Maximizing Profit

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Main Tool: Excel

Technique: Optimization

Industry: Enterprise Analytics

Problem Description and Introduction

Problem Description

A northern hardware company is considering opening a new distribution center in the southeastern region to enhance its market presence and operational efficiency. The company plans to rent a warehouse and an adjacent office, from which it will distribute its primary products: pressure washers, go-karts, generators, and water pumps. This strategic initiative involves significant logistical and financial considerations, necessitating a thorough analysis to optimize inventory levels and maximize profitability.

The company has allocated a monthly purchasing budget of $170,000 to procure these products. The unit costs for the products are as follows: $330 for a pressure washer, $370 for a go-kart, $410 for a generator, and $635 for a case of five water pumps. Correspondingly, the selling prices are $499.99 for a pressure washer, $729.99 for a go-kart, $700.99 for a generator, and $269.99 for a water pump. **The primary objective is to determine the optimal inventory levels that will maximize the company's net profit while adhering to budgetary and spatial constraints.**

The warehouse has 82 shelves, each measuring 30 feet long and 5 feet wide. Storage requirements for the products differ: pressure washers and generators each occupy a 5 by 5-foot pallet, go-karts require an 8 by 5-foot pallet, and four cases of water pumps fit on a 5 by 5-foot pallet. Additionally, the marketing department mandates that at least 30% of the inventory must consist of pressure washers and go-karts combined, and the company must sell at least twice as many generators as water pumps.

This project employs a linear programming model to optimize the allocation of resources and maximize the company's net profit. By solving this model, we aim to provide insights into the most profitable inventory mix and identify the impact of various constraints on the overall profit.

Introduction

Optimizing resource allocation is paramount for achieving operational efficiency and maximizing profitability in any business establishment. This project focuses on applying linear programming techniques to a real-world scenario faced by a northern hardware company planning to expand its distribution network. The company aims to establish a new distribution center in the southeastern United States, requiring a careful analysis of inventory management to ensure optimal utilization of resources.

The primary challenge lies in determining the ideal quantity of each product to stock, given the constraints of a fixed purchasing budget and limited warehouse space. By integrating cost, revenue, and spatial data into a linear programming model, this study seeks to identify the optimal inventory levels that will maximize the company's net profit. The model also incorporates marketing constraints to ensure a balanced inventory mix that aligns with the company's strategic objectives.

Through this analysis, we aim to provide actionable insights that will aid the company in making informed decisions about inventory management, budget allocation, and warehouse space utilization. The results will highlight the trade-offs between different products and offer recommendations on how to adjust purchasing strategies to achieve the desired financial outcomes.

By leveraging the power of linear programming, this project demonstrates the practical application of mathematical optimization techniques in solving complex business problems, thereby contributing to the body of knowledge in operational research and decision science.

Description of Problem Analysis

In analyzing the problem of determining the optimal inventory levels for the northern hardware company's new distribution center, we employed several key analytic concepts and theories from linear programming and operations research. This rigorous approach allows for the formulation of a mathematical model that optimally allocates resources within given constraints to maximize the company's net profit.

Linear Programming Model

Linear programming (LP) is a mathematical method used for optimizing a linear objective function, subject to linear equality and inequality constraints. In this problem, the objective function aims to maximize the net profit from selling four products: pressure washers, go-karts, generators, and water pumps. The constraints include budget limitations, warehouse space availability, marketing requirements, and product ratios.

Question 1

1. In a Word document, write the mathematical formulation of the problem.

In this section, we present the mathematical formulation of the linear programming problem aimed at maximizing the profit for a northern hardware company that plans to open a new distribution center. The formulation incorporates the objective function and several constraints based on the company's operational requirements.

Question 1

Item	X(n)	Cost per Unit ($)	Selling Price Per Unit ($)	Profit (Selling Price- Cost Price) ($)	Area (square feet)
Pressure washer	X1	330	499.99	169.99	5*5 = 25
Go-Kart	X2	370	729.99	359.99	8*5 = 40
Generator	X3	410	700.99	290.99	5*5 = 25
Water Pumps (Case has 5)	X4	127	269.99	142.99	(5*5)/20 = 1.25

