

2020 봄학기 프로그래밍언어

Written Assignment 3 – Sample Solution

다음은 수학적 귀납법을 이용하여 증명하라.

$$\text{For all } n \geq 1, P(n) \triangleq \sum_{k=1}^n 2^k = 2^1 + 2^2 + \cdots + 2^n = 2^{n+1} - 2$$

Proof. By mathematical induction on n .

- Base case) $n = 1$:

$$P(1) \triangleq \sum_{k=1}^1 2^k = 2^1 = 2^{1+1} - 2$$

- Inductive case) $n = m + 1$ where $m \geq 1$:

Induction hypothesis: $P(m) \triangleq \sum_{k=1}^m 2^k = 2^{m+1} - 2$

$$\begin{aligned} P(m+1) &\triangleq \sum_{k=1}^{m+1} 2^k = 2^1 + 2^2 + \cdots + 2^m + 2^{m+1} \\ &= \sum_{k=1}^m 2^k + 2^{m+1} \\ &= \underline{2^{m+1} - 2} + 2^{m+1} \quad \text{by IH} \\ &= 2^{m+2} - 2 \end{aligned}$$

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