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Formula for the contrast sensitivity of the human eye

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ABSTRACT

For design criteria of displayed images and for the judgment of image quality, it is very important to dispose of a trustful formula for the contrast sensitivity of he human eye. The contrast sensitivity function or CSF depends on a number of conditions. Most important are the luminance and the viewing angle of the object, but surround illumination can also play a role. In the paper a practical formula is given for a standard observer. This formula is derived from a more general physical formula for the contrast sensitivity. In this paper also the effects of orientation angle and surround luminance will be treated. The orientation angle will be incorporated in the formula and a correction factor will be given for the dependence on surround luminance.

Keywords: contrast sensitivity, human eye, CSF, standard observer, orientation angle, surround luminance.

1. INTRODUCTION

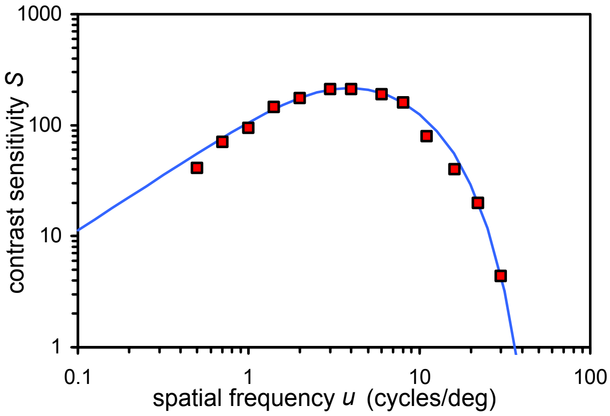

Figure 1. Example of the contrast sensitivity function. with measurement data from Robson[1].

The contrast sensitivity of the human eye plays an important role for the perceptibility of images. Contrast sensitivity is defined as the inverse of the modulation threshold of a sinusoidal luminance pattern. The modulation threshold of this pattern is generally defined by 50% probability of detection. The contrast sensitivity function or CSF gives the contrast sensitivity as a function of spatial frequency. In the CSF, the spatial frequency is expressed in angular units with respect to the eye. It reaches a maximum between 1 and 10 cycles per degree with a fall off at higher and lower spatial frequencies. An example of the CSF is given in Fig. 1 with measurement data from Robson[1]. In the past several attempts have been made to describe the CSF with a mathematical formula[2-4]. These attempts were not very successful, as the CSF appears to depend strongly on luminance and viewing angle, which was not taken into account. An example of the dependence of the CSF on luminance is given in Fig. 2 with measurements from van Meeteren et al.[5] and an example of the dependence on viewing angle is given in Fig. 3 with measurements from Carlson[6]. The author[7] published in 1990 a formula where both effects were taken into account and which had since been used by several authors. It is given by the following equation:

$$S(u) = \frac{1}{m_{\mathrm{t}}(u)} = a\,u\,\mathrm{e}^{-bu}\sqrt{1+c\,\mathrm{e}^{bu}} \tag{1}$$

where S is the contrast sensitivity, u is the spatial frequency in cycles per degree of visual angle, m_{t} is the modulation threshold, and a, b, and c are given by

$$a = \frac{540\,(1+0.7/L)^{-0.2}}{1+\dfrac{12}{X_{\mathrm{o}}(1+u/3)}} \qquad b = 0.3\,(1+100/L)^{0.15} \qquad c = 0.06$$

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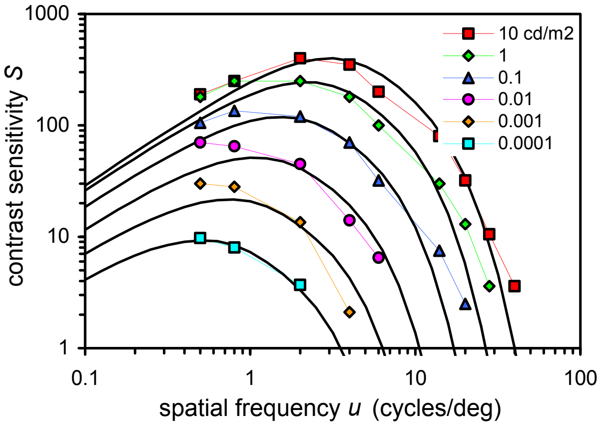

Figure 2. CSF dependence on luminance. Measurements by van Meeteren et al.[5] at a field size of 17° × 11°.

