Some Applications of Lauricella Hypergeometric Function F_A in Performance Analysis of Wireless Communications

Qinghua Shi and Y. Karasawa

***Abstract*—Lauricella hypergeometric function (LHF) F_A is often used in the performance analysis of wireless communications, especially diversity systems. In this letter, F_A is applied to solve two challenging problems: (1) a product of error functions averaged over generic fading channels and (2) error probability of BPSK modulation with equal-gain combining (EGC) in independent Nakagami-m fading channels. Closed-form expressions in terms of F_A are obtained. In addition, a novel and efficient approximation to F_A is presented.**

***Index Terms*—Error probability, Lauricella hypergeometric function, Equal gain combining.**

I. Introduction

LAURICELLA hypergeometric function (LHF) F_A is a multiple variable extension of the Gauss hypergeometric function [1]. F_A of order r is defined as [1]

$$F_A^{(r)}\left(a, b_1, \cdots, b_r; c_1, \cdots, c_r; x_1, \cdots, x_r\right) \triangleq \sum_{n_1,\cdots,n_r=0}^{\infty} (a)_{n_1+\cdots+n_r} \frac{(b_1)_{n_1}\cdots(b_r)_{n_r}}{(c_1)_{n_1}\cdots(c_r)_{n_r}} \frac{x_1^{n_1}}{n_1!}\cdots\frac{x_r^{n_r}}{n_r!} \quad (1)$$

where $|x_1|+\cdots+|x_r|<1$ and $(\beta)_n \triangleq \beta(\beta+1)\cdots(\beta+n-1)$ with $(\beta)_0 \triangleq 1$. It reduces to the Gauss hypergeometric function $F(a,b;c;x)$ [14, 9.100] when $r=1$ and Appell function $F_2\left(a, b_1, b_2; c_1, c_2; x_1, x_2\right)$ [14, 9.180.2] when $r=2$.

F_A has found various applications in the performance analysis of wireless communications, particularly when selection diversity or equal-gain combining (EGC) is employed [2]- [9]. Here we apply F_A to solve two challenging problems. First, we consider a product of N error functions averaged over generic fading channels. This problem can be encountered in the error performance analysis of binary modulation, AM, M-ary QAM, M-ary frequency-shift-keying, and differential encoded QPSK in fading channels [11]. In the literature [12], a product of up to $N=3$ Gaussian Q-functions has been addressed. Second, we attack EGC with N branches in independent, not necessarily identically distributed (i.n.d.) Nakagami-m fading, ending up with closed-form expressions in terms of F_A. EGC is notoriously difficult to deal with. Relevant closed-form results are available only for $N=2,3$ [2], [13]. Recently, a new closed-form expression using another LHF, F_B, has been derived in [8, (28)]. This expression is simple, but its numerical evaluation is intricate in the sense that multiple integration appears to be unavoidable.

Manuscript received December 4, 2011. The associate editor coordinating the review of this letter and approving it for publication was Y. Chen.

The authors are with the Department of Electronic Engineering, University of Electro-Communications, 1-5-1 Chofugaoka, Chofu-shi, Tokyo 182-8585, Japan (e-mail: {qhshi, karasawa}@ee.uec.ac.jp).

Digital Object Identifier 10.1109/LCOMM.2012.030912.112454

As a complicated multivariate special function, F_A is in general difficult to compute. This issue has received little attention so far. According to its definition, F_A of order r can be directly computed from its series expansion, but this method is practical only for small r. Fortunately, one nice feature of F_A is that it has a Laplace-type single-integral representation given by [1, (35), p. 285]

$$F_A^{(r)} = \frac{1}{\Gamma(a)} \int_0^{\infty} e^{-t} t^{a-1} \prod_{i=1}^{r} M\left(b_i, c_i, x_i t\right) dt \quad (2)$$

where $\Gamma(\cdot)$ denotes Gamma function and $M(\cdot)$ confluent hypergeometric function [14]. It is clear that F_A can be calculated via numerical integration. However, when the integrand of F_A has a heavy tail, special care is needed and numerical integration can be time-consuming. In [9], Gauss-Laguerre quadrature (GLQ) is applied to evaluate F_A. We will show that the accuracy of GLQ is not sufficient when general parameters are considered. Based on the semi-infinite Gauss-Hermite quadrature (SI-GHQ) [10], we will present an approximation to F_A with higher accuracy for a broad range of parameters.

