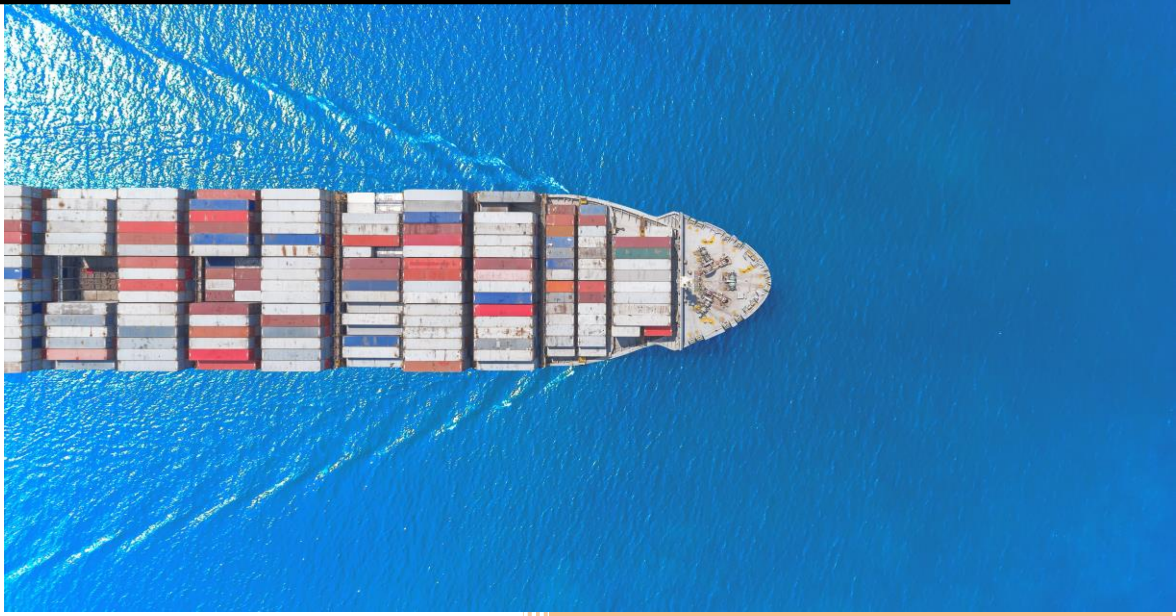
Syed Faizan

Transshipment and Risk Minimization



Main Tool: Excel

Technique: Optimization

Syed Faizan

Introduction and Problem Description

This Project encompasses two distinct optimization problems: a Transshipment Problem and a Risk Minimizing Problem. The objective of this project is to apply optimization techniques to real-world scenarios, thereby demonstrating proficiency in analytical methods and decision-making strategies. This report is structured to address each problem separately, with comprehensive analysis and solutions presented for both cases.

In the first part of the project, we analyze a shipping problem faced by the Rockhill Shipping & Transport Company. The company must determine the most cost-effective way to transport hazardous waste from six manufacturing plants to three disposal sites. This problem is complicated by the need to potentially use intermediate points for dropping and picking up loads to minimize costs. We will explore both direct shipping routes and the use of intermediate points, providing detailed solutions and cost analyses for each scenario.

The second part of the project involves an investment allocation problem. An investor seeks to allocate $10,000 across various asset types to achieve a minimum expected return of 11% while minimizing risk. We will utilize historical data to estimate returns and covariances among the assets, then apply optimization techniques to determine the optimal investment strategy. Additionally, we will analyze the relationship between expected return and risk by plotting various solutions for different baseline return values.

This report is organized according to APA standards and includes an introduction, detailed analysis, and conclusions for each part of the project. The findings and methodologies are meticulously documented to provide a clear understanding of the optimization processes and their outcomes.

Part 1: Rockhill Shipping & Transport Company

A Direct shipping model

	Waste Proposal Site		
Plant:	*Orangeburg*	*Florence*	*Macon*
Denver	$12	$15	$17
Morganton	14	9	10
Morrisville	13	20	11
Pineville	17	16	19
Rockhill	7	14	12
Statesville	22	16	18

Table 1: Shipping costs, per barrel of waste from six plants to three waste disposal sites

Plant:	*Waste per Week (bbl)*
Denver	45
Morganton	26
Morrisville	42
Pineville	53
Rockhill	29
Statesville	38
Total waste generated	233

	Plant					
Plant:	**Denver**	**Morganton**	**Morrisville**	**Pineville**	**Rockhill**	**Statesville**
Denver	$---	$3	$4	$9	$5	$4
Morganton	6	---	7	6	9	4
Morrisville	5	7	---	3	4	9
Pineville	5	4	3	---	3	11
Rockhill	5	9	5	3	---	14
Statesville	4	7	11	12	8	---

Table 2: Shipping costs, per barrel of waste from each plant to another plant

	Waste Proposal Site		
Waste Disposal Site:	**Orangeburg**	**Florence**	**Macon**
Orangeburg	$---	$12	$10
Florence	12	---	15
Macon	10	15	---

Table 3: Shipping costs, per barrel of waste between the three waste disposal sites

Figure 1 Costs and Capacity of the Rockhill Shipping company.

Problem statement: The Rockhill Shipping & Transport Company is faced with the challenge of optimizing its shipping routes for hazardous chemical waste generated by Chimotoxic, a chemical manufacturing company. The task involves transporting waste from six different plants to three designated waste disposal sites. The complexity of this problem is heightened by the hazardous nature of the waste, which necessitates careful handling, potential route diversions due to municipal regulations, and the consideration of using intermediate points to minimize costs.

The primary objective is to determine the most cost-effective shipping routes that minimize Rockhill's total transportation cost. The company must decide whether it is cheaper to **ship waste directly from the plants to the disposal sites** or to **utilize intermediate drop-and-pickup points**. The shipping costs between each plant and disposal site, as well as between the plants and among the disposal sites, are provided. Additionally, each plant produces a specific amount of waste weekly, and each disposal site has a maximum capacity it can handle.

The optimization problem involves calculating the optimal routes and associated costs for both direct and intermediate shipping scenarios. This includes determining the number of barrels to be transported from each source to each destination, ensuring that the solution adheres to the capacity constraints of the disposal sites and minimizes the total shipping cost. The findings from this analysis will inform the contract proposal to be submitted to Chimotoxic, aiming to balance cost-efficiency with safety and regulatory compliance.

