Coding Review (I2P 2019)

Basic Recursions with C v1.1

Review of basic recursions. This part is more conceptual than the previous sections.

Steps for Writing Recursion Code

Recursion in programming is like induction in mathematics. So, if you have trouble understanding recursion, maybe you can start from reviewing the concept of mathematical induction.

> Every good recursion code writing consists of four parts or steps. The first part is called "The Goal". The programmer defines a function prototype that can solve the problem once implemented correctly. But of course... the function isn't implemented yet. The second step is called "The Basic Case". The programmer solves the simplest case to ensure the termination of recursion calls in later steps. Then move on to the third step, "The Inductive Case". The programmer solve other complicated cases by breaking it down to simpler cases that utilize the defined function as if it is already fully implemented. This takes the ordinary basic case and use it to do something extraordinary. The code works now, but, the programmer doesn't understand why it actually works yet. He or she should think over the code and convince himself or herself why the code works. That's why every recursion code writing should have the fourth step, the most important but so easily forgotten part, the part we call "The Recursion".
>
> -- Quote imitating Christopher Priest in the movie, The Prestige

Steps	**Recursion**	**Induction**
The Goal	What does the function do	$f(n)$ have some certain properties
The Basic Case	`if (n == 1) return ...;`	prove $f(1)$ satisfies the properties
The Inductive Case	`else func(n-1); return ...;`	prove $f(n)$ satisfies, by assuming $f(k)$ works $\forall k \in [1, n-1]$
The Recursion	Rethink the intuitive	meaning on why it works

Let's take the classic Hanoi tower as example.

Hanoi Tower

The objective of the puzzle is to move the entire stack from rod-1 to rod-3, obeying the following simple rules:

- Only one disk can be moved at a time.
- Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack or on an empty rod.

- No larger disk may be placed on top of a smaller disk.

Now, we want to output the solution in text format, let's follow the 4 steps.

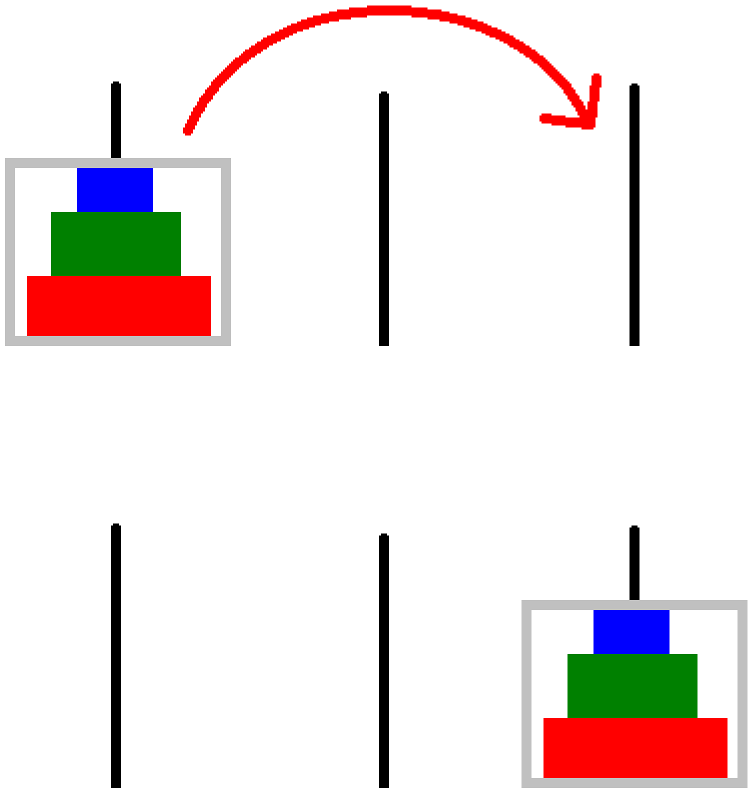
1. "The Goal": Define a function prototype that can solve the problem once implemented correctly.

   Define the function prototype and what does it do. Answer:

   ```
   void move_hanoi(int n, int from, int to, int temp);
   ```

   This function moves n disks on the top of the `from` rod to the `to` rod legally. (without violating the rules) The `temp` rod only have disks that are larger than the top n disks, so it can be used to temporarily store disks as if the rod is empty.

