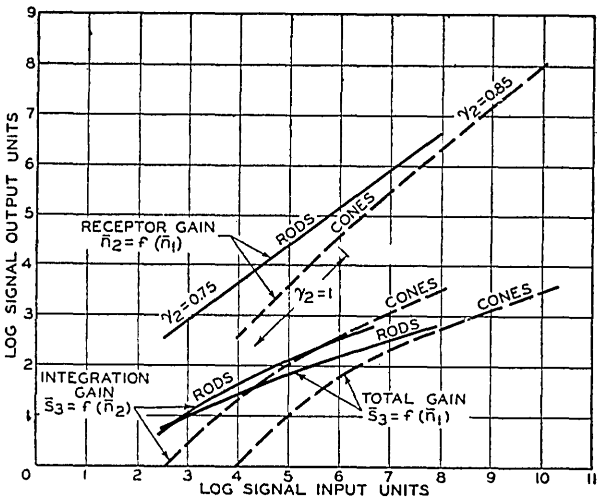

FIG. 15. Intermediate transfer functions of the visual system (gain characteristics).

is further restricted by a number of system design principles. It is unlikely that the relative gain functions $\bar{n}_2=f(\bar{n}_1)$ of the photoreceptors shown in Fig. 15, would exhibit an erratic behavior, depart from a substantially constant slope at low energy values, or increase their slope at high energy values, which would disagree with photochemical properties and energy supply mechanisms of the liquid photoconductive materials (rhodopsin, iodopsin?). Since substantial portions of these latter functions are determined by known values, it is apparent that the extrapolated characteristics $N_{e(r)}=f(E_r)$ are far from arbitrary, because they have a powerful control (square) over the known course of other functions. It is pointed out that normalized units (s.u. per mm^2 per sec) must be used to judge and obtain "smooth" functions. Repeated trials and progressively smaller corrections resulted in the final system of functions shown in Figs. 14 and 15 and listed in Table IV, which appear to fill all system requirements and do not contradict observations as far as the author is aware.

4. Discussion of Characteristics

Maximum Signals and Pulse Frequency

At near maximum signal ($E_r=4\times10^4$), the receptor output signal $\bar{n}_2$ should agree with the observed pulse frequency limit of the receptors, which is in the order of 500 cps. The value $\bar{n}_2\simeq1.4\times10^8$ s.u. (see Fig. 14) multiplied by the area of one cone (2.2 diameter) gives a frequency of 535 units/sec. The maximum signal in the optic nerve is $\bar{s}_3\simeq3800$ s.u. of which 85% is suppressed by feedback ($W-/W$ in Fig. 9) leaving 570 s.u./sec. These values indicate that the storage time T at high illumination is not likely to be shorter than the values used (a factor considered in estimating T).

Gamma, Gain, and Negative Feedback

Gamma values less than unity can be caused by an additive constant, by saturation effects or by negative feedback. The decreasing $\dot{\gamma}_1$ of the primary characteristics ($\bar{n}_1$) at high signal levels indicate a loss of energy by material or energy supply limitations which occurs earlier in the rods (bleaching of rhodopsin). It is significant that the receptor gain characteristic $\bar{n}_2=f(\bar{n}_1)$ of the rods has a constant and lower gamma ($\gamma_2=0.75$) than the gain characteristic of the cones, which has an initial gamma of unity ($\gamma_2=1$) decreasing to a lower value ($\gamma_2=0.85$) at normal signal intensities. These gain functions are normal properties of photoconductors. It is possible, however, that the lamellated structure of the photoreceptors has something to do with the sublinearity of their gain, by letting the light penetrate progressively deeper into the receptor with increased illumination, thus "moving" in effect the output "connection."

