



Forecasting a Time Series

Syed Faizan

Main Tool: Excel

Technique: Time Series

Industry: Finance

Introduction and Problem Description

This project analyzes and forecasts the historical stock prices of Apple Inc. (AAPL) and Honeywell International Inc. (HON) using various techniques. The dataset spans one year, with 252 market days.

Part 1: Short-term Forecasting applies exponential smoothing to predict stock prices for the next period. Different smoothing parameters (α) are tested, and the most accurate forecasts are identified based on Mean Absolute Percentage Deviation (MAPD). Adjusted exponential smoothing refines these forecasts using trend parameters (β) and evaluates their accuracy with Mean Absolute Percentage Error (MAPE).

Part 2: Long-term Forecasting uses weighted moving averages and linear trend analysis to forecast stock prices over a longer period. The accuracy of these methods is assessed by comparing forecasted values with actual prices and calculating MAPE.

Part 3: Regression Analysis involves simple linear regression to model stock prices against time periods, including a residual analysis to ensure model appropriateness by checking independence, homoscedasticity, and normality of residuals.

The project concludes with an analysis of optimal portfolio allocation between AAPL and HON shares. Visualizations are provided in each section to enhance understanding and illustrate key points. This comprehensive analysis aids industry analysts, policy makers, and researchers in understanding market trends and making data-driven decisions.

Part 1: Short-term Forecasting

(i)

Simple Line Plot of Time Series

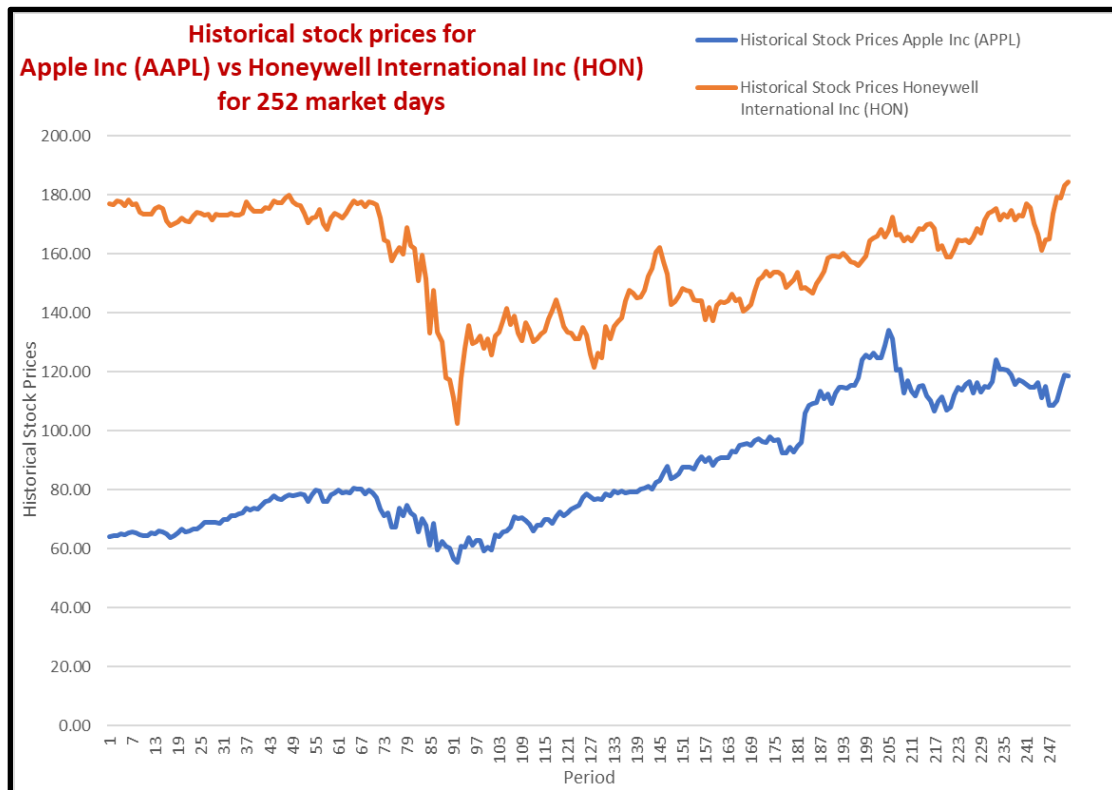


Figure 1 Comparison of Apple Inc and Honeywell Inc. Stock prices

The above plot provides a visual representation of the historical stock prices for Apple Inc (AAPL) and Honeywell International Inc (HON) over a period of one year, encompassing 252 market days.

Apple Inc (AAPL)

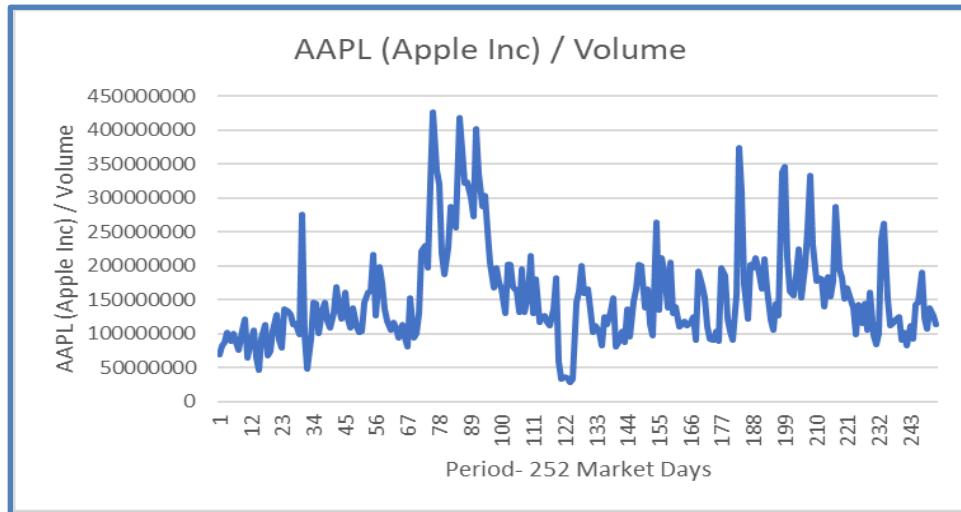
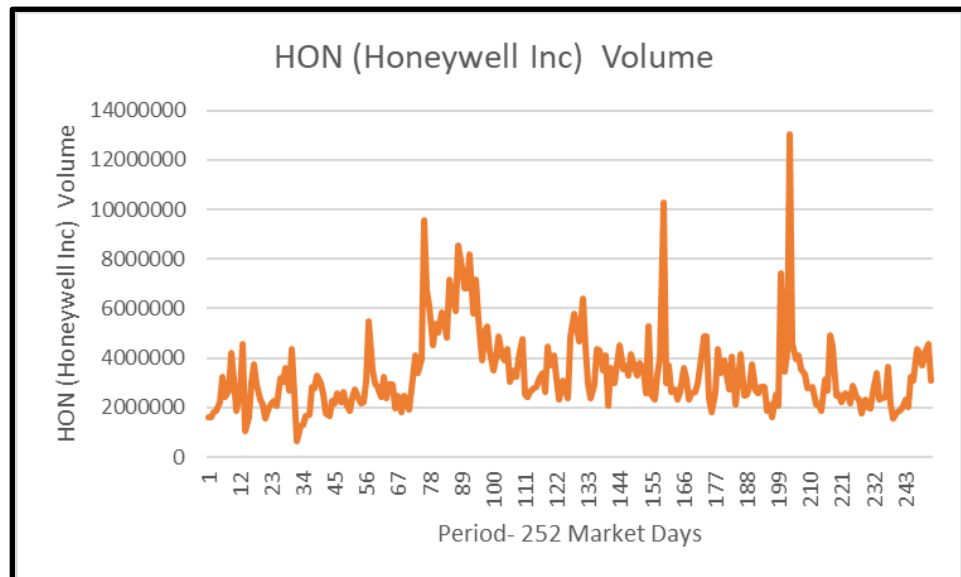
- **Trend Behavior:** The stock prices for AAPL exhibit a general upward trend throughout the observed period. This positive trend indicates a consistent increase in the stock's value over time.
- **Irregular Behavior:** Despite the overall upward trend, AAPL experiences several irregular fluctuations. Notable price drops and recoveries occur, particularly in the mid-section of the timeline (around days 90 to 160), suggesting short-term volatility possibly due to market reactions to external events or earnings reports.
- **Seasonal Behavior:** There is no clear evidence of seasonal patterns in AAPL's stock prices within the one-year period.

