

# Decentralized Blockchain Based Dynamic Spectrum Acquisition for Wireless Downlink Communications

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**Abstract**—Wireless network virtualization is a promising solution to improve the spectrum efficiency. For a wireless downlink communication system with multiple mobile virtual network operators (MVNOs), we propose a decentralized blockchain based dynamic spectrum acquisition scheme. Our proposed scheme aims to minimize the sum transmit power at all MVNOs while satisfying the average data transmission rate thresholds. For each MVNO, the required wireless spectrum to provide customized services to the mobile users (MUs) is predicted using the half-range Gauss-Hermite quadrature. Based on the predicted values, all the MVNOs carry out a blockchain-based distributed alternative direction method of multipliers to obtain the global optimal solution to the aforementioned sum transmit power minimization problem. To examine the effectiveness of our proposed scheme, with known system parameters, we also theoretically derive the semi-closed-form solution to the actual required sum transmit power minimization problem subject to data transmission rate constraints. Simulation results illustrate that our proposed dynamic spectrum acquisition scheme achieves almost the same minimum sum power as the non-causal scheme which assumes the number of active MUs in all cells and all the channels are known non-causally for the optimal dynamic spectrum allocation.

**Index Terms**—Alternating direction method of multipliers (ADMM), blockchain, dynamic spectrum acquisition, wireless network virtualization.

## I. INTRODUCTION

With the popularity of various smart phones, an exponential traffic growth is generated by wireless applications in next generation 5G networks and beyond [1]. To cope with the dramatic traffic growth, network densification is expected to increase by 1000 times ranging from macrocells to femtocells [2]. Thus, a serious spectrum shortage problem should be properly addressed. To solve the spectrum shortage problem, either we release more licensed bandwidth for next generation 5G networks and beyond, or we improve the spectrum efficiency by employing the efficient spectrum management schemes.

To improve the spectrum efficiency, wireless network virtualization is a promising solution [3]–[5]. Wireless network virtualization is able to facilitate multiple network operators to share common resources, e.g., licensed spectrum. The virtualized wireless networks (VWNs) commonly consist of multiple mobile network operators (MNOs) and multiple mobile virtual network operators (MVNOs) [6]. The MNO owns the

physical cellular infrastructure and radio resources, it executes the virtualization by leasing the isolated virtualized network resources to the MVNO. The MVNO leases the resources from the MNO and then assigns the resources to the MUs. The spectrum resources which belong to one or more MNOs are virtualized and spitted into slices. The MVNO utilizes the slices leased from MNOs depending on the quality-of-service (QoS) of MUs. Recently, wireless network virtualization had attracted an increasing attention from the research community [7]–[10]. In [7], an active sharing of physical infrastructure and the spectrum was proposed, where MNOs share the network resources and provide wholesale access to MVNOs, allowing them to provide voice and data services using part of the available resources. The work in [8] introduced a novel scheme for slicing and scheduling for VWNs by developing an efficient resource allocation scheme. In [9], an efficient low-complexity scheme was put forward to virtualize the wireless resource blocks and share them between MUs of multiple MNOs. The scheme aims to maximize the throughput while maintaining access proportional fairness among MUs as well as MNOs. In [10], a two-stage spectrum leasing problem with the goal of maximizing the average profit of a VMNO was investigated.

Aforementioned dynamic spectrum allocation schemes in VWNs improve the spectrum efficiency in a centralized manner which requires a central node. If the central node is under cyberattack, all the MVNOs are vulnerable. In addition, if the MVNOs belong to different MNOs, the information ~~ex-changing~~ among different nodes should be as less as possible. Furthermore, the management overhead increases with the increase of the number of MVNOs which causes that the wireless communication system is not scalable. Therefore, in this paper, we propose a decentralized blockchain based dynamic spectrum acquisition scheme.

Blockchain, which was invented by Satoshi Nakamoto in 2008, is an emerging technology to build consensus between disparate individuals [11]. The intuition is to solve the double-spending problem in the cryptocurrency bitcoin without the use of a trusted central utility. Blockchain consists of lists of blocks which are linked by the hash algorithm. Each block includes collections of signed transactions. A new block can be appended to the end of the current blockchain after solving a proof-of-work (PoW) puzzle which is to find a number whose hash value is less than the current target. PoW can create distributed consensus between all participants and solve the ~~double-spend~~ problem. The data on blockchain is shared and saved by all participants on peer-to-peer networks. All participants have right to access but cannot tamper the

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data on blockchain. The pillars of blockchain are state-of-the-art cryptography and PoW based distributed consensus mechanism. This kind of system may introduce some computational cost but can provide decentrality, security, transparency and robustness. The blockchain technologies have been extensively used in many areas [12]–[17].

In this paper, we propose a decentralized blockchain based dynamic spectrum acquisition scheme for a wireless downlink communication system with multiple MVNOs. With wireless network virtualization operation, the whole communication processes are divided into multiple periods, each measured in minutes. At the beginning of each period, MVNOs should predict the required wireless spectrum to provide customized services to the MUs in their respective service cells. Based on the predicted values, MVNOs acquire wireless spectrum by our proposed decentralized blockchain based dynamic spectrum acquisition scheme. Our proposed scheme aims to minimize the sum transmit power at all MVNOs while satisfying the average data transmission rate thresholds. We propose to employ the half-range Gauss-Hermite quadrature (HR-GHQ) to simplify the optimization problem. Then, we propose a blockchain-based distributed alternative direction method of multipliers (ADMM) to obtain the global optimal solution to aforementioned sum transmit power minimization problem.

To fairly compare the performance of our proposed scheme with the fixed spectrum allocation scheme and other schemes, with known system parameters, we propose to investigate the actual required minimum sum transmit power of all MVNOs subject to that the data transmission rate thresholds for all MUs in all cells are satisfied. We theoretically derive semi-closed-form expressions for the optimal power and spectrum allocation for each MVNO.

The remainder of the paper is organized as follows. The system model and dynamic spectrum acquisition optimization problem formulation are described in Section II and Section III, respectively. We propose the blockchain based distributed ADMM algorithm for dynamic spectrum acquisition in Section IV. With known system parameters, the optimal power and spectrum allocation is theoretically derived in Section V. Section VI provides simulation results to validate the effectiveness of our proposed scheme. Section VII concludes this paper.

