PRACTICE EXAM #3

This review sheet only gives you the guideline on what kind of problems will be on the exam, the real exam will be very different from it. The final exam will cover chapter 1 to chapter 8 (up to section 8.4) except those I mentioned in the last email. There might be more practice problems listed in this sheet after the last lecture. Please come back to check it on May 2nd.

1. True or False? (Give a reason for your choice.)

(1.) The curve traced by $< \cos^2(t), \sin^2(t) >$ is a circle.
(2.) The plane $3x + 2y - z = 0$ is perpendicular to the line $x = 3t$, $y = 2t$, $z = -t$.
(3.) If the acceleration is constant, then the trajectory must be a straight line.
(4.) If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, then $\vec{b} = \vec{c}$
(5.) The cross product of two unit vectors is a unit vector.
(6.) If $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$ is C^2, then $\nabla \cdot (\nabla \times \vec{F}) = 0$

2. Calculate by changing to polar coordinates.

$$\int_0^1 \int_{-\sqrt{x-x^2}}^{\sqrt{x-x^2}} (x^2 + y^2)\, dydx$$

3. Let C be the unit circle in the xy plane oriented counterclockwise, let

$$\vec{F} = (-y^3 + \sin(\sin x), x^3 + \sin(\sin y)).$$

Compute the line integral $\int_C \vec{F} \cdot d\vec{s}$

4. Find the area of the domain D which is in the first quadrant and bounded by

$$xy = 1,\ xy = 3,\ y = x,\ y = 3x.$$

5. Let D be the $y-$simple domain in the xy plane defined by

$$D = \{(x, y) : 0 \le x \le 2\pi, 0 \le y \le 2 + \cos x\}.$$

Let $\vec{F} =< xe^x - y^2, \sin y >$, Evaluate $\int_C \vec{F} \cdot d\vec{s}$ using Green's theorem, where C is the boundary of D oriented counterclockwise.

6. Evaluate the integral $\int_0^4 \int_{\sqrt{y}}^2 \cos(1 + x^3)\, dxdy$.

7. Analyze the behavior of the following functions at the indicated points.

$$z = x^2 + y^2 + Cxy. \quad for\ (x, y) = (0, 0).$$

(Hint: Your answer may depend on the constant C.)

8. Let C be the oriented upper half unit circle with initial point $(-1, 0)$ and terminal point $(1, 0)$. Let

$$P = x + ye^{xy} + \sin x \text{ and } Q = x + xe^{xy} + \cos y.$$

There was a typo for Q function before.

Evaluate the path integral $\int_C P dx + Q dy$.

9. Let S be the upper half unit sphere with outward normal vectors. Compute the surface integral $\iint_S \vec{F} \cdot d\vec{S}$ where

$$\vec{F} =< x + xye^{yz}, xye^{xz}, 1 - e^{xz} - e^{yz} > .$$

10. Consider surface $z^3 = xyz - 4$.

(10a) What is the intersection of this surface with xy plane? with xz plane? with yz plane?

(10b) Find an equation of the tangent plane to this surface at the point $(2, 3, 2)$.

(10c) Use the tangent plane determined in (10b) to get an approximation near $z = 2$ to the equation $z^3 = (1.95)(3.05)z - 4$.

11. Find the area of the graph of the function $f(x, y) = \frac{2}{3}(x^{\frac{3}{2}} + y^{\frac{3}{2}})$ over the unit square $0 \le x \le 1$, $0 \le y \le 1$.

12. Evaluate $\iiint_D \exp[(x^2 + y^2 + z^2)^{\frac{3}{2}}]\, dxdydz$ where D is the region defined by $1 \le x^2 + y^2 + z^2 \le 2$ and $z \ge 0$.

13. Sketch the region for the integral $\int_0^1 \int_0^x \int_0^y f(x, y, z)\, dzdydx$ and interchange the order to $dydxdz$.