		Waste Disposal Site		
	LP Primal	**Orangeburg (P)**	**Florence (Q)**	**Macon (R)**
Plants	**Denver (A)**	XAP	XAQ	XAR
	Morganton (B)	XBP	XBQ	XBR
	Morrisville (C)	XCP	XCQ	XCR
	Pineville (D)	XDP	XDQ	XDR
	Rockhill (E)	XEP	XEQ	XER
	Statesville (F)	XFP	XFQ	XFR
	Let P be the Plant	**Let D be Destination**		

CONSTRAINTS FORMULATION			
1	X (P1, D1)+X (P1, D2)+X (P1, D3)	=	45
2	X (P2, D1)+X (P2, D2)+X (P2, D3)	=	26
3	X (P2, D3)+X (P3, D2)+X (P3, D3)	=	42
4	X (P4, D1)+X (P4, D2)+X (P4, D3)	=	53
5	X (P5, D1)+X (P5, D1)+ X (P5, D3)	=	29
6	X (P6, D1)+X (P6, D2)+X (P6, D3)	=	38
7	X (P1,D1) + X(P2, D1) + X(P3, D1) + X(P4, D1) + X(P6, D1)	≤	65
8	X (P1,D2) + X(P2, D2) + X(P3, D2) + X(P4, D2) + X(P6, D2)	≤	80
9	X (P1,D3) + X(P2, D3) + X(P3, D3) + X(P4, D3) + X(P6, D3)	≤	105
10	X(Pij, Dij)	≥	0

Figure 2 Mathematical Modelling of the Transshipment problem for direct shipping.

Mathematical Modelling: The shipping cost optimization problem for the Rockhill Shipping & Transport Company involves determining the most cost-effective routes for transporting hazardous waste from six chemical plants to three designated disposal sites. This problem can be modeled using a linear programming approach, where the objective is to minimize the total transportation cost while satisfying various constraints.

The decision variables represent the **number of barrels** transported from each plant to each disposal site. The objective function to be minimized is **the total transportation cost**, which is the sum of the products of the number of barrels transported and the corresponding shipping costs between plants and disposal sites.

The model includes several constraints to ensure feasibility and adherence to capacity limits. Firstly, the **supply constraints** ensure that the total number of barrels shipped from each plant does not exceed the plant's weekly waste production. Secondly, the **demand constraints** ensure that the total number of barrels received at each disposal site does not exceed its maximum weekly **capacity**. Additionally, there are constraints to ensure **non-negativity**, meaning that the number of barrels transported cannot be negative.

Moreover, the model (in the second section of Part 1) shall also consider **intermediate shipping points,** where loads can be dropped at one plant or disposal site and picked up by another truck to reach the final destination. This scenario introduces additional decision variables and constraints, accounting for the possibility of using intermediate points to further reduce costs.

By solving this linear programming model, the optimal shipping routes can be identified, minimizing the total transportation cost while ensuring safe and efficient waste disposal in compliance with regulatory and capacity constraints. This comprehensive approach provides Rockhill Shipping & Transport Company with a robust plan for negotiating a cost-effective and operationally feasible contract with Chimotoxic.

The first model: Direct Shipping from Plants to Disposal sites

				Part 1		
			Question 2	Direct shipping		
Decision Variables		Waste Disposal Site				
	Orangeburg (P)	Florence (Q)	Macon (R)	Total ship from P	sign	Waste made
Plants: Denver (A)	36	9	0	45	=	45
Plants: Morganton (B)	0	0	26	26	=	26
Plants: Morrisville (C)	0	0	42	42	=	42
Plants: Pineville (D)	0	53	0	53	=	53
Plants: Rockhill (E)	29	0	0	29	=	29
Plants: Statesville (F)	0	18	20	38	=	38
Total ship to D	65	80	88		Sum Waste	233
sign	≤	≤	≤	Sum Capacity		
Capacity	65	80	105	250		
				MINIMIZE	Total Cost	2988

Figure 3 Results of a linear programming model for direct shipping.

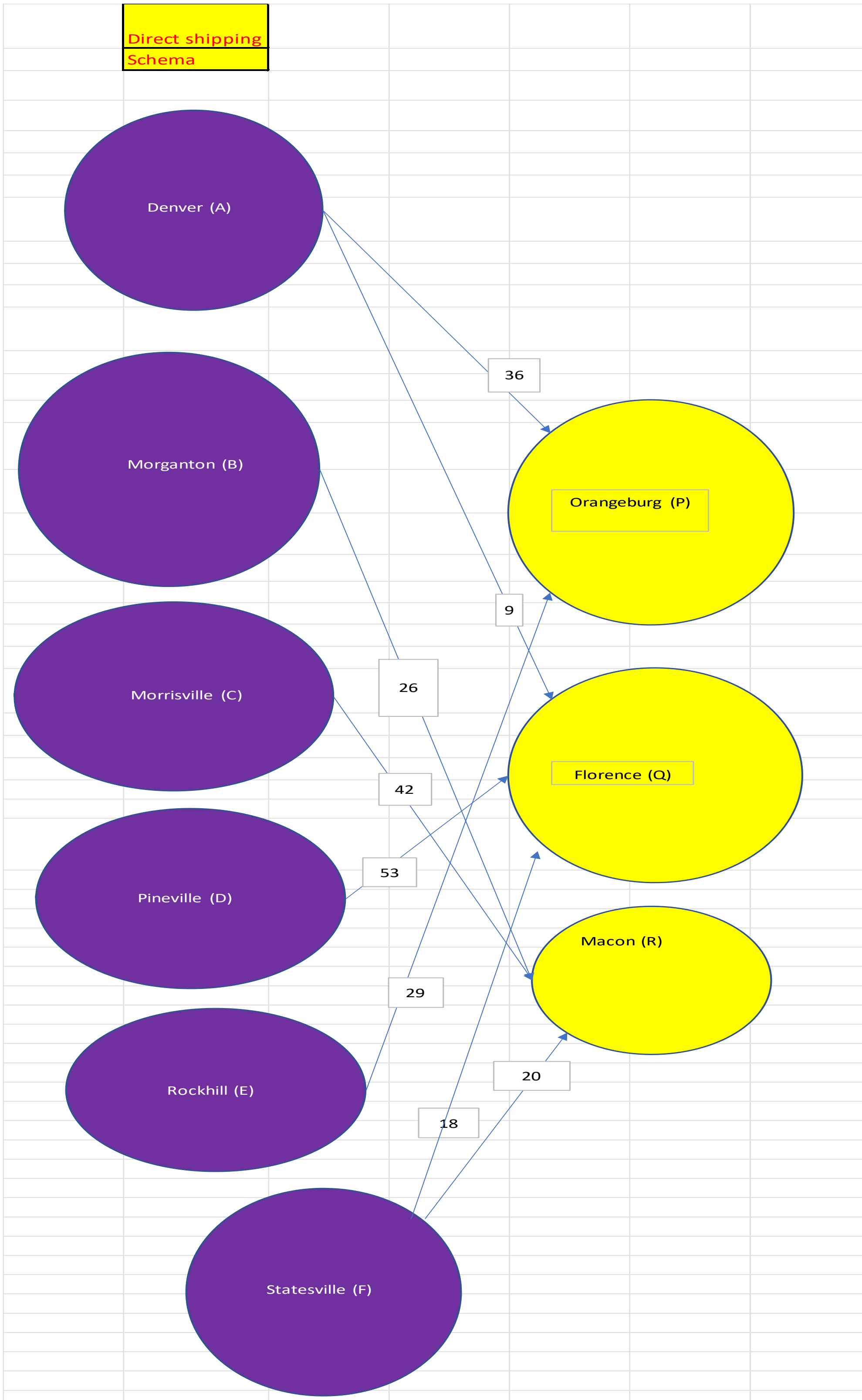


Figure 4 Schematic representation of the optimized direct shipping model.

