A Two-stage Spectrum Leasing Optimization Framework for Virtual Mobile Network Operators

Yingxiao Zhang*, Suzhi Bi†, and Ying Jun (Angela) Zhang*
*Department of Information Engineering and Shenzhen Research Institute, The Chinese University of Hong Kong, Shatin, N. T., Hong Kong, E-mail: {zy012, yjzhang}@ie.cuhk.edu.hk
†College of Information Engineering, Shenzhen University, Shenzhen, Guangdong, China 518060, E-mail: bsz@szu.edu.cn

***Abstract*—Wireless network virtualization (WNV) allows mobile network operators (MNOs) to lease its network infrastructure or licensed spectrum to virtual mobile network operators (VMNOs). A VMNO in WNV pays the MNOs for leasing wireless resources and receives payments from its mobile subscribers (MSs) based on the qualities of communication services. In this paper, we propose a novel two-stage spectrum leasing framework to maximize the average profit of a VMNO. Specifically, a VMNO can first make *long-term* spectrum lease based on the prediction of the average user traffic intensity over a long time period. Besides, the VMNO can flexibly make additional *short-term* leases based on the actual realizations of MS locations. We adopt a general $alpha$-fairness utility function to evaluate the qualities of downlink services to MSs. Within the proposed framework, we derive a closed-form expression for the optimal short-term leasing strategy for the VMNO, based on which we then propose an efficient algorithm to calculate the optimal long-term lease strategy. Simulation results show that the proposed two-stage spectrum leasing strategies can effectively increase the profit of VMNOs.**

I. Introduction

Wireless network virtualization (WNV) is a recent innovation in mobile communication industry that decouples the ownership of mobile network infrastructures and the provision of mobile services, which are conventionally unified at one entity [1], [2]. In particular, it allows mobile network operators (MNOs) to lease their network infrastructures, e.g., licenced spectrum, base stations and network equipments, to virtual mobile network operators (VMNOs), who then provide customized services to their mobile subscribers (MSs). WNV can essentially benefit all parties of the industry. Specifically, the MNOs can save their cost on launching new services and focus on providing physical infrastructure; small VMNOs without spectrum licences or network resource can now participate in the open market competition; MSs can also benefit from tailor-made services at potentially lower price due to the competition. Overall, WNV can promote business innovations and reduce the capital cost on infrastructure/maintenance per MNO, thanks to the more efficient sharing of wireless resource among the VMNOs.

The potential benefits of WNV have recently attracted rising research and industrial attentions. Several research projects have been launched to implement WNV in CDMA- and LTE-based cellular networks [3]–[7]. Two common methods to create wireless virtual resources by MNOs are to slice the wireless network infrastructure and spectrum into multiple virtual slices. In this paper, we focus on spectrum virtualization that MNOs lease licenced spectrum to VMNOs. Most of the previous studies are from the perspective of MNOs, especially on the dynamic spectrum allocation method to fulfil the prescribed spectrum leasing contracts between the VMNOs. For instance, [5] proposed a fair spectrum allocation method that ensures each VMNO can obtain at least a prescribed portion of spectrum indicated by individual service contract. Similarly, [6] proposed a scheduling method that each VMNO can receive on average a portion of resource blocks by contract, and the obtained resource blocks of a VMNO are equally shared by its MSs. Besides, [7] also introduced a VMNO admission control mechanism that the MNO decides whether to accept each VMNO's request of spectrum based on the estimated user traffic intensities, and then leaving each admitted VMNO to individually allocate the obtained spectrum to the MSs.

In this paper, we study the spectrum virtualization problem from the perspective of a VMNO. Specifically, we propose a *two-stage* spectrum leasing framework that allows the VMNO to make *long-term* spectrum lease based on the average user traffic intensity within a long time period, and also additional *short-term* lease based on the actual locations of MSs within a short time period. In particular, we intend to address an important question that concerns a VMNO: how much spectrum to lease in both long term and short term to maximize its profit. Towards this end, the detailed contributions of this paper are as follows:

- We propose a two-stage spectrum leasing framework that allows a VMNO to flexibly lease spectrum based both on long-term user traffic statistics and short-term realizations. A profit maximization problem is formulated to *jointly* optimize the long-term and short-term leasing strategies.
- We adopt a general α-fairness utility maximization method to dynamically allocate the obtained spectrum among the MSs to cope with wireless channel fadings. We derive a closed-form expression of the *optimal short-term* lease strategy of a VMNO, which maximizes its short-term profit given the locations of a set of MSs.
- Based on the obtained optimal short-term lease strategy, we further propose an efficient algorithm to calculate the *optimal long-term* spectrum lease by the VMNO based only on the prediction of the MS traffic intensity.

This work was supported in part by General Research Funding (Project number 14209414) from the Research Grants Council of Hong Kong and by the National Basic Research Program (973 program Program number 2013CB336701). The work of S. Bi is supported in part by the National Natural Science Foundation of China (project no. 61501303) and the Foundation of Shenzhen City (project no. JCYJ20160307153818306).