Formulae and Mathematical Formulation				
Objective Function	Maximization	Value	Unit	Z=169.99* X1 + 359.99* X2 + 290.99* X3 + 142.99* X4
Constraint 1 Budget	Budget Constraint	170000	Dollars	330* X1 + 370 * X2 + 410 * X3 + 127 * X4 ≤ 170000
Constraint 2 Available space	Space	12300	Square Feet	25 * X1 + 40 * X2 + 25 * X3 + 1.25 * X4 ≤ 12300
Inventory constraint 3	Minimum 30% of the inventory reserved for Pressure washers and Go Karts			X1+ X2 ≥ 0.3 (X1 + X2 + X3+ X4) which gives -0.7X1 -0.7 X2 + 0.3X3+ 0.3X4 ≤ 0
Constraint 4 Selling prop	Twice as many generators as water pumps to be sold			X3 ≥ 2* X4 which gives -1X3+ 2X4 ≤0
Constraint 5 Positivity	Positivity constraint			X1, X2, X3, X4 ≥ 0

Figure 1 Mathematical model of the problem.

Objective Function

The objective function is designed to maximize the total profit derived from selling four main products: pressure washers, go-karts, generators, and water pumps. The profit for each product is the difference between the selling price and the cost price. The objective function can be expressed as follows:

$$\textbf{Maximize } Z=169.99X1+359.99X2+290.99X3+142.99X4$$

where:

- X1 is the number of pressure washers,
- X2 is the number of go-karts,

  X3 is the number of generators,
- X4 is the number of units of water pumps (as each case contains 5 water pumps).

Constraints

The constraints ensure that the solution is feasible within the company's budget, warehouse space, and marketing requirements. All the constraints were mathematically formulated into expressions in the normal form with variables on the left-hand side and the constants on the right-hand side.

1. **Budget Constraint**: The total cost of purchasing the products must not exceed the allocated monthly budget of $170,000.

$$\mathbf{330X1+370X2+410X3+127X4 \leq 170000}$$

Note that we have arrived at the value 127 for each unit of water pump as each case of water pump has 5 units and costs 635. Therefore, each unit of water pump is 635 divided by 5, that is, 127.

2. **Space Constraint**: The total space occupied by the products in the warehouse must not exceed the available space of 12,300 square feet.

$$\mathbf{25X1+40X2+25X3+1.25X4 \leq 12300}$$

Note that we have arrived at the value 1.25 sq ft for each unit of water pump as a 5 ft by 5 ft pallet is used to store four cases of water pumps. Therefore, each unit of water pump is 25 sq ft divided by 20 (four cases of five units each), that is, 1.25.

3. **Marketing Constraint**: At least 30% of the total inventory must consist of pressure washers and go-karts combined.

$$\mathbf{X1+X2 \geq 0.3(X1+X2+X3+X4)}$$

Simplifying, we get:

$$\mathbf{-0.7X1-0.7X2+0.3X3+0.3X4 \leq 0}$$

4. **Product Ratio Constraint**: The company must sell at least twice as many generators as cases of water pumps.

$$\mathbf{X3 \geq 2X4}$$

Simplifying, we get:

$$\mathbf{-X3+2X4 \leq 0}$$

5. **Non-negativity Constraint**: The quantities of all products must be non-negative.

$$\mathbf{X1, X2, X3, X4 \geq 0}$$

Summary of the solution of Question 1

Mathematical Formulation

Maximize $Z=169.99X1+359.99X2+290.99X3+142.99X$

Subject to

$$330X1+370X2+410X3+127X4 \leq 170000$$

$$25X1+40X2+25X3+1.25X4 \leq 12300$$

$$-0.7X1-0.7X2+0.3X3+0.3X4 \leq 0$$

$$-X3+2X4 \leq 0$$

Positivity constraint

$$X1, X2, X3, X4 \geq 0$$

The linear programming problem for maximizing the profit of the northern hardware company is defined by the objective function and the set of constraints. The objective function aims to maximize the profit from selling pressure washers, go-karts, generators, and water pumps. The constraints ensure that the solution respects the budget limits, warehouse space availability, marketing requirements, and non-negativity of product quantities. This mathematical formulation provides a structured approach to determining the optimal inventory levels that will maximize the company's profit while adhering to operational constraints.