   We assume this function is implemented correctly. So if the problem is to move 3 disks from rod-1 to rod-3, we can achieve it by a single function call, `move_hanoi(n, 1, 3, 2)`

   This part should be the hardest part since the function definition may be tricky to define. Sometimes we even need to think how to deal with the induction / recursive case and then we can think of this function definition.

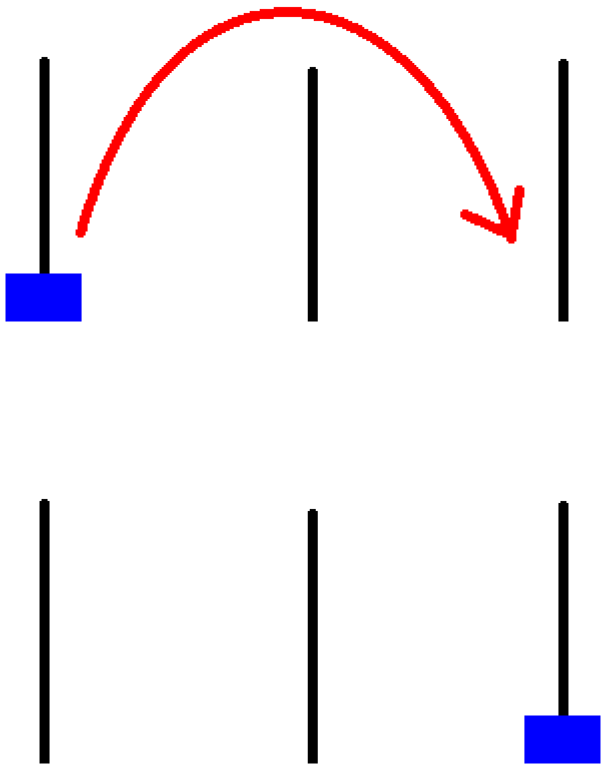
2. "The Basic Case": Solve the simplest case to ensure the termination of recursion calls in later steps.

   Assume that we are only required to move a single disk, how should we move it.

   Answer:

```
// Move the disk directly to the target rod.
printf("Move disk %d from %d to %d\n", 1, from, to);
```

If we have only one disk, any move is legal.



Once this basic case is implemented, we can use it without worrying infinite recursion.

3. "The Inductive Case": Solve other complicated cases by breaking it down to simpler cases that utilize the defined function as if it is already fully implemented.

   Since the basic case is implemented, we now assume that `move_hanoi(1, ...)`, `move_hanoi(2, ...)`, `move_hanoi(n-1, ...)` are all implemented. Now we can use them to move the n disks. It's like strong mathematical induction here.

   Keep in mind that we can use `move_hanoi` as if it's already implemented. No need to worry that it isn't implemented yet, just use it freely like `printf` or `strlen`.

   I think the importance of this thought is ofter underrated. Lots of students may think: "I haven't implemented this function yet, why can I call it?", then start writing codes that can actually be done by a recursion function call, and eventually find out they cannot write all of the codes in this way.

   If you know mathematical induction, you can write the code like writing mathematical proofs. As long as the recursion function calls only use $f(k), \forall k \in [1, n-1]$, the call will be correct.

   Recall the definition of the function.

```
void move_hanoi(int n, int from, int to, int temp);
```

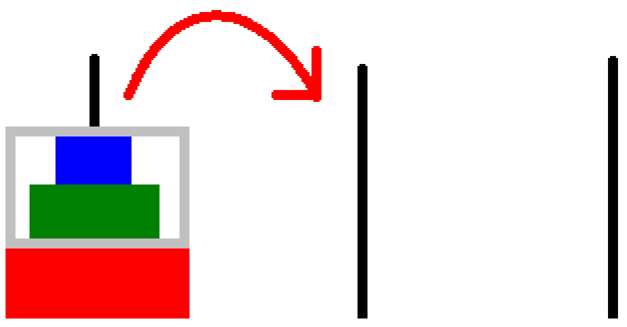
This function moves n disks on the top of the `from` rod to the `to` rod legally. (without violating the rules) The `temp` rod only have disks that are larger than the top n disks, so it can be used to temporarily store disks as if the rod is empty.