II. Product of Error Functions Averaged Over Fading Channels

Write a product of N error functions as

$$\mathcal{G}(\gamma) = \prod_{i=1}^{N} \operatorname{erf}\left(\sqrt{a_i \gamma}\right) \quad (3)$$

where γ denotes the instantaneous SNR and $\{a_i\}_{i=1}^N$ are parameters determined by modulation in use. We assume that γ is a random variable with unit average power and is described by various fading probability density functions (PDFs). By using [15, 7.1.21], (3) can be expressed as

$$\mathcal{G}(\gamma) = B\gamma^{\frac{N}{2}} \exp\left(-\gamma \sum_{i=1}^{N} a_i\right) \prod_{i=1}^{N} M\left(1, \frac{3}{2}; a_i\gamma\right) \quad (4)$$

where $B \triangleq \left(\frac{2}{\sqrt{\pi}}\right)^N \left(\prod_{i=1}^N \sqrt{a_i}\right)$.

A. $\eta-\mu$ Distribution

$\eta-\mu$ distribution represents a general fading channel model, including Rayleigh, Nakagami-m, Nakagami-q (Hoyt), one-sided Gaussian fading as special cases [16]. Under $\eta-\mu$ fading, the PDF of γ is given by [16, (26)]

$$\begin{aligned} p_\gamma^I(\gamma) &= \frac{2\sqrt{\pi} h^\mu \mu^{\mu+\frac{1}{2}}}{\Gamma(\mu) H^{\mu-\frac{1}{2}}} \gamma^{\mu-\frac{1}{2}} e^{-2\mu h\gamma} I_{\mu-\frac{1}{2}}\left(2\mu H\gamma\right) \\ &= \frac{2\sqrt{\pi} h^\mu \mu^{2\mu}}{\Gamma(\mu)\Gamma(\mu+\frac{1}{2})} \gamma^{2\mu-1} e^{-2\mu(h+H)\gamma} M\left(\mu, 2\mu; 4\mu H\gamma\right) \end{aligned} \quad (5)$$

where $\eta > 0$ and $\mu > 0$ are two parameters, $h = (2 + \eta^{-1} + \eta)/4$, $H = (\eta^{-1} - \eta)/4$, and [14, 9.238.2] has been invoked. The change of variables $\gamma = \frac{t}{C_1}$ with $C_1 = 2\mu(h + H) + \sum_{i=1}^{N} a_i$ allows us to derive

$$\begin{aligned}\int_0^\infty \mathcal{G}(\gamma)p_\gamma^I(\gamma)d\gamma &= \frac{2\sqrt{\pi}h^\mu\mu^{2\mu}B}{\Gamma(\mu)\Gamma(\mu+\frac{1}{2})}\int_0^\infty e^{-t}\left(\frac{t}{C_1}\right)^{\frac{N}{2}+2\mu-1} \\ &\quad M\left(\mu, 2\mu, 4\mu H\frac{t}{C_1}\right)\prod_{i=1}^{N} M\left(1, \frac{3}{2}; \frac{a_i t}{C_1}\right)\frac{dt}{C_1} \\ &= \frac{2\sqrt{\pi}h^\mu\mu^{2\mu}B}{\Gamma(\mu)\Gamma(\mu+\frac{1}{2})}\frac{\Gamma(\frac{N}{2}+2\mu)}{C_1^{N/2+2\mu}}F_A^{(N+1)}\Big(\frac{N}{2}+2\mu, \mu, [1]_N; \\ &\quad 2\mu, [1.5]_N; \frac{4\mu H}{C_1}, \frac{a_1}{C_1}, \cdots, \frac{a_N}{C_1}\Big) \qquad (6)\end{aligned}$$

where $[x]_N$ means $\underbrace{x, x, \cdots, x}_{N \text{ times}}$. In the following, this notation is heavily used for F_A.