Question 2

2. Set up the linear programming formulation in an Excel workbook or R.

	Question 2			
Product	X(Decision Variable)	Constraints	Z (dollars)	
Pressure washer	0	169.99	142050.703	
Go-Kart	155.179067	359.99		
Generator	237.7692613	290.99		
Water Pumps	118.8846306	142.99		

Matirx	X1	X2	X3	X4
Budget	330	370	410	127
Space	25	40	25	1.25
Inventory	-0.7	-0.7	0.3	0.3
Sales proportion	0	0	-1	2
Positivity constraint Washer	-1	0	0	0
Positivity Go-Kart constraint	0	-1	0	0
Positivity Gen constraint	0	0	-1	0
Water Pumps constraint	0	0	0	-1

Matirx	Ax	Inequality	Constraint RHS	Slack
Budget	170000	≤	170000	0
Space	12300	≤	12300	0
Inventory	-1.629179331	≤	0	1.629179331
Sales	0	≤	0	0
Positivity constraint Washer	0	≤	0	0
Positivity Go-Kart constraint	-155.179067	≤	0	155.179067
Positivity Gen constraint	-237.7692613	≤	0	237.7692613
Water Pumps constraint	-118.8846306	≤	0	118.8846306

Figure 2 Linear programming formulation.

To address the problem of optimizing inventory levels for the northern hardware company, we set up the linear programming model in an Excel workbook. The formulation integrates decision variables, constraints, and the objective function to systematically determine the optimal solution. The above image provides a comprehensive representation of this setup.

Decision Variables and Constraints

The decision variables, constraints, and the objective function are depicted in the image, ensuring clarity and precision in the formulation. The decision variables X1, X2, X3, X4 represent the quantities of pressure washers, go-karts, generators, and water pumps (in cases of 5) respectively. These variables are subject to several constraints that capture the operational limitations of the company.

Objective Function

The objective function is designed to maximize the total profit, expressed as:

$$Z=169.99X1+359.99X2+290.99X3+142.99X4$$

Constraints

The constraints ensure that the solution adheres to budget limits, warehouse space, inventory requirements, and product sales ratios. The constraints are mathematically represented in a matrix format for ease of implementation in Excel Solver.

- **Budget Constraint**: $330X1+370X2+410X3+127X4 \leq 170000$
- **Space Availability Constraint**: $25X1+40X2+25X3+1.25X4 \leq 12300$
- **Inventory Constraint (Marketing Requirement)**: $-0.7X1-0.7X2+0.3X3+0.3X4 \leq 0$
- **Product Sales Constraint (Ratio Requirement)**: $-X3+2X4 \leq 0$
- **Non-negativity Constraint**: $X1, X2, X3, X4 \geq 0$

Slack Variables

The slack variables indicate the unused portion of the constraints, providing insights into the binding nature of each constraint. For instance, the slack for the budget and space availability constraints is zero, indicating that these constraints are fully utilized in the optimal solution.

Matrix Representation

The matrix representation of the linear programming problem is crucial for its implementation in Excel Solver. The coefficients of the decision variables are organized in a tabular format, aligning with the constraints and the objective function. This structured approach facilitates the use of optimization tools in Excel.

- **Coefficient Matrix (Ax)**: The coefficient matrix captures the relationship between decision variables and constraints.

$$\begin{matrix} 330 & 370 & 410 & 127 \\ 25 & 40 & 25 & 1.25 \\ -0.7 & -0.7 & 0.3 & 0.3 \\ 0 & 0 & -1 & 2 \end{matrix}$$

Figure 3 Matrix for the model.

- **Inequality and RHS**: The inequalities and the right-hand side (RHS) values are specified to define the feasible region of the solution.

$$\begin{matrix} \leq & 170000 \\ \leq & 12300 \\ \leq & 0 \\ \leq & 0 \end{matrix}$$

Figure 4 The inequalities and the constants of the model for the matrix.

Conclusion

The linear programming formulation, as depicted in the image of the linear programming formulation, provides a detailed and systematic approach to solving the optimization problem for the northern hardware company. By leveraging Excel Solver, the company can identify the optimal inventory levels that maximize profit, ensuring efficient resource allocation and adherence to operational constraints. This formulation serves as a robust framework for strategic decision-making in inventory management.

Question 3

3. Use the Excel Solver or R to solve the problem and generate a sensitivity report.

	Question 3			
SUM PRODUCT CELLS FOR SOLVER				
Objective Function	142050.703			
Constraint 1	170000	170000		
Constraint 2	12300	12300		
Constraint 3	-1.629179331	0		
Constraint 4	0	0		
X(Decision Variable)	0	155.179067	237.7692613	118.885
Constraints	169.99	359.99	290.99	142.99

Figure 5 Sum Product cells created for usage in Solver.