*Notations:* Boldface lowercase letters denote vectors.  $(\cdot)^T$  denotes the transpose operation.  $\|\cdot\|_1$  and  $\|\cdot\|_2$  represent the  $l_1$  and  $l_2$  norm of a vector, respectively.

## II. SYSTEM MODEL

Consider a wireless downlink communication system with  $M$  MVNOs. Each MVNO serves the MUs in a cell. The  $m$ -th transmission cell,  $m \in \mathcal{M} = \{1, 2, \dots, M\}$ , is assumed to be a fixed circular region, denoted as  $\mathcal{D}_m \in \mathbb{R}^2$ , whose radius is denoted as  $r_m$ . The  $m$ -th MVNO is located at the cell center.

With wireless network virtualization operation, the whole communication processes are divided into multiple periods, each measured in minutes. At the beginning of each period, MVNOs should predict the required wireless spectrum to provide customized services to the MUs in their respective

service cells. Based on the predicted values, MVNOs acquire wireless spectrum. Denote the bandwidth of wireless spectrum acquired by the  $m$ -th MVNO as  $w_m$ , for  $m \in \mathcal{M}$ . We have

$$\sum_{m=1}^M w_m \leq W \quad (1)$$

where  $W$  denotes the total available bandwidth.

To predict the required wireless spectrum in a period, the active MUs in the  $m$ -th cell are modeled as a Poisson point process (PPP) with density  $\lambda_m$ . Suppose that transmitted signals are affected by both large-scale path loss and small-scale flat Rayleigh fading. Thus, the channel between the  $n$ -th MU and its associated  $m$ -th MVNO is denoted by

$$h_{mn} = \frac{g_{mn}}{\sqrt{1 + L_{mn}^\alpha}} \quad (2)$$

where  $L_{mn}$  denotes the distance between the  $m$ -th MVNO and the  $n$ -th MU in the  $m$ -th cell,  $\alpha$  denotes the path-loss decay factor, and  $g_{mn}$  denotes a complex Gaussian random variable with zero mean and unit variance. The instantaneous data rate for the  $n$ -th MU in the  $m$ -th MVNO is given by

$$R_{mn} = b_{mn} \log_2 \left( 1 + \frac{q_{mn} |h_{mn}|^2}{\Gamma b_{mn} \sigma_0^2} \right) \quad (3)$$

where  $b_{mn}$  and  $q_{mn}$  denote the bandwidth and power of the  $n$ -th MU allocated by the  $m$ -th MVNO, respectively,  $\Gamma$  denotes the signal-to-noise ratio (SNR) gap to the information theoretical channel capacity owing to the non-ideal coding and modulation in practice [18], and  $\sigma_0^2$  denotes the power spectral density of additive white Gaussian noise at all MUs.

## III. DYNAMIC SPECTRUM ACQUISITION OPTIMIZATION PROBLEM FORMULATION

For the required wireless spectrum prediction, since we have no information on the channels,  $h_{mn}$ , we employ the uniform bandwidth allocation and power allocation for the MUs in each cell. Thus, in the  $m$ -th cell,

$$b_{mn} = \frac{w_m}{N_m} \text{ and } q_{mn} = \frac{p_m}{N_m} \quad (4)$$

where  $N_m$  denotes the number of active MUs in the  $m$ -th cell and  $p_m$  denotes the total transmission power budget at the  $m$ -th MVNO. Since the active MUs in the  $m$ -th cell are modeled as a PPP with density  $\lambda_m$ , the average number of MUs in the  $m$ -th cell is

$$\mathbb{E}[N_m] = \Lambda_m = \pi r_m^2 \lambda_m. \quad (5)$$

Accordingly, the expected data transmission rate of the  $j$ -th MU in the  $i$ -th cell is expressed as

$$\mathbb{E}[R_{mn}] = \sum_{N_m=1}^{\infty} \frac{\Omega}{N_m} \Pr[N_m] \quad (6)$$

where

$$\Omega = \int_0^\infty w_m \log_2 \left( 1 + \frac{p_m x}{w_m \sigma_0^2} \right) f_{|h_{ij}|^2}(x) dx, \quad (7)$$

$$\Pr[N_m] = \frac{(\Lambda_m)^{N_m}}{N_m!} \exp(-\Lambda_m). \quad (8)$$

In (7),  $\sigma^2 = \Gamma\sigma_0^2$  and  $f_{|h_{ij}|^2}(x)$  is the probability density function (PDF) of the random variable  $|h_{ij}|^2$ . According to [19], we have

$$\sum_{N_m=1}^{\infty} \frac{1}{N_m} \Pr[N_m] = \phi_m \quad (9)$$

where

$$\phi_m = (\text{Ei}(\Lambda_m) - \ln(\Lambda_m) - C) \exp(-\Lambda_m). \quad (10)$$

In (10),  $\text{Ei}(x)$  is the exponential integral function whose series expansion is [19, 5.1.10]

$$\text{Ei}(x) = \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!} + \ln x + C \quad (11)$$

and  $C$  denotes the Euler-Mascheroni constant.

Denote the average data transmission rate threshold for each MU in the  $m$ -th cell as  $\bar{R}_m$ , for  $m \in \mathcal{M}$ . Note that with limited  $w_m$ , to ensure that

$$\mathbb{E}[R_{mn}] \geq \bar{R}_m, \quad (12)$$

we may increase  $p_m$ . Thus, to reasonably allocate wireless spectrum among different MVNOs, we should minimize the sum transmit power at all MVNOs while satisfying the average data transmission rate thresholds, which is formulated as

$$\begin{aligned} \min_{\mathbf{p}, \mathbf{w}} \quad & \sum_{m=1}^M p_m \\ \text{s.t.} \quad & (1), (12), p_m \geq 0, w_m \geq 0, \forall m \in \mathcal{M} \end{aligned} \quad (13)$$

where  $\mathbf{p} = [p_1, p_2, \dots, p_M]^T$  and  $\mathbf{w} = [w_1, w_2, \dots, w_M]^T$ . To proceed, we will first introduce the following proposition.