B. Rician Shadowed Distribution

Rician shadowed distribution can model both fading and shadowing effects [11]. In particular, shadowing is described by a Gamma distribution. Defining the Rician factor K as the ratio of the power of the specular component to the power of scattered components, and assuming the total average power is normalized to 1, we can obtain from [11, (2.67)]

$$p_\gamma^{II}(\gamma) = \frac{(1+K)m^m}{(m+K)^m}e^{-(1+K)\gamma}M\left(m, 1; \frac{K(K+1)}{m+K}\gamma\right) \quad (7)$$

where $m \geq 0$ reflects shadowing. Similarly, the change of variable $\gamma = \frac{t}{C_2}$ with $C_2 = K + 1 + \sum_{i=1}^{N} a_i$ yields

$$\begin{aligned}\int_0^\infty \mathcal{G}(\gamma)p_\gamma^{II}(\gamma)d\gamma &= \frac{m^m(1+K)B}{(m+K)^m}\int_0^\infty e^{-t}\left(\frac{t}{C_2}\right)^{N/2} \\ &\quad M\left(m, 1; \frac{K(K+1)}{m+K}\frac{t}{C_2}\right)\prod_{i=1}^{N} M\left(1, \frac{3}{2}; \frac{a_i t}{C_2}\right)\frac{dt}{C_2} \\ &= \frac{m^m(1+K)B}{(m+K)^m}\frac{\Gamma(\frac{N}{2}+1)}{C_2^{N/2+1}}F_A^{(N+1)}\Big(\frac{N}{2}+1, m, [1]_N; \\ &\quad 1, [1.5]_N; \frac{(K+1)K}{(K+m)C_2}, \frac{a_1}{C_2}, \cdots, \frac{a_N}{C_2}\Big). \qquad (8)\end{aligned}$$

III. Equal-Gain Combining

Consider BPSK modulation and without loss of generality, assume +1 is transmitted. The decision variable for a coherent receiver with N-branch EGC can be expressed as

$$\Delta = \sum_{n=1}^{N}\beta_n + \sum_{n=1}^{N}\eta_n \qquad (9)$$

where β_n denotes a fading amplitude on the nth branch and η_n is the corresponding Gaussian noise with zero mean and variance $\frac{N_0}{2}$. We assume β_n is subject to i.n.d. Nakagami-m fading characterized by PDF

$$f_{\beta_n}(x) = \frac{2}{\Gamma(m_n)}\left(\frac{m_n}{\Omega_n}\right)^{m_n}x^{2m_n-1}e^{-\frac{m_n}{\Omega_n}x^2}, 1 \leq n \leq N \quad (10)$$

where $m_n \geq 0.5$ is the fading severity parameter and Ω_n the average power of β_n.

Assuming $\{\beta_n\}_{n=1}^N$ and $\{\eta_n\}_{n=1}^N$ are all independent of each other, the characteristic function (CF) of the decision variable Δ can be written as

$$\Phi_\Delta(\omega) = \prod_{n=1}^{N}\Phi_{\beta_n}(\omega)\Phi_{\eta_n}(\omega) \qquad (11)$$

where the CF of β_n and the CF of η_n are given by [2]

$$\begin{aligned}\Phi_{\beta_n}(\omega) &= \exp\left(-\frac{\Omega_n\omega^2}{4m_n}\right)\Bigg\{M\left(\frac{1}{2}-m_n, \frac{1}{2}; \frac{\Omega_n\omega^2}{4m_n}\right) \\ &\quad + j\omega\frac{\Gamma(m_n+\frac{1}{2})}{\Gamma(m_n)}\sqrt{\frac{\Omega_n}{m_n}}M\left(1-m_n, \frac{3}{2}; \frac{\Omega_n\omega^2}{4m_n}\right)\Bigg\}, \quad (12) \\ \Phi_{\eta_n}(\omega) &= \exp\left(-\frac{\omega^2}{4}N_0\right). \qquad (13)\end{aligned}$$

The bit error rate (BER) can be calculated by [13, (14)-(15)]

$$P_e = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\{\Phi_\Delta(\omega)\}}{\omega}d\omega \qquad (14)$$

where Im(x) denotes the imaginary part of x.

