Exercise Set 6

AS.150.498: Modal Logic and Its Applications
Johns Hopkins University, Spring 2017

Hard copy due in class on May 4. **[53 points total]**

6.1 Consider the class of frames of the form $\mathcal{F} = \langle \mathcal{W}, \{\mathcal{R}_a\}_{a \in \text{Agt}} \rangle$ where for each $a \in \text{Agt}$, $\mathcal{R}_a$ is reflexive and transitive—that is, $\forall w(w\mathcal{R}_a w)$ and $\forall w, v, u((w\mathcal{R}_a v \wedge v\mathcal{R}_a u) \supset w\mathcal{R}_a u)$. Which of the following sentences, if any, is valid on each frame in this class? Justify your answers. **[5 points each]**

a. $\text{E}_{\text{Agt}} A \supset \text{E}_{\text{Agt}}\text{E}_{\text{Agt}} A$

b. $\text{S}_{\text{Agt}} A \supset \text{S}_{\text{Agt}}\text{S}_{\text{Agt}} A$

c. $\text{D}_{\text{Agt}} A \supset \text{D}_{\text{Agt}}\text{D}_{\text{Agt}} A$

6.2 Agents a, b, and c are sitting around a table on which a playing card lies face down.

a. The agents each know that the card is one of the four Jacks but they do not know which one. Draw an epistemic model that represents the knowledge in this situation. **[6 points]**

b. Now suppose that agent a picks up the card and (without showing it to the other agents) looks at the Jack. He then returns the card face down to the table. Draw an epistemic model that represents the knowledge in this situation. **[6 points]**

c. Now suppose that agent a announces that he will tell agent b the color of the card. He then pulls b aside and tells him the color. Agent c sees this all happening but remains in the dark about the color of the card. Draw an epistemic model that represents the knowledge in this situation. **[6 points]**

6.3 Describe what will happen in these variations on the Muddy Children Puzzle. Draw sequences of epistemic models to represent how the knowledge state of each child changes as new information comes to light. **[10 points each]**

a. The mother first announces that at least one of the children is clean and then proceeds, as before, to repeatedly ask each of the children to step forward if they know whether they have mud on their forehead.

b. The mother first announces that child a is dirty and then proceeds, as before, to repeatedly ask each of the children to step forward if they know whether they have mud on their forehead.

6.4 Extra Credit Problem. Translate the following sentences into the language $\mathcal{L}_{PAL}$. Which of the sentences, if any, are true at the actual world @ in the initial epistemic model $\mathcal{M}$ for the Muddy Children Puzzle? Justify your answers by appealing to the truth clauses for the dynamic epistemic operators in $\mathcal{L}_{PAL}$. **[5 points each]**

a. After a true announcement that at least one of the children is dirty and at least one of the children is clean, a knows that he is dirty.

b. After a true announcement that at least one of the children is dirty and at least one of the children is clean, c knows that he is clean.

c. After a true announcement that all three children are dirty, all three children know that they are dirty.