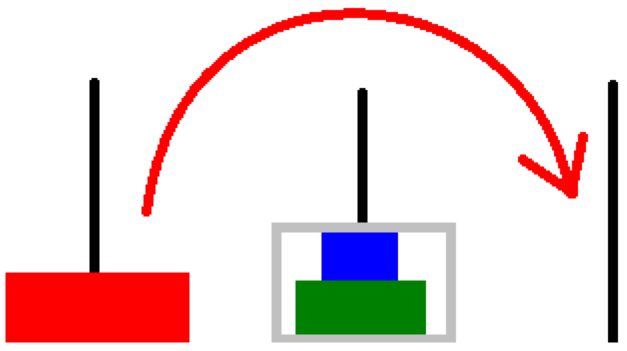
And now, for the inductive / recursive case:

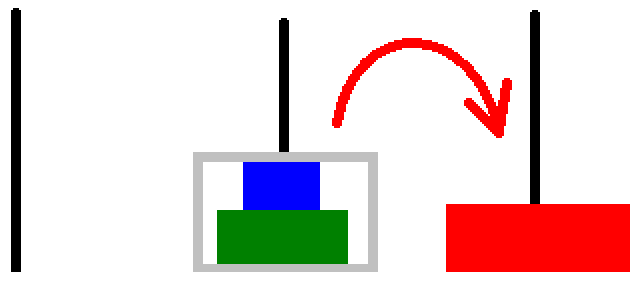
Answer:

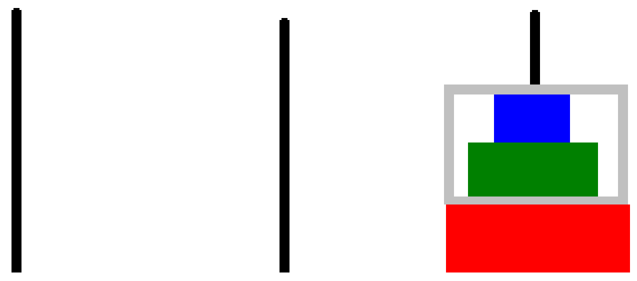
```
// Keep the largest disk below and move the upper disks to the temporary
rod.
move_hanoi(n - 1, from, temp, to);
// Move the largest disk to the target destination.
printf("Move disk %d from %d to %d\n", n, from, to);
// Stack the upper disks above the largest disk.
move_hanoi(n - 1, temp, to, from);
```

The `temp` rod may not be empty, but we can still temporarily store disks on it since the disks on it are larger than the disks that we're currently moving.









4. "The Recursion": The code works now, but, the programmer doesn't understand why it actually works yet. He or she should think over the code and convince himself or herself why the code works.

   Type the entire code and convince yourself of its correctness.

   Answer:

```
// This function moves $n$ disks on the top of the `from` rod to the `to`
rod legally. (without violating the rules) The `temp` rod only have disks
that are larger than the top $n$ disks, so it can be used to temporarily
store disks as if the rod is empty.
void move_hanoi(int n, int from, int to, int temp);
void move_hanoi(int n, int from, int to, int temp) {
    if (n == 1)
        // If there are only one disk to move,
        // Move the disk directly to the target rod.
        printf("Move disk %d from %d to %d\n", 1, from, to);
    else {
        // If there are multiple disks to move,
        // Keep the largest disk below and move the upper disks to the
temporary rod.
        move_hanoi(n - 1, from, temp, to);
        // Move the largest disk to the target destination.
        printf("Move disk %d from %d to %d\n", n, from, to);
        // Stack the upper disks above the largest disk.
        move_hanoi(n - 1, temp, to, from);
    }
}
```

Think over the entire process and find out that the code we write is equivalent to proving the correctness through mathematical induction.

5. Bonus Section: Time Complexity Analysis

   Analyze how many movement occurs.