To proceed with (11) and (14) efficiently, it is necessary to expand a product of N complex numbers explicitly. This can be down by the following expression.

$$\prod_{n=1}^{N}(A_n + jB_n) = \sum_{l=0}^{N} j^{\,l} \sum_{S_B^{(l)} \subseteq S_N}\left(\prod_{k \in S_A^{(N-l)}} A_k\right)\left(\prod_{i \in S_B^{(l)}} B_i\right) \qquad (15)$$

where $\{A_n\}_{n=1}^N$ and $\{B_n\}_{n=1}^N$ are real numbers, $S_N \triangleq \{1, 2, \cdots, N\}$, $S_A^{(N-l)} \triangleq \{k_1, k_2, \cdots, k_{N-l}\}$, $S_B^{(l)} \triangleq \{i_1, i_2, \cdots, i_l\}$, and $S_A^{(N-l)} = S_N \setminus S_B^{(l)}$, $0 \leq l \leq N$. Since $S_B^{(l)}$ is a subset of S_N, i.e., $S_B^{(l)} \subseteq S_N$, the number of choices for $S_B^{(l)}$ is $\binom{N}{l}$. It is worth noting that $S_A^{(N-l)}$ is completely determined by $S_B^{(l)}$ because $S_A^{(N-l)} \bigcup S_B^{(l)} = S_N, 0 \leq l \leq N$.

Using a change of variables $\frac{\Omega_T}{4}\omega^2 = t$ ($\Omega_T = N_0 N + \sum_{n=1}^{N}\frac{\Omega_n}{m_n}$) in (14), we obtain (16) (at the top of next page), which is a BER expression applicable for general i.n.d. Nakagami-m fading channels. From (16), it is convenient to consider some special cases.

- i.i.d. Nakagami-m fading
  In this case, all diversity branches have the same fading parameter m and average power Ω. (16) reduces to (17).
- Rayleigh fading
  Under Rayleigh fading, (16) can be simplified to (18).

Since for $N = 2, 3$, closed-form expressions are available, we intend to verify (16) by examining the case of $N = 3$.

- $N = 3$, Rayleigh fading
  In this case, l can be 1 or 3. For $l = 1$, $S_B^{(1)} = \{1\}$, $\{2\}$, or $\{3\}$. For $l = 3$, $S_B^{(3)} = \{1, 2, 3\}$. It follows from (18)

$$P_e^{i.n.d.} = \frac{1}{2} - \frac{1}{\pi} \sum_{l=1,\ l\ odd}^{N} (-1)^{\frac{l-1}{2}} 2^{l-1} \Gamma\left(\frac{l}{2}\right) \sum_{S_B^{(l)} \subseteq S_N} \left(\prod_{i \in S_B^{(l)}} \frac{\Gamma\left(m_i + \frac{1}{2}\right)}{\Gamma(m_i)} \sqrt{\frac{\frac{\Omega_i}{m_i}}{\Omega_T}} \right)$$
$$F_A^{(N)}\left(\frac{l}{2}, \frac{1}{2} - m_{k_1}, \cdots, \frac{1}{2} - m_{k_{N-l}}, 1 - m_{i_1}, \cdots, 1 - m_{i_l}; \left[\frac{1}{2}\right]_{N-l}, \left[\frac{3}{2}\right]_l; \frac{\frac{\Omega_{k_1}}{m_{k_1}}}{\Omega_T}, \cdots, \frac{\frac{\Omega_{k_{N-l}}}{m_{k_{N-l}}}}{\Omega_T}, \frac{\frac{\Omega_{i_1}}{m_{i_1}}}{\Omega_T}, \cdots, \frac{\frac{\Omega_{i_l}}{m_{i_l}}}{\Omega_T} \right) \quad (16)$$