978-1-5090-3423-9/16/$31.00 ©2016 IEEE

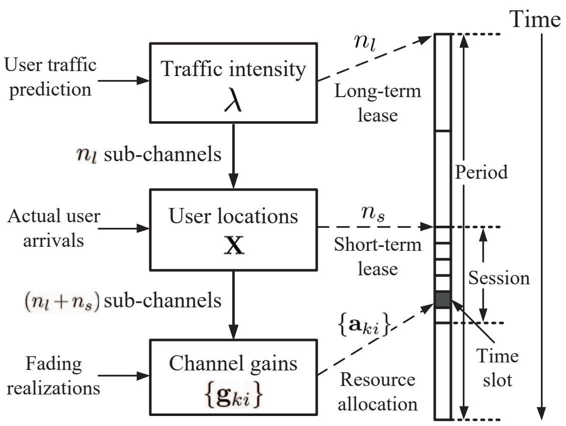

Fig. 1: The proposed two-stage spectrum leasing framework.

With the proposed framework, a VMNO can optimally adjust its leases of spectrum from the MNO in two timescales, and make use of the spectrum to serve MSs. The proposed methods have low computational complexity. Simulation results show that the proposed two-stage spectrum lease framework can effectively increase the profit of a VMNO compared to other non-trivial benchmark methods.

II. System Model

A. Network Model

We consider a VMNO who rents wireless spectrum from an MNO and programs on the obtained spectrum to provide customized services to its MSs. In particular, we focus on the downlink communication in a single OFDMA cell, where the spectrum owned by the MNO is divided into a number of sub-channels (SCs) each with bandwidth B. We assume that the number of available SCs is assumed to be sufficiently large compared to the number of VMNOs and their MSs in the system, such that the total number of SCs can be leaseed by the VMNO is considered unconstrained. Moreover, we assume that the price of SCs is decided by the MNO instead of the competition between VMNOs.

Without loss of generality, we assume that the base station (BS) is located at the origin o. At a tagged time instant, the locations of the MSs, denoted by $\mathbf{X} = \left\{\mathbf{x}_k \in \mathbb{R}^2 \mid k = 1, \ldots, K_x\right\}$, are modeled as a realization of a Poisson point process (PPP) with intensity λ in a fixed circular region $\mathcal{D}(o, D) \subset \mathbb{R}^2$, where K_x denotes the random number of MSs in $\mathbf{X}$ and D denotes the cell radius. Throughout this paper, we assume that the distribution of wireless fadings on each SC between the BS and an MS depends only on the location of the MS, and the fadings are ergodic given the MS location. In this case, the VMNO is only interested in the number of SCs needed, instead of the indices of the SCs, in the sense of achievable average communication performance.

In this paper, we study a two-stage wireless spectrum leasing system that operates in three different timescales, namely a *period*, a *session*, and a *time slot*. As shown in Fig. 1, a period is a duration, often measured in hours, during which the user density λ is considered constant while the MS locations $\mathbf{X}$ may vary with user arrivals and departures; a session, often measured in minutes, is the duration where $\mathbf{X}$ is considered fixed such that the wireless channel fadings are ergodic within each session; a time slot is the smallest unit of fixed length, during which the wireless channel fading remains constant but may vary in the coming slot. The conversion of the three timescales is that each period has T_m sessions and each session has t_m time-slots.

In particular, in the t-th time slot of a tagged session, we denote $\ell(\mathbf{x}_k)g_{ki}[t]$ as the instantaneous channel gain of the i-th SC from the BS to the k-th MS, where $\ell(\mathbf{x}_k) \triangleq \|\mathbf{x}_k\|^{-\beta}$ denotes the path loss gain ($\beta > 2$ denotes the path-loss exponent) and $g_{ki}[t]$ denotes the Rayleigh fading gain of unit transmission power. Then, the achievable data rate of the k-th MS on the i-th SC is denoted as

$$r_{ki}[t] \triangleq B \log\left(1 + \frac{P_t \ell(\mathbf{x}_k) g_{ki}[t]}{\Gamma N_0}\right), \tag{1}$$

where P_t and B denote respectively the fixed transmission power and the bandwidth of a SC, N_0 denotes the Gaussian noise power at receiver, and $\Gamma \geq 1$ denotes the capacity margin. We assume that $r_{ki}[t]$'s are known by the VMNO at the beginning of the t-th time slot through channel feedback from the MSs. As the traffic intensity, MS locations and the channel conditions change in different timescales, the VMNO could update its operations accordingly to maximize its profit. The three-timescale framework operates as follows.

B. Business Model

1) Long-term lease: As shown in Fig. 1, at the beginning of a period, the VMNO rents n_l SCs from the MNO at the cost of c_l per SC to maximize its average profit in the period, i.e.,

$$\underset{n_l \in \mathbb{Z}_+}{\text{maximize}} \; -c_l n_l + \mathbf{E}_{\mathbf{X}}\left[Q\left(\mathbf{X}, n_l\right)\right]. \tag{2}$$

Here, $Q\left(\mathbf{X}, n_l\right)$ denotes the profit achieved by the VMNO given MS locations $\mathbf{X}$ in a session and the period's long-term lease n_l, which is to be explicitly defined later in (3). Notice that once the long-term lease is settled, the VMNO cannot release the SCs back to the MNO until the end of the period.

2) Short-term lease: At the beginning of a session, the VMNO is allowed to make short-term lease in addition to the long-term lease after knowing the MSs in the session, e.g., when a large number of MSs request services. For a set of MS locations $\mathbf{X}$, the VMNO chooses to lease another n_s SCs at a price c_s per SC to maximize its average profit within the session, i.e.,

$$Q\left(\mathbf{X}, n_l\right) = \underset{n_s \in \mathbb{Z}_+}{\text{maximize}} \; -c_s n_s + c_g G\left(\mathbf{X}, n_l + n_s\right). \tag{3}$$

Here, $G\left(\mathbf{X}, n\right)$ denotes the average utility, e.g., total data rate, obtained by serving the MSs $\mathbf{X}$ with n SCs, and c_g denotes a fixed scaling parameter that converts the utility into its equivalent dollar value. Notice that the average utility is taken over the wireless channel fadings of all the t_m time slots within the session.