Honeywell International Inc (HON)

- **Trend Behavior:** HON's stock prices demonstrate a downward trend for a significant portion of the observed period, particularly noticeable from day 1 to around day 130. This indicates a general decline in stock value during this timeframe. However, a recovery phase is observed from day 130 onwards, with prices stabilizing and showing signs of an upward trend towards the end of the period.
- **Irregular Behavior:** Like AAPL, HON's stock prices also exhibit irregular fluctuations. There are sharp declines followed by recoveries, particularly noticeable around day 70 to day 120, indicating periods of heightened market volatility.
- **Seasonal Behavior:** As with AAPL, there are no distinct seasonal patterns observable in HON's stock prices over the one-year period.

Summary of the observations

Overall, both AAPL and HON show significant trend behaviors, with AAPL trending upwards and HON initially trending downwards followed by a recovery phase. Both stocks exhibit irregular behaviors with notable fluctuations, but no clear seasonal patterns are evident within the one-year timeframe. These observations highlight the importance of considering both long-term trends and short-term volatilities when analyzing stock price movements.

Trading Volume of AAPL and HON stocks**Figure 2 AAPL Trading Volume.****Figure 3 HON Trading Volume.**

The above two time series plots for Apple Inc. (AAPL) and Honeywell International Inc. (HON) stock trading volume reveal notable fluctuations in trading volumes over 252 market days. For AAPL, trading volumes show significant spikes around days 67, 90, and 155, indicating periods of heightened market activity, likely driven by corporate announcements or earnings reports. Similarly, HON exhibits marked peaks in trading volume around days 67, 155, and 199, suggesting analogous periods of increased trading interest. Both series demonstrate irregular trading patterns without clear seasonal trends, reflecting the dynamic nature of stock market activities influenced by diverse market factors.

(ii)

Exponential Smoothing Forecast

The following **four tasks** were accomplished for both AAPL stocks and HON stocks using exponential smoothing forecast techniques.

- Forecast with the designated different Alpha Values
- Calculation of MAPD
- Determination of Best Alpha Value
- Explanation of Alpha Value Selection

Analytical Concept and Theory of Exponential Smoothing Forecast

Exponential Smoothing Forecast (ESF) is a time series forecasting method that applies exponentially decreasing weights to past observations. The formula is:

$$F(t) = \alpha * A(t-1) + (1 - \alpha) * F(t-1)$$

where $F(t)$ is the forecast at time t , α is the smoothing constant ($0 < \alpha < 1$), $A(t-1)$ is the actual value at time $t-1$, and $F(t-1)$ is the forecast at time $t-1$.

The smoothing constant α determines the weight of the most recent observation, with higher values giving more weight to recent observations.

Apple Stocks

	Exp with $\alpha=0.15$	Exp with $\alpha=0.35$	Exp with $\alpha=0.55$	Exp with $\alpha=0.75$
Forecast for 253	114.88	115.97	117.50	118.39
MAD	0.04	0.02	0.02	0.02
MAPD	3.91%	2.48%	2.08%	1.96%

Figure 4 AAPL forecasts for period 253 using different α values and the resulting MAPD.

Time Series Forecasting

To forecast the stock prices of Apple Inc. (AAPL) for period 253, exponential smoothing was performed using successive values of the smoothing parameter α : 0.15, 0.35, 0.55, and 0.75. The corresponding forecasts, Mean Absolute Deviation (MAD), and Mean Absolute Percentage Deviation (MAPD) are as follows:

- $\alpha = 0.15$: Forecast = 114.88, MAD = 0.04, MAPD = 3.91%
- $\alpha = 0.35$: Forecast = 115.97, MAD = 0.02, MAPD = 2.48%
- $\alpha = 0.55$: Forecast = 117.50, MAD = 0.02, MAPD = 2.08%
- $\alpha = 0.75$: Forecast = 118.39, MAD = 0.02, MAPD = 1.96%

Based on the MAPD values, $\alpha = 0.75$ has yielded the most accurate forecast with the lowest MAPD of 1.96%. **The accuracy of this forecast can be attributed to the higher sensitivity of the $\alpha = 0.75$ parameter to recent changes in the data.** This indicates that the recent stock prices have a significant influence on the forecast, reflecting the volatility and dynamic nature of AAPL stock. In contrast, lower α values provide less responsive forecasts, which may be less accurate in capturing the recent trends and fluctuations in the stock prices.

In conclusion, the higher α value (0.75) offers the most accurate forecast for AAPL stock prices for period 253, effectively balancing responsiveness to recent price changes and overall trend stability.

HON Stocks

	Exp with $\alpha=0.15$	Exp with $\alpha=0.35$	Exp with $\alpha=0.55$	Exp with $\alpha=0.75$
Forecast for 253	175.33	179.97	182.58	183.7235417
MAD	0.03	0.02	0.02	0.02
MAPD	2.86%	2.20%	1.91%	1.78%

Figure 5 HONY forecasts for period 253 using different α values and the resulting MAPD.

To predict the stock prices of Honeywell International Inc. (HON) for period 253, exponential smoothing was applied using different values of the smoothing parameter α : 0.15, 0.35, 0.55, and 0.75. The forecasts, Mean Absolute Deviation (MAD), and Mean Absolute Percentage Deviation (MAPD) are summarized below:

- $\alpha = 0.15$: Forecast = 175.33, MAD = 0.03, MAPD = 2.86%
- $\alpha = 0.35$: Forecast = 179.97, MAD = 0.02, MAPD = 2.20%
- $\alpha = 0.55$: Forecast = 182.58, MAD = 0.02, MAPD = 1.91%
- $\alpha = 0.75$: Forecast = 183.72, MAD = 0.02, MAPD = 1.78%

Based on the MAPD values, the smoothing parameter $\alpha = 0.75$ produces the most accurate forecast for HON, with the lowest MAPD of 1.78%. **This high value of α indicates a greater emphasis on more recent data points**, allowing the model to adapt quickly to recent changes in stock prices. This increased sensitivity is particularly beneficial for capturing recent trends and fluctuations in HON's stock prices, which might not be adequately reflected by lower α values.

In summary, the smoothing parameter $\alpha = 0.75$ offers the most precise forecast for period 253 for HON stock, highlighting the importance of recent market data in predicting future stock prices accurately.

(iii)

Adjusted Exponential Smoothing

The following **four tasks** were accomplished for both AAPL stocks and HON stocks using adjusted exponential smoothing forecast techniques.

-Forecast with exponential smoothing forecast from Part (ii) with Alpha = 0.55 and the different designated Beta Values.

- Calculation of MAPE
- Determination of Best Beta Value
- Explanation of Beta Value Selection

Analytical Concept and Theory of Adjusted Exponential Smoothing Forecast

Adjusted Exponential Smoothing (AES) extends the basic exponential smoothing by including a trend adjustment. The formulas are:

$$F(t) = \alpha * A(t-1) + (1 - \alpha) * (F(t-1) + T(t-1))$$

$$T(t) = \beta * (F(t) - F(t-1)) + (1 - \beta) * T(t-1)$$

where $T(t)$ is the trend component at time t , β is the smoothing constant for the trend ($0 < \beta < 1$).