The optimization results for the shipping cost problem indicate that Rockhill Shipping & Transport Company can achieve a minimized total cost by strategically allocating the shipment of hazardous waste from the six plants to the three disposal sites. The total shipping cost was minimized to 2988 units (Dollars $).

From the Denver plant, 36 barrels are shipped to the Orangeburg Waste Disposal Site, and 9 are shipped to the Florence Waste Disposal Site, making a total of 45 barrels. From the Morganton plant, 26 barrels are shipped to the Macon Waste Disposal Site. From the Morrisville plant, 42 barrels are shipped to the Macon Waste Disposal Site. From the Pineville plant, 53 barrels are shipped to the Florence Waste Disposal Site. From the Rockhill plant, 29 barrels are shipped to the Orangeburg Waste Disposal Site. From the Statesville plant, 18 barrels are shipped to the Florence Waste Disposal Site, and 20 barrels to the Macon Waste Disposal Site, making a total of 38 barrels from Statesville. Consolidating the shipments to each disposal site, the Orangeburg Waste Disposal Site receives a total of 65 barrels (36 from Denver and 29 from Rockhill), the Florence Waste Disposal Site receives a total of 80 barrels (9 from Denver, 53 from Pineville, and 18 from Statesville), and the Macon Waste Disposal Site receives a total of 88 barrels (26 from Morganton, 42 from Morrisville, and 20 from Statesville). In total, 233 barrels are shipped from all plants.

Optimization ensures that the capacity constraints of each disposal site are respected, with the total waste shipped from the plants matching the sum of the capacities of the disposal sites. The results show an optimal distribution that leverages direct shipping routes to minimize transportation costs while adhering to the given constraints.

These findings provide a detailed and cost-effective shipping plan, ensuring that hazardous waste is managed efficiently and safely, meeting the operational requirements of both Rockhill Shipping & Transport Company and Chimotoxic.

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [ALY6050_MOD6_Project_FaizanS.xlsx]Part 1
Report Created: 6/22/2024 7:17:35 AM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
V6	Denver (A) Orangeburg (P)	36	0	12	4	0
W6	Denver (A) Florence (Q)	9	0	15	0	4
X6	Denver (A) Macon (R)	0	0	17	1E+30	0
V7	Morganton (B) Orangeburg (P)	0	9	14	1E+30	9
W7	Morganton (B) Florence (Q)	0	1	9	1E+30	1
X7	Morganton (B) Macon (R)	26	0	10	1	1E+30
V8	Morrisville (C) Orangeburg (P)	0	7	13	1E+30	7
W8	Morrisville (C) Florence (Q)	0	11	20	1E+30	11
X8	Morrisville (C) Macon (R)	42	0	11	7	1E+30
V9	Pineville (D) Orangeburg (P)	0	4	17	1E+30	4
W9	Pineville (D) Florence (Q)	53	0	16	1	1E+30
X9	Pineville (D) Macon (R)	0	1	19	1E+30	1
V10	Rockhill (E) Orangeburg (P)	29	0	7	0	1E+30
W10	Rockhill (E) Florence (Q)	0	4	14	1E+30	4
X10	Rockhill (E) Macon (R)	0	0	12	1E+30	0
V11	Statesville (F) Orangeburg (P)	0	9	22	1E+30	9
W11	Statesville (F) Florence (Q)	18	0	16	1	0
X11	Statesville (F) Macon (R)	20	0	18	0	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
V12	Total ship to D Orangeburg (P)	65	-5	65	9	17
W12	Total ship to D Florence (Q)	80	-2	80	20	17
X12	Total ship to D Macon (R)	88	0	105	1E+30	17
Y7	Morganton (B) Total ship from P	26	10	26	17	26
Y11	Statesville (F) Total ship from P	38	18	38	17	20
Y10	Rockhill (E) Total ship from P	29	12	29	17	9
Y8	Morrisville (C) Total ship from P	42	11	42	17	42
Y6	Denver (A) Total ship from P	45	17	45	17	9
Y9	Pineville (D) Total ship from P	53	18	53	17	20

Figure 5 Sensitivity Report for direct shipping model.

The sensitivity report for the shipping cost optimization problem reveals detailed insights into the stability and flexibility of the optimal solution concerning the three waste disposal sites: Orangeburg, Florence, and Macon.

The sensitivity report indicates that the Denver plant's shipment to Orangeburg has a final value of 36 and a reduced cost of 0. Notably, the Denver plant has a reduced cost of 0 for all three of its shipments, showing that it is already optimal and needs no further alteration. Conversely, the Morganton plant exhibits a reduced cost of 9 for shipments to Orangeburg and 1 for shipments to Florence. This suggests that for Morganton to Orangeburg to become significant in the optimization model, a minimum of 23 shipments (9 plus 14) would be required. Morganton to Macon, however, shows a reduced cost of 0. On the other hand, Morrisville has a reduced cost of 7 for shipments to Orangeburg and 11 for shipments to Florence.

Focusing on the capacity constraints, each shipment to the Orangeburg Waste Disposal Site has a shadow price of minus 5, indicating that every barrel shipped to Orangeburg reduces the final total cost by 5, thus marking it as an economical route. Similarly, each shipment to the Florence Waste Disposal Site reduces the cost by minus 2. In contrast, shipments to Morganton, Statesville, Rockhill, Morrisville, Denver, and Pineville all have a positive shadow price, implying that for each barrel shipped to these destinations, the total cost increases.

Based on these findings, the **following recommendations** are proposed. Rockhill Shipping & Transport Company **should maintain the current capacities at Orangeburg, Florence, and Macon as they are largely sufficient for the current shipping needs and any increases would not result in cost savings**. The company should focus on monitoring the costs associated with each route to ensure they remain within the allowable ranges identified in the sensitivity analysis. Additionally, Rockhill should periodically review and update the shipping cost data to ensure the model remains accurate and reflective of any changes in transportation costs or operational conditions. This proactive approach will help the company maintain an optimal and cost-effective shipping strategy over time.

Optimal Routes and Costs for Direct Shipping

The following is a summary of the optimal routes and its cost according to our model

From Denver:

- To Orangeburg: 36 barrels at $12 per barrel.
- To Florence: 9 barrels at $15 per barrel.

From Morganton:

- To Macon: 26 barrels at $10 per barrel.

From Morrisville:

- To Macon: 42 barrels at $11 per barrel.

From Pineville:

- To Florence: 53 barrels at $16 per barrel.

From Rock Hill:

- To Orangeburg: 29 barrels at $7 per barrel.