   Answer:

   Assume we have n disks, and each move requires c time.

$$
\begin{aligned}
T(0) &= 0 \\
T(1) &= c \\
T(n) &= 2 \cdot T(n-1) + c \\
&= 2 \cdot 2 \cdot T(n-2) + 2 \cdot c + c \\
&= 2 \cdot 2 \cdot 2 \cdot T(n-3) + 4 \cdot c + 2 \cdot c + c \\
&= 2^n \cdot T(0) + \sum_{k=0}^{n-1} (2^k \cdot c) \\
&= c \cdot \sum_{k=0}^{n-1} 2^k
\end{aligned}
$$

   Sum of geometric sequence:

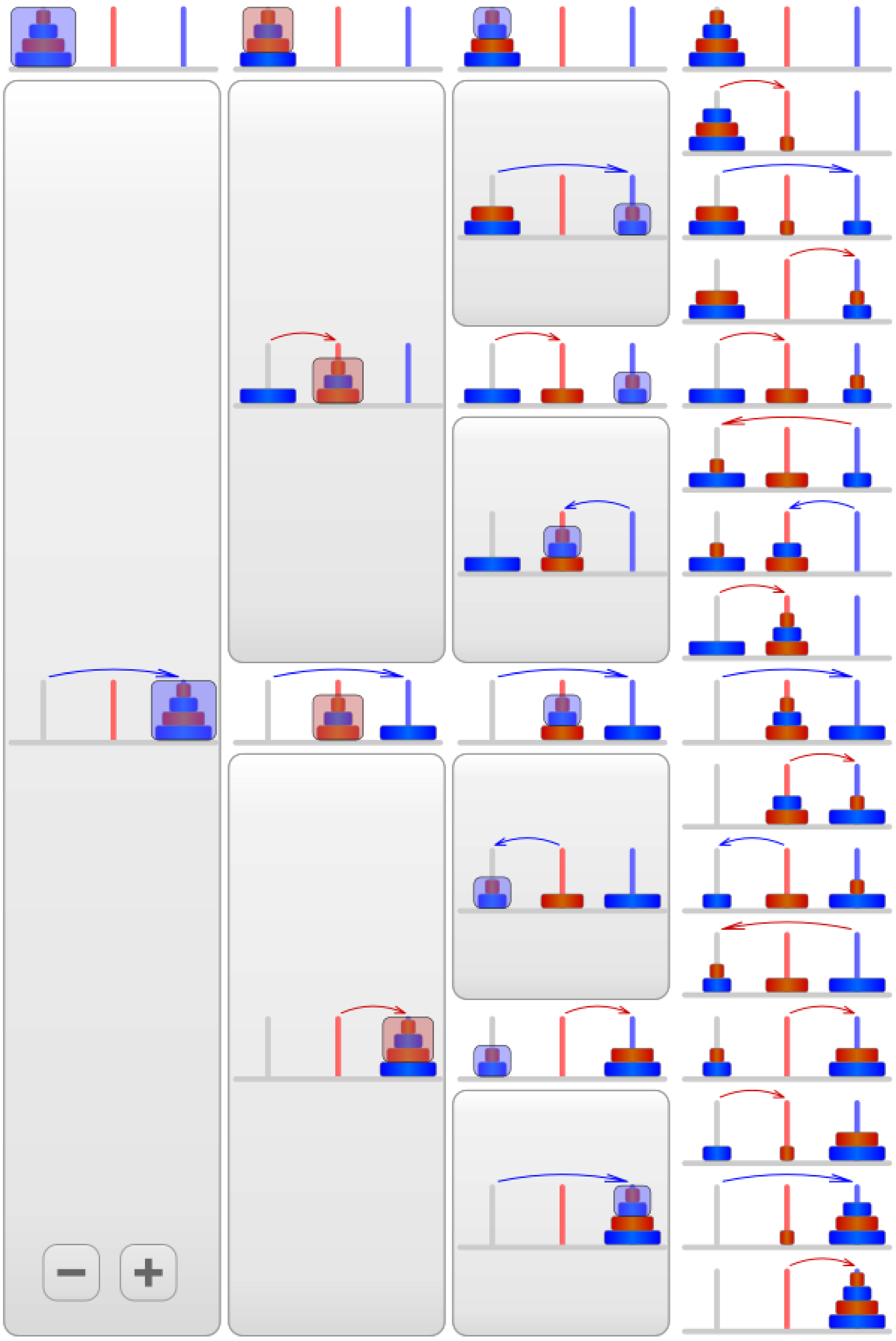
$$
\sum_{k=0}^{n-1} 2^k = \frac{2^0 \cdot (1 - 2^{n-1})}{1-2} = 2^n - 1
$$

$$
T(n) = (2^n - 1) \cdot c
$$

   We proved that the time complexity is $O(2^n)$, total $2^n - 1$ moves.

