Math 202 Practice Final Exam Spring 2011

I agree to complete this exam without unauthorized assistance.

Name: __________________________ Date: ____________________

Section:_____________ TA Name: ______________________________

You have **3 hours** to complete the exam which consists of 12 problems (problems 1 and 4 have two parts). This is plenty of time to calmly complete the exam, showing all work, with time enough to check your work. This is very important as **there will be limited part credit for a wrong answer**. That is **you will receive at most half credit if you have done most of the problem correctly and no credit otherwise**. Therefore it is extremely important to do the problems perfectly. All answers must be justified. Do each problem in the space provided (all of the problems can be done without excessive calculation). Show all your work in a neat, clear and logical manner and **circle your answers**. There are 16 pages (including this cover page) with 3 pages of scrap paper at the end. Check to see if any pages are missing.

Problem	Worth	Your Score	Problem	Worth	Your Score
1	20		7	15	
2	15		8	15	
3	15		9	15	
4	20		10	15	
5	15		11	15	
6	20		12	20	

1a. (10pts) Find the equation of plane passing through the points (2,0,0), (0,-1,0), (0,0,3).

b. (10pts) Find the distance from the origin to this plane.

2. (15pts) Let C be the curve in R^3 which is the image of $\vec{c}(t) =< t^2, t^2, 2t >$, $0 \leq t \leq 1$. Let $f(x, y, z) = z\frac{x^4+1}{y^4+1}$. Find $\int_C f \; ds$. Simplify your answer.

3. (15pts) Let $\vec{F}(x, y, z) =< 2x - y^2 + z, x - y - z^3 >$ be a differentiable mapping from R^3 to R^2 and let $\vec{c}(t)$ ibe a curve in R^3 with $\vec{c}(0) =< 1, 1, 2 >$, $\vec{c}'(0) =< 0, -1, 3 >$. Compute $D[\vec{F} \circ \vec{c}](0)$.

4. Let $h(x, y) = \frac{15}{x^2+2y^2+2}$ denote the height function of a mountain at the point (x,y) of the xy plane.

a. (10pts) What is the equation of the tangent plane at the climber's location?

b. (10pts) If a climber is at the point (1,1,3), and wants to descend most rapidly, in what horizontal (unit) direction should he go?

5. (15pts) Let $D = \{(x,y) : x^2 + \frac{y^2}{4} = 1,\ x \geq 0, y \geq 0\}$ be the part of the interior of the ellipse in the first quadrant and let C be the boundary of D oriented counter-clockwise. Evaluate directly $\int_C \vec{F} \cdot \vec{ds}$ where $\vec{F}(x,y) =< y, -x >$.

6. (20pts) Let $f(x, y) = 3x^2 + \frac{3}{2}y^2 + yx^2$. Find all critical points of f(x,y) and classify them as local max, local min or saddle point. Justify .

7. (15pts) Let $\vec{F} =< x+yz, y+xz, z+xy >$ and let $\vec{c}(t) =< \cos \pi t, 2\sin \pi t, (1+t)^2 >$, $0 \leq t \leq 1$. Calculate the line integral $\int_C \vec{F} \cdot \vec{ds}$.

8. (15pts) Let D be the y-simple domain in the xy plane defined by

$$D = \{(x, y) : 0 \leq x \leq 2\pi,\ 0 \leq y \leq 2 + \cos x\} \ .$$

Let $\vec{F}(x, y) =< xe^x - y^2, \sin y >$. Evaluate $\int_C \vec{F} \cdot d\vec{s}$ using Green's theorem, where C is the boundary of D oriented counterclockwise.

9. (15pts) Let W be the solid region consisting of the part of the unit ball $x^2+y^2+z^2 \leq 1$ in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$). Let S be the boundary of W oriented by the outward unit normal. Calculate

$$\int\int_S \vec{F} \cdot d\vec{S} \text{ where } \vec{F}(x,y,z) =< -xyz, y^2z + x, e^x > .$$

Hint: Use the divergence theorem to convert this to a triple integral and then use spherical coordinates.

10. (15 pts) Find the area of the part of the cylinder $x^2 + y^2 = 1$ that lies above the plane z=0 and below the surface $z = 4+x^2-y^2$. Hint: Recall $\cos^2\theta = \frac{1+\cos 2\theta}{2}$, $\sin^2\theta = \frac{1-\cos 2\theta}{2}$.

11. (15pts) Find the area of the graph of the function $f(x, y) = \frac{2}{3}(x^{\frac{3}{2}} + y^{\frac{3}{2}})$ over the unit square $0 \leq x \leq 1$, $0 \leq y \leq 1$.

12. (20 pts) Let $\vec{F} =< x^2+y-4, 3xy, 2xz+z^2$ and let S be the hemisphere $x^2+y^2+z^2 = 16$, $z \geq 0$ with S oriented by the upward normal. Use Stokes' theorem to calculate $\int\int_S \nabla \times \vec{F} \cdot d\vec{S}$.

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