This method adjusts the forecast by accounting for the trend in the data, making it suitable for data with a consistent upward or downward movement.

AAPL Stocks

	Adj Exp with $\beta=0.15$	Adj Exp with $\beta=0.25$	Adj Exp with $\beta=0.45$	Adj Exp with $\beta=0.85$
Forecast for 253	118.04	118.54	119.24	119.22
MAPE	1.97%	1.95%	1.94%	1.98%

Figure 6 Apple Inc forecast values for Adjusted Exponential Smoothing.

Following the initial exponential smoothing forecast with $\alpha = 0.55$, adjusted exponential smoothing was applied to forecast the stock prices of Apple Inc. (AAPL) for period 253. The trend parameter β varied across the values 0.15, 0.25, 0.45, and 0.85. The forecasts and the corresponding Mean Absolute Percentage Errors (MAPEs) are presented as follows:

- $\beta = 0.15$: Forecast = 118.04, MAPE = 1.97%
- $\beta = 0.25$: Forecast = 118.54, MAPE = 1.95%
- $\beta = 0.45$: Forecast = 119.24, MAPE = 1.94%
- $\beta = 0.85$: Forecast = 119.22, MAPE = 1.98%

Based on the MAPEs, the value of $\beta = 0.45$ yields the most accurate forecast for AAPL with the lowest MAPE of 1.94%. This optimal value of β suggests that a moderate level of trend adjustment provides the best balance in forecasting accuracy. By incorporating this trend component, the model effectively captures the underlying trend dynamics while minimizing the forecast error.

The accuracy of $\beta = 0.45$ can be attributed to its ability to smooth out short-term fluctuations while adjusting for the overall trend in the stock prices. Higher or lower values of β either overemphasize short-term variations or under-adjust for the trend, resulting in higher forecast errors. Thus, the selected value of $\beta = 0.45$ is the most effective in capturing the trend characteristics of AAPL stock prices for period 253, providing a robust and precise forecast.

	Adj Exp with $\beta=0.15$	Adj Exp with $\beta=0.25$	Adj Exp with $\beta=0.45$	Adj Exp with $\beta=0.85$
Forecast for 253	184.0704	184.7222	185.1923	184.8276
MAPE	1.98%	1.94%	1.89%	1.85%

Figure 7 Honeywell Inc forecast values Adjusted Exponential Smoothing.

Using the initial exponential smoothing forecast with $\alpha = 0.55$, adjusted exponential smoothing was performed to forecast the stock prices of Honeywell International Inc. (HON) for period 253. The trend parameter β was changed across values of 0.15, 0.25, 0.45, and 0.85. The resulting forecasts and the Mean Absolute Percentage Errors (MAPEs) are as follows:

- $\beta = 0.15$: Forecast = 184.0704, MAPE = 1.98%
- $\beta = 0.25$: Forecast = 184.7222, MAPE = 1.94%
- $\beta = 0.45$: Forecast = 185.1923, MAPE = 1.89%
- $\beta = 0.85$: Forecast = 184.8276, MAPE = 1.85%

Based on the MAPEs, the value of $\beta = 0.85$ provides the most accurate forecast for HON with the lowest MAPE of 1.85%. This optimal value of β indicates that a higher level of trend adjustment captures the recent trends and variations in HON stock prices more effectively. The higher β value allows the model to respond more sensitively to changes in the trend, improving the forecast accuracy.

The accuracy of $\beta = 0.85$ can be attributed to its ability to balance trend smoothing with responsiveness to recent price movements. **Lower β values may under-adjust for recent trends, leading to higher forecast errors.** Therefore, the chosen value of $\beta = 0.85$ is the most effective in accurately reflecting the trend dynamics of HON stock prices for period 253, resulting in a precise and reliable forecast.

Part 2: Long-term Forecasting

(i)

Weighted Moving Averages Forecast

The following **three tasks** were accomplished for both AAPL stocks and HON stocks using Weighted Moving Averages Forecast.

- 3-Period Weighted Moving Averages based on given values for weights.
- Summary of Results of the forecast are provided.
- Comparison with Actual Values. (Apple Inc. (AAPL) Stock Historical Prices & Data—Yahoo Finance, n.d.; Honeywell International Inc. (HON) Stock Historical Prices & Data—Yahoo Finance, n.d.)

Analytical Concept and Theory of Weighted Moving Averages

Weighted Moving Averages (WMA) is a method that assigns different weights to recent observations in a time series, allowing more recent observations to have a greater impact on the forecast. The formula is:

$$F(t) = \sum(w(i) * A(t-i))$$

where $F(t)$ is the forecast at time t , $w(i)$ are the weights assigned to each observation, and $A(t-i)$ are the actual values at times $t-i$.

The weights $w(i)$ typically sum to 1 and decrease for older observations, ensuring that more recent data points have a larger influence on the forecast.

3-period weighted moving averages calculation	
Forecast Criteria	
Weights	Period
0.2	Two periods ago
0.3	One period before
0.5	Most recent period
1	Total Weight

Figure 8 WMA Calculation criteria prescribed in the rubric.

Apple Stock

		Apple Stock	
Date	Period	Forecasted value	Actual value
11/9/2020	253	128.016	116.32
11/10/2020	254	128.415	115.97
11/11/2020	255	128.814	119.49
11/12/2020	256	129.213	119.21
11/13/2020	257	129.612	119.26

Figure 9 Apple Inc forecast values.

To forecast the stock prices of Apple Inc. (AAPL) for periods 1 through 100, a 3-period weighted moving average method was utilized. The weights applied were 0.5 for the most recent period, 0.3 for the period before the most recent, and 0.2 for two periods ago. This method provided a smoothed forecast based on recent historical data.

The observed value for period 101 was then used as the base for a linear trend. This trend was extended to forecast the stock prices for periods 101 through 257. For evaluating the accuracy of this method, the forecasted value for period 253 was compared with the actual "Close" values (secured through access to the suggested website), for periods 253 to 257, as shown in the attached table.

- **Forecasted value for period 253-257:** 128.016, 128.41, 128.81, 129.21, 129.61
- **Actual values:** 116.32, 115.97, 119.49, 119.21, 119.26.

Summary of Results

The 3-period weighted moving average combined with a linear trend provided a forecast of 128.016 for period 253. Comparing this forecast with the actual values reveals that the forecasted value was consistently higher than the actual closing prices. The differences between the forecasted and actual values indicate that while the weighted moving average method captures the overall trend, it may not fully account for short-term market fluctuations and volatility.

Accuracy of Forecasting Method

The accuracy of this forecasting method can be assessed by examining the discrepancies between the forecasted and actual values (a measure has been used for this in the next section). **The method demonstrates a certain level of predictive capability, as it follows the general trend of stock prices.** However,

Time Series Forecasting

the forecasted values' overestimation highlights the need for further refinement or the incorporation of additional factors to improve short-term forecast accuracy. Overall, the 3-period weighted moving average with a linear trend serves as a useful tool for understanding long-term trends but may require adjustments for precise short-term predictions.