   This part is a bonus section, you'll learn more about it in the Design and Analysis of Algorithm course.

Here's a nice image from wikipedia.



The leftmost column is "The Goal". It defines the function and use it like a black box, we only need to know what it does, without knowing how it does it.

The second column (counting from left) is "The Recursion Case". It uses the function as if it is implemented, but only use $f(k), \forall k \in [1, n-1]$. (Does not use $f(n)$)

The right lower part is "The Basic Case", we can just move the disk directly.

For other parts, it enumerates each move. It is almost impossible to enumerate this in our head for a large enough n. So we can use recursion to abstract this part and prove its correctness.

There are many other solutions that can solve the Hanoi Tower problem, feel free to come up with other solutions on your own. The solution shown here is the most simplest and straightforward one in my opinion.

The Variable Scopes and Function Call Stack

Note that all variables in each recursive function call is a new local variable with its own scope.

For the first move in a $n = 3$ Hanoi tower problem, we can visualize the call stack as below.

```
move_hanoi(3, 1, 3, 2);
```

```
void move_hanoi_depth1(int n1=3, int from1=1, int to1=3, int temp1=2) {
    move_hanoi(n1 - 1, from1, temp1, to1);
    // The call above is shown as 'move_hanoi_depth2` below.
    printf("Move disk %d from %d to %d\n", n1, from1, to1);
    move_hanoi(n1 - 1, temp1, to1, from1);
}
void move_hanoi_depth2(int n2=2, int from2=1, int to2=2, int temp2=3) {
    move_hanoi(n2 - 1, from2, temp2, to2);
    // The call above is shown as 'move_hanoi_depth3` below.
    printf("Move disk %d from %d to %d\n", n2, from2, to2);
    move_hanoi(n2 - 1, temp2, to2, from2);
}
void move_hanoi_depth3(int n3=1, int from3=1, int to3=3, int temp3=2) {
    printf("Move disk %d from %d to %d\n", 1, from3, to3);
    // This function will return to 'move_hanoi_depth2' above, and continue
running the subsequent codes.
}
```

With the knowledge of variable scopes, function stacks and mathematical induction, you should have the ability to write recursion codes for most problems.

Replace Loops with Recursion

`for` loops and recursion is interchangeable. However, recursion has higher overhead due to many function calls, you'll learn more about it in Computer Architecture course.

The following code is the code of Selection Sort using `for` loop.

```
#include <stdio.h>
#define MAX 10
#define swap(a, b) if (a != b) {(*a)^=(*b); (*b)^=(*a); (*a)^=(*b);}
```

```c
void selection_sort(int* a, int size);
int a[MAX] = {3, 7, 9, 1, 4, 0, 8, 6, 2, 5};

int main(void) {
    selection_sort(a, MAX);
    for (int i = 0; i < MAX; i++) {
        printf("%d ", a[i]);
    }
    return 0;
}

void selection_sort(int* a, int size) {
    int i, j, min_idx;
    // Put the i-th smallest number in a[i].
    for (i = 0; i < size - 1; i++) {
        // Scan through the remaining array and pick the smallest element.
        min_idx = i;
        for (j = i+1; j < size; j++) {
            if (a[min_idx] > a[j])
                min_idx = j;
        }
        // Swap the smallest number to the left.
        swap(&a[i], &a[min_idx]);
    }
}
```

Modify the code to use recursion without using any explicit loops (`while`, `for`).