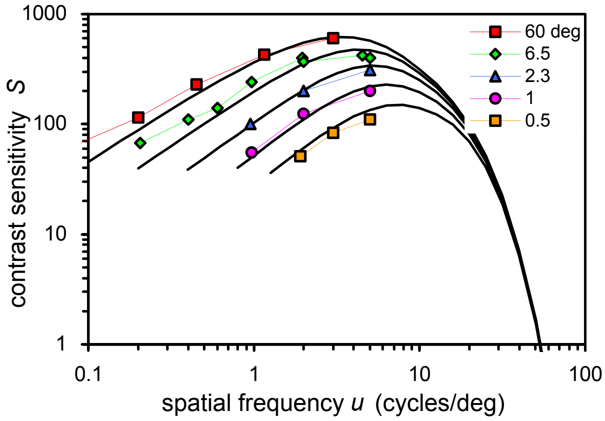

Figure 3. CSF dependence on viewing angle. Measurements by Carlson[6] at a luminance of 108 cd/m^2.

where L is the luminance in cd/m^2, and X_0 is the angular size of the object in degrees calculated from the square root of the picture area. This equation was used for the continuous curves in Figs. 2 and 3.

2. PHYSICAL MODEL FOR THE CSF

The above given formula is only a mathematical approximation of published measurements. In reality, the contrast sensitivity depends on a large number of physical quantities, like the quality of the eye lens, the sensitivity of the photoreceptors and a number of neural characteristics that determine the behavior of the visual system. In 1999, the author published a physical model where all these properties were taken into account[8]. This model is based on the assumption that the contrast sensitivity of the eye is partly determined by noise and partly by the optical MTF of the eye and lateral inhibition. A block diagram of the model is given in Figure 4. The model results in the following equation for the contrast sensitivity function:

$$S(u) = \frac{1}{m_t} = \frac{M_{opt}(u)/k}{\sqrt{\frac{2}{T}\left(\frac{1}{X_o^2}+\frac{1}{X_{max}^2}+\frac{u^2}{N_{max}^2}\right)\left(\frac{1}{\eta p E}+\frac{\Phi_0}{1-e^{-(u/u_0)^2}}\right)}} \qquad (2)$$

where $M_{opt}(u)$ is the optical MTF of the eye, k is the signal to noise ratio, T is the integration time of the eye, X_o is the angular size of the object, X_{max} is the maximum angular size of the integration area of the noise, N_{max} is the maximum number of cycles over which the eye can integrate the information, η is the quantum efficiency of the eye, E is the

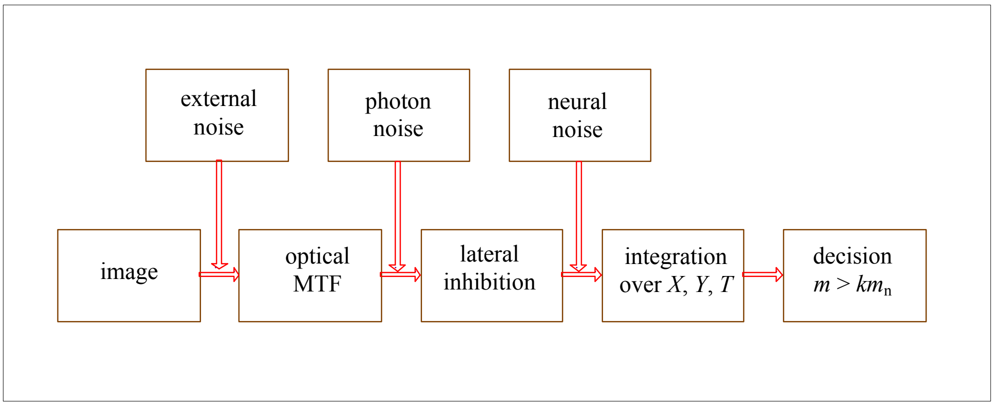

Figure 4. Block diagram of the contrast sensitivity model used for Eq. (2). m is the modulation of the signal, m_n is the average modulation of the internal noise, and k is the signal to noise ratio.

