

Mathematics 202

NAME _____

Practice Final Examination

April 30, 2009

This exam consists of **10** problems, numbered **1–10**. For partial credit **you must present your work clearly and understandably; no credit will be given for unsupported answers.**

If more space than allotted is needed, use the back of the previous page and make note of this. Please make your final answer clear by circling it. **Except for your name, please refrain from writing anywhere else on this page.**

During the exam, no calculators, computers, or other electronic aids are allowed, nor are you allowed to refer to any written notes or source material, nor to communicate with other students.

Please switch off all mobile phones

Check this examination booklet before you start. There should be 10 problems on 16 pages (including this one).

Question	Points	Score
1	20	
2	15	
3	20	
4	30	
5	20	
6	10	
7	20	
8	30	
9	15	
10	20	
Total	200	

1. [20 points]

Answer **TRUE** or **FALSE** to the following questions. Each question is worth 4 points. You do not need to provide an explanation of your answer.

(a) If $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ is C^2 then $\nabla \cdot \nabla f = 0$.

(b) If $\mathbf{F} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is C^2 then $\nabla \cdot (\nabla \times \mathbf{F}) = 0$.

(c) Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a C^2 function with a critical point at (x_0, y_0) . If the determinant of the Hessian of f at (x_0, y_0) is negative then f has a local maximum at (x_0, y_0) .

(d) A constant vector field is conservative.

(e) The iterated integral

$$\int_0^{2\pi} \int_0^2 \int_{2r}^4 r \, dz \, dr \, d\theta$$

represents the volume enclosed by the cone of height 4 and radius 2.

2. [15 points]

Compute the second-order Taylor expansion of the function

$$f(x, y) = \sin(xy) + \cos(xy)$$

about the point $(x_0, y_0) = (\frac{1}{4}\pi, 1)$.

3. [20 points]

(a) (5 points)

State Green's theorem.

(b) (15 points)

Let C be the unit circle in the xy plane oriented anti-clockwise, and let

$$\mathbf{F} = (-y^3 + \sin(\sin x), x^3 + \sin(\sin y))$$

Use Green's theorem to compute $\int_C \mathbf{F} \cdot d\mathbf{s}$.

4. [30 points]

(a) (5 points)

State Stokes' theorem.

(b) (25 points)

Verify Stokes' theorem for the case where $S = \{(x, y, z) : x^2 + y^2 = 25, 1 \leq z \leq 3\}$ is the curved surface of a cylinder with outwards pointing normal vector, and \mathbf{F} is the vector field $\mathbf{F}(x, y, z) = (-zy, 2zx, x^2)$.

Note: To get full credit for this question you must compute both sides of Stokes' theorem and show that they are equal.

(Extra space for problem 4)

(Extra space for problem 4)

5. [20 points]

Let D be the region bounded by $xy = 1$, $xy = 4$, $y = 1$ and $y = 2$.

(a) (5 points) Sketch the region D .

(b) (5 points) Consider the co-ordinate change $u = xy$, $v = y$. Solve for (x, y) in terms of (u, v) , compute the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$, and sketch the region in the uv -plane corresponding to the region D in the xy -plane.

(c) (10 points) Compute the double integral

$$\iint_D \frac{1}{x^2y^2 + 1} dx dy$$

6. [10 points] Let C be the curve given by

$$\mathbf{c}(t) = (2t, t^2, t^3/3) \quad \text{for } 0 \leq t \leq 1.$$

- (a) Find the total arc length of C . (Hint: $(t^2 + 2)^2 = t^4 + 4t^2 + 4$.)

7. [20 points]

Use Lagrange multipliers to find the dimensions of a solid cylinder (i.e. find the radius and height) with fixed surface area A and maximum volume.

Note: The surface area of the solid cylinder is the surface area of the curved side, plus the surface area of the two ends.

8. [30 points]

Let B be the inverted paraboloid $B = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq z \leq 4 - x^2 - y^2\}$, and let $\mathbf{F}(x, y, z) = (x, y, z - x^2)$.

(a) (5 points) State Gauss' theorem

(b) (5 points) Sketch the region B in \mathbb{R}^3 .

(c) (20 points) Verify Gauss' theorem for this case.

Note: To get full credit for this question you must compute both sides of Gauss' theorem and show that they are equal.

(Extra space for problem 8)

9. [15 points]

This question concerns the following double integral.

$$\int_0^1 \left(\int_y^1 e^{x^2} dx \right) dy$$

(a) (5 points) Sketch the region of integration.

(b) (10 points) Evaluate the integral by integrating with respect to y first.

10. [20 points]

The torus T can be parametrised by the function $\mathbf{X} : D \rightarrow \mathbb{R}^3$, where

$$\mathbf{X}(s, t) = ((R + \cos s) \cos t, (R + \cos s) \sin t, \sin s),$$

$R > 1$ is fixed, and D is the rectangle $[0, 2\pi] \times [0, 2\pi]$.

Show that the surface area is $(2\pi)^2 R$.