There is much evidence that the lower than unity gamma of the retinal integration characteristics $\bar{s}_3=f(\bar{n}_2)$ shown in Fig. 15 is controlled by negative feedback. An A.G.C. (automatic gain control) action can usually be recognized by its time delay. Although it may be fast, it requires a certain time to build up to its full effect, explaining initial signal overshoots and "adaptation" periods. The A.G.C. for the (d)-bipolars (blue) is slowest, causing an initial 5 to 1 overshoot in the rise of visual sensation at moderate light and the highest flicker sensitivity. The (f)-bipolars are faster, and the (e)-bipolars (red) have the fastest A.G.C., counteracting rapid intensity changes and exhibiting the lowest flicker sensitivity. The buildup time (α time constant)[1] of 20 to 100 milliseconds produces no time-delay problems in the simultaneous system of the eye as compared to a sequential system, where wide-band negative feedback control is hampered by time delay difficulties. There are other longer electrochemical time constants (β,γ) which are properties of the energy supply system, controlling adaptation periods and stabilizing signal integration. The A.G.C. is, of course, not uniform over the image area (as indicated by $N_{e(e-)}$) but forms effectively a negative feedback image. It does, therefore, not only change the

TABLE V. Relative luminance Y of Munsell lightness scale V.

V	0.1	0.2	0.5	1	2	3	4	5	6	7	8	9	10
Y	0.12	0.237	0.58	1.21	3.126	6.56	12.0	19.77	30.05	43.06	59.1	78.66	102.56

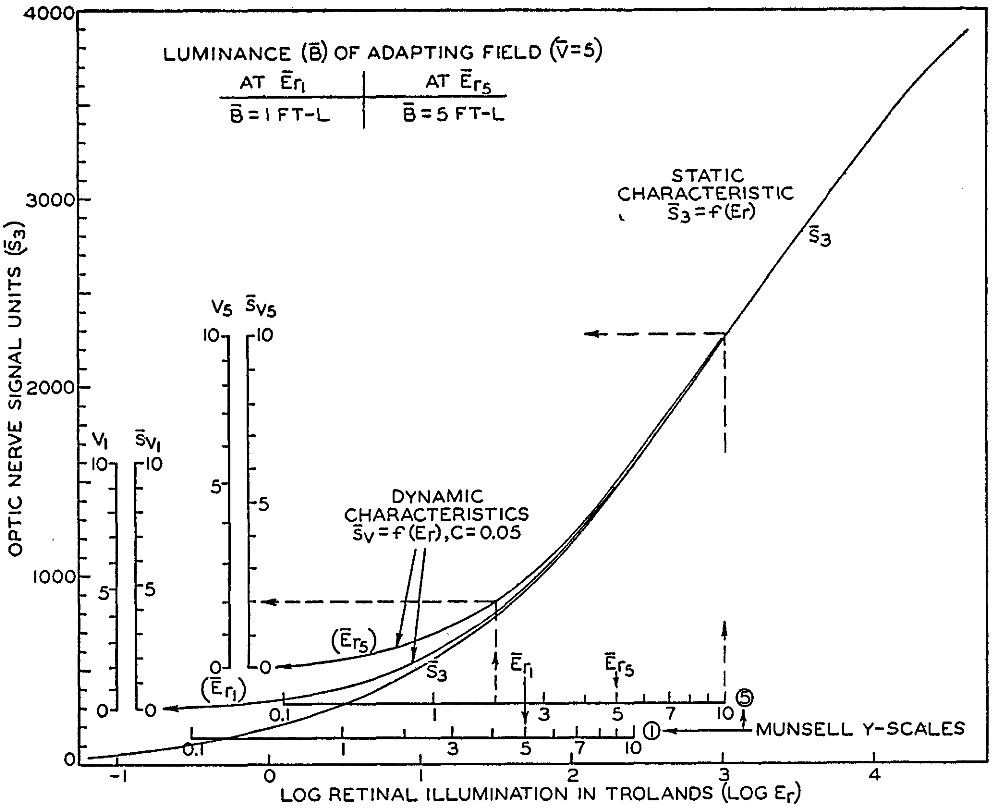

FIG. 16. Construction of operating characteristics and lightness scales.

steady gain (transfer constant) but also reduces the gamma of the transfer characteristic $\bar{s}_3 = f(E_r)$ under dynamic conditions to values ($\dot{\gamma}_1\dot{\gamma}_2\dot{\gamma}_3$) which are considerably lower than the gamma ($\dot{\gamma}_1\dot{\gamma}_2$) of the receptor characteristic $\bar{n}_2 = f(E_r)$.