**Proposition 1:** The cumulative distribution function (CDF) of  $|h_{mn}|^2$  is

$$F_{|h_{mn}|^2}(x) = 1 - \Psi\left(\frac{2}{\alpha}, 1 + \frac{2}{\alpha}, -xr_m^\alpha\right) e^{-x} \quad (14)$$

where  $\Psi(a, b, z)$  denotes the Kummer's function [19].

*Proof:* See Appendix A.  $\blacksquare$

Using Proposition 1, by taking the first-order derivative with respect to  $x$ ,  $f_{|h_{mn}|^2}(x)$  is

$$\begin{aligned} f_{|h_{mn}|^2}(x) = & \Psi\left(\frac{2}{\alpha}, 1 + \frac{2}{\alpha}, -xr_m^\alpha\right) e^{-x} \\ & - \frac{d\Psi\left(\frac{2}{\alpha}, 1 + \frac{2}{\alpha}, -xr_m^\alpha\right)}{dx} e^{-x}. \end{aligned} \quad (15)$$

From [19, 13.4.8], we know

$$\frac{d\Psi(s, t, y)}{dy} = \frac{s}{t} \Psi(s+1, t+1, y). \quad (16)$$

Thus, we have

$$f_{|h_{mn}|^2}(x) = e^{-x} \kappa(x) \quad (17)$$

where

$$\begin{aligned} \kappa(x) = & \Psi\left(\frac{2}{\alpha}, 1 + \frac{2}{\alpha}, -xr_m^\alpha\right) \\ & + \frac{2r_m^\alpha}{2+\alpha} \Psi\left(1 + \frac{2}{\alpha}, 2 + \frac{2}{\alpha}, -xr_m^\alpha\right). \end{aligned} \quad (18)$$

Substituting (17) into (12), we obtain

$$\xi \geq \bar{R}_m \quad (19)$$

where

$$\xi = \phi_m \int_0^\infty w_m \log_2 \left(1 + \frac{p_m x}{w_m \sigma^2}\right) e^{-x} \kappa(x) dx. \quad (20)$$

It is noted that on the left-hand side of (19), the integral result  $\xi$  is difficult to obtain. Furthermore, when the integrand has a non-negligible tail, even the numerical integration is complicated and time-consuming. In this paper, we propose to apply the half-range Gauss-Hermite quadrature (HR-GHQ) to approximate the integral on the left-hand side of (19) with high accuracy [20], [21].

Based on [21], a  $K$ -point HR-GHQ can be written as

$$\int_0^\infty e^{-t^2} \zeta(t) dt \approx \sum_{k=1}^K a_k \zeta(t_k) \quad (21)$$

where both the weights  $\{a_k\}_{k=1}^K$  and abscissas  $\{t_k\}_{k=1}^K$  are real numbers. By letting  $x = t^2$ , we have

$$\xi = \phi_m \int_0^\infty 2w_m t \log_2 \left(1 + \frac{p_m t^2}{w_m \sigma^2}\right) e^{-t^2} \kappa(t^2) dt. \quad (22)$$

Applying the  $K$ -point HR-GHQ, we obtain

$$\xi \approx \sum_{k=1}^K w_m \eta_{mk} \log_2 \left(1 + \frac{p_m t_{mk}^2}{w_m \sigma^2}\right) \quad (23)$$

where  $\eta_{mk} = 2\phi_m a_{mk} t_{mk} \kappa(t_{mk}^2)$ , and both the weights  $\{a_{mk}\}$  and abscissas  $\{t_{mk}\}$  are real numbers.

Substituting (23) into problem (13), we have the following optimization problem

$$\begin{aligned} \min_{\mathbf{p}, \mathbf{w}} \quad & \sum_{m=1}^M p_m \\ \text{s.t.} \quad & \sum_{k=1}^K \frac{w_m \eta_{mk}}{N_m} \log_2 \left(1 + \frac{p_m t_{mk}^2}{w_m \sigma^2}\right) \geq \bar{R}_m, \\ & (1), p_m \geq 0, w_m \geq 0, \forall m \in \mathcal{M}. \end{aligned} \quad (24)$$

Problem (24) is convex in terms of optimization variables  $\mathbf{p}$  and  $\mathbf{w}$ . It can be solved efficiently using the interior point method [22]. However, to solve problem (24) in a centralized manner requires a central node. If the central node is under cyberattack, all the  $M$  MVNOs are vulnerable. In addition, since the  $M$  MVNOs may belong to different network providers, the information exchanging among different nodes should be as less as possible. Furthermore, the management overhead increases with the increase of  $M$  which causes that the wireless downlink communication system is not scalable. To deal with the aforementioned problems, we propose a blockchain-based distributed ADMM algorithm [23], [24] for dynamic spectrum acquisition in this paper.

#### IV. BLOCKCHAIN-BASED DISTRIBUTED ADMM ALGORITHM FOR DYNAMIC SPECTRUM ACQUISITION

By introducing auxiliary variable  $\mathbf{z} = [z_1, z_2, \dots, z_M]^T$ , problem (24) is reformulated as

$$\min_{\mathbf{p}, \mathbf{w}, \mathbf{z}} \sum_{m=1}^M p_m \quad (25a)$$

$$\text{s.t.} \quad \sum_{k=1}^K \frac{w_m \eta_{mk}}{N_{m\mathbf{z}}} \log_2 \left( 1 + \frac{p_m t_{mk}^2}{w_m \sigma^2} \right) \geq \bar{R}_m, \quad (25b)$$

$$\sum_{m=1}^M w_m \leq W, \quad (25c)$$

$$\mathbf{w} - \mathbf{z} = \mathbf{0}, \quad (25d)$$

$$p_m \geq 0, w_m \geq 0, z_m \geq 0, \forall m \in \mathcal{M}. \quad (25e)$$

Denote the feasible region of constraint (25c) and  $z_m \geq 0$ ,  $\forall m \in \mathcal{M}$  as  $\mathcal{Z}$ , its indicator function is defined as

$$\mathbb{I}_{\mathcal{Z}}(\mathbf{z}) = \begin{cases} 0, & \text{if } \mathbf{z} \in \mathcal{Z}, \\ \infty, & \text{otherwise.} \end{cases} \quad (26)$$