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [ALY6050_MOD5Project_FaizanS.xlsx]Question 1-3
Report Created: 6/18/2024 2:56:50 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
Q10	X(Decision Variable)	0	-110.0715237	169.99	110.0715237	1E+30
R10	X(Decision Variable)	155.179067	0	359.99	205.8402439	76.73878564
S10	X(Decision Variable)	237.7692613	0	290.99	98.20490541	131.8664063
T10	X(Decision Variable)	118.8846306	0	142.99	196.4098108	89.11965734

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Q5	Constraint 1	170000	0.557648341	170000	428.8	56225
Q6	Constraint 2	12300	3.841502841	12300	6078.378378	30.94688222
Q7	Constraint 3	-1.629179331	0	0	1E+30	1.629179331
Q8	Constraint 4	0	33.68339104	0	974.1201949	27.91666667

Figure 6 The Solver Sensitivity Report.

In this analysis, we utilized Excel Solver (Srinivas, n.d.) to solve the linear programming problem for maximizing the profit of a northern hardware company. The problem involved determining the optimal quantities of four products—

pressure washers, go-karts, generators, and water pumps—while adhering to constraints on budget, warehouse space, and inventory requirements.

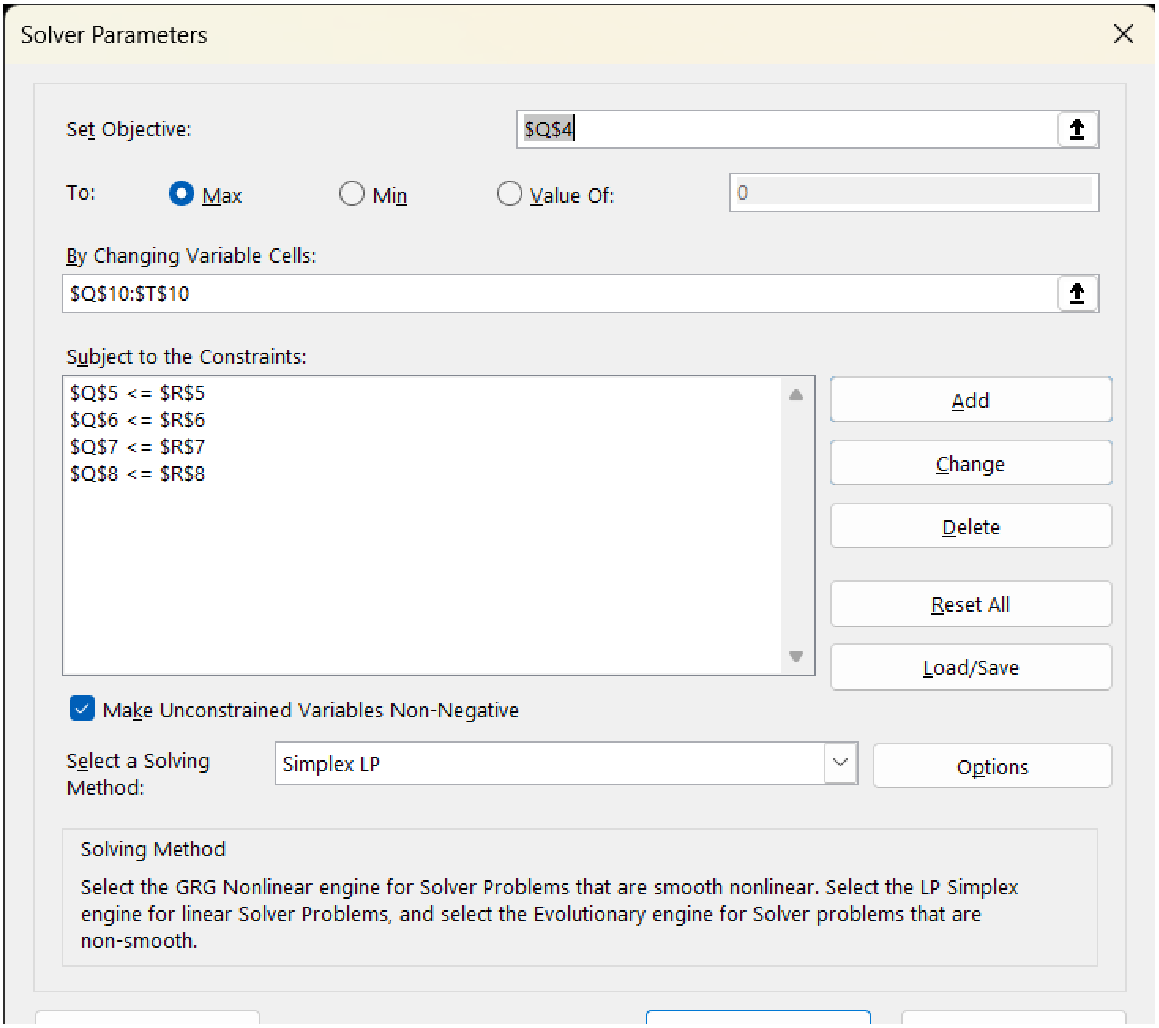

Figure 7 A screenshot of the solver window used.

To solve this problem using Excel Solver, we followed these steps:

1. **Input Data**: We entered the coefficients of the decision variables, constraints, and the right-hand side (RHS) values into the Excel worksheet. The objective function and constraints were organized in a tabular format as shown in the above images.

2. **Set Objective**: In Solver, we set the objective cell to the cell containing the formula for the total profit Z and chose "Maximize."

3. **Add Constraints**: We added the budget, space availability, inventory, and product sales constraints in Solver.

4. **Solve**: We clicked "Solve" to find the optimal values of the decision variables X1, X2, X3, X4 that maximized the profit while satisfying all constraints using the simplex LP model option. George. B. Dantzig is the famous inventor of the linear programming model.

Results and Sensitivity Report

The Solver output provided the optimal solution for the decision variables and generated a sensitivity report. The sensitivity report contained crucial information, such as shadow prices, reduced costs, and allowable increases and decreases for both the objective coefficients and constraints.

Optimal Solution:

- **Pressure Washers (X1)**: 0
- **Go-Karts (X2)**: 155.179067
- **Generators (X3)**: 237.7692613
- **Water Pumps (X4)**: 118.8846306

Total Profit: Z= **$142,050.703**

This configuration results in a total profit of $142,050.703. The solution suggests that, given the current constraints on budget and warehouse space, it is not profitable to stock pressure washers, while go-karts, generators, and water pumps should be stocked in the specified quantities to maximize profit.

Interpretation of the Sensitivity Report

The sensitivity report provided insights into the robustness of the optimal solution and the impact of changes in the constraints and coefficients.

1. **Shadow Prices**:
    - The shadow price for the budget constraint was 0.5576483441. This indicated that for every additional dollar added to the budget, the profit increased by approximately $0.56, provided all other constraints remained unchanged.
    - The shadow price for the space availability constraint was 3.841502841, meaning that for every additional square foot of space, the profit increased by approximately $3.84.
2. **Reduced Costs**:

- The reduced cost for pressure washers was -110.0715237. This meant that the selling price of pressure washers would need to increase by $110.07 for it to become profitable to include them in the optimal solution.

3. **Allowable Increases and Decreases**:

    - These values indicated the range within which the coefficients could change without altering the optimal basis. For example, the allowable increase for the budget was $428.8, indicating that the budget could be increased by this amount before a new optimal solution had to be recalculated. It is also notable that the optimal solution does n ot change even if the budget is reduced by $56225.

Conclusion

By utilizing Excel Solver, we determined the optimal inventory levels for the northern hardware company, maximizing the profit while adhering to budget and space constraints. The sensitivity report further provided valuable insights into the impact of potential changes in constraints and coefficients, guiding strategic decisions for resource allocation and operational adjustments. This robust approach ensured that the company could efficiently manage its inventory and maximize profitability.

Question 4

Describe the optimal solutions obtained in the Word document. These will consist of the inventory level for all four products and the optimal monthly profit.