$$P_e^{i.i.d.} = \frac{1}{2} - \frac{1}{\pi} \sum_{l\ odd}^{N} (-1)^{\frac{l-1}{2}} 2^{l-1} \binom{N}{l} \Gamma\left(\frac{l}{2}\right) \left(\frac{\Gamma\left(m + \frac{1}{2}\right)}{\Gamma(m)} \sqrt{\frac{\frac{\Omega}{m}}{\Omega_T}} \right)^l F_A^{(N)}\left(\frac{l}{2}, \left[\frac{1}{2} - m\right]_{N-l}, [1-m]_l; \left[\frac{1}{2}\right]_{N-l}, \left[\frac{3}{2}\right]_l; \left[\frac{\frac{\Omega}{m}}{\Omega_T}\right]_N \right) \quad (17)$$

$$P_e^{Ray} = \frac{1}{2} - \frac{1}{\pi} \sum_{l\ odd}^{N} (-1)^{\frac{l-1}{2}} 2^{l-1} \Gamma\left(\frac{l}{2}\right) \sum_{S_B^{(l)} \subseteq S_N} \left(\prod_{i \in S_B^{(l)}} \frac{\sqrt{\pi}}{2} \sqrt{\frac{\Omega_i}{\Omega_T}} \right) F_A^{(N-l)}\left(\frac{l}{2}, \left[-\frac{1}{2}\right]_{N-l}; \left[\frac{1}{2}\right]_{N-l}; \frac{\Omega_{k_1}}{\Omega_T}, \cdots, \frac{\Omega_{k_{N-l}}}{\Omega_T} \right) \quad (18)$$

that

$$P_e = \frac{1}{2} - \frac{1}{\pi}(\mathcal{I}_1 + \mathcal{I}_2 + \mathcal{I}_3 + \mathcal{I}_4) \quad (19)$$

$$\mathcal{I}_1 = \frac{\pi}{2}\sqrt{\frac{\Omega_1}{\Omega_T}} F_2\left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}; \frac{1}{2}, \frac{1}{2}; \frac{\Omega_2}{\Omega_T}, \frac{\Omega_3}{\Omega_T}\right), \quad (20)$$

$$\mathcal{I}_2 = \frac{\pi}{2}\sqrt{\frac{\Omega_2}{\Omega_T}} F_2\left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}; \frac{1}{2}, \frac{1}{2}; \frac{\Omega_1}{\Omega_T}, \frac{\Omega_3}{\Omega_T}\right), \quad (21)$$

$$\mathcal{I}_3 = \frac{\pi}{2}\sqrt{\frac{\Omega_3}{\Omega_T}} F_2\left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}; \frac{1}{2}, \frac{1}{2}; \frac{\Omega_1}{\Omega_T}, \frac{\Omega_2}{\Omega_T}\right), \quad (22)$$

$$\mathcal{I}_4 = -\frac{\pi^2}{4}\sqrt{\frac{\Omega_1}{\Omega_T}\frac{\Omega_2}{\Omega_T}\frac{\Omega_3}{\Omega_T}} \quad (23)$$

where $\Omega_T = 3N_0 + \Omega_1 + \Omega_2 + \Omega_3$. Using the reduction formula [14, 9.182.3], we can express the Appell function F_2 in terms of the Gauss hypergeometric function. Then (19)-(23) reduce to [13, (21)-(22)] as expected.