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retinal illuminance in Troland, p is the photon conversion factor, which gives the number of photons per second per square degree per Troland, Φ_0 is the spectral density of the neural noise, and u_0 is the spatial frequency above which the lateral inhibition ceases. The formula holds for bilateral viewing and for equal dimensions of the object in x and y direction. For monocular vision, the contrast sensitivity is a factor √2 smaller and for non-equal dimensions of the object, X_o has to be replaced by the square root of the angular object size. The optical MTF of the eye used in the model is given by the following equation:

$$M_{opt}(u) = e^{-2\pi^2\sigma^2 u^2} \tag{3}$$

where σ depends on the pupil size. σ is given by

$$\sigma = \sqrt{\sigma_0^2 + (C_{ab} d)^2} \tag{4}$$

where σ_0 is a constant, C_{ab} is a constant that describes the increase of σ at increasing pupil size, and d is the diameter of the pupil. The pupil size depends on the average luminance of the observed area. It can be calculated with the following approximation given by Le Grand[9]:

$$d = 5 - 3 \tanh(0.4 \log L) \tag{5}$$

where the pupil diameter d is expressed in mm and the luminance L in cd/m^2. The constants used in the model have the following typical values:

$k = 3.0$	$T = 0.1$ sec	$\eta = 0.03$
$\sigma_0 = 0.5$ arc min	$X_{max} = 12°$	$\Phi_0 = 3 \times 10^{-8}$ sec deg^2
$C_{ab} = 0.08$ arc min/mm	$N_{max} = 15$ cycles	$u_0 = 7$ cycles/deg

The value of the photon conversion factor p depends on the light source. For usual luminance conditions, $p \approx 1.2 \times 10^6$ photons/sec/deg^2/Td. With the model given by Eq. (2), a large range of contrast sensitivity measurements published in literature can be described. For individual subjects, only the values of k, σ_0, and η have to be adapted.

3. GENERAL FORMULA FOR THE CSF

Although the physical formula given by Eq. (2) enables a very good description of published measurement data, it is too complicated to be used as a standard for the CSF. Such a standard should be easy applicable and need to be valid only for a standard observer and for usual viewing conditions. Such a formula, can be derived from Eq. (2) by the use of the typical values for the constants and the introduction of simplified conditions for the variables that influence the results. To obtain this, Eq. (2) is first written in the form

$$S(u) = \frac{1/k}{\sqrt{\frac{2}{T}\frac{\Phi_0}{X_{max}^2}}} \frac{M_{opt}(u)}{\sqrt{(1 + \frac{X_{max}^2}{X_o^2} + \frac{X_{max}^2}{N_{max}^2} u^2)(\frac{1}{\eta p E \Phi_0} + \frac{1}{1 - e^{-(u/u_0)^2}})}} \tag{6}$$

By using the typical values for k, T, Φ_0, X_{max}, and N_{max}, this equation reduces to

$$S(u) = \frac{5200 M_{opt}(u)}{\sqrt{(1 + \frac{144}{X_o^2} + 0.64 u^2)(\frac{1}{\eta p E \Phi_0} + \frac{1}{1 - e^{-(u/u_0)^2}})}} \tag{7}$$

For a large range of luminance levels, the retinal illuminance E can be approximated by

$$E = 14.7 L^{0.83} \tag{8}$$

where E is expressed in Troland and the luminance L is expressed in cd/m^2. By further using the typical values for η, Φ_0, and u_0 and using for p the mentioned value of 1.2×10^6 photons/sec/deg^2/Td, Eq. (7) is reduced to

$$S(u) = \frac{5200 M_{opt}(u)}{\sqrt{(1 + \frac{144}{X_o^2} + 0.64 u^2)(\frac{63}{L^{0.83}} + \frac{1}{1 - e^{-0.02 u^2}})}} \tag{9}$$

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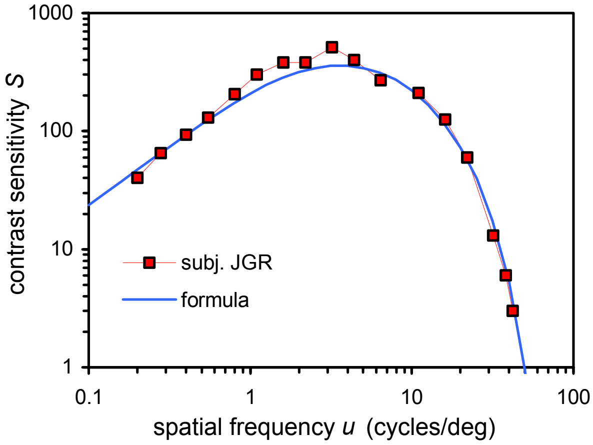

Figure 5. CSF measurements by Campbell et al.[10]. Luminance 500 cd/m², field size 10º × 10º, constant 3700 (monocular).