Inventory Levels and Monthly Profit

The linear programming model formulated for the northern hardware company aimed at maximizing profit by determining the optimal inventory levels for four products: pressure washers, go-karts, generators, and water pumps. The optimization was subject to constraints on budget, warehouse space, and inventory requirements. The following results were obtained:

Optimal Inventory Levels:

- **Pressure Washers (X1)**: 0 units
- **Go-Karts (X2)**: 155.179067 units
- **Generators (X3)**: 237.7692613 units

- **Water Pumps (X4)**: 118.8846306 cases

Total Monthly Profit: Z=$142,050.703

Interpretation of Results

1. **Pressure Washers (X1)**:
    - The optimal solution suggests that no pressure washers should be stocked. This decision is based on the constraints and the relative profitability of pressure washers compared to the other products.
2. **Go-Karts (X2)**:
    - The model recommends stocking approximately 155.18 go-karts. This quantity is optimal given the selling price, cost, and space they occupy.
3. **Generators (X3)**:
    - The optimal inventory level for generators is approximately 237.77 units. Generators contribute significantly to the profit while fitting within the budget and space constraints.
4. **Water Pumps (X4)**:
    - The company should stock approximately 118.88 cases of water pumps. The water pumps, along with generators and go-karts, form a profitable mix that maximizes the total profit.

Sensitivity Analysis

The sensitivity report generated by Excel Solver provides valuable insights into the robustness of the optimal solution and the impact of changes in the constraints and coefficients. Key findings include:

1. **Shadow Prices**:
    - The shadow price for the budget constraint is 0.5576483441. This indicates that for every additional dollar added to the budget, the profit increases by approximately $0.56, provided all other constraints remain unchanged.
    - The shadow price for the space availability constraint is 3.841502841. This means that for every additional square foot of space, the profit increases by approximately $3.84.

2. **Reduced Costs**:

   - The reduced cost for pressure washers is -110.0715237. This means that the selling price of pressure washers would need to increase by $110.07 for them to become profitable to stock.

3. **Allowable Increases and Decreases**:

   - These values indicate the range within which the coefficients can change without altering the optimal basis. For example, the allowable increase for the budget is $428.8, indicating that the budget can be increased by this amount before a new optimal solution must be recalculated.

Conclusion

The optimal inventory levels and the corresponding total profit were determined using linear programming. The solution provides a clear strategy for the company to maximize its profit while adhering to budgetary and spatial constraints. The sensitivity analysis further offers insights into how changes in the constraints and parameters can impact the solution, guiding strategic decisions for future adjustments in inventory management. This rigorous approach ensures that the company can efficiently allocate resources and achieve its financial objectives.

Question 5

5. One of the decision variables has an optimal value of zero. Use the Solver sensitivity report to determine the smallest selling price for that item so that this optimal zero solution value changes to a non-zero value.

Product	X(Decision Variable)	New Selling Price	Z (dollars)
Pressure washer	434.0998152	280.07	142054.3826
Go-Kart	0	359.99	
Generator	56.48798521	290.99	
Water Pumps	28.24399261	142.99	

Matirx	X1	X2	X3	X4
Budget	330	370	410	127
Space	25	40	25	1.25
Inventory	-0.7	-0.7	0.3	0.3
Sales proportion	0	0	-1	2
Positivity constraint Washer	-1	0	0	0
Positivity Go-Kart constraint	0	-1	0	0
Positivity Gen constraint	0	0	-1	0
Water Pumps constraint	0	0	0	-1

Matirx	Ax	Inequality	Constraint RHS	Slack
Budget	170000	≤	170000	0
Space	12300	≤	12300	1.81899E-12
Inventory	-278.4502773	≤	0	278.4502773
Sales proportion	0	≤	0	0
Positivity constraint Washer	-434.0998152	≤	0	434.0998152
Positivity Go-Kart constraint	0	≤	0	0
Positivity Gen constraint	-56.48798521	≤	0	56.48798521
Water Pumps constraint	-28.24399261	≤	0	28.24399261

Figure 8 Model formulation for the smallest selling price of Pressure washers to figure in the inventory.

SUM PRODUCT CELLS FOR SOLVER				
Objective Function	142054.3826			
Constraint 1	170000	170000		
Constraint 2	12300	12300		
Constraint 3	-278.4502773	0		
Constraint 4	0	0		
X(Decision Variable)	434.0998152	0	56.48798521	28.244
constraints	280.07	359.99	290.99	142.99
SUM OF OBJECTIVE COEFFICIENT AND ALLOWABLE INCREASE				
NEW SELLING PRICE=	280.0615237	Choosing a value 0.01 above this new selling price in the model we secured a non-zero inventory entry for Pressure Washers		

Figure 9 Sum product cells for use in the solver model.