3) Dynamic resource allocation: Within each session, the VMNO allocates dynamically the $n = n_l + n_s$ SCs to a set of MSs $\mathbf{X}$ in each time slot to maximize the utility. Let $a_{ki}[t] \in [0, 1]$ denote the fraction of airtime of the i-th SCs allocated to the k-th MS at the t-th time slot. Then, the average throughput of the k-th MS in this session is given by

$$\bar{r}_k = \frac{1}{t_m}\sum_{t=1}^{t_m}\sum_{i=1}^{n} a_{ki}[t] r_{ki}[t]. \tag{4}$$

We assume that all users are infinitely back-logged and assigned a throughput-based utility $U(\bar{r}_k)$ to measure the quality of service (QoS) received by the k-th MS. We further assume that $U(\bar{r}_k)$ which is a continuous increasing differentiable concave function of the average throughput $\bar{r}_k$ [8]. Then, the resource allocation problem within the session can be formulated as follows:

$$G\left(\mathbf{X}, n\right) = \underset{\mathbf{A}}{\text{maximize}} \; \sum_{k=1}^{K_x} U(\bar{r}_k) \tag{5a}$$

$$\text{s.t.} \; \sum_{k=1}^{K_x} a_{ki}[t] = 1, \; \forall i, \, t, \tag{5b}$$

$$a_{ki}[t] \geq 0, \; \forall k, \, i, \, t, \tag{5c}$$

where $\mathbf{A} \triangleq \{a_{ki}[t], k = 1, \ldots, K_x, i = 1, \ldots, n, t = 1, \ldots, t_m\}$. Specially, $G(\mathbf{X}, n) = 0$ if $\mathbf{X} = \emptyset$. The objective is to maximize the total utility of the MSs, and the constraint (5b) indicates that the airtime of each SC is shared by the MSs in each time slot. Notice that, at the t-th time slot, the VMNO only has the causal information $r_{ki}[\tau]$'s for $\tau \leq t$, thus can only decide the allocation of SCs in the current time slot, i.e., $\mathbf{a}[t] \triangleq \{a_{ki}[t], k = 1, \ldots, K_x, i = 1, \ldots, n\}$. Moreover, $G(\mathbf{X}, n)$ may be negative to represent the penalty.

C. Problem Statement

In the above two-stage spectrum leasing system, the key challenges are to determine respectively

- the optimal SC allocation $\mathbf{a}[t]$ in each time slot t,
- the optimal short-term lease n_s^* in (3) in each session,
- and the optimal long-term lease n_l^* in (2) in a period.

In the next section, we address all the above challenges for a general utility function $U(\cdot)$, and use α-fairness utility function as an example to illustrate its operation in practice.

III. Optimal System Operation

A. Optimal SC Allocation

We first consider the problem of optimal resource allocation problem in (5), whose Lagrangian is given by

$$\begin{aligned} L\left(\mathbf{A}, \boldsymbol{\nu}, \boldsymbol{\mu}\right) = &\sum_{k=1}^{K_x} U\left(\bar{r}_k\right) + \sum_{i=1}^{n}\sum_{t=1}^{t_m} \nu_i[t]\left(1 - \sum_{k=1}^{K_x} a_{ki}[t]\right) \\ &+ \sum_{i=1}^{n}\sum_{k=1}^{K_x}\sum_{t=1}^{t_m} \mu_{ki}[t] a_{ki}[t], \end{aligned} \tag{6}$$

where $\nu_i[t]$'s and $\mu_{ki}[t]$'s are the corresponding Lagrangian dual variables. Let $\nabla U(r)$ denote the first-lease derivative of the function $U(x)$ evaluated at $x = r$, i.e.,

$$\nabla U(r) = \left.\frac{\partial U(x)}{\partial x}\right|_{x=r}. \tag{7}$$

Let $a_{ki}^*[t]$'s denote the optimal solutions, and $\bar{r}_k^*$'s denote the corresponding optimal average throughput. The following KKT conditions hold:

$$\left.\frac{\partial L}{\partial a_{ki}[t]}\right|_{a_{ki}^*[t]} = \nabla U(\bar{r}_k^*)\frac{r_{ki}[t]}{t_m} - \nu_i[t] + \mu_{ki}[t] = 0, \forall i, k, t; \tag{8a}$$

$$\sum_{k=1}^{K_x} a_{ki}^*[t] = 1, \forall i, t; \tag{8b}$$

$$a_{ki}^*[t] \geq 0, \; \mu_{ki}[t] \geq 0, \; \mu_{ki}[t] a_{ki}^*[t] = 0, \forall k, i, t. \tag{8c}$$

We can infer from (8c) that, if $a_{ki}^*[t] > 0$ for some MS k, then $\mu_{ki}[t] = 0$ must hold. Therefore, we can further see from (8a) that for the i-th SC,

$$\nabla U(\bar{r}_k^*) r_{ki}[t] = t_m \nu_i[t] \geq \nabla U(\bar{r}_j^*) r_{ji}[t], \; \forall j \neq k. \tag{9}$$