IV. Numerical Results

A. Evaluation of F_A

Apart from numerical integration of (2), F_A can readily be computed via GLQ, as suggested in [9, (44)]. Based on SI-GHQ, we present a novel approximation to F_A. Specifically, using the Kummer transformation [15, 13.1.27] and the change of variables $t = \frac{y^2}{C}$ with $C \triangleq 1 - \sum_{i=1}^r x_i$ in (2), we get

$$F_A^{(r)} = \frac{2}{\Gamma(a)} \int_0^\infty e^{-y^2} \frac{y^{2a-1}}{C^a} \prod_{i=1}^r M\left(c_i - b_i, c_i, -\frac{x_i}{C} y^2\right) dy. (24)$$

Note that in (2), without loss of generality [1], we assume $x_i > 0, 1 \le i \le N$. After the Kummer transformation is performed, the confluent hypergeometric functions in (24) now do not possess any exponential factor as $y \to \infty$ [15, 13.1.5]. This is numerically desirable.

According to [10], an L-point SI-GHQ can be described by

$$\int_0^\infty e^{-x^2} f(x) dx \approx \sum_{l=1}^L \omega_l f(\xi_l), \quad (25)$$

[1] In case of $x_{i'} < 0, 1 \le i' \le N$, the Kummer transformation is not needed for the corresponding confluent hypergeometric function.

TABLE I
Relative error of approximate $F_A^{(r)}$ for $\{b_i < c_i\}_{i=1}^r$
($r = 5, L = 15;\ b_i = 1, c_i = 2, x_i = \frac{1}{6}, i = 1, 2, 3, 4, 5$).

	$F_A^{(r)}$ Numerical Integration	Relative Error GLQ	Relative Error SI-GHQ
$a = 0.5$	1.327314067459637e+000	9.44e-002	8.37e-009
$a = 1.0$	1.779045213385593e+000	2.03e-009	5.74e-009
$a = 1.5$	2.409187299291844e+000	8.14e-004	4.92e-009
$a = 2.0$	3.298137115887973e+000	7.14e-008	5.73e-008
$a = 3.0$	6.401011402744171e+000	1.19e-006	1.70e-007
$a = 4.0$	1.307089550859672e+001	1.26e-005	5.62e-008
$a = 5.0$	2.822899713752880e+001	9.39e-005	5.94e-007
$a = 6.0$	6.479999999999996e+001	5.26e-004	1.04e-006
$a = 8.0$	4.162319999999994e+002	8.05e-003	5.92e-007
$a = 10.0$	3.505630679999993e+003	5.55e-002	9.69e-008

where the weights $\{\omega_l\}_{l=1}^L$ and abscissas $\{\xi_l\}_{l=1}^L$ are real numbers, given in Table II of [10]. Applying (25) to (24) yields

$$F_A^{(r)} \approx \frac{2}{C^a \Gamma(a)} \sum_{l=1}^L \omega_l \xi_l^{2a-1} \prod_{i=1}^r M\left(c_i - b_i, c_i, -\frac{x_i}{C}\xi_l^2\right). \quad (26)$$

The motivation behind this approximation is twofold: (1) the Kummer transformation allows us to extract an exponential factor from a confluent hypergeometric function, which asymptotically dominates the confluent hypergeometric function and is desirably included in the weight function e^{-x^2}; (2) SI-GHQ is more efficient than GLQ in convergence speed because the weight function e^{-x^2} (for SI-GHQ) decreases to zero much faster than e^{-x} (for GLQ) does.

The accuracy of (26) in terms of relative error is examined in Table I for $\{b_i < c_i\}_{i=1}^r$ and Table II for $\{b_i > c_i\}_{i=1}^r$. $L = 15$ is adopted for both GLQ and SI-GHQ. It can be seen that for the parameters considered, SI-GHQ shows a higher and more consistent accuracy than GLQ, particularly when a becomes relatively large or in the case of $\{b_i > c_i\}_{i=1}^r$. Note that SI-GHQ and GLQ have similar computational complexity.