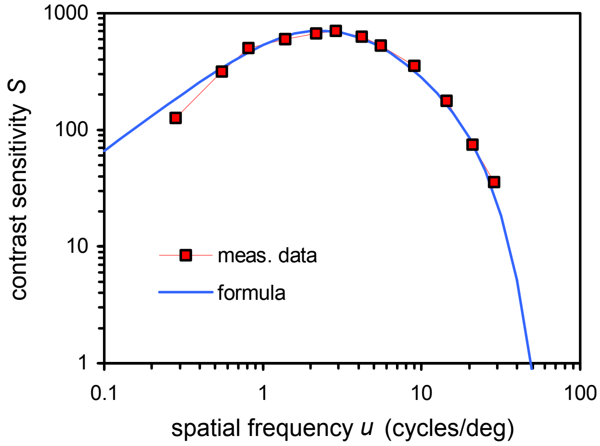

Figure 6. CSF measurements by Watanabe et al.[11]. Luminance 34 cd/m², field size 19º × 14º, constant 5800.

In this equation $M_{opt}(u)$ is given by Eqs. (3), (4) and (5), which means that it depends not only on σ_0 and C_{ab}, but also on the luminance L. This dependence can be approximated by the following equation, where further use has been made of the typical values for σ_0 and C_{ab}:

$$M_{\mathrm{opt}}(u) = \mathrm{e}^{-0.0016\, u^2 (1+100/L)^{0.08}} \qquad (10)$$

This leads to the following general formula for the CSF:

$$S(u) = \frac{5200\,\mathrm{e}^{-0.0016\, u^2 (1+100/L)^{0.08}}}{\sqrt{\left(1+\dfrac{144}{X_o^2}+0.64\,u^2\right)\left(\dfrac{63}{L^{0.83}}+\dfrac{1}{1-\mathrm{e}^{-0.02u^2}}\right)}} \qquad (11)$$

This equation contains only the luminance and viewing angle, like Eq. (1), but is further more accurate and trustful. In this equation, u is the spatial frequency in cycles/degree, L is the luminance in cd/m², and X_0^2 is the angular object area in square degrees. The constant 5200 in the numerator is valid for binocular viewing. At monocular viewing, it is a factor √2 smaller, or 5200/√2 = 3700. The results will be valid for a standard observer. This means an observer with good vision and with an age between 20 and 30 years. For a comparison with published measurement data, the constant of the numerator has to be adapted for individual subjects or different measurement conditions.

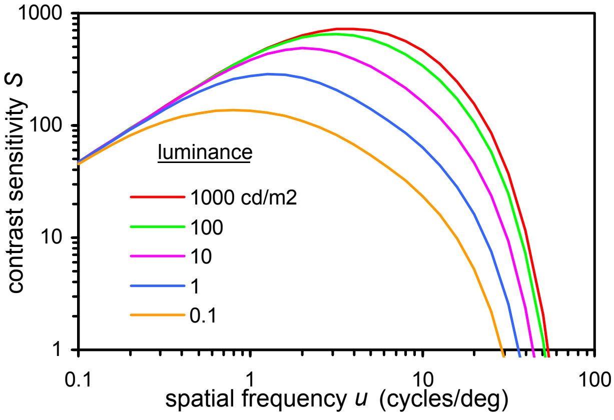

Figure 7. CSF dependence on luminance calculated with Eq. (11) for a field size of 10º × 10º.

