

1. If the diameter of a network with 100 nodes is 1, what is the minimum number of links in this network?

Because every node have to directly connected to any other nodes, so that every node have 99 links which connected to the other nodes, and have to divide by two because two node only connect by one link. So the solution of this problem is  $\frac{100 \times 99}{2} = 4950$

2. If the diameter of a network with 100 nodes is 2, what is the minimum number of links in this network?

Cause every node have to connected with any other nodes at most two distance, so we can think as have one node in the center and others all connect one link to the center node, so that we can only use 99 links, then we can create a network. So the answer is 99

3. For a network of 100 nodes, if the degree of every node is at most 2, what is the minimum diameter of that network?

By Moore graph  $1 + d \sum_{i=0}^{k-1} (d-1)^i = \text{the upper bound of vertices}$ , d is degree, k is diameter. By the problem

$$1 + 2 \sum_{i=0}^{k-1} (1)^i \geq 100$$

So that we know  $k \geq 50$  and k is the number of diameter, so we know that the minimum of diameter is 50.

4. For a network of 100 nodes, if the degree of every node is at most 3, is it possible that the diameter of this network is not greater than 5?

By Moore graph  $1 + d \sum_{i=0}^{k-1} (d-1)^i = \text{the upper bound of vertices}$ , d is degree, k is diameter. By the problem

$$1 + 3 \sum_{i=0}^4 (2)^i = 1 + 3(1 + 2 + 4 + 8 + 16) = 1 + 3 \times 31 = 94 < 100$$

So that we know the maximum number of node that the degree of every node is at most 3 and also the diameter of the network is not greater than 5 is 94 nodes, then we know the answer is Not Possible.