From Statesville:

- To Florence: 18 barrels at $16 per barrel.
- To Macon: 20 barrels at $18 per barrel.

From	To	Barrels	Cost per Barrel ($)	Total Cost ($)
Denver	Orangeburg	36	12	432
Denver	Florence	9	15	135
Morganton	Macon	26	10	260
Morrisville	Macon	42	11	462
Pineville	Florence	53	16	848
Rock Hill	Orangeburg	29	7	203
Statesville	Florence	18	16	288
Statesville	Macon	20	18	360
	Total Weekly Cost			2988

Figure 6 Tabular summary of optimal routes and costs.

Calculating the total cost for each shipment route and the overall cost:

1. Denver to Orangeburg: 36 barrels × $12/barrel = $432
2. Denver to Florence: 9 barrels × $15/barrel = $135
3. Morganton to Macon: 26 barrels × $10/barrel = $260
4. Morrisville to Macon: 42 barrels × $11/barrel = $462
5. Pineville to Florence: 53 barrels × $16/barrel = $848
6. Rock Hill to Orangeburg: 29 barrels × $7/barrel = $203
7. Statesville to Florence: 18 barrels × $16/barrel = $288
8. Statesville to Macon: 20 barrels × $18/barrel = $360

Total Weekly Shipping Cost: $432 + $135 + $260 + $462 + $848 + $203 + $288 + $360 = $2,988

In summary, the total weekly cost of shipping barrels from the various plants to their respective destinations **is $2,988.**

Part 1: Section 2, An intermediate stop and pick-up model

Intermediate Model

From	To								
	Denver (A)	Morganton (B)	Morrisville (C)	Pineville (D)	Rockhill (E)	Statesville (F)	Orangeburg (P)	Florence (Q)	Macon (R)
Denver (A)	XAA	XAB	XAC	XAD	XAE	XAF	XAP	XAQ	XAR
Morganton (B)	XBA	XBB	XBC	XBD	XBE	XBF	XBP	XBQ	XBR
Morrisville (C)	XCA	XCB	XCC	XCD	XCE	XCF	XCP	XCQ	XCR
Pineville (D)	XDA	XDB	XDC	XDD	XDE	XDF	XDP	XDQ	XDR
Rockhill (E)	XEA	XEB	XEC	XED	XEE	XEF	XEP	XEQ	XER
Statesville (F)	XFA	XFB	XFC	XFD	XFE	XFF	XFP	XFQ	XFR
Orangeburg (P)	XPA	XPB	XPC	XPD	XPE	XPF	XPP	XPQ	XPR
Florence (Q)	XQA	XQB	XQC	XQD	XQE	XQF	XQP	XQQ	XQR
Macon (R)	XRA	XRB	XRC	XRD	XRE	XRF	XRP	XRQ	XRR

Constraint Number	Formulae
1	Xij ≥ 0
2	ΣXAj - ΣXiA = 45
3	ΣXBj - ΣXiB = 26
4	ΣXCj - ΣXiC = 42
5	ΣXDj - ΣXiD = 53
6	ΣXEj - ΣXiE = 29
7	ΣXFj -ΣXiF= 38
8	ΣXPi ≤ 65
9	ΣXQi ≤ 80
10	ΣXRi ≤ 105

Figure 7 Mathematical model of the drop and pick up model.

Mathematical model: The drop and pick-up shipping cost optimization problem for the Rockhill Shipping & Transport Company is formulated to minimize the total transportation cost of hazardous waste from six plants to three disposal sites, considering the possibility of using intermediate points. The primary objective is to determine the optimal routes that minimize transportation costs while adhering to supply and demand constraints.

In this model, the decision variables represent the number of barrels transported from each plant to every other plant, as well as from each plant to each disposal site and between disposal sites. The costs associated with each of these routes are defined, and the model seeks to minimize the total transportation cost.

The constraints in this model ensure that the total amount of waste shipped from each plant does not exceed the waste generated at that plant. Additionally, the constraints ensure that the total amount of waste received at each disposal site does not exceed its capacity. For intermediate shipping, the constraints guarantee that the amount of waste arriving at an intermediate point equals the amount of waste leaving that point, maintaining a balance.

The model also includes non-negativity constraints, which ensure that the number of barrels transported is non-negative. This comprehensive set of constraints ensures that the solution adheres to practical operational limits and regulatory requirements, making the solution feasible and implementable.

By solving this mathematical model, the company can identify the most cost-effective routes for transporting hazardous waste, including the potential benefits of using intermediate points. This approach provides a more flexible and potentially cheaper solution compared to direct shipping, as it allows for optimization of routes and better utilization of available resources. The result is an efficient and cost-effective transportation plan that meets all operational and safety requirements.

	To								
From	Denver (A)	Morganton (B)	Morrisville (C)	Pineville (D)	Rockhill (E)	Statesville (F)	Orangeburg (P)	Florence (Q)	Macon (R)
Denver (A)	$ 100,000	$ 3	$ 4	$ 9	$ 5	$ 4	$ 12	$ 15	$ 17
Morganton (B)	$ 6	$ 100,000	$ 7	$ 6	$ 9	$ 4	$ 14	$ 9	$ 10
Morrisville (C)	$ 5	$ 7	$ 100,000	$ 3	$ 4	$ 9	$ 13	$ 20	$ 11
Pineville (D)	$ 5	$ 4	$ 3	$ 100,000	$ 3	$ 11	$ 17	$ 16	$ 19
Rockhill (E)	$ 5	$ 9	$ 5	$ 3	$ 100,000	$ 14	$ 7	$ 14	$ 12
Statesville (F)	$ 4	$ 7	$ 11	$ 12	$ 8	$ 100,000	$ 22	$ 16	$ 18
Orangeburg (P)	$ 100,000	$ 100,000	$ 100,000	$ 100,000	$ 100,000	$ 100,000	$ 100,000	$ 12	$ 10
Florence (Q)	$ 100,000	$ 100,000	$ 100,000	$ 100,000	$ 100,000	$ 100,000	$ 12	$ 100,000	$ 15
Macon (R)	$ 100,000	$ 100,000	$ 100,000	$ 100,000	$ 100,000	$ 100,000	$ 10	$ 15	$ 100,000
		We input high values							

Decision Variables

	To								
From	Denver (A)	Morganton (B)	Morrisville (C)	Pineville (D)	Rockhill (E)	Statesville (F)	Orangeburg (P)	Florence (Q)	Macon (R)
Denver (A)	0	45	0	0	0	0	0	0	0
Morganton (B)	0	0	0	0	0	0	0	42	46
Morrisville (C)	0	0	0	0	0	0	0	0	42
Pineville (D)	0	17	0	0	36	0	0	0	0
Rockhill (E)	0	0	0	0	0	0	65	0	0
Statesville (F)	0	0	0	0	0	0	0	38	0
Orangeburg (P)	0	0	0	0	0	0	0	0	0
Florence (Q)	0	0	0	0	0	0	0	0	0
Macon (R)	0	0	0	0	0	0	0	0	0

Constraints	Left HS	Sign	Right HS
ΣXAj - ΣXiA = 45	45	=	45
ΣXBj - ΣXiB = 26	26	=	26
ΣXCj - ΣXiC = 42	42	=	42
ΣXDj - ΣXiD = 53	53	=	53
ΣXEj - ΣXiE = 29	29	=	29
ΣXFj -ΣXiF= 38	38	=	38
ΣXPi ≤ 65	65	≤	65
ΣXQi ≤ 80	80	≤	80
ΣXRi ≤ 105	88	≤	105

Total Minimum Cost	$ 2,674

Figure 8 Solver used to solve the drop and pick-up model of optimization.