In other words, the i-th SC is only allocated to the MS(MSs) that has(have) the largest $\nabla U(\bar{r}_k^*) r_{ki}[t]$. As $r_{ki}[t]$ is drawn from a continuous distribution, the probability that two or more MSs have the same value of $\nabla U(\bar{r}_k^*) r_{ki}[t]$ is zero. Therefore, each SC is exclusively allocated to a single MS in each time slot. The optimal allocation policy is described by:

$$a_{ki}^*[t] = \begin{cases} 1, & k = \arg\max_v \nabla U(\bar{r}_v^*) r_{vi}[t], \\ 0, & \text{otherwise}, \end{cases} \tag{10}$$

for all i, k, t. A point to notice is that to perform the SC allocation policy in (10), one must obtain beforehand the optimal average throughput of all the MSs, i.e., $\bar{r}_k^*$'s. We propose in the following an iterative method to calculate $\bar{r}_k^*$'s given the MS locations $\mathbf{X}$ and the number of SCs n within a session.

B. Average Throughput within a Session

With the SC allocation policy in (10), we have

$$\bar{r}_k^* = \frac{1}{t_m}\sum_{t=1}^{t_m}\sum_{i=1}^{n} r_{ki}[t] \mathbf{1}_{\left\{\nabla U(\bar{r}_k^*) r_{ki}[t] > \nabla U_j(\bar{r}_j^*) r_{ji}[t], \forall j \neq k\right\}} \tag{11}$$

for $k = 1, \ldots, K_x$, where $\mathbf{1}_{\{\cdot\}}$ denotes an indicator function with value 1 if the input argument is true, and 0 otherwise. We assume t_m is sufficiently large such that the wireless channels are ergodic during a session, i.e.,

$$\begin{aligned} \bar{r}_k^* &= \sum_{i=1}^{n}\frac{1}{t_m}\sum_{t=1}^{t_m} r_{ki}[t] \mathbf{1}_{\left\{\nabla U(\bar{r}_k^*) r_{ki}[t] > \nabla U_j(\bar{r}_j^*) r_{ji}[t], \forall j \neq k\right\}} \\ &= \sum_{i=1}^{n} \mathbf{E}\left[R_{ki} \mathbf{1}_{\left\{\nabla U(\bar{r}_k^*) R_{ki} > \nabla U_j(\bar{r}_j^*) R_{ji}, \forall j \neq k\right\}}\right], \end{aligned} \tag{12}$$

where R_{ki} denotes the random data rate for the k-th user on the i-th SC. As the n SCs are i.i.d., we have from (12) that

$$\begin{aligned} \bar{r}_k^* &= n \mathbf{E}_{R_k}\left[R_k \prod_{j \neq k} \mathbf{Pr}\left[\nabla U(\bar{r}_k^*) R_k > \nabla U(\bar{r}_j^*) R_j \middle| R_k\right]\right] \\ &= n \int_0^\infty x \prod_{j \neq k} F_{R_j}\left(\frac{\nabla U(\bar{r}_k^*)}{\nabla U(\bar{r}_j^*)} x\right) f_{R_k}(x) \, dx, \end{aligned} \tag{13}$$

where $f_{R_k}(\cdot)$ and $F_{R_k}(\cdot)$ denote the PDF and CDF of the k-th MS's data rate on a single SC, respectively. For instance, for Rayleigh fading channel, we have

$$\begin{aligned} F_{R_k}(x) &= \mathbf{Pr}\left[B \log\left(1 + \frac{P\ell(\mathbf{x}_k) g_k}{\Gamma N_0}\right) \leq x\right] \\ &= 1 - \exp\left(-\left(e^{\frac{x}{B}} - 1\right)\rho_k\right), \end{aligned} \tag{14}$$

where $\rho_k \triangleq \frac{\Gamma N_0}{P\ell(\mathbf{x}_k)}$, and

$$f_{R_k}(x) = \frac{\rho_k}{B}\exp\left(\frac{x}{B} - \left(e^{\frac{x}{B}} - 1\right)\rho_k\right). \tag{15}$$

In general, $F_{R_k}(x)$ and $f_{R_k}(x)$ can be calculated given the MS location $\mathbf{x_k}$.

Let $\bar{\mathbf{r}}^* \triangleq [\bar{r}_1^*, \ldots, \bar{r}_{K_x}^*]$ denote the optimal throughputs for MSs in the session. From (13), we can see that $\bar{\mathbf{r}}^*$ is the solution of the following system of equations

$$\mathbf{r} = [nH_1(\mathbf{r}), \ldots, nH_{K_x}(\mathbf{r})], \tag{16}$$

where $\mathbf{r}$ is a K_x-dimension vector and $H_k(\mathbf{r}) \triangleq \int_0^\infty x \prod_{j\neq k} F_{R_j}\left(\nabla U(r_k)/\nabla U(r_j)x\right) f_{R_k}(x)\,dx$. The uniqueness of the solution to the system in (16) is omitted due to the page limit. Given the MS locations $\mathbf{X}$, we can obtain $\bar{\mathbf{r}}^*$ numerically with existing algorithms. For example, in Section IV, we use Quasi-Newton method which converges to the unique solution quickly. Then the VMNO can easily implement the SC allocation policy in (10) in each time slot of a session.

C. Optimal Short-term Lease

From the above analysis, the VMNO can calculate the total utility $G(\mathbf{X}, n)$ in (5) if $n = n_l + n_s$ SCs are leaseed in the session. Notice that n_l is considered as a parameter in each session, and thus the VMNO needs to decide the optimal n_s at beginning of the session by solving the problem in (3).