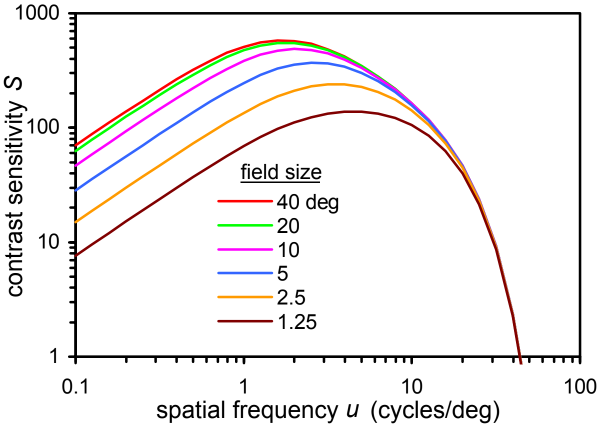

Figure 8. CSF dependence on field size calculated with Eq. (11) for a luminance of 10 cd/m².

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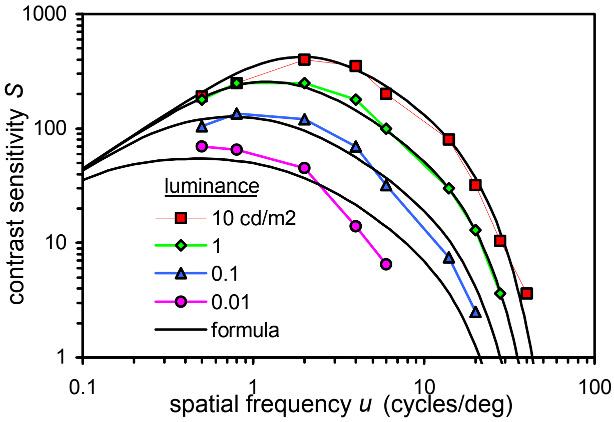

Figure 9. CSF measurements by van Meeteren et al.[5] at different luminance for a field size of 17º × 11º. Constant 4200.

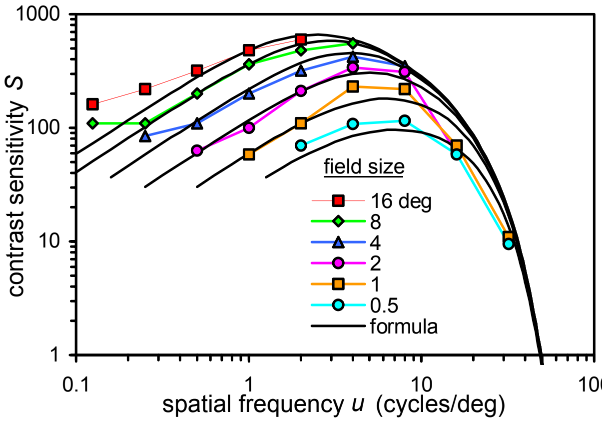

Figure 10. CSF measurements by Rovamo et al.[12] at different field sizes for a luminance of 50 cd/m^2. Constant 5200.

Figures 5 and 6 show the so obtained CSF formula together with measurement data from Campbell et al.[10] and from Watanabe et al.[11], respectively. The measurements of Fig. 5 were made monocular, so that the constant 3700 for monocular vision was used and in Fig. 6 a value of 5800 was used to adapt to the measurements. Both figures show that the formula gives a good description of the measurements. The dependence of the CSF on luminance and field size is shown in figures 7 and 8, respectively. Figure 7 clearly shows the flattening of the CSF at lower luminance levels and a shift of the maximum of the curve to lower special frequencies. Figure 8 shows that a smaller viewing angle mainly affects the CSF at low spatial frequencies. These effects are also shown in figures 9 and 10, respectively, with measurement data from van Meeteren et al.[5] for figure 9, and from Rovamo et al.[12] for figure 10. In figure 9 the constant was adapted to 4200; in figure 10 the value of 5200 was used.