Similarly, denote the feasible region of constraint (25b),  $p_m \geq 0$  and  $w_m \geq 0$ ,  $\forall m \in \mathcal{M}$  as  $\mathcal{C}$ , its indicator function can be defined as

$$\mathbb{I}_{\mathcal{C}}(\mathbf{p}, \mathbf{w}) = \begin{cases} 0, & \text{if } (\mathbf{p}, \mathbf{w}) \in \mathcal{C}, \\ \infty, & \text{otherwise.} \end{cases} \quad (27)$$

With (26) and (27), problem (25) can be rewritten in ADMM form as follows

$$\min_{\mathbf{p}, \mathbf{w}, \mathbf{z}} \sum_{m=1}^M p_m + \mathbb{I}_{\mathcal{Z}}(\mathbf{z}) + \mathbb{I}_{\mathcal{C}}(\mathbf{p}, \mathbf{w}) \quad (28a)$$

$$\text{s.t.} \quad \mathbf{w} - \mathbf{z} = \mathbf{0}. \quad (28b)$$

Thus, the augmented Lagrangian (using the scaled dual variable) of problem (28) is given by

$$\mathcal{L}_{\nu}(\mathbf{p}, \mathbf{w}, \mathbf{z}, \mathbf{u}) = \sum_{m=1}^M p_m + \mathbb{I}_{\mathcal{Z}}(\mathbf{z}) + \mathbb{I}_{\mathcal{C}}(\mathbf{p}, \mathbf{w}) + \frac{\nu}{2} \|\mathbf{w} - \mathbf{z} + \mathbf{u}\|_2^2 \quad (29)$$

where  $\mathbf{u} = [u_1, u_2, \dots, u_N]^T$  is the dual variable for the constraint (28b) and  $\nu > 0$  is the penalty parameter. From problem (28), it is observed that the variables can be split into two groups:  $\{\mathbf{p}, \mathbf{w}\}$  and  $\mathbf{z}$ . Furthermore, the objective function can be separable accordingly. Therefore, the ADMM algorithm can be applied to solve problem (28) by iteratively updating  $\mathbf{p}, \mathbf{w}, \mathbf{z}$  and  $\mathbf{u}$ .

In the  $(l+1)$ -th iteration, given  $\{\mathbf{p}^{(l)}, \mathbf{w}^{(l)}, \mathbf{z}^{(l)}, \mathbf{u}^{(l)}\}$  which is optimal in the  $l$ -th iteration, the optimization variables are updated sequentially as follows.

Step 1: Given  $\{\mathbf{z}^{(l)}, \mathbf{u}^{(l)}\}$ , we solve the following optimization problem

$$\{\mathbf{p}^{(l+1)}, \mathbf{w}^{(l+1)}\} = \arg \max_{\mathbf{p}, \mathbf{w}} \mathcal{L}_{\nu}(\mathbf{p}, \mathbf{w}, \mathbf{z}^{(l)}, \mathbf{u}^{(l)}). \quad (30)$$

Problem (30) is equivalent to

$$\min_{\mathbf{p}, \mathbf{w}} \sum_{m=1}^M p_m + \frac{\nu}{2} \sum_{m=1}^M \left( w_m - z_m^{(l)} + u_m^{(l)} \right)^2 \quad (31)$$

$$\text{s.t.} \quad (25b), w_m \geq 0, p_m \geq 0, \forall m \in \mathcal{M}.$$

It is noted that problem (31) can be decomposed into  $M$  subproblems. Each subproblem solves

$$\min_{p_m, w_m} p_m + \frac{\nu}{2} \left( w_m - z_m^{(l)} + u_m^{(l)} \right)^2$$

$$\text{s.t.} \quad \sum_{k=1}^K \frac{w_m \eta_{mk}}{N_{m\mathbf{z}}} \log_2 \left( 1 + \frac{p_m t_{mk}^2}{w_m \sigma^2} \right) \geq \bar{R}_m,$$

$$w_m \geq 0, p_m \geq 0. \quad (32)$$

Problem (32) is a convex problem and can be solved efficiently using interior point method [22].

Step 2: Given  $\{\mathbf{p}^{(l+1)}, \mathbf{w}^{(l+1)}, \mathbf{u}^{(l)}\}$ , we solve the following optimization problem

$$\mathbf{z}^{(l+1)} = \arg \max_{\mathbf{z}} \mathcal{L}_{\nu}(\mathbf{p}^{(l+1)}, \mathbf{w}^{(l+1)}, \mathbf{z}, \mathbf{u}^{(l)}). \quad (33)$$

Problem (33) is equivalent to

$$\min_{\mathbf{z}} \left\| \mathbf{w}^{(l+1)} - \mathbf{z} + \mathbf{u}^{(l)} \right\|_2^2 \quad (34)$$

$$\text{s.t.} \quad \|\mathbf{z}\|_1 \leq W, z_m \geq 0, \forall m \in \mathcal{M}. \quad (35)$$

Problem (34) is a convex  $\ell_1$  ball projection problem [25]. Its closed-form solution is obtained as follows.

If  $\|\mathbf{w}^{(l+1)} + \mathbf{u}^{(l)}\|_1 > W$ , we have

$$z_m^{(l+1)} = \max \left\{ w_m^{(l+1)} + u_m^{(l)} - \beta, 0 \right\} \quad (36)$$

where

$$\beta = \frac{1}{\tau} \left( \sum_{m=1}^{\tau} \varphi_m - W \right), \quad (37)$$

$$\tau = \max_{1 \leq i \leq M} \left\{ i \left| \varphi_i - \frac{1}{i} \left( \sum_{m=1}^i \varphi_m - W \right) > 0 \right. \right\}, \quad (38)$$

and  $\varphi = [\varphi_1, \varphi_2, \dots, \varphi_M]^T$  denotes the vector obtained by sorting  $\mathbf{w}^{(l+1)} + \mathbf{u}^{(l)}$  in a descending order.

