Exercise Set 5

AS.150.498: Modal Logic and Its Applications
Johns Hopkins University, Spring 2017

Hard copy due in class on Apr 25. **[66 points total]**

5.1 Provide a pointed neighborhood model for the deontic language $\mathcal{L}_d$ in which $\mathrm{O}A \wedge \mathrm{O}\neg A$ is true. **[4 points]**

5.2 Prove that the following arguments are not valid over neighborhood models for $\mathcal{L}_d$. **[4 points each]**

(P1) $\mathrm{O}A$

(C) $\mathrm{O}(A \vee B)$

(P1) $\mathrm{P}A$

(C) $\mathrm{P}(A \vee B)$

5.3 Determine whether each of the following deontic principles involving conditional obligation are valid. Justify your answer by appealing to the counterfactual-style truth conditions for $\mathrm{O}(\psi/\varphi)$ or by providing a countermodel. **[8 points each]**

a. Factual Detachment: $(\varphi \wedge \mathrm{O}(\psi/\varphi)) \supset \mathrm{O}(\psi/\neg\bot)$

b. Deontic Detachment: $(\mathrm{O}(\varphi/\neg\bot) \wedge \mathrm{O}(\psi/\varphi)) \supset \mathrm{O}(\psi/\neg\bot)$

5.4 Translate the following arguments into the language of counterfactuals $\mathcal{L}_{cf}$ and determine whether each argument is valid over frames with the properties of Well-Foundedness and Centering. Justify each of your answers by appealing to the truth conditions for $\Box\!\!\rightarrow$ and $\Diamond\!\!\rightarrow$ or by providing a countermodel. **[10 points each]**

a. If Dean is in his office then Yitzhak might be in his office. If Michael is in his office then Meredith might be in her office. Thus, if both Dean and Michael are in their offices, then Yitzhak and Meredith might be in their offices.

b. If it were not the case that both Nandi and Hilary teach at Hopkins, then no one would study ethics. Thus, if it were not the case that Nandi teaches at Hopkins, then no one would study ethics.

c. If Aristotle were still alive today, then Richard would be ecstatic. If Aristotle studied medicine instead of philosophy, then Aristotle would still be alive. Thus, if Aristotle studied medicine instead of philosophy, then Richard would be ecstatic.

5.5 Prove the following correspondence result: **[8 points]**

$\models_{\mathcal{F}} (\varphi \Box\!\!\rightarrow \psi) \supset (\varphi \supset \psi)$ if and only if every world-order $\leq_w$ in frame $\mathcal{F}$ is weakly centered.