B. Error Rate Performance

16-QAM in η-μ fading is considered first. From [11, (8.10)], the symbol error rate (SER) of Q-ary QAM, conditioned on the fading amplitude x, is given by

$$P_e|_x = 2q - q^2 - 2q(1-q)\mathrm{erf}(cx) - q^2 \mathrm{erf}^2(cx) \quad (27)$$

TABLE II
RELATIVE ERROR OF APPROXIMATE $F_A^{(r)}$ FOR $\{b_i > c_i\}_{i=1}^r$
$(r = 5, L = 15;\ b_i = 2, c_i = 1, x_i = \frac{1}{6}, i = 1, 2, 3, 4, 5)$.

	$F_A^{(r)}$ Numerical Integration	Relative Error GLQ	SI-GHQ
$a = 0.5$	2.255826959872377e+002	3.72e-002	5.40e-011
$a = 1.0$	1.955999999999995e+003	6.06e-002	5.81e-016
$a = 1.5$	1.171943058123716e+004	9.25e-002	2.79e-015
$a = 2.0$	5.871599999999982e+004	1.33e-001	4.09e-015
$a = 3.0$	1.092095999999996e+006	2.39e-001	5.54e-015
$a = 4.0$	1.599393599999993e+007	3.70e-001	4.89e-015
$a = 5.0$	2.017560959999991e+008	5.12e-001	2.07e-015
$a = 6.0$	2.294122175999989e+009	6.46e-001	1.25e-015
$a = 8.0$	2.395149212159986e+011	8.48e-001	9.30e-015
$a = 10.0$	2.058214211481586e+013	9.50e-001	1.48e-014

where $q = 1 - \frac{1}{\sqrt{Q}}, c = \sqrt{\frac{1.5}{Q-1}\frac{E_s}{N_0}}$, and $\frac{E_s}{N_0}$ denotes the SNR. The average SER of 16-QAM in η-μ fading is shown in Fig. 1, where numerical integration is performed by the MATLAB function "quadl". It can be seen that SER results evaluated via (26) agree well with the results obtained from numerical integration. This observation suggests that our approximation (26) is accurate in evaluating F_A.

In Fig. 2, we show BER performance of EGC with N branches in i.n.d. Nakagami-m fading channels. The good agreement between analytical and simulation results demonstrates that the general expression (16) is correct and the approximation formula (26) is accurate enough.

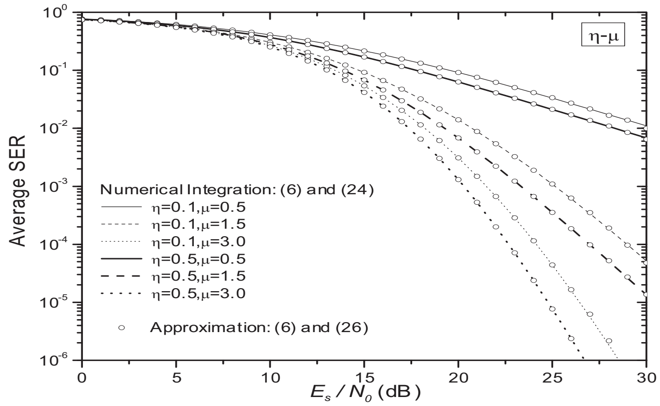

Fig. 1. SER versus SNR for 16-QAM in η-μ fading ($L = 15$).

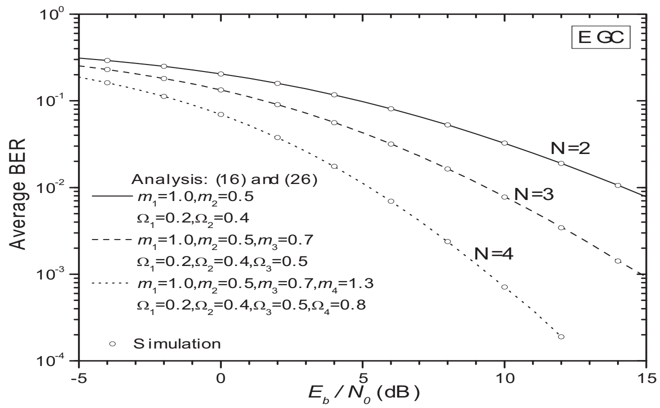

Fig. 2. BER of BPSK with EGC in Nakagami-m fading channels ($L = 15$).