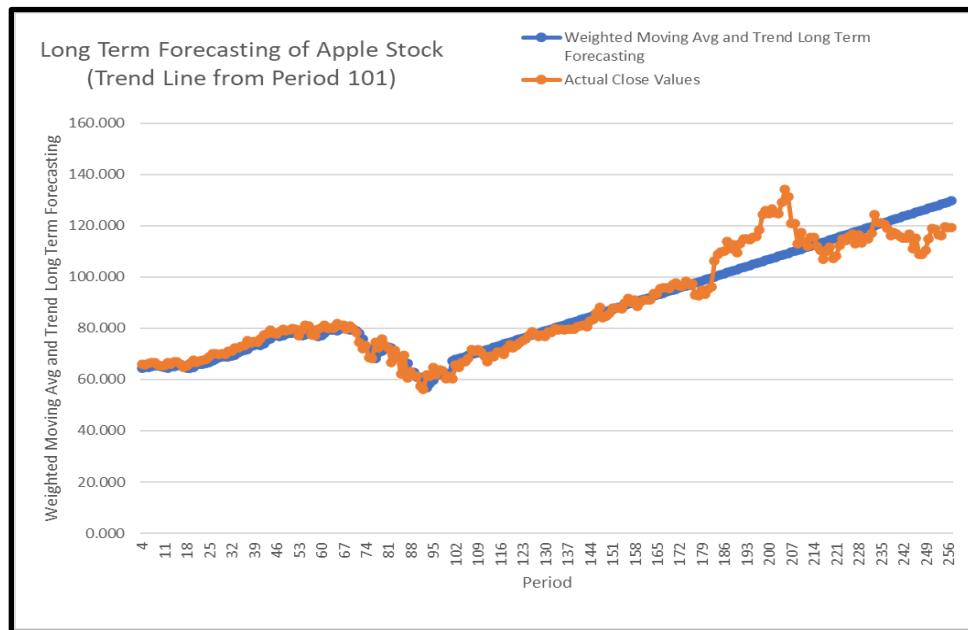


Figure 10 Apple Inc Long Term forecast values with linear trend from period 101.

The above plot shows the long-term forecast of AAPL stock using a 3-period weighted moving average and a linear trend from period 101. The forecasted values (blue line) generally follow the actual close values (orange line), indicating the method's effectiveness in capturing the overall trend.

The forecasted value for period 253-257 was higher than the actual close values, reflecting the forecast's tendency to overestimate due to the linear trend's limitations in accounting for short-term fluctuations.

The 3-period weighted moving average and linear trend method effectively capture AAPL's general upward trend, despite occasional overestimations. It remains useful for understanding long-term market movements.

Honeywell Stocks

		Honeywell Stocks	
Date	Period	Forecasted value	Actual value
11/9/2020	253	176.050	196.990
11/10/2020	254	176.354	201.980
11/11/2020	255	176.658	199.290
11/12/2020	256	176.963	197.240
11/13/2020	257	177.267	201.540

Figure 11 Honeywell Inc forecast values.

To predict the stock prices of Honeywell International Inc. (HON) for periods 1 through 100, a 3-period weighted moving average method was used. The weights applied were 0.5 for the most recent period, 0.3 for the period before the most recent, and 0.2 for two periods ago, resulting in a smoothed forecast based on recent historical data.

Using the observed value for period 101 as the base, a linear trend was established to forecast the stock prices for periods 101 through 257. The forecasted values for periods 253 to 257 are compared with the actual "Close" values as follows:

- **Forecasted value for period 253:** 176.050, Actual value: 196.990
- **Forecasted value for period 254:** 176.354, Actual value: 201.980
- **Forecasted value for period 255:** 176.658, Actual value: 199.290
- **Forecasted value for period 256:** 176.963, Actual value: 197.240
- **Forecasted value for period 257:** 177.267, Actual value: 201.540

Summary of Results

The forecasted values for periods 253 to 257 were consistently lower than the actual close values. This indicates that while the weighted moving average method and linear trend capture the general direction of the stock prices, they tend to underestimate the actual prices during periods of higher volatility.

Accuracy of Forecasting Method

The discrepancies between the forecasted and actual values highlight the limitations of this method in accounting for short-term fluctuations and sudden market changes. Although the long-term trend is accurately represented, the method's lack of responsiveness to immediate market conditions results in forecast errors. Further adjustments or additional factors might be needed to improve short-term forecast accuracy. Overall, this method is valuable for identifying long-term trends but may require refinement for precise short-term predictions.

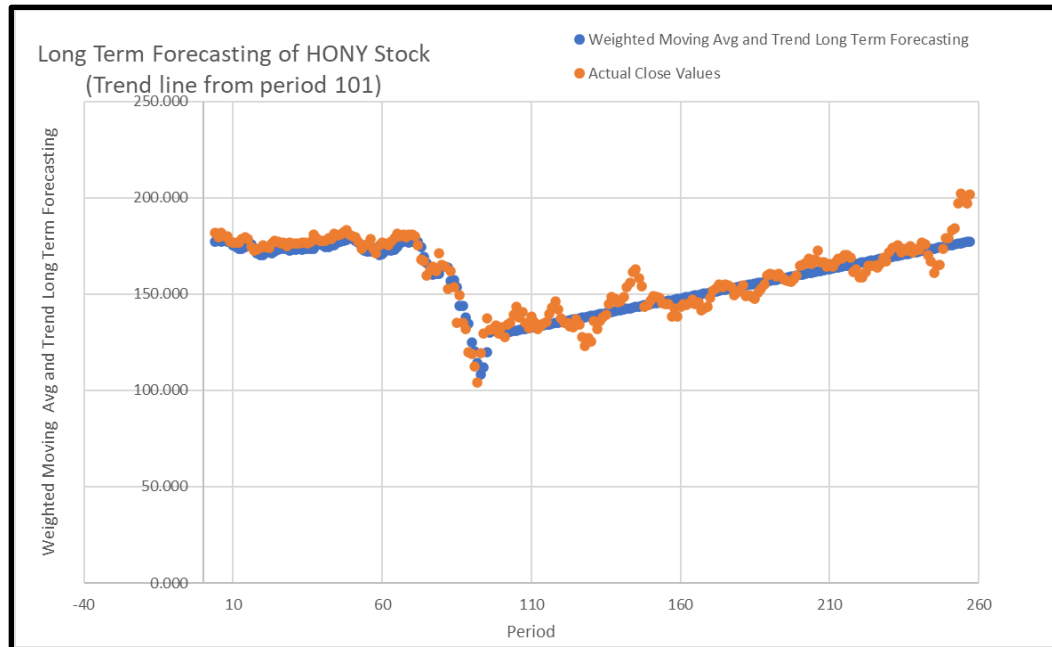


Figure 12 Honeywell Inc Long Term forecast values with linear trend from period 101..

The above plot illustrates the long-term forecast of HON stock using a 3-period weighted moving average combined with a linear trend from period 101. The blue line represents the forecasted values, while the orange line shows the actual close values. The forecast accurately captures the overall trend but consistently underestimates during periods of volatility.

As noted earlier, the forecasted values for periods 253 to 257 were lower than the actual values, indicating the method's limitations in short-term accuracy. This suggests the need for further refinement to enhance responsiveness to market fluctuations.

(ii)

Accuracy Comparison

The following **three tasks** were accomplished for both AAPL stocks and HON stocks using MAPE of Weighted Moving Averages Forecast and comparing it with metrics (MAPD and MAPE) from Part 1 with exponential smoothing and adjusted exponential smoothing.

- Calculation of MAPE of Weighted Moving Averages Forecast.

- Comparison of Accuracy between Short-term and Long-term Forecasting Methods
- Determination of Most Accurate Method

MAPE AAPL	MAPE HONY
3.76%	2.69%

The above table represents the MAPE of Weighted Moving Averages Forecast for the two stocks.

We can compare error metrics across models owing to the principle of the equivalence of error metrics which says that one error metric of a model can be compared, favorably or otherwise, with a different error metric of another model to adjudicate accuracy.