If  $\|\mathbf{w}^{(l+1)} + \mathbf{u}^{(l)}\|_1 \leq W$ , we have

$$z_m^{(l+1)} = w_m^{(l+1)} + u_m^{(l)}. \quad (39)$$

Step 3: Given  $\{\mathbf{w}^{k+1}, \mathbf{z}^{k+1}\}$ , we solve the following optimization problem

$$\mathbf{u}^{(l+1)} = \arg \max_{\mathbf{u}} \mathcal{L}_{\nu}(\mathbf{p}^{(l+1)}, \mathbf{w}^{(l+1)}, \mathbf{z}^{(l+1)}, \mathbf{u}). \quad (40)$$

The closed-form solution to problem (40) is

$$\mathbf{u}^{k+1} = \mathbf{w}^{k+1} - \mathbf{z}^{k+1} + \mathbf{u}^k. \quad (41)$$

Thus, the whole blockchain-based distributed ADMM algorithm for dynamic spectrum acquisition is summarized in Algorithm 1.

*Remark:* Blockchains provide a method for providing a transparent, trustless platform for data computation. This makes a blockchain the perfect platform for conducting the aggregation step of ADMM, allowing all participants to audit

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**Algorithm 1** Blockchain-Based Distributed ADMM Algorithm for Dynamic Spectrum Acquisition
 

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1: **Initialize:**  $z_m^{(0)} = 0$  and  $u_m^{(0)} = 0 \forall m \in \mathcal{M}; \nu = 0; l = 0;$   
 2: **Repeat:**  
   **Local Computation:**  
     Get  $z_m^{(l)}$  and  $u_m^{(l)}$  from blockchain;  
     Obtain  $p_m^{(l+1)}$  and  $w_m^{(l+1)}$  by solving (32);  
     Send  $w_m^{(l+1)}$  to ADMM aggregator on blockchain;  
   **Blockchain Computation:**  
     Obtain  $z_m^{(l+1)}$  and  $u_m^{(l+1)}$ ;  
      $l = l + 1;$   
**Until** Convergence.

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the progress of the algorithm, the accuracy of the solution, and the veracity of their scheduled commitments. Further, ADMM is a natural fit for implementation on a blockchain, as it guarantees convergence yet has a computationally cheap aggregation step (minimizing the burden of verification) [16].

## V. OPTIMAL POWER AND SPECTRUM ALLOCATION WITH KNOWN SYSTEM PARAMETERS

In this paper, we propose a dynamic spectrum acquisition scheme. To fairly compare the performance of our proposed scheme with the fixed spectrum allocation scheme and other schemes, with known system parameters, we propose to investigate the actual required minimum sum transmit power of  $M$  MVNOs subject to that the data transmission rate thresholds for all MUs in  $M$  cells are satisfied. With our proposed dynamic spectrum acquisition scheme and other spectrum allocation schemes, the bandwidth of wireless spectrum of the  $m$ -th MVNO, i.e.,  $w_m$ , is known. Thus, the aforementioned optimization problem is decoupled into  $M$  transmit power minimization subproblems. For the transmit power minimization subproblem of the  $m$ -th MVNO, with known system parameters, we should study the power and spectrum allocation optimization problem among all MUs in the  $m$ -th cell subject to that the data transmission rate thresholds for all MUs in the  $m$ -th cell are satisfied.

In the  $m$ -th cell, when the actual number of MUs,  $N_m$ , and the channel between the  $n$ -th MU and its associated  $m$ -th MVNO,  $h_{mn}$ , are known for  $n \in \mathcal{N}_m = \{1, 2, \dots, N_m\}$ , the power and spectrum allocation optimization problem is formulated as

$$\min_{\mathbf{q}_m, \mathbf{b}_m} \sum_{n=1}^{N_m} q_{mn} \quad (42a)$$

$$\text{s.t.} \quad \sum_{n=1}^{N_m} b_{mn} \leq w_m, \quad (42b)$$

$$b_{mn} \log_2 \left( 1 + \frac{q_{mn} |h_{mn}|^2}{b_{mn} \sigma^2} \right) \geq \tilde{R}_{m,n} \quad (42c)$$

$$q_{mn} \geq 0, b_{mn} \geq 0, \forall n \in \mathcal{N}_m \quad (42d)$$

where  $\tilde{R}_m$  denotes the data transmission rate threshold for each MU in the  $m$ -th cell,  $\mathbf{q}_m = [q_{m1}, q_{m2}, \dots, q_{m, N_m}]^T$ ,

and  $\mathbf{b}_m = [b_{m1}, b_{m2}, \dots, b_{m, N_m}]^T$ . To continue, we need the following proposition.

*Proposition 2:* For problem (42), the optimal  $\mathbf{q}_m$  and  $\mathbf{b}_m$  should satisfy that constraint (42c) are all active, i.e.,

$$b_{mn} \log_2 \left( 1 + \frac{q_{mn} |h_{mn}|^2}{b_{mn} \sigma^2} \right) = \tilde{R}_{m,n}, \forall n \in \mathcal{N}_m \quad (43)$$

*Proof:* See Appendix B. ■

From Proposition 2, problem (42) is equivalently written as

$$\begin{aligned} \min_{\mathbf{b}_m} \quad & \sum_{n=1}^{N_m} \frac{b_{mn} \sigma^2}{|h_{mn}|^2} \left( 2^{\frac{\tilde{R}_{m,n}}{b_{mn}}} - 1 \right) \\ \text{s.t.} \quad & \sum_{n=1}^{N_m} b_{mn} \leq w_m, b_{mn} \geq 0, \forall n \in \mathcal{N}_m. \end{aligned} \quad (44)$$

Problem (44) is convex. In the following proposition, we propose to solve problem using the bisection search.