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [ALY6050_MOD5Project_FaizanS.xlsx]Question 5
Report Created: 6/18/2024 3:12:43 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
Q10	X(Decision Variable)	434.0998152	0	280.07	73.57390244	0.008476279
R10	X(Decision Variable)	0	-0.023711645	359.99	0.023711645	1E+30
S10	X(Decision Variable)	56.48798521	0	290.99	0.020297468	75.41325
T10	X(Decision Variable)	28.24399261	0	142.99	0.040594937	113.90325

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Q5	Constraint 1	170000	0.557584104	170000	32257.30193	7640
Q6	Constraint 2	12300	3.842689834	12300	578.7878788	1961.683509
Q7	Constraint 3	-278.4502773	0	0	1E+30	278.4502773
Q8	Constraint 4	0	33.68672828	0	138.280543	95.5

Figure 10 Sensitivity model of the new selling price model for the Pressure Washers.

In the optimal solution obtained from the linear programming model, the inventory level for pressure washers (X1) is zero. To determine the smallest selling price for pressure washers that would result in a non-zero inventory level, we analyzed the Solver sensitivity report and made necessary adjustments to the selling price.

Current Scenario

In the original optimal solution:

- **Pressure Washers (X1)**: 0 units
- **Total Profit (Z)**: $142,050.703

The reduced cost for pressure washers is -110.0715237, indicating that the current selling price is not sufficient to make stocking pressure washers profitable. The selling price must be increased to overcome this negative reduced cost.

Adjusted Selling Price Calculation

The new selling price (**P_{new}**) is determined by adding the absolute value of the reduced cost to the current profit margin (selling price - cost price).

Given:

- Current profit margin for pressure washers: $169.99
- Reduced cost: -$110.0715237

The new selling price required for pressure washers to be included in the optimal solution is calculated as:

$\mathbf{P}_{\mathbf{new}}$=169.99+110.0715237=280.0615237

To secure a non-zero inventory level for pressure washers, we set the new selling price slightly above this threshold, at $280.07.

New Selling Price and Impact

By setting the selling price of pressure washers to $280.07, we reassessed the linear programming model:

- **Pressure Washers (X1)**: Non-zero inventory level is achieved
- **New Selling Price for Pressure Washers**: $280.07

Summary of the solution to Question 5

Adjusting the selling price of pressure washers to $280.07 ensures that pressure washers become part of the optimal inventory solution, thus changing the decision variable X1 from zero to a non-zero value. This adjustment allows the company to include pressure washers in its inventory while still maximizing total profit, adhering to budget and spatial constraints. The recalibration of the selling price demonstrates the importance of sensitivity analysis in optimizing decision-making processes in inventory management.

Question 6

In the word document explain whether, in addition to the $170,000 allocated to the purchasing budget during the first month, the company should allocate additional money. If yes, how much additional investment do you recommend, and how much should the company expect its net monthly profit to increase as a consequence of this increase?

To determine whether the company should allocate additional money beyond the $170,000 budget and how much profit would increase as a result, we need to interpret the shadow price from the sensitivity report.

Interpretation of the Shadow Price

1. **Shadow Price**:
   - The shadow price (or dual value) for the budget constraint (Constraint 1) is $0.5576483441.
   - This means that for every additional dollar added to the budget, the company's profit will increase by approximately $0.56, provided other constraints remain unchanged.
2. **Allowable Increase**:
   - The allowable increase for the budget constraint is $428.8.
   - This means that the shadow price remains valid for an increase in the budget of up to $428.8. Beyond this point, the shadow price may change, and a new optimal solution would need to be recalculated.

Additional Investment Recommendation

Given the above interpretations, the company can consider increasing the budget by up to $428.8 for the first month.

Expected Increase in Net Monthly Profit

To calculate the expected increase in net monthly profit due to the additional budget:

Increase in Profit=Shadow Price×Additional Budget

Using the shadow price of $0.5576483441:

Increase in Profit=0.5576483441×428.8≈239.04

Therefore, by increasing the budget by $428.8, the company can expect the net monthly profit to increase by approximately $239.04.

Total Budget and Profit After Increase

- **Current Budget**: $170,000
- **Additional Budget**: $428.8
- **Total Budget**: $170,000 + $428.8 = $170,428.8
- **Current Profit**: $142,050.703
- **Increase in Profit**: $239.04

- **Total Profit**: $142,050.703 + $239.04 = $142,289.743

Conclusion

- **Additional Budget**: The company should consider allocating an additional $428.8 to the purchasing budget. However, the space constraint must also be borne in mind, as the space constraint of the warehouse size might limit profitability despite additional budgeting.
- **Total Budget**: This would increase the total budget to $170,428.8.
- **Increase in Profit**: This additional investment would result in an approximate increase in net monthly profit of $239.04.
- **Total Profit**: The total expected profit after the budget increase would be approximately $142,289.743.