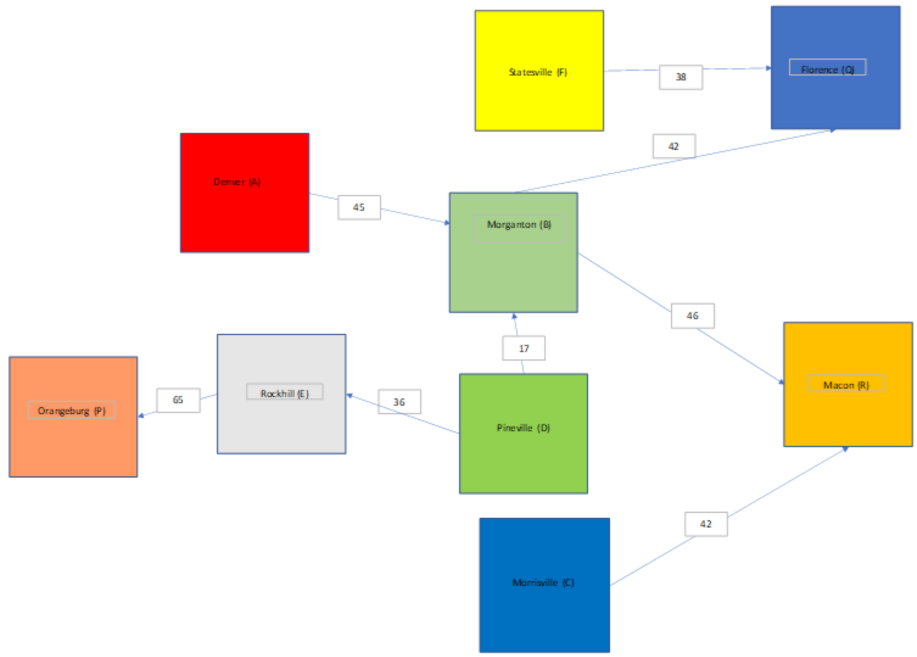


Figure 9 Schematic representation of the drop and pick-up model of transshipment.

Optimal Routes and Costs for Intermediate Shipping explaining how many barrels will be transported each week from a source to a destination: From Denver, 45 barrels will be shipped directly to Morganton. This route has been chosen to leverage intermediate shipping, reducing overall costs. Morganton, serving as an intermediate point, will send 42 barrels to Florence and 46 barrels to Macon. The direct shipments from Morganton to Florence and Macon utilize the intermediate point strategy effectively, ensuring cost efficiency. Morrisville will ship 42 barrels directly to Macon. This direct route from Morrisville to Macon is optimal in terms of minimizing costs while meeting the capacity constraints of the disposal site. Pineville, another crucial intermediary point in this model, will deliver 17 barrels to Morganton and ship an additional 36 barrels to Rockhill. This will optimize the direct shipment route to meet both capacity and cost constraints. In addition, Rockhill will ship 65 barrels to Orangeburg, optimizing the route through direct shipping without utilizing any intermediate points in order to reduce the overall costs. Statesville, without utilizing any intermediate points, will directly ship 38 barrels to Florence. This direct shipping strategy from Statesville ensures adherence to capacity constraints and cost minimization.

From	To	Barrels	Cost per Barrel ($)	Total Cost ($)
Denver	Morganton	45	3	135
Morganton	Florence	42	9	378
Morganton	Macon	46	10	460
Morrisville	Macon	42	11	462
Pineville	Morganton	17	4	68
Pineville	Rockhill	36	3	108
Rockhill	Orangeburg	65	7	455
Statesville	Florence	38	16	608

Total Minimal Cost: $2674

Figure 10 Summary of the pick-up and drop model.

The total minimal cost for the transshipment model using the pick-up and drop intermediary stops model is $ 2,674. Statesville shipments bear the most cost and Rockhill ships the largest number of barrels according to this model.