Operating Characteristics

Operating characteristics $\bar{s}_v = f(E_r)$ are dynamic transfer characteristics which describe large area contrast and signal relations in the image. They differ in first approximation from static characteristics by an additive constant caused by scattered light, lens flare, diffusion of charges, retinal coupling, etc., which introduces a sensation "bias" and a subjective black level. If the total effect can be represented by an equivalent light bias $c\bar{E}_r$ which is a constant fraction of the average illumination ($\bar{E}_r$), the operating characteristic $\bar{s}_v = f(E_r)$ is equal to $\bar{s}_3 = f(E_r - c\bar{E}_r)$ as illustrated in Fig. 16. Two operating characteristics have been constructed for the average values $\bar{E}_{r1} = 50$ and $\bar{E}_{r5} = 200$ trolands and the constant $c = 0.05$.

The subjective black levels $\bar{s}_V = 0$ occur at $E_r = c\bar{E}_r$ and correspond to the values $\bar{s}_3 = 290$ and $\bar{s}_3 = 510$, respectively. The *Munsell* notation[22] assigns a 10-unit lightness scale V to the total subjective range $\bar{s}_V = 0$ to $\bar{s}_{V\,\max}$ obtained with a gray adaptation background $\bar{E}_r$. Letting $\bar{E}_r$ coincide with the center lightness scale value $V = 5$, fixes the maximum at $V = 10$ and the position of the relative Munsell luminance scales Y_1 and Y_5 (see Table V) which are supposed to give a linear sensation scale $V = \bar{s}_V$ when projected over the operating characteristic $\bar{s}_V = f(E_r)$. It is known and evident from Fig. 16 that the Munsell scale relation can be accurate only at one particular illumination and for specified viewing conditions. With the constant $c = 0.05$, the linear scale $\bar{s}_{V1}$ (see Fig. 16) matches the projected Munsell scale V_1 quite well for the average illumination $\bar{E}_{r1}$ (corresponding to $\bar{B} \simeq 1$ ft-L and $B_{\max} \simeq 5$ ft-L), the Munsell scale being but slightly compressed towards the black, while the higher value $\bar{E}_{r5}$ (corresponding to $\bar{B} \simeq 5$ ft-L and $B_{\max} \simeq 25$ ft-L) stretches the black end of the Munsell scale V_5. It is evident that a decrease of the adapting background level to $\bar{E}_r < Y = 5$ or of the constant c causes a lower black level, an expansion of dark steps, and a longer subjective scale, while a high background level ($\bar{E}_r > Y = 5$) compresses the dark steps and the subjective scale. A general increase of illumination compresses the white scale units, while a decrease compresses the dark scale units as known from observation and expected from the nature of the basic transfer characteristics which become increasingly more linear and less logarithmic at reduced illumination.

[22] Deane B. Judd, Natl. Bur. Standards Circ. 478 (1950).

5. Noise Perception

All imaging systems contain one or several sources of random deviations or fluctuations termed "noise," which introduce a granular structure into the image. The spectrum of a noise source is generally a "flat" spectrum of constant amplitude. The spectrum of the granular structure or of the deviations in an imaging system, however, is modified by the passband of the system elements through which the "noise" is observed. In certain television systems, for example, the principal noise source is an amplifier stage located between a low-pass filter and a correcting high-pass filter, which "differentiates" the noise, and changes its spectrum to have amplitudes proportional to frequency ("peaked" noise), causing unidirectional impulses (grains) to become pulse doublets (black and white grain doublets). The character of any mixture or modification of noise can be described in general by the noise spectrum and its equivalent passband (N_e). It is apparent that integration by a sampling area or low-pass filter reduces the energy of "peaked" noise much more rapidly than that of "flat" noise, as readily seen by multiplication of the noise spectrum with the spectrum of the sampling area. The relative amplitude of the noise at a point q in a system is specified by the signal-to-(rms)-noise ratio $[R]_q$ and is related, in all cases, to the signal-to-noise ratio $[R]_m$ at an earlier point in the system by

$$[R]_q N_{e(q)}=[R]_m N_{e(m)}/\dot{\gamma}_{(m\to q)}, \tag{29}$$

where $N_{e(q)}$ and $N_{e(m)}$ are the equivalent system passbands between the noise source and points q and m respectively, and $\dot{\gamma}_{(m-q)}$ is the point-gamma between points m and q.[6] The signal-to-noise ratio $[R]$ is a reciprocal contrast measure of the grain structure, and Eq. (29) shows that integration of many randomly distributed grains or impulses reduces the contrast in proportion to the *diameter* of the equivalent sampling area ($1/N_e$) and not in proportion to the area as for single objects. The "computer" of the visual system compares the transduced external "noise" with its own system noise and frame time, and threshold perception of the transduced external noise can be expected to occur when the internal noise level is increased by a constant factor.