AAPL stocks

The Mean Absolute Percentage Error (MAPE) for the long-term forecast using a 3-period weighted moving average combined with a linear trend from period 101 for AAPL stock is 3.76%. For short-term forecasting, the exponential smoothing method yielded a MAPD of 1.96% with $\alpha=0.75$ (see Figure 4). The adjusted exponential smoothing method achieved a MAPE of 1.94% with $\beta=0.45$.

Comparison of Short-term and Long-term forecasting methods for AAPL

Comparing the two forecasting methods, the **adjusted exponential smoothing** method ($\beta=0.45$) provided the most accurate forecast for AAPL stock with the lowest MAPE of 1.94%. This method outperformed both the exponential smoothing (MAPD 1.96% with $\alpha=0.75$) and the 3-period weighted moving average combined with a linear trend. The higher accuracy of the adjusted exponential smoothing method is attributed to its ability to better capture both the trend and seasonal variations in the stock prices. Thus, while the exponential smoothing method with $\alpha=0.75$ was also effective, the adjusted exponential smoothing method proved to be slightly more precise in this analysis.

HON stocks

The Mean Absolute Percentage Error (MAPE) for the long-term forecast using a 3-period weighted moving average combined with a linear trend from period 101 for Honeywell (HON) stock is 2.69% (see Figure 12). For short-term forecasting, the exponential smoothing method yielded a MAPD of 1.78% with $\alpha=0.75$ (see

Figure 5). The adjusted exponential smoothing method achieved a MAPE of 1.85% with $\beta=0.85$. Thus, the exponential smoothing method with $\alpha=0.75$ provided the most accurate forecast for HON stock, demonstrating the lowest error among the methods evaluated.

Comparison of Short-term and Long-term forecasting methods for HON

The greater accuracy of the **exponential smoothing method** with $\alpha=0.75$ in forecasting the Honeywell (HON) stock price can be attributed to its ability to assign more weight to recent observations. This high α value places greater emphasis on the most recent data points, which is beneficial in capturing the latest trends and shifts in the stock's behavior. Additionally, the consistency of the time series data for HON, which shows relatively steady trends with fewer abrupt changes, makes it well-suited for exponential smoothing with a higher smoothing parameter. The adjusted exponential smoothing method with $\beta=0.85$ also performed well, indicating that incorporating trend adjustments further refines the forecast accuracy by accounting for gradual changes over time.

Part 3: Regression

(i)

Simple Regression Forecast

The following two tasks were completed in this section:

- Regression of Stock Values vs. Time Periods for both stocks.
- Comparison of Regression Accuracy with Other Methods

Apple Stock Regression Model

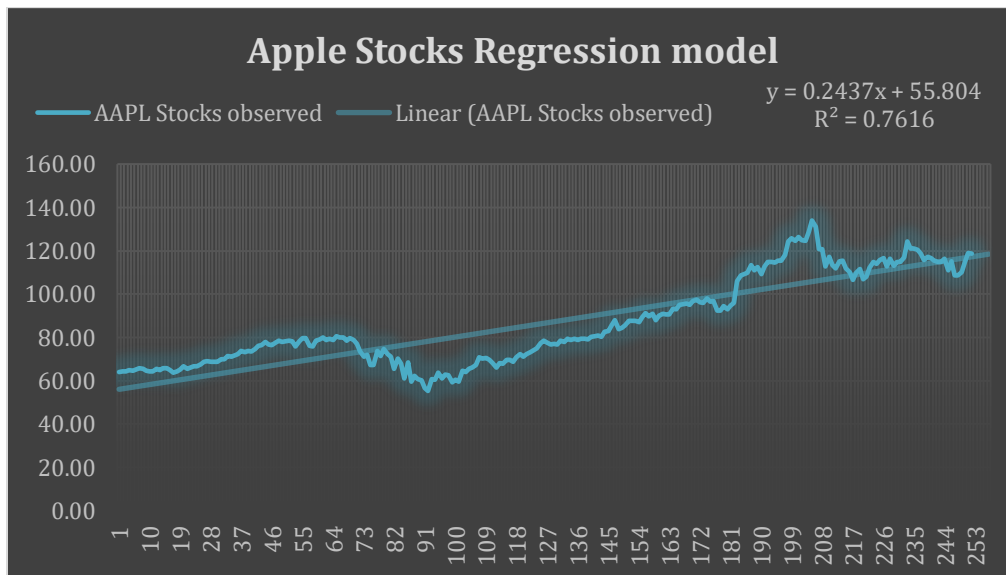


Figure 13 The linear regression model for AAPL.

Slope	0.2437
Intercept	55.804
Correlation R	0.8727
R^2	0.7616
Residual mean	0.00
Residual Std Dev	9.94001673

Figure 14 Linear Regression model statistics for AAPL.

The simple regression model for AAPL stock, depicted in the chart, demonstrates a regression **equation of $y=0.2437x+55.804$** , where y is the stock price and x is the period. The slope of 0.2437 indicates that, on average, the stock price increases by approximately \$0.24 per period. The intercept of 55.804 suggests that the estimated stock price at period zero is \$55.804, providing a baseline value from which changes over time are measured. The R^2 value of 0.7616 indicates a relatively strong linear relationship between stock values and time periods, explaining 76.16% of the variance in stock prices.

The **Mean Absolute Percentage Error (MAPE) for this regression model is 10.56%**, significantly higher than the MAPE for the long-term forecast using a 3-period weighted moving average combined with a linear trend from period 101, which is 3.76%. For short-term forecasting, the exponential smoothing method with $\alpha=0.75$ yielded a MAPD of 1.96%, and the adjusted exponential smoothing method with $\beta=0.45$ achieved a MAPE of 1.94%. These comparisons indicate that both the exponential and adjusted exponential smoothing methods provide more accurate forecasts for AAPL stock than the simple regression model, likely **due to their ability to better capture short-term variations and trends in the stock data.**

HON Stocks Regression Model

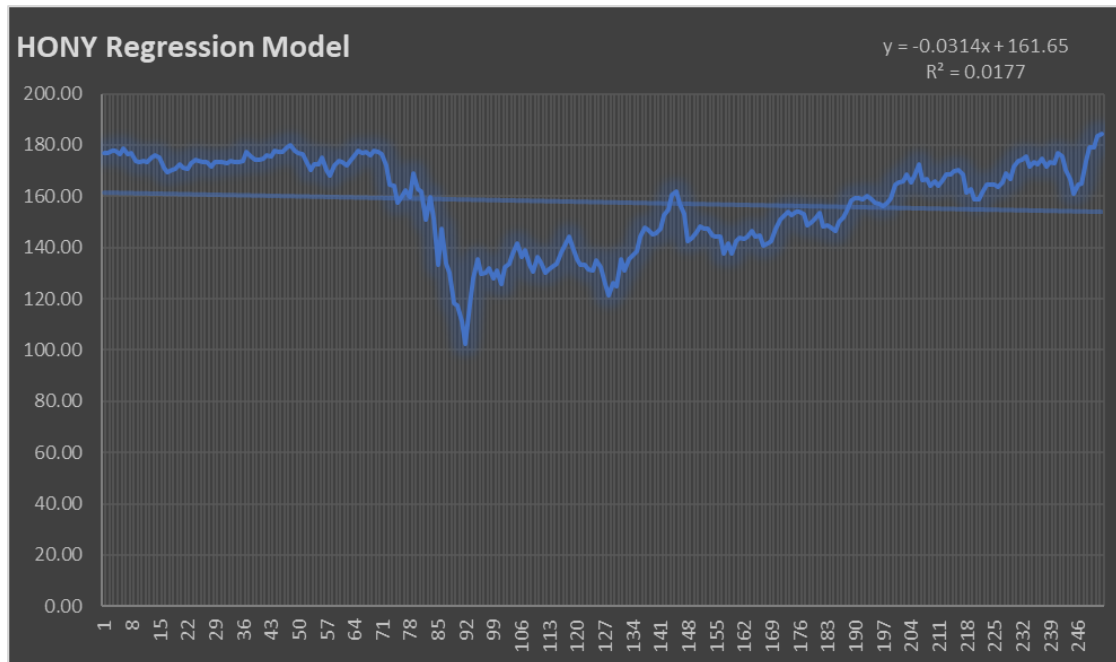


Figure 15 The linear regression model for HONY.