4. ORIENTATION DEPENDENCE OF THE CSF

The contrast sensitivity measurements, on which the given formula is based, were all made with vertically or horizontally oriented sinusoidal luminance patterns, where no difference appeared between the horizontal and vertical direction. However, a difference would have been observed for diagonal directions. Kelly[13] mentioned in 1975 that it has been known already for at least 50 years, that horizontal and vertical line patterns are more visible than diagonal ones. He says: "At high spatial frequencies, the contrast sensitivity decreases considerably for oblique gratings. This

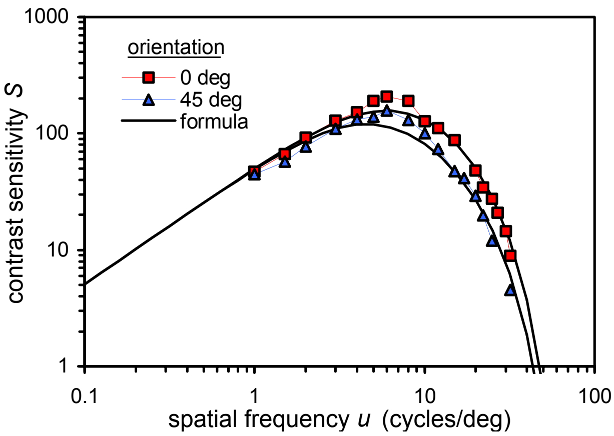

Figure 11. CSF measurements by Campbell et al.[15] at 0º and 45º orientation. The given data are the average for three subjects.

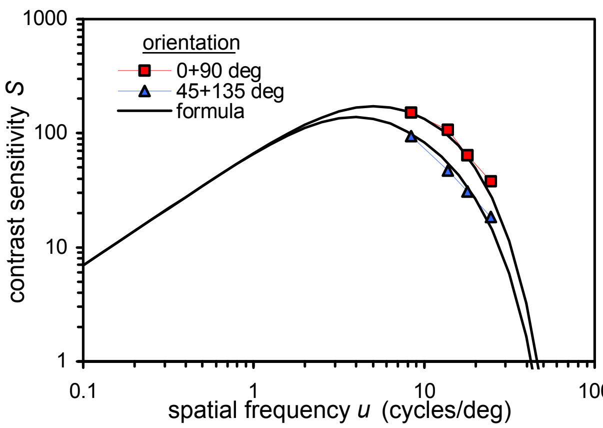

Figure 12. CSF measurements by Mitchell et al.[16]. The given data are the average for 0º and 90º, and 45º and 135º orientation for one subject.

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effect must be caused by a neural mechanism, because similar results are obtained with grating patterns formed in the plane of the retina by interference between two coherent light sources, which eliminates all optical forms of anisotropy." We found some years ago that this effect can be described by a reduction of the maximum number of integration cycles N_{max} with a factor 2 for the diagonal directions[14]. This would mean a multiplication of the constant 0.64 in the denominator of Eq. (11) with a factor 4 for these directions. To make this factor a continuous function of the orientation angle, we will write it in the form (1+ 3 $\sin^2(2\varphi)$), where φ is the orientation angle. By introducing this factor in Eq. (11), the CSF formula gets the following form for different orientation angles:

$$S(u) = \frac{5200\,\mathrm{e}^{-0.0016\,u^2\,(1+100/L)^{0.08}}}{\sqrt{(1+\dfrac{144}{X_o^2}+0.64\,(1+3\sin^2(2\varphi))u^2)(\dfrac{63}{L^{0.83}}+\dfrac{1}{1-\mathrm{e}^{-0.02\,u^2}})}} \qquad (12)$$

Figure 11 gives a comparison of the so obtained CSF with measurements by Campbell et al.[15] for a rotation angle of 0º and 45º. The measurements were made monocular with interference fringes projected on the retina using an artificial pupil of 2.8 mm. The given data are the average for three subjects. Figure 12 gives the CSF for similar measurements made by Mitchell et al.[16]. The given data are the average for 0º and 90º orientation and for 45º and 135º orientation, respectively, obtained from one subject. Figure 13 shows the dependence on orientation angle of these measurements on a relative scale at 15º intervals for a spatial frequency of 18 cycles/degree. The given data are the average for two subjects. Similar measurements are shown in figure 14. They were made by Watanabe et al.[11] with normal sinusoidal patterns with a circular field and a spatial frequency of 11.4 cycles/degree. The measurement data are from one subject. The given figures show that the simple orientation dependence that was introduced in Eq. (12) is in very good agreement with published measurement data.