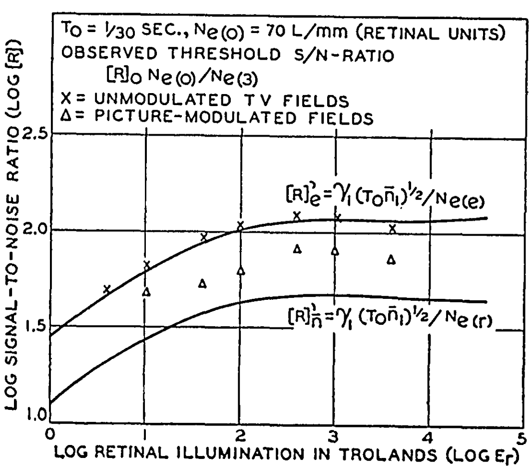

FIG. 17. To the perception of random fluctuations in external images (see text).

The external signal-to-noise ratio $[R]_0$ is changed according to Eq. (29) at the optic nerve to the following values:

for a single steady frame:

$$[R]_3=[R]_0 N_{e(0)}/N_{e(3)}\dot{\gamma}_1\dot{\gamma}_2\dot{\gamma}_3; \tag{30}$$

for a sequence of frames with $T/T_0 \geqq 1$:

$$[R]_3=(T/T_0)^{\frac{1}{2}} N_{e(0)}/N_{e(3)}\dot{\gamma}_1\dot{\gamma}_2\dot{\gamma}_3, \tag{31}$$

where

$N_{e(3)}$ = equivalent passband of spectrum between external noise source and optic nerve,

$N_{e(0)}$ = equivalent passband of spectrum between external noise source and external image,

T/T_0 = ratio of frame times of eye to external process.

The internal signal-to-noise ratio in the optic nerve is

$$[R]_{\bar{n}}=(T\bar{n}_1)^{\frac{1}{2}}/N_{e(r)}\dot{\gamma}_2\dot{\gamma}_3. \tag{32}$$

The "computer" (brain), however, may associate the over-all passband $N_{e(e)}$ and several nerve lines with the sampling of internal noise, as it does for images and change the internal signal-to-noise ratio to

$$[R]_e=(T\bar{n}_1)^{\frac{1}{2}}/N_{e(e)}\dot{\gamma}_2\dot{\gamma}_3. \tag{33}$$

The gamma values $\dot{\gamma}_2\dot{\gamma}_3$ cancel out in the ratios $[R]_3/[R]_{\bar{n}}=p$ which can, therefore, be written more conveniently:

$$[R]_3=p\bar{n}[R]_n' \quad \text{and} \quad [R]_3'=p_e[R]_e'. \tag{34}$$

External noise source:

$$[R]_3'=[R]_0 N_{e(0)}/N_{e(3)} \text{ (all cases)}. \tag{35}$$

Internal noise source:

$$\left.\begin{aligned}[R]_{\bar{n}}&=\dot{\gamma}_1(T\bar{n}_1)^{\frac{1}{2}}/N_{e(r)}\\ [R]_e'&=\dot{\gamma}_1(T\bar{n}_1)^{\frac{1}{2}}/N_{e(e)}\end{aligned}\right\}\text{single image} \tag{36}$$

$$\left.\begin{aligned}[R]_{\bar{n}}'&=\dot{\gamma}_1(T_0\bar{n}_1)^{\frac{1}{2}}/N_{e(r)}\\ [R]_e'&=\dot{\gamma}_1(T_1\bar{n}_1)^{\frac{1}{2}}/N_{e(e)}\end{aligned}\right\}\text{sequential frame } T/T_0\geqq 1. \tag{37}$$