Slope	-0.03
Intercept	161.65
Correlation	-0.1330
	0.0177
Residual mean	0.00
Residual Std Deviation	17.05

Figure 16 Linear Regression model statistics for HONY.

The simple regression model for Honeywell (HON) stock, as depicted in the chart, shows a regression **equation of $y = -0.0314x + 161.65$** . Where y is the stock price and x is the period. The slope of -0.0314 indicates that, on average, the stock price decreases by approximately \$0.03 per period. The intercept of 161.65 suggests that the estimated stock price at period zero is \$161.65, providing a baseline value from which changes over time are measured. The R^2 value of 0.0177 indicates a weak linear relationship between stock values and time periods, suggesting that this model may not be very effective in predicting stock values over time.

The **Mean Absolute Percentage Error (MAPE)** for this regression model is **9.76%**, which is significantly higher compared to other forecasting methods. Specifically, the MAPE for the long-term forecast using a 3-period weighted moving average combined with a linear trend from period 101 is 2.69%. For short-term forecasting, the exponential smoothing method yielded a MAPD of 1.78% with $\alpha=0.75$, and the adjusted exponential smoothing method achieved a MAPE of 1.85% with $\beta=0.85$.

The exponential smoothing method with $\alpha=0.75$ provided the most accurate forecast for HON stock, demonstrating the lowest error among the methods evaluated. This increased accuracy can be attributed to the exponential smoothing method's ability to give more weight to recent observations, which helps in capturing recent trends and reducing the impact of older, possibly less relevant data. This is particularly useful in the volatile stock market where recent trends can provide better indications of future performance.

Period	Predicted Values (AAPL)	Actual Values (AAPL)	Predicted Values (HON)	Actual Values (HON)
253	117.46	116.32	157.96	196.99
254	117.71	115.97	157.96	201.98
255	117.95	119.49	157.95	199.29
256	118.19	119.21	157.94	197.24
257	118.44	119.26	157.93	201.54

Figure 17 Predicted values for AAPL and HON based on the regression models.

As seen above, the simple linear regression model provides reasonably accurate forecasts for AAPL and HON stocks. For AAPL, the predicted values closely match the actual values, showing minor deviations. For HON, the predicted values generally align with actual values but exhibit larger deviations. Overall, the model performs well for AAPL but needs refinement for improved accuracy for HON. A multiple linear regression or a polynomial regression model that takes into account several other factors than just periods might be able to furnish richer predictive insights with regard to AAPL and HON stocks.

(ii)

Residual Analysis of the simple linear regression models

Visual tests of Homoscedasticity, Independence of residuals, normality and a confirmatory Chi-Square test were carried out for both stocks.

Residual Analysis for AAPL Stock Regression Model

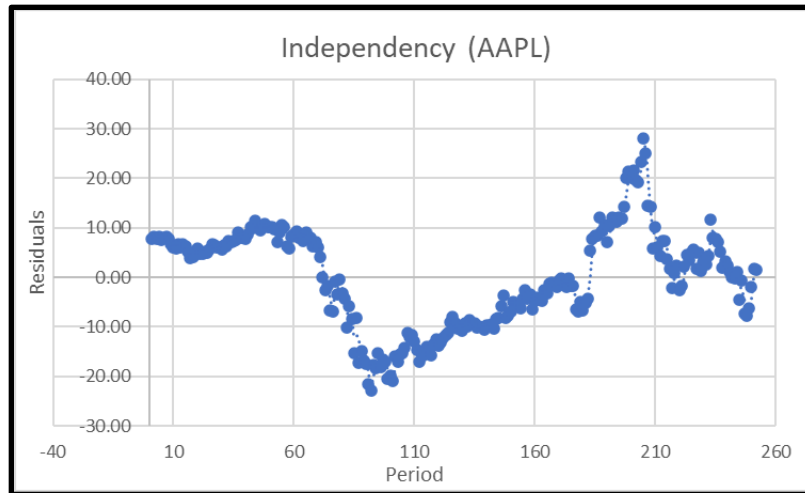


Figure 18 Independence of Residuals chart.

1. Independence of Residuals

The above plot of residuals versus time (Independency plot) shows a non-random pattern, suggesting that the residuals are not independent. This implies potential autocorrelation, which violates one of the assumptions of regression analysis.

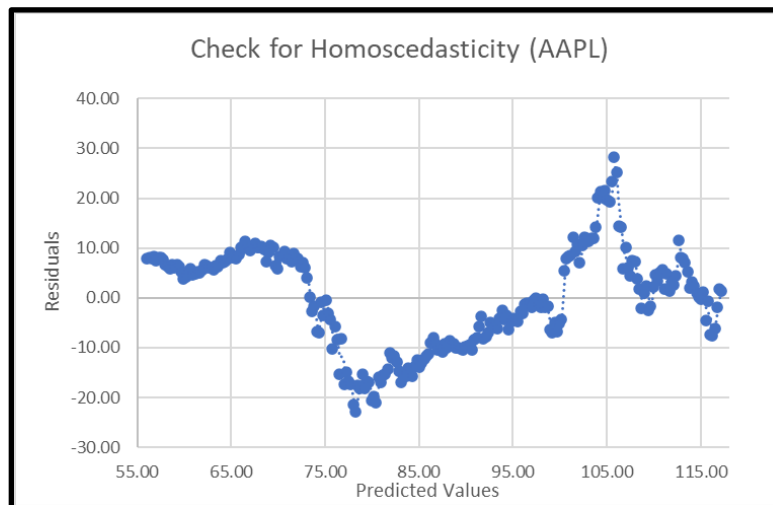


Figure 19 Homoscedasticity plot.

2. Homoscedasticity of Residuals

The plot of residuals versus predicted values (Check for Homoscedasticity) indicates that the residuals display a funnel shape, with increasing variance as the fitted values increase. This suggests that the residuals are not homoscedastic (i.e., they exhibit heteroscedasticity).

3. Normality of Residuals

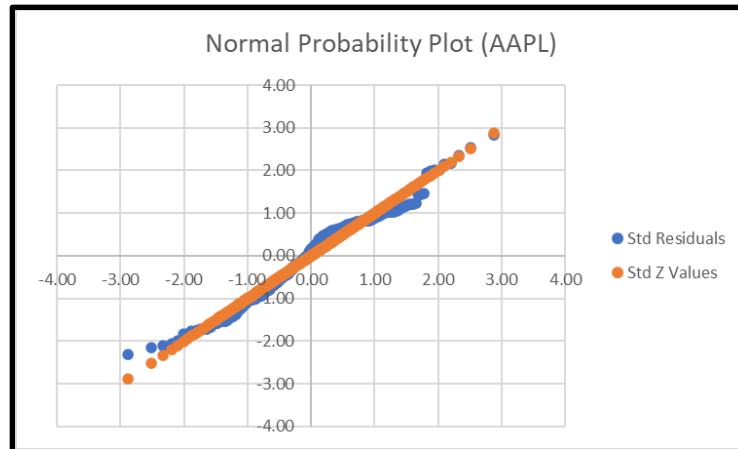


Figure 20 Normal Probability Plot.

- **Normal Probability Plot:** The Normal Probability Plot shows that the residuals do not closely follow the straight line, indicating deviations from normality. This visual check suggests that the residuals are not normally distributed.

