Lecture I:

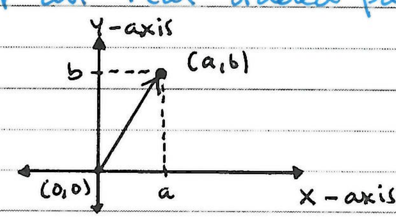
- Discuss course structure.
  - Introduce myself.
  - Talk about syllabus.
  -

- The goal of this course is to study calculus for real and vector-valued functions in 2 or 3 variables, extending the theory of differentiation & integration to this setting.

Vectors in $\mathbb{R}^2$, $\mathbb{R}^3$, and $\mathbb{R}^n$ $(n>3)$:

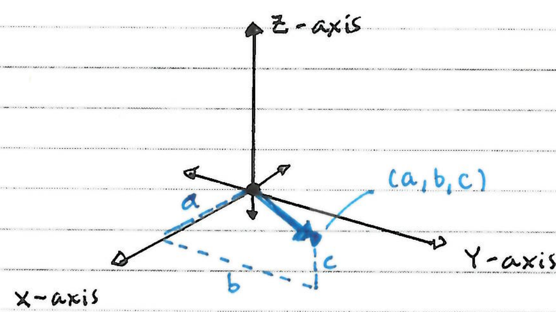
$\mathbb{R}^2$ = set of all real ordered pairs (a,b):

"Can represent Geometrically"



(a,b) = the vector with x-coordinate a and y-coordinate b.

$\mathbb{R}^3$ = set of all real ordered triples (a,b,c)

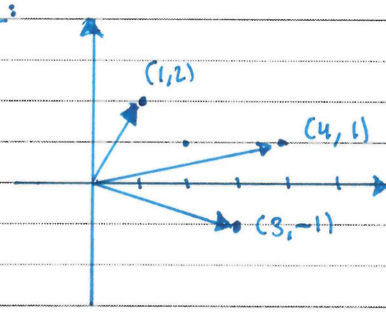


$n>3$, $\mathbb{R}^n$ = set of real ordered n-tuples $(a_1,\dots,a_n)$

operations: addition and scalar multiplication

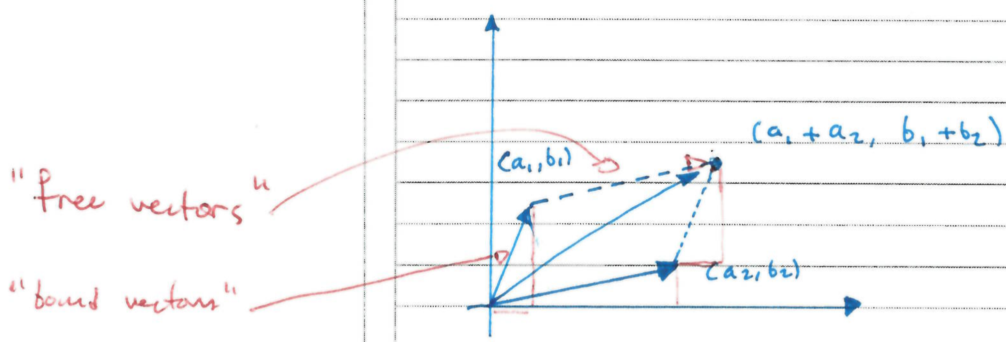
In $\mathbb{R}^2$: $(a_1, b_1) + (a_2, b_2) = (a_1 + a_2, b_1 + b_2)$

Example:



$(1,2) + (3,-1) = (4,1)$

Geometric interpretation of addition.



scalar multiplication:

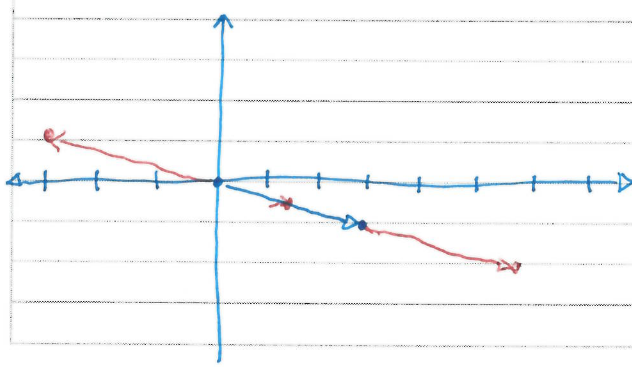
λ a real #

$$\lambda \cdot (a, b) = (\lambda a, \lambda b)$$

Examples: $2 \cdot (3, -1) = (6, -2)$

$\frac{1}{2} \cdot (3, -1) = \left(\frac{3}{2}, \frac{-1}{2}\right)$

$-1 \cdot (3, -1) = (-3, 1)$



~~Addition & scalar mult. in $\mathbb{R}^n$~~

"Basic properties"

Properties:

① λ_1, λ_2 scalars

$$(\lambda_1\lambda_2)\cdot(a, b) = (\lambda_1\lambda_2 a, \lambda_1\lambda_2 b)$$
$$= \lambda_1 \cdot (\lambda_2 a, \lambda_2 b)$$
$$= \lambda_1 \cdot (\lambda_2 \cdot (a, b)).$$

