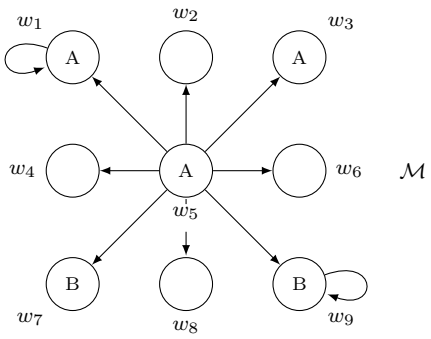
Exercise Set 2

AS.150.498: Modal Logic and Its Applications
Johns Hopkins University, Spring 2017

Hard copy due in class on Mar 7. **[48 points total]**

2.1 What is the filtration of $\mathcal{M}$ through $A \wedge \Diamond B$? **[8 points]**



2.2 Determine whether each of the following sentences is valid. Justify your answers by providing a proof in **K** or a pointed countermodel that falsifies the sentence. **[5 points each]**

a. $(\Box A \vee \Box B) \supset \Box(\Box A \vee \Box B)$

b. $\Diamond(A \supset B) \supset (\Box A \supset \Diamond B)$

c. $\Diamond(A \supset B) \vee \Box(B \supset A)$

d. $\Diamond\Box A \wedge \Diamond\Box\neg A$

e. $\Diamond\neg\bot \supset (\Box A \supset \Diamond A)$

2.3 Prove the following facts about any maximal **K**-consistent set Γ: **[5 points each]**

a. $\varphi \in \Gamma$ iff $\neg\varphi \notin \Gamma$.

b. If $\varphi \in \Gamma$ and $\{\varphi\} \vdash_{\mathbf{K}} \psi$, then $\psi \in \Gamma$.

c. $\varphi, \psi \in \Gamma$ iff $\varphi \wedge \psi \in \Gamma$.