We consider an α-fair utility function [8] which is defined by

$$U(r) = \begin{cases} \frac{r^{1-\alpha}}{1-\alpha}, & \text{if } \alpha \geq 0, \alpha \neq 1, \\ \ln r, & \text{if } \alpha = 1. \end{cases} \tag{17}$$

Specially, $\alpha = \{0, 1, 2, \infty\}$ correspond to sum throughput, proportional fair, sum delay and max-min fairness optimizations, respectively. Besides, we have $\nabla U(r) = r^{-\alpha}, \forall \alpha \geq 0$.

With a bit abuse of notation, we denote $\bar{\mathbf{r}}^*(n)$ as the optimal throughputs achieved for MSs when in total n SCs are rent within the session. From (16), we have

$$\begin{aligned} \bar{\mathbf{r}}^*(n)/n &= [H_1(\bar{\mathbf{r}}^*(n)), \ldots, H_{K_x}(\bar{\mathbf{r}}^*(n))] \\ &= [H_1(\bar{\mathbf{r}}^*(n)/n), \ldots, H_{K_x}(\bar{\mathbf{r}}^*(n)/n)] \end{aligned} \tag{18}$$

where the last equation holds because

$$\begin{aligned} H_k(\mathbf{r}/n) &= \int_0^\infty x \prod_{j\neq k} F_{R_j}\left(\frac{\nabla U(r_k/n)}{\nabla U(r_j/n)}x\right) f_{R_k}(x)\,dx \\ &= \int_0^\infty x \prod_{j\neq k} F_{R_j}\left(\frac{n^{\alpha-1}r_k^{-\alpha}}{n^{\alpha-1}r_j^{-\alpha}}x\right) f_{R_k}(x)\,dx \\ &= \int_0^\infty x \prod_{j\neq k} F_{R_j}\left(\frac{\nabla U(r_k)}{\nabla U(r_j)}x\right) f_{R_k}(x)\,dx \\ &= H_k(\mathbf{r}). \end{aligned} \tag{19}$$

From (18), we can see that $\bar{\mathbf{r}}^*(n)/n$ is the solution of the system $\mathbf{r} = [H_1(\mathbf{r}), \ldots, H_{K_x}(\mathbf{r})]$, which can be denoted by $\bar{\mathbf{r}}(1)$. Therefore, we have

$$\bar{\mathbf{r}}^*(n)/n = \bar{\mathbf{r}}^*(1). \tag{20}$$

In other words, the throughput of each MS is a linear function of the number of SCs n.

With (20), the total utility as defined in (5) is given by

$$G(\mathbf{X}, n) = \sum_{k=1}^{K_x} U\left(\bar{r}_k^*(n)\right) = U(n)G(\mathbf{X}) + G(\mathbf{X}, 1)\mathbf{1}_{\alpha=1}, \tag{21}$$

where $G(\mathbf{X}) \triangleq \sum_{k=1}^{K_x} \bar{r}_k^*(1)^{1-\alpha}$. Note that for $\alpha > 1$, $G(\mathbf{X}, 0) = -\infty$ which means that the VMNO would receive severe penalty if no SCs are leaseed to serve its MSs. Moreover, the total utility $G(\mathbf{X}, n)$ is negative for $\alpha > 1$ which indicates that the overall profit is negative. In reality, the VMNO would charge its MSs with constant monthly rate and use negative utility to denote the penalties for unsatisfied services. Hence, the VMNO is still profitable even with negative utilities.

Then the short-term lease problem in (3) becomes

$$Q(\mathbf{X}, n_l) = \max_{n_s \in \mathbb{Z}_+} \; -c_s n_s + c_g U(n_s + n_l)G(\mathbf{X}) + G(\mathbf{X}, 1)\mathbf{1}_{\alpha=1}, \tag{22}$$

where both $G(\mathbf{X})$ and the last term are independent of n_s. By relaxing the integer constraint, (22) becomes a convex optimization problem and the optimal solution is given by

$$\tilde{n}_s^* = \begin{cases} 0, & G(\mathbf{X}) \leq \frac{c_s}{c_g} n_l^\alpha; \\ \left(\frac{c_g}{c_s} G(\mathbf{X})\right)^{1/\alpha} - n_l, & \text{otherwise.} \end{cases} \tag{23}$$

If $\tilde{n}_s^*$ is an integer, $n_s^* = \tilde{n}_s^*$ and the original integer programming problem is solved; otherwise, n_s^* is selected from $\{\lfloor \tilde{n}_s^* \rfloor, \lceil \tilde{n}_s^* \rceil\}$ which ever achieves a larger objective. In general, we can infer from (23) that the VMNO makes more short-term lease when the price c_s is lower or when few SCs are reserved in long-term lease within the period. In particular, for $\alpha = 0$, the optimal short-term lease is either zero or infinite, depending on the MS locations $\mathbf{X}$. This is because the objective in (22) is a linear function of n_s and the slope depends on $G(\mathbf{X})$. In real cases, with finite number of SCs on sale, the optimal short-term lease is to rent as much SCs as possible when $G(\mathbf{X})$ is large, e.g., a large number of MSs are close to the BS. Moreover, for the special case $\alpha = 1$, we have $G(\mathbf{X}) = K_x$ and thus the short-term lease depends on the number of MSs in the session instead of their average throughput. In the following analyses, we focus on general cases and omit the special cases $\alpha = 0$ and $\alpha = 1$ due to page limits.