*Proposition 3:* The closed-form optimal solution to problem (44), denoted by  $b_{mn}^o$ , is

$$b_{mn}^o = \frac{\tilde{R}_m \ln 2}{1 + \mathcal{W} \left( \frac{\mu^o |h_{mn}|^2 - \sigma^2}{e \sigma^2} \right)}, \forall n \in \mathcal{N}_m \quad (45)$$

where  $\mathcal{W}(x)$  denotes the Lambert  $\mathcal{W}$  function of  $x$  [26] and  $\mu^o > 0$  is a constant which satisfies  $\sum_{n=1}^{N_m} b_{mn} = w_m$ . The value of  $\mu^o$  can be found by the bisection search over  $[\mu_{\min}, \mu_{\max}]$  with

$$\mu_{\min} = \frac{\chi}{\max |h_{mn}|^2} \text{ and } \mu_{\max} = \frac{\chi}{\min |h_{mn}|^2} \quad (46)$$

where

$$\chi = \sigma^2 \left( \frac{N_m \tilde{R}_m \ln 2}{w_m} - 1 \right) \exp \left( \frac{N_m \tilde{R}_m \ln 2}{w_m} \right) + \sigma^2. \quad (47)$$

*Proof:* See Appendix C. ■

## VI. SIMULATION RESULTS

In this section, we evaluate the performance of our proposed dynamic spectrum acquisition through simulations. We consider a wireless downlink communication system with  $M = 6$  MVNOs. Each MVNO serves the MUs in a cell. Since each MVNO uses different wireless spectrum for transmission, there is no co-channel interference. Thus, the overlap of different cells will not cause a problem. We assume that the radii and the corresponding user densities of the  $M = 6$  cells are ( $r_1 = 80$  m,  $\lambda_1 = 2400$  persons/km<sup>2</sup>), ( $r_2 = 80$  m,  $\lambda_2 = 1600$  persons/km<sup>2</sup>), ( $r_3 = 100$  m,  $\lambda_3 = 1600$  persons/km<sup>2</sup>), ( $r_4 = 100$  m,  $\lambda_4 = 2000$  persons/km<sup>2</sup>), ( $r_5 = 120$  m,  $\lambda_5 = 2000$  persons/km<sup>2</sup>), and ( $r_6 = 120$  m,  $\lambda_6 = 2400$  persons/km<sup>2</sup>). The total available bandwidth is 100 MHz. The path-loss decay factor is 3.76 [27]. We assume that  $\sigma^2 = \Gamma \sigma_0^2 = 2$  dBm/Hz. In (21), a  $K = 500$ -point HR-GHQ is employed. If not specified, the average data transmission rate threshold in (12) and the data transmission rate threshold in (42) are  $\tilde{R}_m = \tilde{R}_m = \tilde{R}_{\min}, \forall m \in \mathcal{M}$ .

In Fig. 1, we show the convergence performance of Algorithm 1 when  $R_{\min} = 1$  Mbps. From Fig. 1, it is observed that



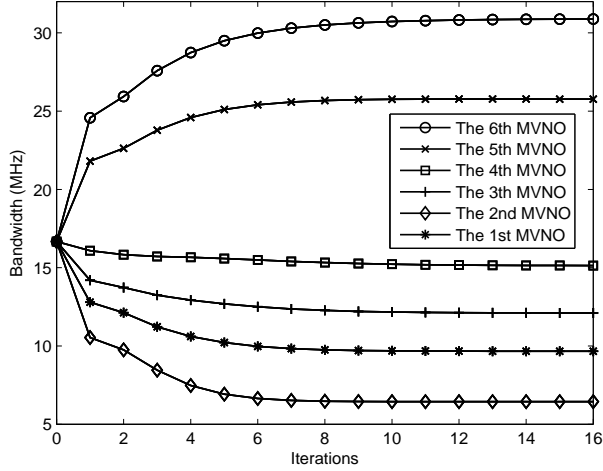


Fig. 1. Bandwidth versus iterations; the convergence performance of Algorithm 1 when  $R_{\min} = 1$  Mbps.

our proposed Algorithm 1 converges for about 10 iterations. Furthermore, it is found that since  $\bar{R}_m = R_{\min} = 1$  Mbps  $\forall m \in \mathcal{M}$ , the more bandwidth is needed when the average number of MUs in the cell,  $\mathbb{E}[N_m]$ , is large.

In Fig. 2, we present the minimum sum power comparison of different schemes. In the legend, “DSA” denotes our proposed dynamic spectrum acquisition scheme. “Uniform” denotes that the total available bandwidth is uniformly allocated to  $M = 6$  MVNOs. “Proportional” denotes that the total available bandwidth is allocated to MVNOs proportionally according to  $\mathbb{E}[N_m]R_{\min}$ . “Non-Causal” denotes that assuming the number of active MUs in all cells and all the channels,  $h_{mn}$ ,  $\forall n \in \mathcal{N}_m$ ,  $m \in \mathcal{M}$ , are known non-causally, we solve the joint sum transmit power minimization and bandwidth allocation optimization problem subject to that the data transmission rate thresholds for all MUs in  $M$  cells are satisfied. From Fig. 2, it is observed that our proposed dynamic spectrum acquisition scheme achieves almost the same minimum sum power as the “Non-Causal” scheme.

In Fig. 3, we present the bandwidth allocation comparison of different schemes when  $R_{\min} = 2$  Mbps. From Fig. 3, it is found that the allocated bandwidth of our proposed dynamic spectrum acquisition scheme is almost the same as that of the “Non-Causal” scheme.

In Fig. 4, when  $R_{\min} = 2$  Mbps, we vary the value of  $\lambda_6$  while remaining  $\lambda_1, \lambda_2, \dots, \lambda_5$  to be constant and present the minimum sum power comparison of different schemes. In Fig. 5, we present the corresponding bandwidth allocation of our proposed dynamic spectrum acquisition scheme for all MVNOs. From Fig. 4, it is shown that almost the same minimum sum power is obtained by our proposed dynamic spectrum acquisition scheme and the “Non-Causal” scheme.