Except for some observations of threshold signal-to-noise ratios of television noise[23] no data are available giving all necessary constants for plotting observed threshold signal-to-noise ratios as a function of illumination. The internal noise functions Eq. (37), applying to television noise ($T_0=1/30$ sec), are shown in Fig. 17. The external noise function [Eq. (35)] computed from

[23] O. H. Schade, RCA Rev. 9, 5–37, 245–286, 490–530, 653–686 (1948).

values $[R]_e N_{e(0)}$ observed at a 4 to 1 viewing ratio in *unmodulated fields* coincide substantially with the internal function $[R]_e$, furnishing the constants $p_{\bar{n}} \simeq 2.5$ and $p_e \simeq 1.05$. The combined signal-to-noise ratio is computed with

$$\left.\begin{array}{l}[R]_{\text{tot}}=[R]_e'/(1+1/p_e^2)^{\frac{1}{2}} \\ \text{or} \\ [R]_{\text{tot}}=[R]_{\bar{n}}'/(1+1/p_{\bar{n}}^2)^{\frac{1}{2}}\end{array}\right\}; \tag{38}$$

the *noise level* is hence increased by the factors

$$\left.\begin{array}{l}K_e=(1+1/p_e^2)^{\frac{1}{2}}=1.38 \\ \text{or} \\ K_{\bar{n}}=(1+1/p_{\bar{n}}^2)^{\frac{1}{2}}=1.08\end{array}\right\}. \tag{39}$$

It appears from these values that the "computer" operates with the effective band width $N_{e(e)}$ because an 8% increase is difficult to reconcile with the much larger threshold ratios K computed earlier for signal increments.

When the image field is modulated by picture signals, the ratio p_e decreases as indicated by the respective observed values in Fig. 17, and the threshold for the perception of external noise is higher. The probable reason is the *masking* of the grain structure by "interfering" structures such as a television line structure and the picture structure itself. The author observed that the presence of a pronounced line-raster structure in a kinescope image, for example (very fine spaced lines), reduces noise visibility considerably (and also the visibility of detail signals). Elimination of the raster structure by defocusing of the kinescope beam or by simply increasing the viewing distance, causes a marked increase of the apparent noise level and better detail visibility in spite of the fact that such measures reduce the band width of the system!

CONCLUSIONS

The evaluation of effective sampling apertures determined from sine-wave response data has permitted a relatively accurate quantitative synthesis of the basic constants and functions for the "luminance channel" of the visual system from data of its threshold performance. Experimental evidence may change details of the analog system, particularly in the color signal processing, which has only been indicated. The control of spatial integration (parallel operation of receptors) as a function of signal intensity by inhibition or negative feedback in the simultaneous system of the eye, is a basically simple and efficient means of adjusting signal levels, exchanging in one operation definition for signal strength to maintain optimum signal-to-noise ratios at prevailing energy levels and, in addition, suppress continuous redundant signals from larger areas, after the feedback time delay has permitted transmission of an initial signal burst. Evidence of this action is found in the quantitative evaluation of the negative feedback area $(1/N_{e(e-)})$ and its varying intensity with the time of observation. In the process of accommodating a photon signal input range of more than 10^{11} s.u. per mm^2/sec to a system capable of transmitting approximately 2×10^7 s.u. per mm^2 per sec in 3.7×10^4 nerve lines [Eq. (28)], approximately one order of signal magnitude is lost by saturation in the cones, two orders of magnitude by a sublinear receptor gain, (see Fig. 14) and less than one order is suppressed by negative feedback.