D. Optimal Long-term Lease

For the convenience of analysis, we use (23) as the optimal short-term lease to solve the long-term lease problem in (2). In fact, the approximation error in the optimal value $Q(\mathbf{X}, n_l)$ is negligible when $\tilde{n}_s^*$ is large or close to an integer. In Section IV, all simulations are using integer solutions, which demonstrates the effectiveness of integer approximation. By substituting (22) into (3), we have

$$Q(\mathbf{X}, n_l) = \begin{cases} c_g G(\mathbf{X}) \frac{n_l^{1-\alpha}}{1-\alpha}, & G(\mathbf{X}) \leq \frac{c_s}{c_g} n_l^\alpha; \\ c_s n_l + \frac{c_s \alpha}{(1-\alpha)} \left(\frac{c_g}{c_s} G(\mathbf{X})\right)^{1/\alpha}, & \text{otherwise.} \end{cases} \tag{24}$$

Accordingly, the total profit in (2) achieved from making n_l long-term lease is

$$\begin{aligned} J(n_l) &= -c_l n_l + \mathbf{E}_{\mathbf{X}}\left[Q(\mathbf{X}, n_l)\right] \\ &= -c_l n_l + \frac{c_s \alpha}{(1-\alpha)} \int_{\frac{c_s}{c_g} n_l^\alpha}^\infty \left(\frac{c_g}{c_s} x\right)^{1/\alpha} f_G(x)dx \\ &\quad + c_s n_l F_G^c\left(\frac{c_s}{c_g} n_l^\alpha\right) + \frac{c_g n_l^{1-\alpha}}{1-\alpha} \int_0^{\frac{c_s}{c_g} n_l^\alpha} x f_G(x)dx, \end{aligned} \tag{25}$$

where $f_G(\cdot)$ and $F_G^c(\cdot)$ are the PDF and CCDF of $G(\mathbf{X})$, respectively. Given the distribution of $\mathbf{X}$, we can obtain the approximated $f_G(\cdot)$ and $F_G^c(\cdot)$ by numerical sampling. As $J(n_l)$ is a concave function of n_l, the optimal solution of (2) without the non-negative integer constraint can be obtained by finding the root of $\partial J(n_l)/\partial n_l = 0$, denoted by $\tilde{n}_l^*$. The optimal long-term lease n_l^* can be computed by the positive floor or ceiling of $\tilde{n}_l^*$ accordingly. As $f_G(\cdot)$ and $F_G^c(\cdot)$ are obtained numerically, we have no closed-form expression of n_l^*. We will present the properties of n_l^* with numerical results in the next section.

E. Benchmark methods

For comparison, we consider long-term lease only and short-term lease only as two benchmark methods. On one hand, the VMNOs could sign contracts with the MNO to reserve spectrum resources for a long period. In this case, the optimal purchase can be obtained by solving the following problem

$$\max_{n\in\mathbb{Z}_+} \; J_{lo}(n) = -c_l n + c_g \mathbf{E}_{\mathbf{X}}\left[G(\mathbf{X}, n)\right]. \tag{26}$$

From (21), we have

$$J_{lo}(n) = -c_l n + c_g U(n) \mathbf{E}_{\mathbf{X}}\left[G(\mathbf{X})\right], \tag{27}$$

for $\alpha > 0$, $\alpha \neq 1$. As $J_{lo}(n)$ is a convex function of n, we can obtain the optimal solution n_{lo}^* and the optimal value J_{lo}^* as follows:

$$n_{lo}^* = \left(\frac{c_g}{c_l}\mathbf{E}_{\mathbf{X}}\left[G(\mathbf{X})\right]\right)^{1/\alpha}, \tag{28a}$$

$$J_{lo}^* = \frac{c_l \alpha}{1-\alpha}\left(\frac{c_g}{c_l}\mathbf{E}_{\mathbf{X}}\left[G(\mathbf{X})\right]\right)^{1/\alpha}. \tag{28b}$$

On the other hand, the VMNO could only make short-term lease after the MSs show up. In this case, the problem of finding the optimal purchase for a session with MS locations $\mathbf{X}$ can be formulated as (3) with $n_l = 0$. Therefore, the optimal short-term lease is given by

$$n_{so}^*(\mathbf{X}) = \left(\frac{c_g}{c_s}G(\mathbf{X})\right)^{1/\alpha}, \tag{29a}$$

$$J_{so}^* = \frac{\alpha c_s^{1-1/\alpha} c_g^{1/\alpha}}{(1-\alpha)}\mathbf{E}_{\mathbf{X}}\left[G(\mathbf{X})^{1/\alpha}\right], \tag{29b}$$

for $\alpha \neq 1$. The case for $\alpha = 1$ can be obtained similarly and thus is omitted. In Section IV, we will compare the performances of the proposed two-stage lease strategy with these two benchmark methods.