② $(\lambda_1 + \lambda_2)\cdot(a,b) = ((\lambda_1+\lambda_2)a, (\lambda_1+\lambda_2)b)$
$$= (\lambda_1 a + \lambda_2 a, \lambda_1 b + \lambda_2 b)$$
$$= (\lambda_1 a, \lambda_1 b) + (\lambda_2 a, \lambda_2 b)$$
$$= \lambda_1 \cdot (a, b) + \lambda_2 \cdot (a, b).$$

③ $\lambda\cdot[(a_1, b_1) + (a_2, b_2)]$
$$= \lambda\cdot(a_1 + a_2, b_1 + b_2)$$
$$= (\lambda(a_1+a_2), \lambda(b_1+b_2))$$

④ $\lambda\cdot(0,0) = (0,0)$

⑤ $0\cdot(a,b) = (0,0)$

⑥ $1\cdot(a,b) = (a,b)$.

Addition & scalar mult. in $\mathbb{R}^n$ $(n = 2, 3, 4, \ldots)$:

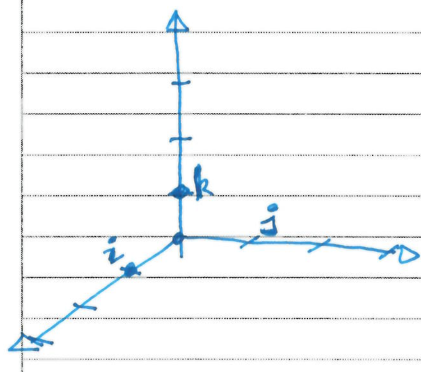
$$(a_1, \ldots, a_n) + (b_1, \ldots, b_n) \overset{\text{def}}{=} (a_1+b_1, \ldots, a_n+b_n).$$

$$\lambda\cdot(a_1, \ldots, a_n) = (\lambda a_1, \ldots, \lambda a_n).$$

Properties analogous to (1) – (6) above also hold in $\mathbb{R}^n$.

In $\mathbb{R}^3$, the geometric interpretation of addition & scalar mult. is analogous to the $\mathbb{R}^2$ situation.

The standard basis vectors in $\mathbb{R}^3$:



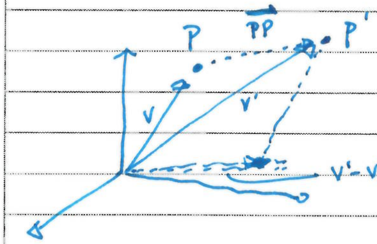
$\hat{i} = (1,0,0)$
$\hat{j} = (0,1,0)$
$\hat{k} = (0,0,1)$

$(a_1, a_2, a_3) = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$.

Example: $(-1,2,1) = (-1)\hat{i} + 2\hat{j} + 1\hat{k}$.

"Basis" means every vector in $\mathbb{R}^3$ can be written in a unique way as a linear combination of $\hat{i}, \hat{j}, \hat{k}$.

The free vector joining two points:



$$\overrightarrow{PP'} \overset{def}{=} v' - v,$$

since $v + (v' - v) = v'$.

components:

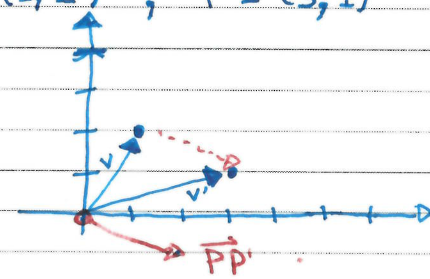
$v = (a_1, a_2, a_3)$ $\quad$ $v' = (b_1, b_2, b_3)$

$\Rightarrow \overrightarrow{PP'} = v' - v = (b_1 - a_1, b_2 - a_2, b_3 - a_3).$

(similar situation for $\mathbb{R}^2$).

Example

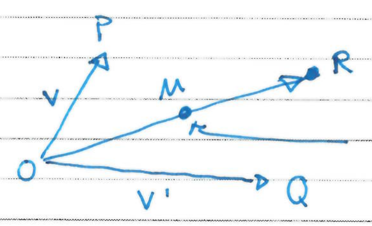
- $P = (1, 2)$, $P' = (3, 1)$

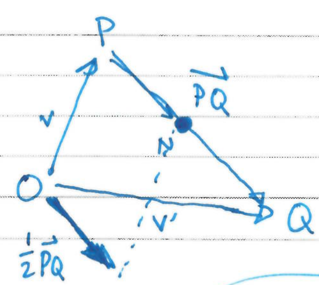


$\overrightarrow{PP'} = (3-1, 1-2) = (2, -1)$

Application: The diagonals of a parallelogram bisect each other.

why?





Put $N = v + \frac{1}{2}\overrightarrow{PQ}$

$N \overset{?}{=} M.$

$N = v + \frac{1}{2}(v' - v) = \frac{1}{2}v + \frac{1}{2}v'$

$= \frac{1}{2}(v + v')$. ✓

END LEC 1