## VII. CONCLUSION

In this paper, we have proposed a decentralized blockchain based dynamic spectrum acquisition scheme for a wireless downlink communication system with multiple MVNOs. It is

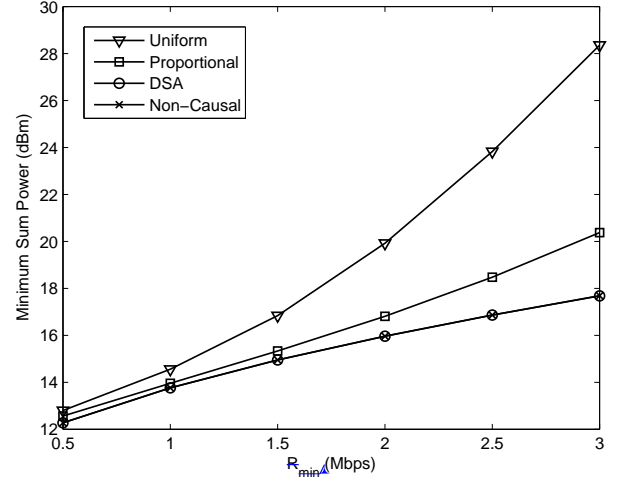


Fig. 2. Minimum sum power versus  $R_{\min}$ ; comparison of our proposed dynamic spectrum acquisition scheme, the “Uniform” scheme, the “Proportional” scheme, and the “Non-Causal” scheme.

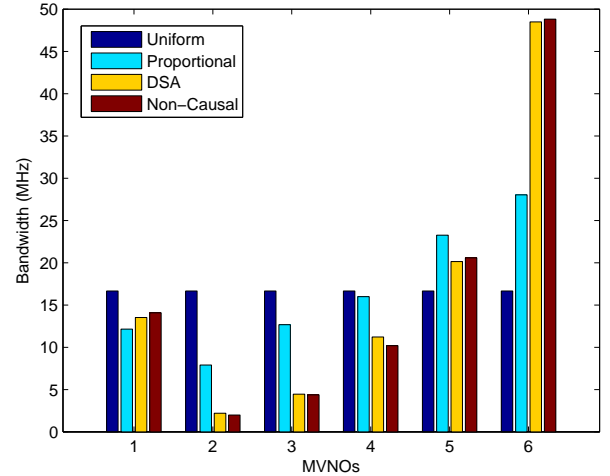


Fig. 3. Bandwidth allocation comparison of our proposed dynamic spectrum acquisition scheme, the “Uniform” scheme, the “Proportional” scheme, and the “Non-Causal” scheme when  $R_{\min} = 2$  Mbps.

shown through simulation results that our proposed dynamic spectrum acquisition scheme achieves almost the same minimum sum power as the non-causal scheme which assumes the number of active MUs in all cells and all the channels are known non-causally for optimal dynamic spectrum allocation.

## APPENDIX A PROOF OF PROPOSITION 1

According to the definition of the CDF, we have

$$F_{|g_{mn}|^2}(x) = \Pr(|g_{mn}|^2 \leq x(1 + L_{mn}^\alpha)). \quad (48)$$

Since  $g_{mn}$  is a complex Gaussian random variable with zero mean and unit variance,  $|g_{ij}|^2$  is an exponentially distributed random variable. Furthermore, the location of the  $n$ -th MU in the  $m$ -th cell is uniformly distributed in  $\mathcal{D}_m$  with the PDF of

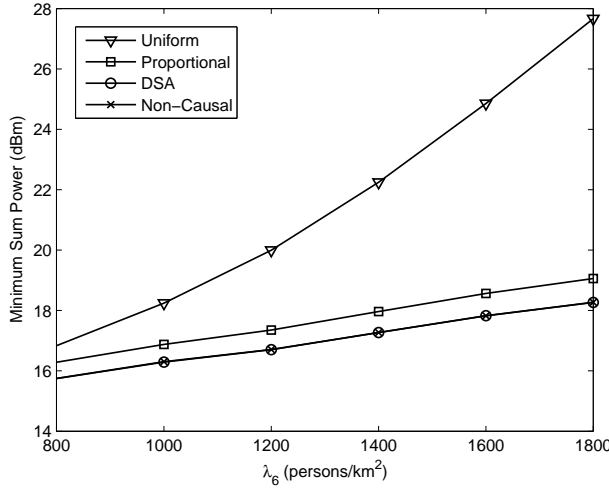


Fig. 4. Minimum sum power versus  $\lambda_6$ ; comparison of our proposed dynamic spectrum acquisition scheme, the “Uniform” scheme, the “Proportional” scheme, and the “Non-Causal” scheme when  $R_{\min} = 2$  Mbps.

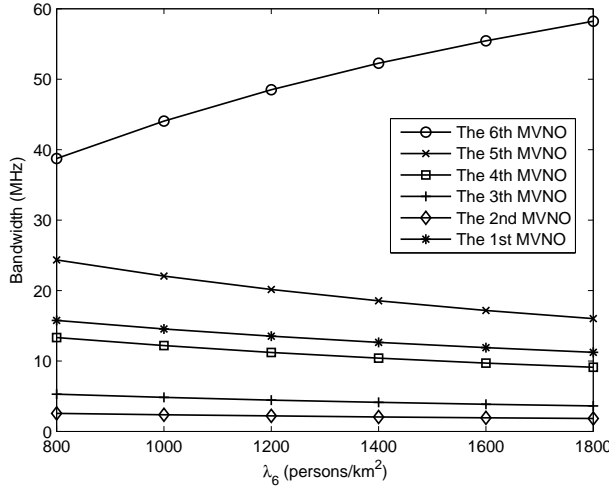


Fig. 5. Bandwidth allocation of all MVNOs versus  $\lambda_6$ ; our proposed dynamic spectrum acquisition scheme when  $R_{\min} = 2$  Mbps.

