Exercise Set 4

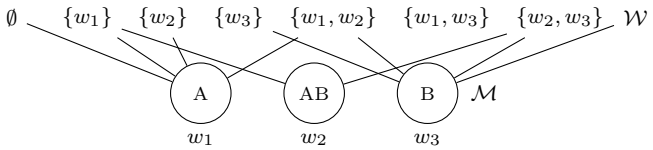
AS.150.498: Modal Logic and Its Applications
Johns Hopkins University, Spring 2017

Hard copy due in class on Apr 11. **[70 points total]**

4.1 Consider the following neighborhood model $\mathcal{M}$:

$\mathcal{W} = \{w_1, w_2, w_3\}$
$\mathcal{N}(w_1) = \{\emptyset, \{w_1\}, \{w_2\}, \{w_1, w_2\}\}$
$\mathcal{N}(w_2) = \{\{w_1\}, \{w_2, w_3\}\}$
$\mathcal{N}(w_3) = \{\{w_3\}, \{w_1, w_2\}, \{w_2, w_3\}, \mathcal{W}\}$
$\mathcal{V}(A, w_1) = \mathcal{V}(A, w_2) = T$, $\mathcal{V}(A, w_3) = F$
$\mathcal{V}(B, w_1) = F$, $\mathcal{V}(B, w_2) = \mathcal{V}(B, w_3) = T$

$\mathcal{M}$ can be represented as follows:



In which worlds are the following sentences true? **[3 points each]**

a. $\Box B$
b. $\Box\Box(A \wedge \neg B)$
c. $\Box(A \supset (B \supset A))$
d. $\Diamond(\Box A \vee \Box B)$
e. $\Box(\Box\Box(A \wedge \neg B) \wedge \Diamond(\Box A \vee \Box B))$

4.2 Prove the following facts: **[6 points each]**

a. The axiom (K) is *not* valid over all neighborhood frames.
b. The rule (Nec) does *not* preserve validity over all neighborhood frames.
c. The rule (RE) preserves validity over all neighborhood frames.

4.3 Prove the following correspondence results: **[8 points each]**

a. Given $\mathcal{F} = \langle \mathcal{W}, \mathcal{N} \rangle$, $\models_{\mathcal{F}} (\Box\varphi \wedge \Box\psi) \supset \Box(\varphi \wedge \psi)$ if and only if $\mathcal{N}$ is *closed under intersections*—that is, $X, Y \in \mathcal{N}(w) \supset X \cap Y \in \mathcal{N}(w)$.
b. Given $\mathcal{F} = \langle \mathcal{W}, \mathcal{N} \rangle$, $\models_{\mathcal{F}} \Box\neg\bot$ if and only if $\mathcal{N}$ *contains the unit*—that is, $\mathcal{W} \in \mathcal{N}(w)$.

4.4 Fill in the blank: **[6 points]**

$\models_{\mathcal{F}} \Box\varphi \supset \Box\Box\varphi$ if and only if _____.

4.5 Prove that **EMCN**=**K**. **[15 points]**