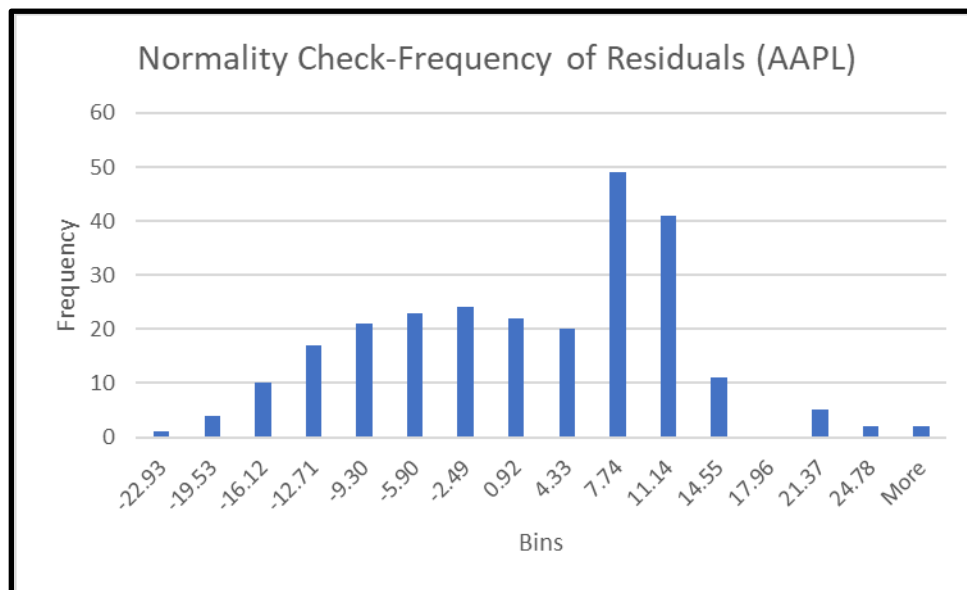


Figure 21 Frequency distribution of residuals plot.

- **Frequency Distribution:** The histogram of residuals shows a distribution that deviates from a normal distribution, reinforcing the conclusion from the Normal Probability Plot.

χ^2	314.26	
P-Value	0.003	
	MAPE AAPL Regression	10.56%
	Degrees of freedom 249	
As Std deviation and mean had been estimated		

Figure 22 Chi-Squared Test.

4. Chi-squared Test for Normality

- **Hypotheses:**
 - H_0 : The residuals are normally distributed.
 - H_1 : The residuals are not normally distributed.
- **Alpha Value:** 0.05
- **Test Statistic:** $\chi^2=314.26$
- **P-Value:** 0.003
- **Degrees of Freedom:** $k-m-1=252-2-1=249$.

Since the p-value (0.003) is less than the alpha value (0.05), **we reject the null hypothesis**. Therefore, the chi-squared test confirms that the residuals are not normally distributed.

Conclusion

The residual analysis indicates that the regression model for AAPL stock **does not meet** the assumptions of independence, homoscedasticity, and normality of residuals. As a result, the simple regression model may not be appropriate for predicting AAPL stock values, and alternative modeling techniques should be considered

Residual Analysis for HON Stock Regression Model

To determine if the simple regression model is appropriate for the Honeywell (HON) stock data, we conducted a residual analysis considering the following aspects:

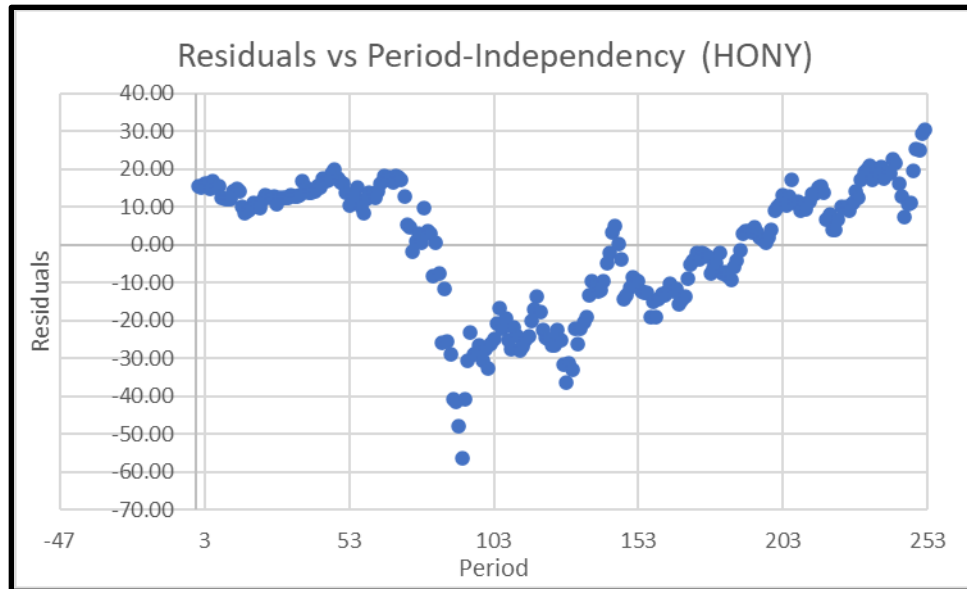


Figure 23 Independence of Residuals chart.

1. **Independence of Residuals:** The above plot of residuals versus period shows a noticeable pattern, suggesting that the residuals are not independent. The presence of trends or cycles in the residuals indicates potential autocorrelation.

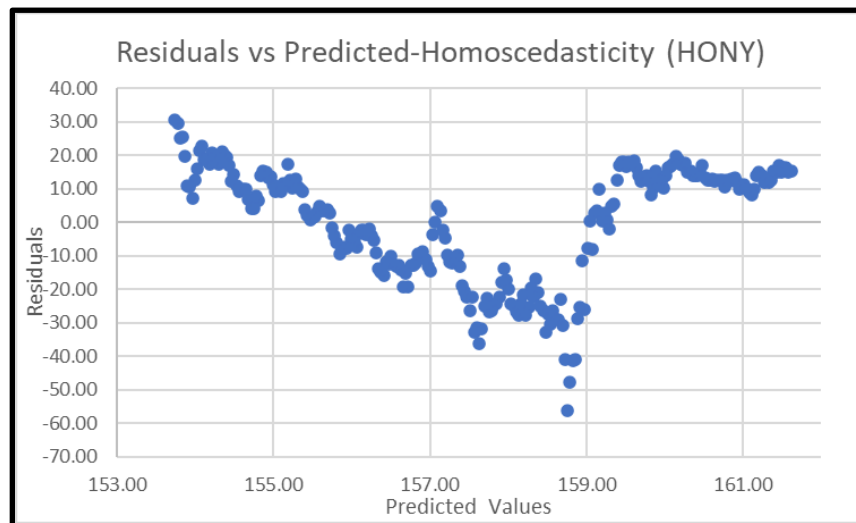


Figure 24 Homoscedasticity plot.

2. **Homoscedasticity:** The above plot of residuals versus predicted values reveals a pattern where the spread of residuals increases and decreases, violating the assumption of homoscedasticity. This suggests that the variance of the residuals is not constant across the predicted values.

