

# Exercise Set 1

AS.150.498: Modal Logic and Its Applications  
Johns Hopkins University, Spring 2017

Hard copy due in class on Feb 16. **[56 points total]**

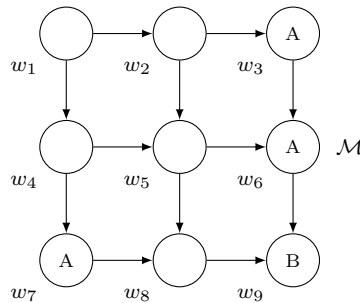
- 1.1** Consider a polymodal language with sentence letters  $E$  and  $M$  that designate ‘There is intelligent life on Earth’ and ‘There is intelligent life on Mars’ respectively, the Boolean constants  $\neg$  and  $\wedge$ , and the following modal operators:

G: Going forward in time, it will always be that...  
F: Going forward in time, it will sometime be that...  
H: Going backward in time, it will always be that...  
P: Going backward in time, it will sometime be that...

Translate the following English sentences into this formal language:  
**[2 points each]**

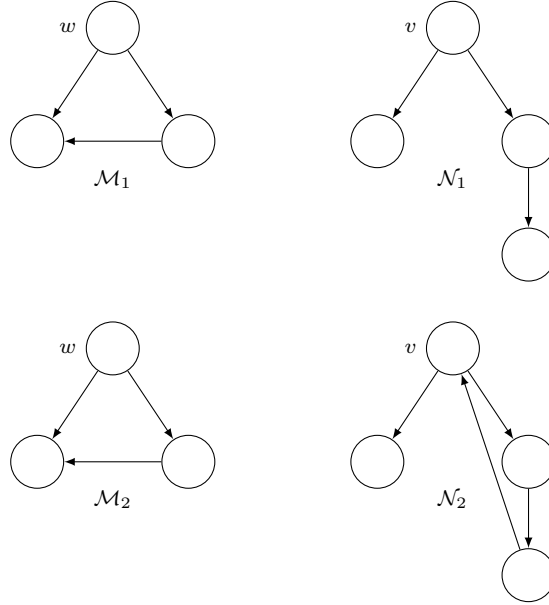
- There will be intelligent life on Mars.
- There was intelligent life on Earth but there was never intelligent life on Mars.
- There will never be intelligent life simultaneously on both Earth and Mars.
- There will have been intelligent life on Earth.

- 1.2** (From van Benthem [2010]) Consider the following Kripke model:



- In which worlds are the following sentences true? **[2 points each]**
  - $\Diamond B$
  - $\Diamond \Box B$
  - $\Diamond A$
  - $\Box \Diamond A$
- For each world, find a sentence that is true only at this world.  
**[1 point each]**

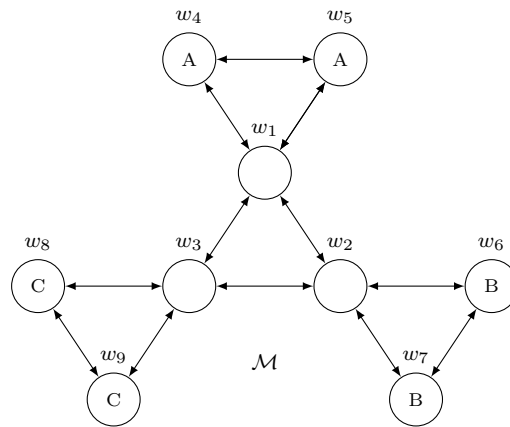
- 1.3** For each pair of pointed models, determine whether these models are bisimilar. Justify each of your answers by providing a bisimulation between the pointed models or by appealing to the Invariance Lemma. [4 points each]



- 1.4** Prove that this ‘difference’ operator is undefinable in  $\mathcal{L}$ : [8 points]

$$\llbracket \mathcal{D}\varphi \rrbracket_{\mathcal{M}}^w = T \quad \text{iff} \quad \exists v (w \neq v \wedge \llbracket \varphi \rrbracket_{\mathcal{M}}^v = T)$$

- 1.5** Illustrate the following:



- The submodel of  $\mathcal{M}$  generated from  $w_1$ . [5 points]
- The bisimulation contraction of  $\mathcal{M}$ . [5 points]
- The tree unraveling of the submodel from (a) around  $w_1$ . [5 points]