V. CONCLUSION

We have applied F_A to solve two difficult problems: (1) a product of error functions averaged over generalized fading channels and (2) error probability of BPSK modulation with EGC in i.n.d. Nakagami-m fading channels. In both cases, we were able to obtain closed-form expressions in terms of F_A. In the case of EGC, our closed-form expressions include existing results as special cases. Moreover, based on SI-GHQ and Kummer transformation, an accurate approximation to F_A was presented.

REFERENCES

[1] H. M. Srivastava and P. W. Karlsson, *Multiple Gauss Hypergeometric Series*. E. Horwood, 1985.

[2] A. Annamalai, C. Tellambura, and V. K. Bhargava, "Equal-gain diversity receiver performance in wireless channels," *IEEE Trans. Commun.*, vol. 48, no. 10, pp. 1732–1745, Oct. 2000.

[3] O. C. Ugweje, "Selection diversity for wireless communications in Nakagami-fading with arbitrary parameters," *IEEE Trans. Veh. Technol.*, vol. 50, no. 6, pp. 1437–1448, Nov. 2001.

[4] R. Annavajjala, A. Chockalingam, and L. B. Milstein, "Performance analysis of coded communication systems on Nakagami fading channels with selection combining diversity," *IIEEE Trans. Commun.*, vol. 52, no. 7, pp. 1214–1220, July 2004.

[5] P. R. Sahu and A. K. Chaturvedi, "Performance analysis of predetection EGC in exponentially correlated Nakagami-m fading channel," *IEEE Trans. Wireless Commun.*, vol. 5, no. 7, pp. 1634–1638, July 2006.

[6] Z. Du, J. Cheng, and N. C. Beaulieu, "Accurate error-rate performance analysis of OFDM on frequency-selective Nakagami-m fading channels," *IEEE Trans. Commun.*, vol. 54, no. 2, pp. 319–328, Feb. 2006.

[7] R. M. Radaydeh and M. M. Matalgah, "Results for infinite integrals involving higher-order powers of the Gaussian Q-function with application to average," *IEEE Trans. Wireless Commun.*, vol. 7, no. 3, pp. 793–798, Mar. 2008.

[8] V. A. Aalo and G. P. Efthymoglou, "On the MGF and BER of linear diversity schemes in Nakagami fading channels with arbitrary parameters," in *Proc. 2009 IEEE VTC – Spring*.

[9] J. M. Romero-Jerez and A. J. Goldsmith, "Performance of multichannel reception with transmit antenna selection in arbitrarily distributed Nagakami fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 2006–2013, Apr. 2009.

[10] N. M. Steen, G. D. Byrne, and E. M. Gelbard, "Gaussian quadratures for the integrals $\int_0^\infty e^{-x^2}f(x)dx$ and $\int_0^b e^{-x^2}f(x)dx$," *Mathematics of Computation*, vol. 23, no. 107, pp. 661–671, 1969.

[11] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels: A Unified Approach to Performance Analysis*, 2nd edition. Wiley, 2005.

[12] Y. Chen and N. C. Beaulieu, "Solutions to infinite integrals of Gaussian Q-function products and some applications," *IEEE Commun. Lett.*, vol. 11, no. 11, pp. 853–855, Nov. 2007.

[13] Q. T. Zhang, "Probability of error for equal-gain combiners over Rayleigh channels: some closed-form solutions," *IEEE Trans. Commun.*, vol. 45, no. 3, pp. 270–273, Mar. 1997.

[14] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th edition. Academic, 2007.

[15] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 4th edition. Dover, 1970.

[16] M. D. Yacoub, "The κ-μ distribution and the η-μ distribution," *IEEE Antennas Propag. Mag.*, vol. 49, no. 1, pp. 68–81, Feb. 2007.