The equivalent video passband Δf_e of the analog system is given by the number of nerve lines multiplied by the frame frequency. It is, therefore, $\Delta f_e = 3.7\times10^4 \times 21.4 \simeq 0.8$ megacycles/mm^2 for the central high-definition section of the retinal image. The nerve line density in the eye, however, and hence the possible number of independent signals per unit area is nonuniform and decreases rapidly away from the foveal center. Assuming uniform density, one would obtain an internal video band width Δf_e of approximately 19 megacycles for a 4.25×5.66 millimeter image which is obtained when viewing a picture frame having a 4×3 width to height ratio at a distance four times its height. Because of the nonuniform definition in this large field ($14.3°\times19°$ approx), the total information content is considerably lower than indicated by this band width. If one assumes that 80% of the 1 million fibers in the optic nerve serve for transmission of video signals from the entire retina, the total equivalent band width of the video system does not exceed 17 megacycles. In terms of over-all performance, which must include the degradation in the optical image, an optimistic figure is again obtained by assuming uniformity of definition. At an average luminance of 40 foot-lamberts the equivalent passband of the eye is $N_{e(e)} = 56.7$ lines/mm, and the equivalent rectangular system passband for the above image area ($A = 4.25\times5.65$ mm^2) is given by $\Delta f_{(e)} = AN_{e(e)}^2/T = 1.65$ megacycles. To prevent transients, the actual spectrum must be given a more gradual cutoff $N_c > N_e$ in all directions, which decreases the band width utilization factor, increases resolution, but has a minor effect on contour sharpness. A ratio $N_c/N_e = 3$ substantially eliminates transients and requires a band width $\Delta f \simeq 15$ megacycles while a ratio $N_c/N_e = 6$ and a band width $\Delta f = 60$ megacycles would provide a uniform resolving power in the entire image field, equaling the axial resolving power of the eye. Since the resolving power of the eye decreases to 40% at 0.25 mm distance from the foveal center and to 25% at 2 mm distance, the average ratio N_c/N_e is probably smaller than 2.5, indicating a band width $\Delta f < 10$ megacycles for duplication of the actual nonuniform resolving power in the picture area. These figures do not include the band width reduction obtainable by eliminating redundancy in the signal transmission.

System thresholds are determined in the entire luminance range by statistical fluctuations in the primary photoelectric conversion process. The sampling areas or equivalent passbands (N_e) of both, optic and retina, are functions of illumination and have a strong control over the numerical value of the image area and signal-to-noise ratio which determines acuity and halftone discrimination. The threshold signal-to-noise ratio is found to be a constant having a value $K=1.6$ to 2 for a long observation time and a 50% probability of seeing the object. The constant increases for a shorter observation time and a higher probability of seeing.

All data give the unexpected result that the maximum statistical transfer ratio μ_{s1} of the receptors occurs in the cones near $E_r=5$ trolands, and that it exceeds the maximum rod transfer ratio by a factor of 1.86! This result indicates that rod excitation requires at least two photons.

One of the objects for constructing an analog is its use for obtaining visual evaluations of image characteristics by calculation, to eliminate subjective observations. This calculation is done by computing the degradation in visual response when an external process is inserted between the object and the eye. The degradation in resolution, for example, is given by the ratio of the two line numbers obtained at a given small response factor; one with the eye alone (Figs. 6 to 7), and the other for the eye in cascade with the external imaging process. The total degradation may be rated by the logarithm of the ratio of the equivalent passbands[24]:

$$\log[N_{e(e+p)}/N_{e(e)}], \tag{40}$$

where $N_{e(e)}$ is the normal visual passband, and the degraded passband is computed from the response factor products of the eye and the external process:

$$N_{e(e+p)}=\int_{N=0}^{\infty}(r_{\bar{\psi}(e)}r_{\bar{\psi}(p)})^2dN.$$

A quantitative unit for the minimum perceptible sharpness difference can be determined by subjective tests. It is likely to be a function of illumination. A liminal sharpness unit for motion pictures, determined by Baldwin,[25] for normal illumination levels is in good agreement with Eq. (40), and indicates a unit of $\log[N_{e(e+p)}/N_{e(e)}]=0.015$, which corresponds to a change of 3.5% in equivalent band width ($\Delta N_e/N_e$), effective retinal point-image diameter, or effective length of edge transitions.

It is apparent that much detail can be added to the analog by increasing the scope of subjective observations, where the analog concepts can serve as a guide in the design of experiments to isolate primary functions and help to reduce the confusion arising from data depending on specific combinations of many variables.

[24] "Wide angle high definition television systems," Contract N6 onr-23605 August, 1952, RCA Laboratories Division, Princeton, New Jersey, for Office of Naval Research Special Devices Center, Port Washington, New York.

[25] M. W. Baldwin, Jr., Bell System Tech. J. 19, 563–587 (1940).