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [ALY6050_MOD6_Project_FaizanS.xlsx]Part 1
Report Created: 6/22/2024 1:38:04 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
AE108	Denver (A) Denver (A)	0	100000	100000	1E+30	100000
AF108	Denver (A) Morganton (B)	45	0	3	2	0
AG108	Denver (A) Morrisville (C)	0	2	4	1E+30	2
AH108	Denver (A) Pineville (D)	0	10	9	1E+30	10
AI108	Denver (A) Rockhill (E)	0	3	5	1E+30	3
AJ108	Denver (A) Statesville (F)	0	8	4	1E+30	8
AK108	Denver (A) Orangeburg (P)	0	3	12	1E+30	3
AL108	Denver (A) Florence (Q)	0	3	15	1E+30	3
AM108	Denver (A) Macon (R)	0	4	17	1E+30	4
AE109	Morganton (B) Denver (A)	0	9	6	1E+30	9
AF109	Morganton (B) Morganton (B)	0	100000	100000	1E+30	100000
AG109	Morganton (B) Morrisville (C)	0	8	7	1E+30	8
AH109	Morganton (B) Pineville (D)	0	10	6	1E+30	10
AI109	Morganton (B) Rockhill (E)	0	10	9	1E+30	10
AJ109	Morganton (B) Statesville (F)	0	11	4	1E+30	11
AK109	Morganton (B) Orangeburg (P)	0	8	14	1E+30	8
AL109	Morganton (B) Florence (Q)	42	0	9	1	0
AM109	Morganton (B) Macon (R)	46	0	10	0	1
AE110	Morrisville (C) Denver (A)	0	7	5	1E+30	7
AF110	Morrisville (C) Morganton (B)	0	6	7	1E+30	6
AG110	Morrisville (C) Morrisville (C)	0	100000	100000	1E+30	100000
AH110	Morrisville (C) Pineville (D)	0	6	3	1E+30	6
AI110	Morrisville (C) Rockhill (E)	0	4	4	1E+30	4
AJ110	Morrisville (C) Statesville (F)	0	15	9	1E+30	15
AK110	Morrisville (C) Orangeburg (P)	0	6	13	1E+30	6
AL110	Morrisville (C) Florence (Q)	0	10	20	1E+30	10
AM110	Morrisville (C) Macon (R)	42	0	11	4	0
AE111	Pineville (D) Denver (A)	0	4	5	1E+30	4
AF111	Pineville (D) Morganton (B)	17	0	4	0	2
AG111	Pineville (D) Morrisville (C)	0	0	3	1E+30	0
AH111	Pineville (D) Pineville (D)	0	100000	100000	1E+30	100000
AI111	Pineville (D) Rockhill (E)	36	0	3	2	1
AJ111	Pineville (D) Statesville (F)	0	14	11	1E+30	14
AK111	Pineville (D) Orangeburg (P)	0	7	17	1E+30	7
AL111	Pineville (D) Florence (Q)	0	3	16	1E+30	3
AM111	Pineville (D) Macon (R)	0	5	19	1E+30	5
AE112	Rockhill (E) Denver (A)	0	7	5	1E+30	7
AF112	Rockhill (E) Morganton (B)	0	8	9	1E+30	8
AG112	Rockhill (E) Morrisville (C)	0	5	5	1E+30	5
AH112	Rockhill (E) Pineville (D)	0	6	3	1E+30	6
AI112	Rockhill (E) Rockhill (E)	0	100000	100000	1E+30	100000
AJ112	Rockhill (E) Statesville (F)	0	20	14	1E+30	20
AK112	Rockhill (E) Orangeburg (P)	65	0	7	3	6
AL112	Rockhill (E) Florence (Q)	0	4	14	1E+30	4
AM112	Rockhill (E) Macon (R)	0	1	12	1E+30	1
AE113	Statesville (F) Denver (A)	0	0	4	1E+30	0
AF113	Statesville (F) Morganton (B)	0	0	7	1E+30	0
AG113	Statesville (F) Morrisville (C)	0	5	11	1E+30	5
AH113	Statesville (F) Pineville (D)	0	9	12	1E+30	9
AI113	Statesville (F) Rockhill (E)	0	2	8	1E+30	2
AJ113	Statesville (F) Statesville (F)	0	100000	100000	1E+30	100000
AK113	Statesville (F) Orangeburg (P)	0	9	22	1E+30	9
AL113	Statesville (F) Florence (Q)	38	0	16	0	8
AM113	Statesville (F) Macon (R)	0	1	18	1E+30	1
AE114	Orangeburg (P) Denver (A)	0	100009	100000	1E+30	100009
AF114	Orangeburg (P) Morganton (B)	0	100006	100000	1E+30	100006
AG114	Orangeburg (P) Morrisville (C)	0	100007	100000	1E+30	100007
AH114	Orangeburg (P) Pineville (D)	0	100010	100000	1E+30	100010
AI114	Orangeburg (P) Rockhill (E)	0	100007	100000	1E+30	100007
AJ114	Orangeburg (P) Statesville (F)	0	100013	100000	1E+30	100013
AK114	Orangeburg (P) Orangeburg (P)	0	100000	100000	1E+30	100000
AL114	Orangeburg (P) Florence (Q)	0	9	12	1E+30	9
AM114	Orangeburg (P) Macon (R)	0	6	10	1E+30	6
AE115	Florence (Q) Denver (A)	0	100012	100000	1E+30	100012
AF115	Florence (Q) Morganton (B)	0	100009	100000	1E+30	100009
AG115	Florence (Q) Morrisville (C)	0	100010	100000	1E+30	100010
AH115	Florence (Q) Pineville (D)	0	100013	100000	1E+30	100013
AI115	Florence (Q) Rockhill (E)	0	100010	100000	1E+30	100010
AJ115	Florence (Q) Statesville (F)	0	100016	100000	1E+30	100016
AK115	Florence (Q) Orangeburg (P)	0	15	12	1E+30	15
AL115	Florence (Q) Florence (Q)	0	100000	100000	1E+30	100000
AM115	Florence (Q) Macon (R)	0	14	15	1E+30	14
AE116	Macon (R) Denver (A)	0	100013	100000	1E+30	100013
AF116	Macon (R) Morganton (B)	0	100010	100000	1E+30	100010
AG116	Macon (R) Morrisville (C)	0	100011	100000	1E+30	100011
AH116	Macon (R) Pineville (D)	0	100014	100000	1E+30	100014
AI116	Macon (R) Rockhill (E)	0	100011	100000	1E+30	100011
AJ116	Macon (R) Statesville (F)	0	100017	100000	1E+30	100017
AK116	Macon (R) Orangeburg (P)	0	14	10	1E+30	14
AL116	Macon (R) Florence (Q)	0	16	15	1E+30	16
AM116	Macon (R) Macon (R)	0	100000	100000	1E+30	100000

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
AE120	ΣXAj - ΣXiA = 45 Left HS	45	13	45	17	45
AE121	ΣXBj - ΣXiB = 26 Left HS	26	10	26	17	46
AE122	ΣXCj - ΣXiC = 42 Left HS	42	11	42	17	42
AE123	ΣXDj - ΣXiD = 53 Left HS	53	14	53	17	17
AE124	ΣXEj - ΣXiE = 29 Left HS	29	11	29	17	17
AE125	ΣXFj -ΣXiF= 38 Left HS	38	17	38	17	38
AE126	ΣXPi ≤ 65 Left HS	65	-4	65	17	17
AE127	ΣXQi ≤ 80 Left HS	80	-1	80	46	17
AE128	ΣXRi ≤ 105 Left HS	88	0	105	1E+30	17

Figure 11 Sensitivity report of the drop and pick-up model of optimization.

In the above sensitivity report we notice that all the intermediary routes used to transfer the barrels of waste have a reduced cost of zero indicated a well optimized model that does not require any substantial alterations given the present constraints and parameters. A judicious adherence to the present model will present a very robust model for the company to reduce costs while preventing any environmental hazard.

Comparative Analysis of Direct vs. Intermediate Shipping

The comparison between the direct and intermediate shipping models reveals a **significant cost saving with the intermediate model**. Specifically, **the total weekly cost is reduced from $2,988 to $2,674, resulting in savings of $314**. This reduction in cost is achieved by leveraging intermediate points, allowing for more flexible and cost-effective routing.

Cost Efficiency

The intermediate shipping model demonstrates superior cost efficiency by utilizing intermediate stops that allow for the redistribution of shipments, optimizing routes, and reducing the overall transportation cost. For example, Denver's shipment to Morganton at $3 per barrel, and subsequent routing from Morganton to Florence and Macon at $9 and $10 per barrel respectively, showcases how intermediate points can lower costs compared to direct routes.

Operational Complexity

While the intermediate shipping model is more cost-effective, it introduces additional operational complexity. The need to manage intermediate stops requires more sophisticated logistics and coordination to ensure that shipments are accurately tracked and efficiently rerouted. This complexity must be balanced against the cost savings to determine the optimal approach for Rockhill Shipping & Transport Company.

Capacity and Constraints

Both models adhere to capacity constraints, ensuring that the disposal sites' capacities are not exceeded. The intermediate model, however, provides more flexibility in managing these constraints by redistributing shipments through intermediate points.