Answer:

First, remove the outer loop.

```c
#include <stdio.h>
#define MAX 10
#define swap(a, b) if (a != b) {(*a)^=(*b); (*b)^=(*a); (*a)^=(*b);}

void selection_sort(int* a, int size);
int a[MAX] = {3, 7, 9, 1, 4, 0, 8, 6, 2, 5};

int main(void) {
    selection_sort(a, MAX);
    for (int i = 0; i < MAX; i++) {
        printf("%d ", a[i]);
    }
    return 0;
}

void selection_sort(int* a, int size) {
    int j, min_idx;
    if (size == 1)
        // No need to sort a single number.
        return;
    // Scan through the remaining array and pick the smallest element.
    min_idx = 0;
    for (j = 1; j < size; j++) {
        if (a[min_idx] > a[j])
            min_idx = j;
    }
    // Swap the smallest number to the first.
    swap(&a[0], &a[min_idx]);
    // Since the first element is smallest, we can sort the remaining array
recursively.
    selection_sort(a+1, size-1);
}
```

Second, remove the inner loop.

```
#include <stdio.h>
#define MAX 10
#define swap(a, b) if (a != b) {(*a)^=(*b); (*b)^=(*a); (*a)^=(*b);}

int get_min_idx(int* a, int size);
void selection_sort(int* a, int size);
int a[MAX] = {3, 7, 9, 1, 4, 0, 8, 6, 2, 5};

int main(void) {
    selection_sort(a, MAX);
    for (int i = 0; i < MAX; i++) {
        printf("%d ", a[i]);
    }
    return 0;
}

int get_min_idx(int* a, int size) {
    if (size == 1)
        return 0;
    int min_idx = get_min_idx(a+1, size-1) + 1;
    if (a[min_idx] > a[0])
        return 0;
    return min_idx;
}

void selection_sort(int* a, int size) {
    int min_idx;
    if (size == 1)
        // No need to sort a single number.
        return;
    // Get the smallest element.
    min_idx = get_min_idx(a, size);
    // Swap the smallest number to the first.
    swap(&a[0], &a[min_idx]);
    // Since the first element is smallest, we can sort the remaining array
recursively.
```

```
    selection_sort(a+1, size-1);
}
```

Replace Recursion with Loops

The following code is Hanoi Tower code using recursion.

```
#include <stdio.h>

void move_hanoi(int n, int from, int to, int temp);

int main(void) {
    move_hanoi(3, 1, 3, 2);
    return 0;
}

void move_hanoi(int n, int from, int to, int temp) {
    if (n == 1)
        // If there are only one disk to move,
        // Move the disk directly to the target rod.
        printf("Move disk %d from %d to %d\n", 1, from, to);
    else {
        // If there are multiple disks to move,
        // Keep the largest disk below and move the upper disks to the temporary
rod.
        move_hanoi(n - 1, from, temp, to);
        // Move the largest disk to the target destination.
        printf("Move disk %d from %d to %d\n", n, from, to);
        // Stack the upper disks above the largest disk.
        move_hanoi(n - 1, temp, to, from);
    }
}
```

Modify the code to use loops without recursions.

Answer:

```
#include <stdio.h>
#define MAX 3

typedef struct {
    int n, from, to, temp;
} Record;
Record stack[MAX];
int top = -1;

void move_hanoi(int n, int from, int to, int temp);

int main(void) {
    move_hanoi(MAX, 1, 3, 2);
    return 0;
}

void move_hanoi(int n, int from, int to, int temp) {
    stack[++top] = (Record) {n, from, to, temp};
    while (top != -1) {
        Record record = stack[top--];
        if (record.n == 1 || record.temp == -1) {
            printf("Move disk %d from %d to %d\n", record.n, record.from,
record.to);
            continue;
        }
        stack[++top] = (Record) {record.n - 1, record.temp, record.to,
record.from};
        stack[++top] = (Record) {record.n, record.from, record.to, -1};
        stack[++top] = (Record) {record.n - 1, record.from, record.temp,
record.to};
    }
}
```

