Foias constant

From Wikipedia, the free encyclopedia

In mathematical analysis, the Foias constant, is a number named after Ciprian Foias.

If $x_1 > 0$ and

$$x_{n+1} = \left(1 + \frac{1}{x_n}\right)^n \text{ for } n = 1, 2, 3, \ldots,$$

then the Foias constant is the unique real number α such that if $x_1 = \alpha$ then the sequence diverges to ∞.[1] Numerically, it is

$$\alpha = 1.187452351126501\ldots$$ A085848.

No closed form is known.

When $x_1 = \alpha$ then we have the limit:

$$\lim_{n\to\infty} x_n \frac{\log n}{n} = 1,$$

where "log" denotes the usual natural logarithm.

A fortuitous observation between the prime number theorem and this constant goes as follows,

$$\lim_{n\to\infty} \frac{x_n}{\pi(n)} = 1,$$

where π is the prime-counting function.[2]

See also

- Mathematical constant

Notes and references

1. Ewing, J. and Foias, C. "An Interesting Serendipitous Real Number." In Finite versus Infinite: Contributions to an Eternal Dilemma (Ed. C. Caluse and G. Păun). London: Springer-Verlag, pp. 119–126, 2000.
2. Ewing, J. and Foias, C. "An Interesting Serendipitous Real Number." In Finite versus Infinite: Contributions to an Eternal Dilemma (Ed. C. Caluse and G. Păun). London: Springer-Verlag, pp. 119–126, 2000.

- Weisstein, Eric W. "Foias Constant". MathWorld.
- S. R. Finch (2003). Mathematical Constants. Cambridge University Press. p. 430. ISBN 0-521-818-052.
- "Sloane's A085848 : Decimal expansion of Foias's Constant". The On-Line Encyclopedia of Integer Sequences. OEIS Foundation.

Categories: Mathematical analysis | Mathematical constants