Conclusion

In conclusion, the intermediate shipping model offers a more cost-effective solution with a total minimal cost of $2,674 compared to the direct shipping model's cost of $2,988. The savings of $314 highlight the potential benefits of utilizing intermediate points for waste transportation. However, the increased operational complexity of the intermediate model must be carefully managed to fully realize these benefits. Rockhill Shipping & Transport Company should consider implementing the intermediate shipping model while investing in the necessary logistics infrastructure to support this more complex but cost-efficient approach.

Recommendations to the Company

Based on the analysis, the following recommendations are proposed for Rockhill Shipping & Transport Company:

1. **Implement the Intermediate Shipping Model:** Given the significant cost savings of $314 per week, it is recommended to adopt the intermediate shipping model. This model optimizes routes and leverages intermediate stops to minimize transportation costs.

2. **Invest in Logistics Infrastructure:** To effectively manage the operational complexity introduced by the intermediate shipping model, the company should invest in advanced logistics infrastructure. This includes sophisticated tracking systems, enhanced coordination mechanisms, and robust communication channels to ensure efficient rerouting and management of shipments.

3. **Training and Capacity Building:** Staff should be trained on the new logistics processes and technologies to handle the intermediate shipping model efficiently. Capacity building initiatives will ensure that the team is well-equipped to manage the increased complexity and deliver cost savings.

4. **Regular Monitoring and Optimization:** The company should regularly monitor the performance of the shipping model and continuously seek opportunities for further optimization. This includes reviewing shipping costs, adjusting routes as necessary, and incorporating feedback from operational experiences.

5. **Contingency Planning:** Develop contingency plans to address potential disruptions in the intermediate shipping model. This includes alternative routing options and emergency response strategies to maintain operational continuity.

By adopting these recommendations, Rockhill Shipping & Transport Company can achieve cost savings while maintaining efficient and effective waste transportation operations.

Part 2: Investment Allocations

(i)

Question i)

	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0004
High tech stocks	0.0003	0.009	0.0004	0.0016	-0.0016	0.0006
Foreign stocks	-0.0003	0.0004	0.008	0.0015	-0.0055	-0.0007
Call options	0.00035	0.0016	0.0015	0.012	-0.0005	0.0008
Put options	-0.00035	-0.0016	-0.0055	-0.0005	0.012	-0.0008
Gold	0.0004	0.0006	-0.0007	0.0008	-0.0008	0.005

Table 1: The Covariance matrix of assets' returns

	Expected Return	Weights	Investment
Bonds (X1)	0.07	0.189806	1898.06
High tech stocks (X2)	0.12	0.108630	1086.30
Foreign stocks (X3)	0.11	0.270828	2708.28
Call options (X4)	0.14	0.047943	479.43
Put options (X5)	0.14	0.254470	2544.70
Gold (X6)	0.09	0.128323	1283.23
	Sum	1	10000
	Sign	=	=
		1	10000

Return	0.11	>=	0.11
Variance	0.000735635		
MINIMIZE Risk	0.027122601		

Figure 12 Solver output of the optimization of the investment allocation.

The above solver output provided depicts the optimal allocation of an investment portfolio aimed at minimizing risk while achieving a minimum expected return of 11%. The portfolio consists of six asset types: Bonds, High-tech stocks, foreign stocks, Call options, Put options, and Gold. The Excel Solver tool has been used to find the optimal weights for these assets in the portfolio.

Covariance Matrix

The covariance matrix of the assets' returns provides the variance (diagonal entries) and the covariance (off-diagonal entries) of the returns between different pairs of assets. The highlighted cells in yellow indicate the covariances between pairs of assets, which are essential in calculating the overall portfolio risk (standard deviation).

Expected Returns and Weights

The expected returns for each asset type are as follows:

- Bonds (X1): 7%
- High-tech stocks (X2): 12%
- Foreign stocks (X3): 11%
- Call options (X4): 14%
- Put options (X5): 14%
- Gold (X6): 9%

The weights section represents the proportions of the total investment allocated to each asset. The solver uses these weights to minimize the portfolio's risk while ensuring the expected return meets or exceeds the target of 11%.

Expected Returns and Investment Weights

Bonds (X1)

- Expected Return: 7%
- Optimal Weight: 0.1898 (Investment: $1,898.06)

High-tech stocks (X2)

- Expected Return: 12%
- Optimal Weight: 0.1086 (Investment: $1,086.30)

Foreign stocks (X3)

- Expected Return: 11%
- Optimal Weight: 0.270828 (Investment: $2,708.28)

Call options (X4)

- Expected Return: 14%
- Optimal Weight: 0.04794 (Investment: $479.43)

Put options (X5)

- Expected Return: 14%
- Optimal Weight: 0.25447 (Investment: $2,544.70)

Gold (X6)

- Expected Return: 9%
- Optimal Weight: 0.1283 (Investment: $1,283.23)

Return, Variance, and Risk Calculation

The Solver output calculates the expected return, variance, and risk of the portfolio:

- Expected Return: The weighted sum of the expected returns of the individual assets.
- Variance: The variance of the portfolio, calculated using the covariance matrix and the weights of the assets. The variance of the optimal portfolio is **0.000735635.**
- Risk: The objective function minimized by the Solver, representing the portfolio's standard deviation or risk. The minimized risk of the portfolio, which is the standard deviation of the variances, is **0.02712.**

Ideal Optimized Portfolio

The ideal **optimized portfolio** based on the Solver output is:

Bonds: 18.98%

High-tech stocks: 10.86%

Foreign stocks: 27.0828%

Call options: 4.79%

Put options: 25.4470%

Gold: 12.83%

This optimized portfolio achieves an expected return that meets the requirement of a minimum 11% expected return while minimizing the portfolio's risk. The portfolio avoids investing in Bonds and Call options, focusing instead on a balanced mix of High-tech stocks, Foreign stocks, Put options, and Gold. This diversification strategy leverages the varying returns and covariances of the assets to achieve the desired return with minimal risk.

(ii)

