# Finding the Missing Number

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## Problem

We are given all integers from 1 to $n$ except one missing number. The input consists of an integer $n$ on the first line and $n-1$ distinct integers (each in the range $1, \ldots, n$) on the second line. The task is to output the missing integer.

**Constraints.** $2 \leq n \leq 2 \cdot 10^5$.

## Solution (mathematical)

Let the given numbers be $a_1, a_2, \ldots, a_{n-1}$, each distinct and in $\{1, 2, \ldots, n\}$. Denote by $m$ the missing number. Observe that the sum of the first $n$ positive integers is

$$S_n = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

Since the list contains all numbers from 1 to $n$ except $m$, we have

$$a_1 + a_2 + \cdots + a_{n-1} = S_n - m.$$

Therefore the missing number $m$ can be recovered as

$$m \;=\; S_n - \sum_{i=1}^{n-1} a_i \;=\; \frac{n(n+1)}{2} - \sum_{i=1}^{n-1} a_i.$$

## Example

For the sample input

$$n = 5, \qquad \text{numbers } = 2, 3, 1, 5,$$

we have

$$S_5 = \frac{5 \cdot 6}{2} = 15, \qquad \sum a_i = 2 + 3 + 1 + 5 = 11,$$

hence the missing number is

$$m = 15 - 11 = 4,$$

which matches the required output.

## Optional Exercises

1. Prove the formula
$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$
You may use induction or a pairing argument.

2. Implement a program in a programming language of your choice that reads the input format described above and outputs the missing number.