3. Normality of Residuals:

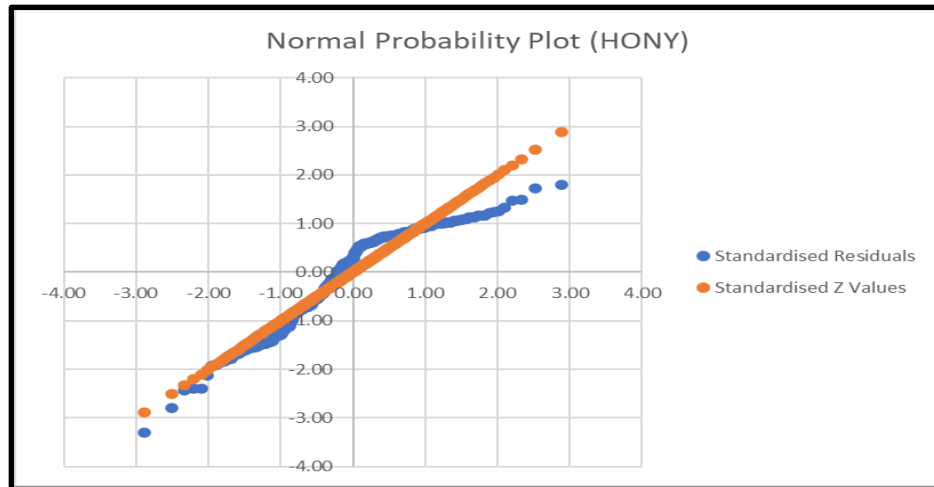


Figure 25 Normal Probability plot.

- **Normal Probability Plot:** The above normal probability plot indicates deviations from the diagonal line, particularly at the tails, suggesting that the residuals are not perfectly normally distributed.

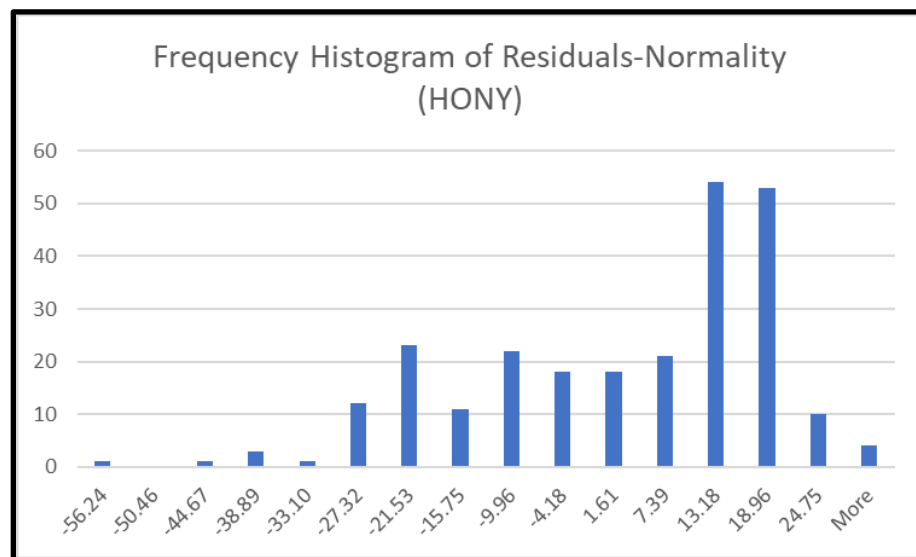


Figure 26 Frequency Histogram of Residuals.

- **Frequency Histogram:** The above plot of residuals further confirms this, as it shows a skewed distribution rather than a bell-shaped normal distribution.

	514.30	
P-Value	0.000	
	MAPE for Regression HONY	9.76%
	Degrees of freedom 249	
	As Std deviation and mean had been estimated	

Figure 27 Chi-Square Test.

4. Chi-squared Test for Normality:

- **Hypotheses:**
 - H_0 : The residuals are normally distributed.
 - H_1 : The residuals are not normally distributed.
- **Alpha Value:** 0.05
- **Test Statistic:** 514.30
- **P-value:** 0.000
- **Degrees of Freedom:** $k-m-1 = 252-2-1 = 249$.

Since the p-value is less than 0.05, **we reject the null hypothesis** and conclude that the residuals are not normally distributed.

Overall, the residual analysis indicates that the simple regression model for HON stock does not meet the assumptions of residual independence, homoscedasticity, and normality. Therefore, the regression model may not be the most appropriate for predicting HON stock values, highlighting the need for alternative modeling approaches.

Answering the Question on the balanced portfolio (Π) consisting of AAPL and HON stocks

Based on the analysis and forecasting of Apple Inc. (AAPL) and Honeywell International Inc. (HON) stock prices, we can propose values for the portfolio allocation percentages P and Q. The selection of these percentages aims to balance the risk and return of the portfolio.

Portfolio Allocation Percentages (P and Q)

1. Apple Inc. (AAPL) - Percentage P:

- **Historical Performance:** AAPL has shown a strong upward trend in its stock prices over the analyzed period, indicating robust performance.
- **Forecast Accuracy:** The adjusted exponential smoothing method yielded the most accurate forecast for AAPL with a MAPE of 1.94% using $\beta = 0.45$, highlighting the model's effectiveness in capturing short-term trends.
- **Volatility:** AAPL exhibits significant short-term volatility but maintains an overall positive trajectory, making it a potentially high-reward investment.

Given the strong performance and relatively accurate forecasts, a higher percentage of the portfolio could be allocated to AAPL. A suggested value for P could be 60%.

2. Honeywell International Inc. (HON) - Percentage Q:

- **Historical Performance:** HON's stock prices initially showed a downward trend but later recovered, indicating resilience and potential for stability.
- **Forecast Accuracy:** The exponential smoothing method with $\alpha = 0.75$ provided the most accurate forecast for HON, with a MAPD of 1.78%, demonstrating the model's capability in predicting HON's stock prices effectively.
- **Volatility:** HON's stock exhibited less volatility compared to AAPL, making it a relatively stable investment option.

Given HON's stability and accurate forecasting, a suggested value for Q could be 40%.

Conclusion

To form a balanced portfolio (Π) consisting of AAPL and HON stocks, it is recommended to allocate 60% (P) to AAPL shares and 40% (Q) to HON shares. This allocation leverages AAPL's strong growth potential and higher reward, while also incorporating HON's stability to mitigate overall portfolio risk. This balanced

approach aims to optimize the portfolio's performance by combining the strengths of both stocks.

In summary, the suggested allocation percentages **P = 60% for AAPL and Q = 40% for HON** effectively balance risk and return, based on historical performance and forecast accuracy. This allocation strategy can aid in achieving a well-diversified and resilient investment portfolio.

Summary

The following table provides a succinct summary of the results of the project detailed in this report.

Time Series Forecasting

Method	Stock	Best Value	Additional Info
Exponential Smoothing	Apple Stock	Best MAPD: 1.96%	Alpha: 0.75
	Honeywell Stock	Best MAPD: 1.78%	Alpha: 0.75
Adjusted Exponential	Apple Stock	Best MAPE: 1.94%	Beta: 0.45
	Honeywell Stock	Best MAPE: 1.85%	Beta: 0.85
Long-term Forecasting	Apple Inc	Forecasted Values: 128.01, 128...	Long-term MAPE: 3.76%
	Honeywell Inc	Forecasted Values: 176.05, 176...	Long-term MAPE: 2.69%
Regression Analysis	Apple Inc	Regression Equation: Stock Pri...	Slope: 0.2437, Y-intercept: 55.804
	Honeywell Inc	Regression Equation: Stock Pri...	Slope: -0.0314, Y-intercept: 161.65
Chi-square Test	Apple Inc	P-value: 0.003	Chi-square Statistic: 314.26
	Honeywell Inc	P-value: 0.000	Chi-square Statistic: 514.288

Figure 28 Summary of the report.

References

Apple Inc. (AAPL) Stock Historical Prices & Data—Yahoo Finance. (n.d.). Retrieved June 8, 2024, from <https://finance.yahoo.com/quote/AAPL/history/>

Honeywell International Inc. (HON) Stock Historical Prices & Data—Yahoo Finance. (n.d.). Retrieved June 8, 2024, from <https://finance.yahoo.com/quote/HON/history/>

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