$1/(\pi r_m^2)$ . By using polar coordinates, we obtain

$$F_{|h_{mn}|^2}(x) = \int_0^{r_m} \int_{-\pi}^{\pi} \frac{1}{\pi r_m^2} (1 - e^{-x(1+y^\alpha)}) y d\theta dy. \quad (49)$$

After some mathematical manipulation, we obtain

$$F_{|h_{mn}|^2}(x) = 1 - \frac{2}{r_m^2} e^{-x} \int_0^{r_m} y e^{-xy^\alpha} dy. \quad (50)$$

Let  $t = xy^\alpha$ .  $F_{|h_{mn}|^2}(x)$  is reexpressed as

$$\begin{aligned} F_{|h_{mn}|^2}(x) &= 1 - \frac{2}{r_m^2} e^{-x} \int_0^{xr_m^\alpha} t^{\frac{1}{\alpha}} x^{-\frac{1}{\alpha}} e^{-t} d\left(t^{\frac{1}{\alpha}} x^{-\frac{1}{\alpha}}\right) \\ &= 1 - \frac{2}{\alpha r_m^2} x^{-\frac{2}{\alpha}} e^{-x} \int_0^{xr_m^\alpha} t^{\frac{2}{\alpha}-1} e^{-t} dt \\ &= 1 - \frac{2}{\alpha} (xr_m^\alpha)^{-\frac{2}{\alpha}} \gamma\left(\frac{2}{\alpha}, xr_m^\alpha\right) e^{-x} \end{aligned} \quad (51)$$

where  $\gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt$  denotes the lower incomplete gamma function. Because  $\Psi(s, 1+s, -x) = sx^{-s}\gamma(s, x)$  [19, 6.5.12], we obtain (14).

## APPENDIX B PROOF OF PROPOSITION 2

We prove Proposition 2 by contradiction. Assume that  $\mathbf{q}_m^* = [q_{m1}^*, q_{m2}^*, \dots, q_{mN_m}^*]^T$  is the optimal solutions to problem (42) such that there exists  $n \in \mathcal{N}_m$  which satisfy

$$b_{mn} \log_2 \left( 1 + \frac{q_{mn} |h_{mn}|^2}{b_{mn} \sigma^2} \right) > \tilde{R}_m. \quad (52)$$

We can choose a proper real scalar  $\Delta > 0$  such that

$$b_{mn} \log_2 \left( 1 + \frac{(q_{mn}^* - \Delta) |h_{mn}|^2}{b_{mn} \sigma^2} \right) = \tilde{R}_m. \quad (53)$$

Note that  $[q_{m1}^*, \dots, q_{m(n-1)}^*, q_{mn}^* - \Delta, q_{m(n+1)}^*, \dots, q_{mN_m}^*]^T$  is also feasible and have smaller objective value than  $\mathbf{q}_m^*$ . This contradicts that  $\mathbf{q}_m^*$  is the optimal solution.

## APPENDIX C PROOF OF PROPOSITION 3

The Lagrangian of problem (44) is given by

$$\tilde{\mathcal{L}} = \sum_{n=1}^{N_m} b_{mn} \sigma^2 \left( e^{\frac{\tilde{R}_m \ln 2}{b_{mn}}} - 1 \right) + \mu \left( \sum_{n=1}^{N_m} b_{mn} - w_m \right) \quad (54)$$

where  $\mu \geq 0$  denotes the Lagrange multiplier associated with the constraint  $\sum_{n=1}^{N_m} b_{mn} \leq w_m$ . Since problem (44) is convex, the Karush-Kuhn-Tucker (KKT) conditions are both necessary and sufficient for the global optimality of problem (44). It can be verified that  $\sum_{n=1}^{N_m} b_{mn} = w_m$  must hold for problem (42). From KKT conditions, we have  $\mu^o > 0$  where  $\mu^o$  denotes the optimal dual solution to problem (44).

Taking the first-order partial derivative of  $\tilde{\mathcal{L}}$  with respect to  $b_{mn}$ , we have

$$\frac{\partial \tilde{\mathcal{L}}}{\partial b_{mn}} = \frac{\sigma^2}{|h_{mn}|^2} \left( \left( 1 - \frac{\tilde{R}_m \ln 2}{b_{mn}} \right) e^{\frac{\tilde{R}_m \ln 2}{b_{mn}}} - 1 \right) + \mu \quad (55)$$

From KKT conditions, we have

$$\left( \frac{\tilde{R}_m \ln 2}{b_{mn}^o} - 1 \right) e^{\frac{\tilde{R}_m \ln 2}{b_{mn}^o} - 1} = \frac{\mu^o |h_{mn}|^2 - \sigma^2}{e \sigma^2}. \quad (56)$$

According to the definition of Lambert  $\mathcal{W}$  function, we obtain the optimal  $b_{mn}^o$  as in (45). In (45), there exists an unknown parameter  $\mu^o$ . The value of  $\mu^o$  should satisfy  $\sum_{n=1}^{N_m} b_{mn} = w_m$ .

In (45), since

$$\frac{\mu^o |h_{mn}|^2 - \sigma^2}{e \sigma^2} \geq -1, \quad (57)$$

$\mathcal{W}(x)$  is a monotonically increasing function. Accordingly,  $b_{mn}^o$  is a monotonically decreasing function of  $|h_{mn}|^2$ . Thus,

we have

$$\begin{aligned} \frac{\tilde{R}_m \ln 2}{1 + \mathcal{W}\left(\frac{\mu^o \max|h_{mn}|^2 - \sigma^2}{e\sigma^2}\right)} &\leq b_{mn}^o \\ &\leq \frac{\tilde{R}_m \ln 2}{1 + \mathcal{W}\left(\frac{\mu^o \max|h_{mn}|^2 - \sigma^2}{e\sigma^2}\right)} \end{aligned} \quad (58)$$

Summing each term in (58) from  $n = 1$  to  $n = N_m$ , we obtain

$$\begin{aligned} \frac{N_m \tilde{R}_m \ln 2}{1 + \mathcal{W}\left(\frac{\mu^o \max|h_{mn}|^2 - \sigma^2}{e\sigma^2}\right)} &\leq w_m \\ &\leq \frac{N_m \tilde{R}_m \ln 2}{1 + \mathcal{W}\left(\frac{\mu^o \max|h_{mn}|^2 - \sigma^2}{e\sigma^2}\right)} \end{aligned} \quad (59)$$

According to the definition of Lambert  $\mathcal{W}$  function, we have

$$\mu_{\min} \leq \mu^o \leq \mu_{\max} \quad (60)$$

where  $\mu_{\min}$  and  $\mu_{\max}$  are defined in (46).

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