Please note that this analysis is based on the current constraints and shadow prices. If constraints or costs change significantly, a new linear programming model would have to be solved to find the updated optimal solution and corresponding shadow prices. Also while allocating additional budget, we must also bear in mind that **the space constraint** limits the profitability despite budget allocation as a limited space is available.

Question 7

In the word document, explain whether you recommend that the company should rent a smaller or a larger warehouse. In any case, indicate the ideal size of your recommended warehouse in square feet, and indicate how much this change in the size of the warehouse will contribute to the monthly profit.

To determine whether the company should rent a smaller or larger warehouse and the ideal size, we need to analyze the shadow price for the space constraint from the sensitivity report.

Shadow Price Analysis

The shadow price for the space constraint (Constraint 2) is $3.841502841. This means that for every additional square foot of space, the company's profit will increase by approximately $3.84, provided other constraints remain unchanged.

Allowable Increase

The allowable increase for the space constraint is 6078.378378 square feet. This indicates that the shadow price remains valid for an increase in the warehouse space of up to 6078.378378 square feet. Beyond this point, the shadow price may change, and a new optimal solution would need to be recalculated.

Recommendation

Given the shadow price and the allowable increase, the company should consider renting a larger warehouse to maximize its profit.

Ideal Size of the Recommended Warehouse

1. **Current Warehouse Size**: 12300 square feet
2. **Recommended Additional Space**: 6078.378378 square feet
3. **Total Recommended Warehouse Size**: 12300 + 6078.378378 = 18378.378378 square feet

Expected Increase in Monthly Profit

To calculate the expected increase in net monthly profit due to the additional warehouse space:

Increase in Profit=Shadow Price×Additional Space

Using the shadow price of $3.841502841:

Increase in Profit=3.841502841×6078.378378≈23350.07

Therefore, by increasing the warehouse space by 6078.378378 square feet, the company can expect the net monthly profit to increase by approximately $23,350.07.

Conclusion

- **Warehouse Size**: The company should rent a larger warehouse.
- **Total Recommended Warehouse Size**: The ideal size of the recommended warehouse is 18378.378378 square feet.

- **Increase in Profit**: This additional space would result in an approximate increase in net monthly profit of $23,350.07.

This analysis is based on the current constraints and shadow prices. If constraints or costs change significantly, a new linear programming model should be solved to find the updated optimal solution and corresponding shadow prices.

Conclusion

I have summarized the conclusions of the analysis in the form of a table, presented below, for convenient viewing by the business stakeholders.

Question	Description	Key Solutions
1	Mathematical formulation of the problem	Objective function and constraints
2	Set up the linear programming formulation in Excel or R	Excel Solver setup
3	Use Excel Solver or R to solve the problem and generate a report	Optimal solution and sensitivity report
4	Describe the optimal solutions obtained	Inventory levels: X1=0, X2=155.18, X3=237.77, X4=118.88, Profit=$142,050.70
5	Determine the smallest selling price for a non-zero inventory level	New selling price for pressure washers: $280.07
6	Recommend additional investment	Increase budget by $428.8, profit increases by $239.04
7	Recommend whether to rent a smaller or larger warehouse	Increase warehouse size by 6078 sq ft, profit increases by $23,350.07

Figure 11 Summary Table.

Recommendations

Based on the comprehensive linear programming analysis and subsequent sensitivity reports, the following recommendations are made for the northern hardware company:

1. **Inventory Management**:
    - Stock go-karts, generators, and water pumps in the specified optimal quantities to maximize profit.
    - Avoid stocking pressure washers unless their selling price is increased to at least $280.07.
2. **Budget Allocation**:
    - Consider increasing the budget by $428.8, which is expected to increase the net monthly profit by approximately $239.04.
3. **Warehouse Space**:
    - Rent a larger warehouse with an additional 6078 square feet. This will likely result in an increase in net monthly profit by approximately $23,350.07.
4. **Profit Maximization**:

- Utilize the linear programming model and sensitivity analysis for continuous optimization of inventory levels and profit margins.

These recommendations are grounded in a detailed quantitative analysis, ensuring that the company can efficiently manage its resources and maximize profitability while adhering to operational constraints.

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