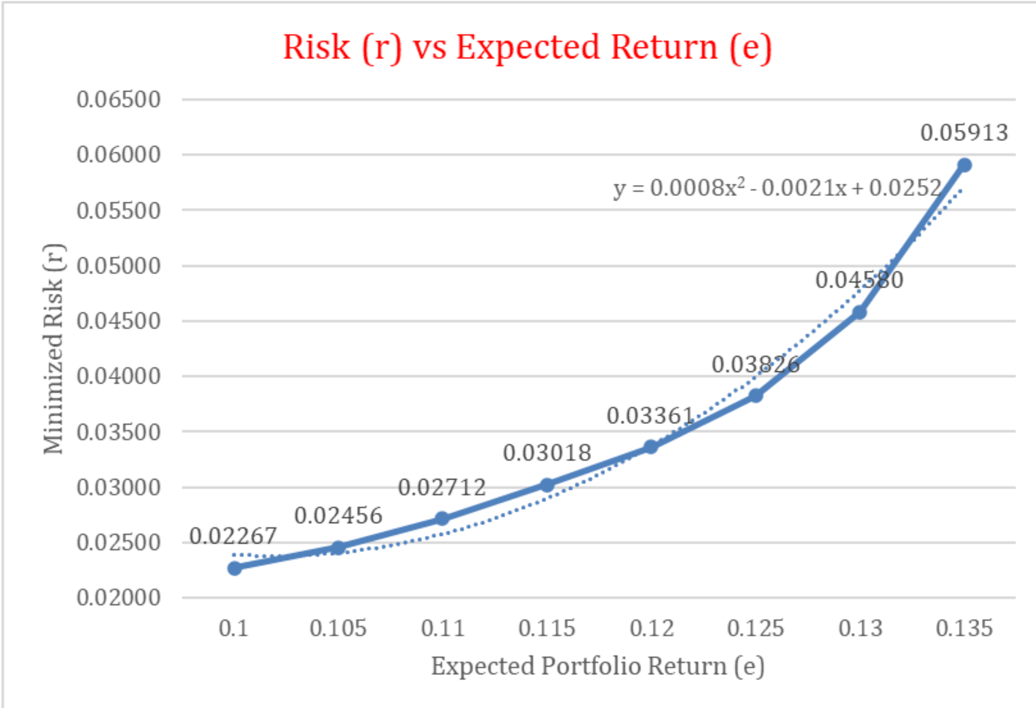
The question requires us to determine the relationship between minimized risk (r) and expected portfolio return (e) for an investment portfolio. Using the Solver, we solve for eight pairs of (r, e) using the baseline return values of 10%, 10.5%, 11%, 11.5%, 12%, 12.5%, 13%, and 13.5%.

Solution Pairs

The following table presents the minimized risk values (r) corresponding to the expected portfolio returns (e):

Question ii)		Minimized Risk (r)	is Standard Deviation
Expected Portfolio Return (e)	Variance	Minimized Risk (r)	
0.1	0.000513907	0.02267	
0.105	0.000603273	0.02456	
0.11	0.000735635	0.02712	
0.115	0.000910995	0.03018	
0.12	0.001129351	0.03361	
0.125	0.001463498	0.03826	
0.13	0.00209792	0.04580	
0.135	0.003496248	0.05913	

Figure 13 Minimized Risk for baseline return values.



Plot: Risk (r) vs. Expected Return (e)

The graph provided illustrates the relationship between the minimized risk and the expected portfolio return. The plot of e versus r shows a clear upward trend, indicating that as the expected return increases, the minimized risk also increases.

Pattern and Mathematical Relationship

Analyzing the plot, it is evident that there exists a quadratic relationship between minimized risk (r) and expected return (e). The relationship can be expressed in the form of a quadratic equation:

$$y=0.0008x^2-0.0021x+0.0252$$

where:

- **y** represents the minimized risk **(r)**
- **x represents** the expected portfolio return **(e)**

The quadratic nature of this relationship suggests that the increase in risk is not linear but accelerates as the expected return rises. This curvature implies that aiming for higher returns results in disproportionately higher risks, which aligns with the fundamental principles of investment where higher potential returns are typically associated with higher levels of risk.

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [ALY6050_MOD6_Project_FaizanS.xlsx]Part 2
Report Created: 6/22/2024 8:10:49 AM

Variable Cells

Cell	Name	Final Value	Reduced Gradient
H15	Bonds (X1) Weights	0.189805874	0
H16	High tech stocks (X2) Weights	0.108630439	0
H17	Foreign stocks (X3) Weights	0.27082789	0
H18	Call options (X4) Weights	0.047942727	0
H19	Put options (X5) Weights	0.254470203	0
H20	Gold (X6) Weights	0.128322867	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
G25	Return Inequality	0.11	0.567279827
H21	Total Sum Weights	1	0
I21	Total Sum Investment	10000	-3.52782E-06

Figure 14 Sensitivity report for this portfolio analysis.

The above sensitivity report confirms that the optimal portfolio allocation involves investing specific proportions in each asset type to achieve a minimum expected return of 11% while minimizing risk. The zero reduced gradients indicate that the solution cannot be further improved within the given constraints. The Lagrange multipliers highlight the importance of the return constraint, showing that any change in the required return would significantly affect the portfolio's risk. This detailed analysis aids the investor in understanding the robustness and implications of the optimal investment strategy.

Mathematical Model

The below table represents the model used to optimize the portfolio allocation task using quadratic programming.

Quadratic Programming Model for Investment Portfolio Optimization

Step	Description
Step 1: Defining Decision Variables	The decision variables, represented by x_i (for i = 1, 2, ..., 6), correspond to the fractions of the total investment allocated to different financial instruments: x_1: Fraction of money invested in Bonds x_2: Fraction of money invested in High Tech Stocks x_3: Fraction of money invested in Foreign Stocks x_4: Fraction of money invested in Call Options x_5: Fraction of money invested in Put Options x_6: Fraction of money invested in Gold
Step 2: Defining the Objective Function	The objective of this model is to minimize the portfolio risk, which is mathematically represented as: Minimize $\sigma^2 = x^T Q x$ where x is the vector of decision variables, and Q is the covariance matrix of the returns of the financial instruments. The term $x^T Qx$ quantifies the portfolio's variance, a measure of risk.
Step 3: Formulation of Constraints	Three primary constraints are applied to ensure a feasible and desirable solution: 1. Total Sum Constraint: The sum of the fractions of the investments must equal 1. $\Sigma x_i = 1$ 2. Expected Return Constraint: The expected return of the portfolio must meet or exceed a specified threshold, here set at 11%. $c^T x \geq 11\%$ where c represents the vector of expected returns for each financial instrument. 3. Non-Negativity Constraint: Each fraction of the investment must be non-negative, meaning no short selling is allowed. $x \geq 0$ for all i

This quadratic programming model balances the trade-off between minimizing risk and achieving a minimum expected return, ensuring a diversified and optimized investment portfolio. The constraints ensure that the solution is practical and adheres to investment guidelines.

Figure 15 Quadratic modelling.

Conclusion

In conclusion, the plot of minimized risk (r) versus expected portfolio return (e) reveals a quadratic relationship. This indicates that as the expected return on the portfolio increases, the risk associated with the portfolio increases at an accelerating rate. This finding is crucial for investors, as it underscores the importance of balancing the desire for higher returns with the acceptance of increased risk. The quadratic model provides a valuable framework for understanding and predicting the trade-offs between risk and return in investment portfolios.

Summary

In this report we have delineated the fulfillment of all the tasks assigned in the Module 6 Optimization project which subsumes two parts, a transshipment model and a portfolio allocation model using optimization techniques. We have arrived at two transshipment models. One involving direct shipments from plants generating waste to the waste disposal sites and another involving intermediary stops. We found the intermediary stop model to be more economical albeit at the expense of simplicity. We also optimized portfolio allocation using quadratic programming techniques in Excel solver. We arrived at a minimized risk value for the portfolio and optimal weights for the various investment options.

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