Exercise Set 3

AS.150.498: Modal Logic and Its Applications
Johns Hopkins University, Spring 2017

Hard copy due in class on Mar 28. [**50 points total**]

3.1 Show the following: [**5 points each**]

a. $\vdash_{\mathbf{K4}} (\Box A \vee \Box B) \supset \Box(\Box A \vee \Box B)$

b. $\vdash_{\mathbf{K5}} \Diamond\Box\Box A \supset \Box\Box A$

3.2 Prove the following correspondence results: [**8 points each**]

a. Given frame $\mathcal{F} = \langle \mathcal{W}, \mathcal{R} \rangle$, $\models_{\mathcal{F}} \Diamond\varphi \supset \Box\varphi$ if and only if $\mathcal{R}$ is a *partial function*—that is, $\forall w, v, u((w\mathcal{R}v \wedge w\mathcal{R}u) \supset v = u)$.

b. Given frame $\mathcal{F} = \langle \mathcal{W}, \mathcal{R} \rangle$, $\models_{\mathcal{F}} \Diamond\Box\varphi \supset \Box\Diamond\varphi$ if and only if $\mathcal{R}$ is *convergent*—that is, $\forall w, v, u((w\mathcal{R}v \wedge w\mathcal{R}u) \supset \exists x(v\mathcal{R}x \wedge u\mathcal{R}x))$.

3.3 The canonical model for **K4** is the model $\mathcal{M}^{\mathbf{K4}}$ where:

$\mathcal{W}^{\mathbf{K4}}$ is the set of all maximal **K4**-consistent sets
$\mathcal{R}^{\mathbf{K4}} = \{\langle \Gamma, \Delta \rangle : \text{for all } \varphi \in S_{\mathcal{L}}, \Box\varphi \in \Gamma \text{ only if } \varphi \in \Delta\}$
$\mathcal{V}^{\mathbf{K4}}(p, \Gamma) = T$ iff $p \in \Gamma$

Prove that $\mathcal{R}^{\mathbf{K4}}$ is transitive. [**8 points**]

3.4 Prove the following facts: [**8 points each**]

a. $\mathbf{K4} < \mathbf{KB5}$

b. $\mathbf{K5} < \mathbf{KB4}$

3.5 **Extra Credit Problem.** Prove that the following first-order property of $\mathcal{R}$ does not have a corresponding modal axiom—that is, prove that there is no $\varphi \in S_{\mathcal{L}}$ such that $\models_{\mathcal{F}} \varphi$ if and only if $\mathcal{R}$ has this property. [**6 points**]

Antisymmetry: $\forall w, v((w\mathcal{R}v \wedge v\mathcal{R}w) \supset w = v)$