APPENDIX

Measurements of Sine-Wave Response of the Eye

The relative accuracy of data taken on a system subject to drifts increases considerably when the time required for a set of observations is reduced. In order to eliminate unnecessary delays between observations, the sine-wave line objects are generated electrically on the screen of a television kinescope by electrical sine-wave modulation of its control grid. Stationary or drifting patterns are obtained by making the sine-wave frequency an integral multiple of the field deflection frequency, or by letting it differ from the integral number by a small amount (the difference in cycles is the drift frequency). By choice of a frame frequency of 60 cps (no interlacing) and a line deflection frequency of 35 to 50 kc (500–800 raster lines), an optically continuous and flicker-free field is obtained at viewing distances greater than 4 times the raster size. The kinescope is mounted behind an opening in a large (white) adapting screen which is illuminated by a frame light projector to match the average luminance B of the kinescope screen. *This adapting luminance* is measured with a Weston eye-corrected meter. Low values are obtained by viewing the assembly through calibrated neutral filter glasses after adequate adaptation periods. *The optical sine-wave amplitude* (contrast) is continuously adjustable from zero to the required threshold value by simply increasing the sine-wave modulating voltage on the kinescope. The modulating voltage is read on a multirange meter calibrated in terms of $\Delta B_{p-p}/B$ on the kinescope screen.

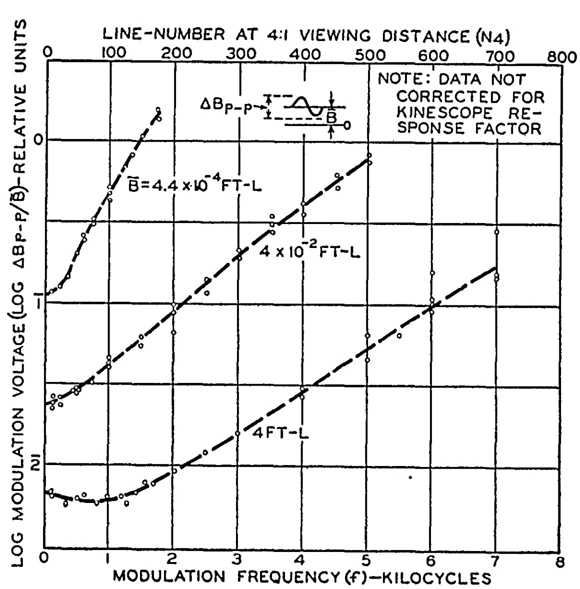

FIG. 18. Typical data obtained from sine-wave response tests.

The procedure of measuring the threshold increment ΔB_{p-p} required for detection of line numbers N at a given luminance B is as follows. The test pattern is faded in by increasing the electrical modulation *at a fixed rate* and observed on the modulation meter; the observer under test gives a signal at the instant he recognizes the line test pattern, and the person conducting the test reads and remembers the corresponding modulation reading. The modulation is returned to zero, and within seconds it is increased again at the same fixed rate to make a new observation. By averaging 10 to 15 readings mentally and recording the average reading directly on graph paper, the function $\Delta B = f(N)$ can be observed in a short time and inconsistencies are discovered immediately and checked by additional observations. Samples of these average readings, uncorrected for kinescope sine-wave response and plotted as a function of the modulating frequency, are shown in Fig. 18. It is observed that readings at the start of a test should be discarded, because a certain time is required until the observer under test has established a stable threshold criterion.

It is obvious that the data-taking process can be made automatic by driving the modulator potentiometer with a motor which is stopped by a relay, actuated by the person under test. The same relay can actuate a recording voltmeter and a reset device to restart the cycle. After correction for the optical sine-wave response factor of the kinescope, the reciprocal values $1/\Delta B$, normalized to unity at the maximum response value, furnish the sine-wave spectrum of the eye. The sine-wave spectrum is by definition independent of the transfer characteristic and must, therefore, be measured with small modulation percentages to assure linearity of the system. The modulation $\Delta B_{p-p}/2B$ is quite small at low line numbers, but increases to unity at the resolving power limit. Response factor readings requiring a modulation $(\Delta B_{p-p}/2B) > 0.5$ are, therefore, subject to distortion and it is more accurate to extrapolate this region of the spectrum.