IV. Numerical Results and Discussions

In this section, we study the properties of the two-stage leasing framework for the VMNO. We consider a circular area with radius 1 km and the BS locates at the cell center. The density of MSs is 10 per km^2. Note that as the services of VMNOs are isolated, we only consider the data request form the tagged VMNO. The average received SNR at the cell edge is -6 dB and the path loss exponent is 3.67. Assuming Rayleigh fading with a unit mean, the distribution of data rate on a single SC can be obtained for each sampled MS. Accordingly, the average throughputs on a single CS for a sampled set of MS can be calculated by solving the fixed point system in (16), which gives the value of $G(\mathbf{X})$. We generate 10000 independent sets of MSs to find the empirical PDF and

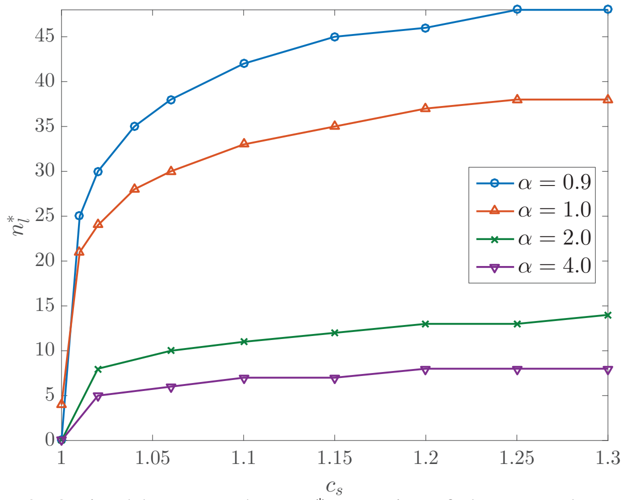

Fig. 2: Optimal long-term lease n_l^* v.s. price of short-term lease c_s. Utility parameter is $\alpha = 0.5, 1, 2$ and the price of long-term lease is $c_l = 1$ per SC.

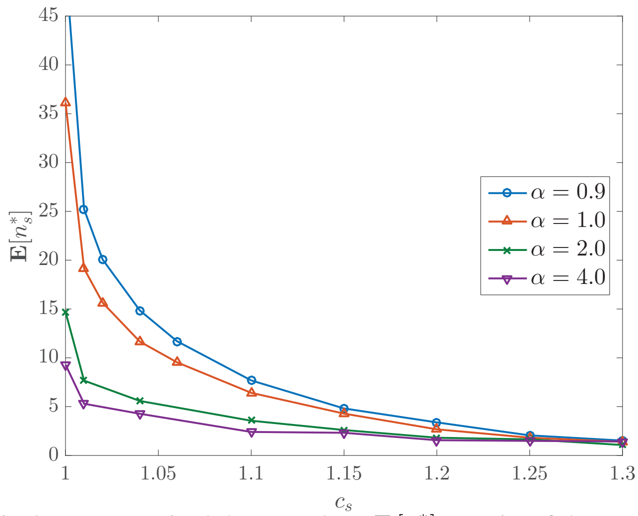

Fig. 3: Average optimal short-term lease $\mathbf{E}\left[n_s^*\right]$ v.s. price of short-term lease c_s.

CCDF of $G(\mathbf{X})$, which are needed in the calculation of the optimal long-term lease n_l^*. Further, we set the price of long-term lease $c_l = 1$ and varies the short-term price c_s to see its impact on the optimal lease strategies. The price for a unit of utility is $c_g = 4$.

Fig. 2 shows the optimal long-term lease n_l^* as a function of the short-term lease price c_s for different utility functions. The plotted lines are not very smooth since n_l^* is an integer. We can see that n_l^* increases with c_s for all α's, which means that the VMNO needs to lease more SCs in advance to reduce possibly high cost short-term leases. An extreme is that the long-term lease equals zero when $c_s = c_l$, indicating that long-term leases have no price advantage over short-term leases.

The optimal short-term lease n_s^* varies with the MSs requesting services at each session. Fig. 3 shows the mean of n_s^* as a function of the short-term lease price c_s for different α's. We can see that $\mathbf{E}\left[n_s^*\right]$ decreases as c_s increases for all α's, which means that the VMNO adds less short-term purchase of

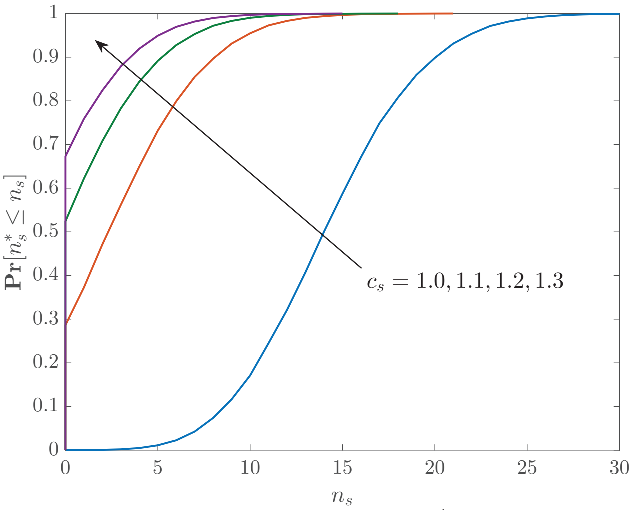

Fig. 4: CDF of the optimal short-term lease n_s^* for short-term lease price $c_s = 1.0$, 1.1, 1.2, 1.3 and utility parameter $\alpha = 2$.

SCs at each session for a higher short-term price. We further plot the CDF of n_s^* in Fig. 4 for $\alpha = 2$ and c_s varies from 1 to 1.3. We can see that the CDF line shifts to the left corner as c_s increases, and, specially, the probability that $n_s^* = 0$ is increasing. This confirms that the VMNO tends to lease less supplementary SCs if the short-term lease is too expensive. In addition, we can see that both n_l^* and n_s^* decreases for larger αs. This is because that the utility with larger α becomes lower for the same c_g.