Local vs. Global Variables in Recursion

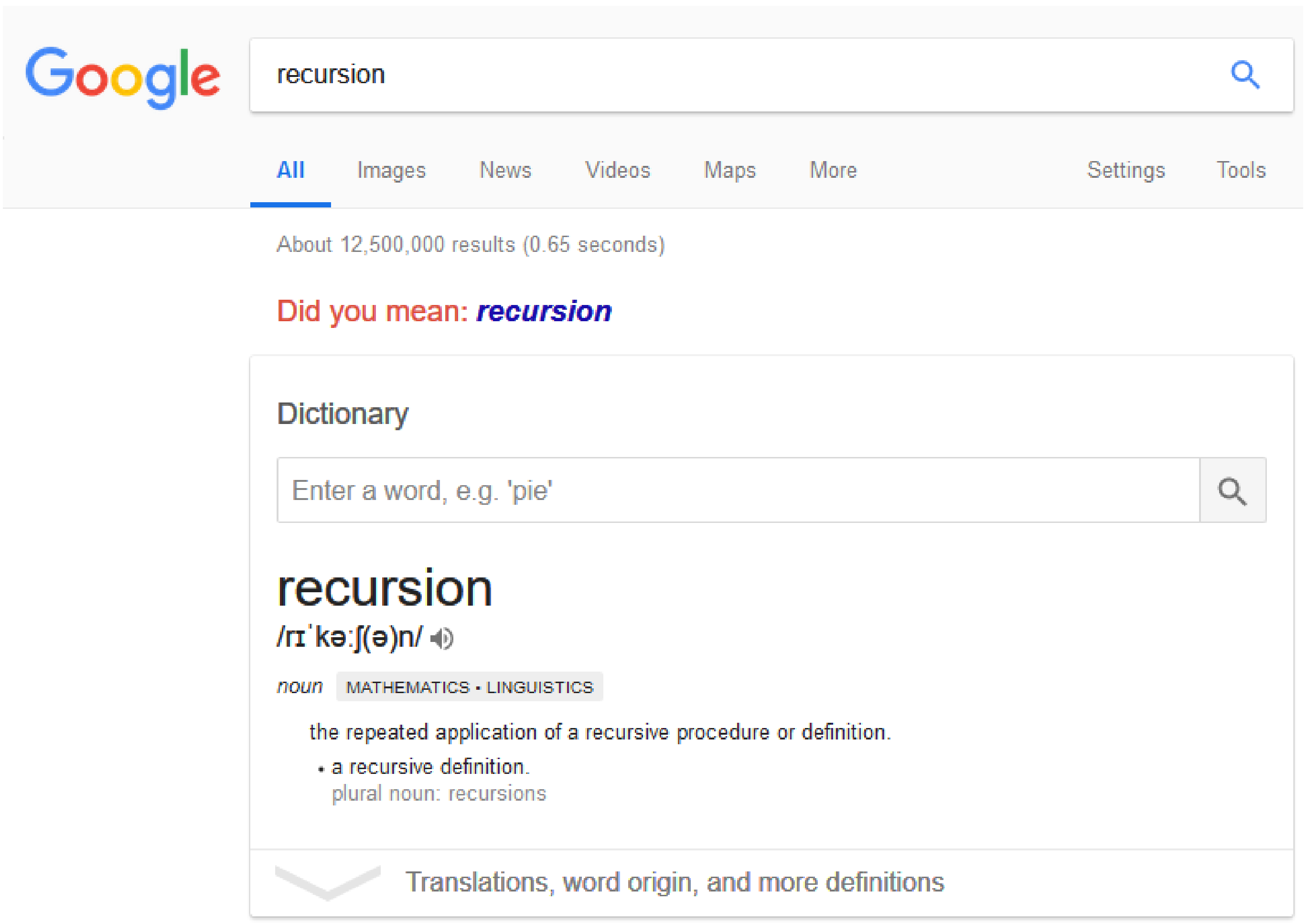
For recursion functions, it's better to only use local variables to avoid misconception. Only using local variables can keep your code clean and keep the structure of mathematical induction.

However, accessing global variables may make your code simpler in some cases, but use it in caution.

For the next assignment, we'll review binary representations.

Epilogue

You can try to learn and implement N-Queens, Binary Search, Merge Sort, Quick Sort, etc by yourself, following the 4 steps. The concept of recursion not only needs a lot of practice, but also requires some critical thinking. After your code got accepted, it's better to think your code all over again and review how it works exactly.


recursion

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Dictionary

Enter a word, e.g. 'pie'

recursion

/rɪˈkəːʃ(ə)n/

noun MATHEMATICS • LINGUISTICS

the repeated application of a recursive procedure or definition.

- a recursive definition.

  plural noun: recursions

Translations, word origin, and more definitions

Photo Credit: Screenshot of Google Search page.

If there's any typo, please discuss on iLMS or email `j3soon@gapp.nthu.edu.tw`, I appreciate your help.