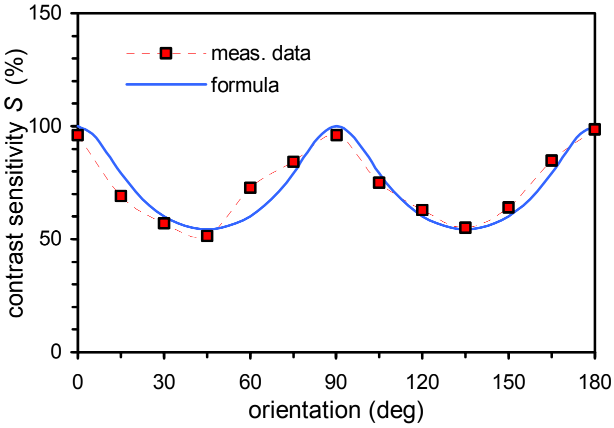

Figure 13. CSF dependence on orientation angle with measurements by Mitchell et al.[16] at a spatial frequency of 18 cycles/degree. The measurement data are the average from two subjects.

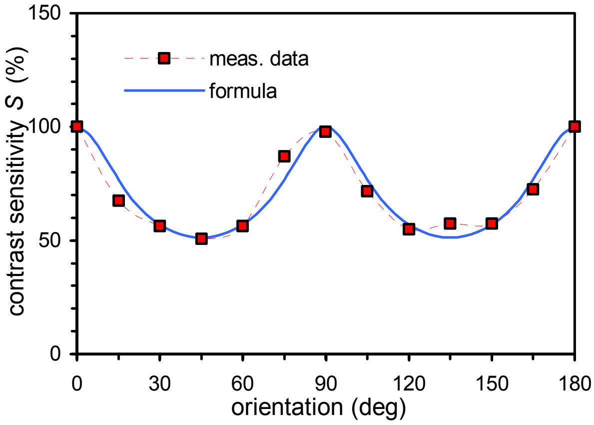

Figure 14. CSF dependence on orientation angle with measurements by Watanabe et al.[11] at a spatial frequency of 11.4 cycles/deg and a circular field. The measurement data are from one subject.

5. EFFECT OF SURROUND ILLUMINATION

Most contrast sensitivity measurements published in literature were made under ideal circumstances with respect to surround illumination. This means a surround illumination at equal luminance or a somewhat lower luminance than the object. It is well known that the visibility of an object, and thus the contrast sensitivity, can considerably be reduced when the object is surrounded by a much higher light level, which causes a blinding of the eye. The same is also true for the opposite situation, when a small object is surrounded by a dark surround. In that case the eye is blinded by its adaptation to the dark surround. These phenomena can be explained by the behavior of the voltage output of the photoreceptors at different adaptation levels. Figure 15 shows the voltage response of cones measured by Normann et al.[17] on the retina of turtles. This figure clearly shows that luminance variations at levels that differ more than one decade from the adaptation luminance arrive in a flat part of the curves so that they can hardly give much signal.

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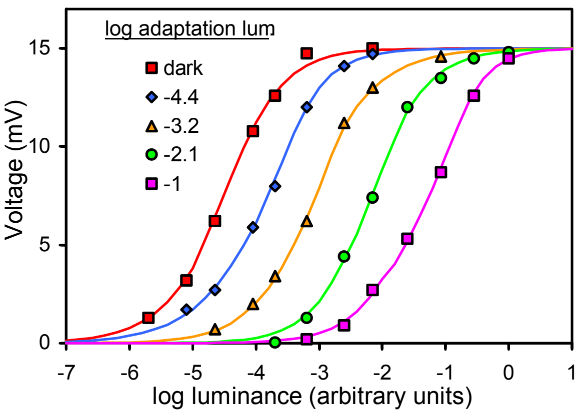

Figure 15. Voltage response cones, measured by Normann et al.[17] on the retina of turtles.

Photoreceptors of the human eye will probably show the same behavior. Rogers and Carel[18] published in a military report the results of an investigation where they measured the CSF at a large range of background levels, extending from 10^{-4} to 10^{4} of the object luminance. The area of the background luminance was 28º × 37º. Although their measurements were only made for very small field sizes up to 4º × 4º and for high spatial frequencies, starting from 4 cycles/deg, and a waffle shaped two-dimensional grating pattern was used instead of a sinusoidal pattern, their results are still very useful for an analysis of the effect of background luminance on the CSF. We found that their measurement results can be described by a single factor by which the CSF has to be multiplied. This means that the surround luminance influences only the signal-to-noise ratio. It could explain the different values of k that can be found at different experiments. It further appeared that this factor can simply be described by a Gaussian function of the logarithm of the surround luminance divided by the object luminance. The standard deviation of this function appeared to be independent of the field size of the investigated objects. The only difference was, that the maximum of the curves shifted to a lower surround luminance at smaller field sizes. The results of this investigation can be described by the following formula:

$$f = e^{-\dfrac{\ln^2\left(\dfrac{L_s}{L}\left(1+\dfrac{144}{X_o^2}\right)^{0.25}\right)-\ln^2\left(\left(1+\dfrac{144}{X_o^2}\right)^{0.25}\right)}{2\ln^2(32)}} \qquad (13)$$

where f is the correction factor by which the CSF has to be multiplied, L is the luminance of the object, L_s is the surround luminance, and X_o^2 is the object area in square degrees of visual angle. In this equation, the factor with X_o in the first term of the numerator describes the shift of the maximum of the Gaussian curve to lower values of L_s/L at smaller field sizes, whereas the second term had to be added to obtain that f =1 for the situation of equiluminous surround. The behavior of this function is shown in figures 16 and 17 for an object size of 0.5º × 0.5º and 4º × 4º, respectively, together with the measurement data of Rogers and Carel for a spatial frequency of 4 cycles/degree. Figure 16 shows that the maximum value of the correction occurs at a surround luminance that is lower than the object luminance. Figure 17 shows that this effect nearly disappears at a higher field size.

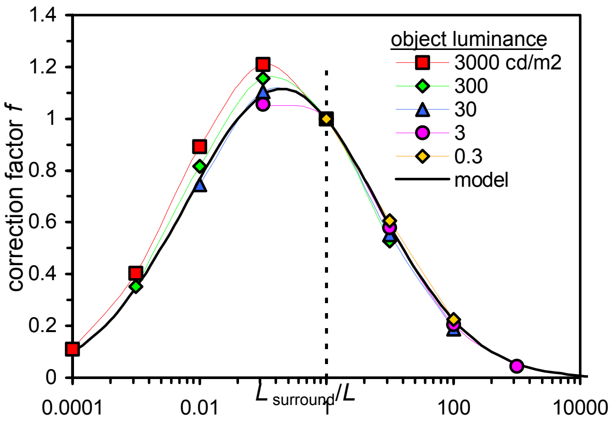

Figure 16. Correction factor for the CSF as a function of surround luminance with measurement data from Rogers et al.[18] for a spatial frequency of 4 cycles/degree. Field size 0.5º × 0.5º.

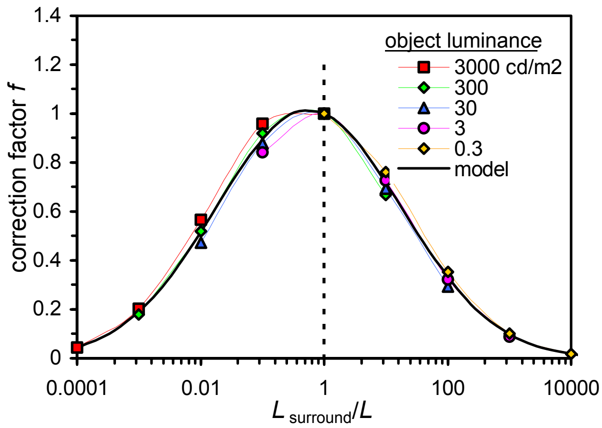

Figure 17. Correction factor for the CSF as a function of surround luminance with measurement data from Rogers et al.[18] for a spatial frequency of 4 cycles/degree. Field size 4º × 4º.

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6. CONCLUSIONS

A practical formula was given for the contrast sensitivity function of the human eye. The formula is valid for a standard observer and can be used for design criteria of displayed images and metrics for image quality. The formula contains only the luminance and the object size of the image and has been derived from a more complicated formula by which a large number of published CSF measurements could be described.

The formula was further extended for use at different orientations of the luminance variation. The so obtained orientation dependence appeared to be in very good agreement with published data.

The effect of surround illumination on the CSF was treated by analyzing published measurements that were made by Rogers and Carel[18] over a wide range of luminance levels. A formula for a correction factor was found by which the CSF has to be multiplied to correct for different conditions of surround illumination.

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