In Fig. 5, we compare the profits achieved by the proposed two-stage lease with that by long-term lease only and short-term lease only. Note that the optimal values for $\alpha = 2$ in Fig. 5(b) is negative, since the utility is the delay penalty. However, the VMNO's profit is positive which equals the optimal value plus a constant subscription fee. We can see that the two-stage lease converges to long-term lease only as c_s becomes large, while converging to short-term purchase only when c_s becomes small. In real cases, a discount for long-term lease would encourage the VMNOs to purchase more SCs beforehand and reduce the trading frequencies with the VMNO. By the proposed two-stage lease framework, the VMNO could take advantages of the price differences while remains the flexibility to dealing with traffic burst.

V. Conclusions

In this paper, we studied a two-stage spectrum leasing problem where the VMNO makes long-term lease for a certain period and flexibly leases additional spectrum for each short session based on the MSs to serve. To find the optimal long-term and short-term leases, we formulated the problem as a stochastic programming which also involves finding the optimal SC allocation for random wireless channels. With a general α-fair utility function, we firstly derived the maximum utility that can be achieved by dynamic resource allocation, which is then used to solve the optimal long-term and short-term leases jointly. Numerical results show that the proposed two-stage spectrum leasing strategies can increase the profit of the VMNOs.

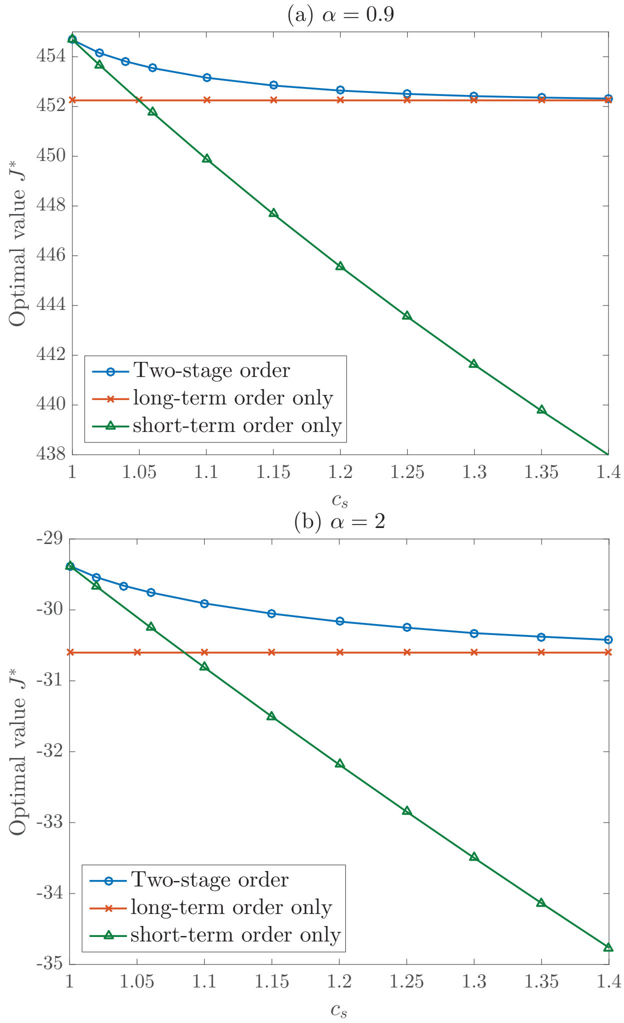

Fig. 5: Comparison of the optimal values achieved by the proposed two-stage lease, long-term lease only, and short-term lease only methods. The price of long-term lease is $c_l = 1$ for each SC.

References

[1] N. Chowdhury and R. Boutaba, “Network virtualization: State of the art and research challenges,” *IEEE Commun. Mag.*, vol. 47, no. 7, pp. 20-26, Jul. 2014.

[2] C. Liang and F. Yu, Wireless network virtualization: a survey, some research issues and challenges, *IEEE Commun. Surveys Tuts.*, vol. 17, no. 1, pp. 358-380, Mar. 2015.

[3] G. Middleton, K. Hooli, A. Tolli, and J. Lilleberg, Inter-operator spectrum sharing in a broadband cellular network, in *Proc. IEEE 9th Int. Symp. Spread Spectrum Tech. Appl.*, pp. 376-380, Aug. 2006.

[4] Y. Zaki, Z. Liang, C. Goerg, and A. Timm-Giel, “LTE wireless virtualization and spectrum management,” in *Proc. 3rd Joint IFIP WMNC*, pp. 1C6, Oct. 2010.

[5] M. I. Kamel, L. Bao Le, and A. Girard. “LTE wireless network virtualization: dynamic slicing via flexible scheduling,” in *Proc. IEEE VTC Fall*, pp. 1-5, Sept. 2014.

[6] M. Kalil, A. Moubayed, A. Shami, and A. Al-Dweik, “Efficient low-complexity scheduler for wireless resource virtualization,” *IEEE Wireless Commun. Letters*, vol. 5, no. 1, pp. 56-59, Feb 2016.

[7] C. Liang and F. R. Yu, “Mobile virtual network admission control and resource allocation for wireless network virtualization: A robust optimization approach,” in *IEEE GLOBECOM*, pp. 1-6, Dec. 2015.

[8] R. Agrawal, A. Bedekar, R. La, and V. Subramanian, “A class and channel-condition based weighted proportionally fair scheduler,” in *Proc. ITC 2001, Salvador, Brazil*, Sept. 2001.