Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
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Tracking Momentum: Modeling and Analysis of "Momentum" in Tennis Players and Match Trends

Summary

In the domain of tennis, scientific investigations into the determinants of match outcomes have advanced considerably, with **Momentum** recognized as a complex yet critical factor attracting significant interest. This study aims to examine the momentum phenomenon in professional tennis, developing models and methodologies to assess and leverage its impact on match results.

Addressing the first issue, after data collection and preprocessing, we selected 18 indicators to quantify player skill levels and match conditions, establishing a **Real-time Momentum Assessment Model**. This model analyzes players' momentum levels from match data, providing immediate feedback. We employed 14 technical indicators as original factors in a **factor analysis evaluation model**, with four situational indicators acting as model hyperparameters. This model allows us to quantitatively evaluate player performance and momentum fluctuations during matches, offering visualized results.

For the second issue, we applied the **Vector Error Correction Model (VECM)** and **Game Outcome Correlation Tests** to examine the randomness of match condition fluctuations and winning streaks, with fluctuations quantified by a **counterfactual analysis model**. VECM results indicate a significant long-term and short-term correlation between match condition fluctuations and player momentum, with no correlation to random momentum sequences. Correlation tests show a high correlation between winning streaks and player momentum, with a negligible chance of randomness at 0.08.

Regarding the third issue, employing four common factors derived from factor analysis and real-time momentum data, we constructed a **Match Condition Prediction Model** using **LSTM Networks**. The **SHAP Model** quantitatively and visually displayed the importance of each influencing factor, revealing player momentum as the most significant determinant of match condition fluctuations, followed by service breaking capacity. Based on these findings, we devised momentum-based match strategy recommendations, considering player rankings among other factors.

For the fourth issue, we validated our model's **generalizability** across different tennis events and surfaces, assessing its adaptability to various match scenarios. Comparative experiments highlighted potential additional factors for model enhancement; incorporating surface type significantly improved single-match prediction accuracy from 74.54% to 91.67%. Extending our predictive methodology to table tennis, we achieved a prediction accuracy of 92% for single matches.

Concluding with a **sensitivity analysis**, we compiled a memorandum summarizing our findings and providing coaches with advice on momentum management and preparing players for events affecting match flow.

This research offers an in-depth analysis and solutions for the momentum phenomenon in professional tennis, aiding coaches and players in understanding and utilizing momentum to enhance competitiveness and win rates. Future studies could expand and optimize our models for broader application across various sporting contexts and complex match scenarios.

Keywords : Tennis, Momentum, Factor analysis,Counterfactual analysis,VECM,LSTM.

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1 Introduction

1.1 Background

Professional tennis matches are controlled by a variety of factors such as competitive elements, environment, experience, etc [1]. When facing different opponents in different matches, even though the players' practice results can be close to the stable value, they still show different levels of play. In addition to the technical level of the players, tactical strategy, venue factors and other factors that we usually consider in determining the winner of a match, ”momentum” is also a potentially huge influence. Athletes who feel their momentum during a match have a higher level of momentum and confidence in their ability and performance.

Tennis matches are often faced with important moments: break points, set points, set points, match points, all of which can be crucial in determining the outcome of a match, as they significantly affect the ”momentum” of the player. [2] Any psychological fluctuations in momentum have an impact on the brain's nervous system and muscle control, which can alter an athlete's performance and, consequently, the outcome of the match. Understanding and effectively utilizing momentum is therefore of great importance in enhancing the on-court performance of tennis players.

1.2 Restatement of the Problem

Based on the above background research and the question requirements of the topic, we will address the following questions:

- Develop a model to capture the momentum of a player during the flow of a match, quantify and visualize how good or bad a player's performance is during any given period of time through his or her momentum.
- Modeling to assess the randomness of game situation fluctuations and players' consecutive scores, and whether they are related to players' momentum.
- Develop a model to predict fluctuations in the game and identify the factors that have the greatest impact on such fluctuations.
- Based on our concept of ”momentum”, we suggest strategies for athletes facing a new game.
- Evaluate the generalization ability of our model, identify some other factors that may be incorporated into the model, and try to generalize model to different scenarios and different sports.
- Based on the results of our modeling analysis, we should provide advice for coaches on the role of “momentum”, and how to prepare players to respond to events that impact the flow of play during a tennis match.

1.3 Our Approach

For Problem 1, this paper first preprocesses the data provided in the Appendix and the data collected additionally, and then selects 18 indicators about players' skills and field situation to build a real-time momentum evaluation model. Among them, we used 14 indicators of player skills to construct the factor analysis evaluation model, and set 4 indicators of field situation as model hyperparameters. Quantitative analysis and visualization of player momentum are performed through the model.

For Problem 2, this paper adopts the methods of VECM and outcome correlation test to test the stochasticity of match situation fluctuation and consecutive victories, respectively. Among them, the field situation fluctuation is evaluated by the fluctuation of the player's real-time win probability, while the real-time win probability is computed by the counterfactual analysis model we constructed.

For Problem 3, this paper uses the common factors and real-time momentum obtained from the factor analysis in Problem 1 as inputs, constructs a prediction model for the match situation based on LSTM network, and quantifies and visualizes the importance of the influencing factors through the SHAP model. Finally, we formulated a momentum-based match strategy proposal based on the analysis results and with reference to other factors such as players' strength rankings.

For Problem 4, we test the generalization performance of the model using data from tennis matches of different types and venues, and add new factors to optimize and enhance the model to adapt to different match scenarios and different sport types.

Finally, we tested the model for sensitivity and prepared a memo summarizing our results with advice for coaches on the role of "momentum", and how to prepare players to respond to events that impact the flow of play during a tennis match.

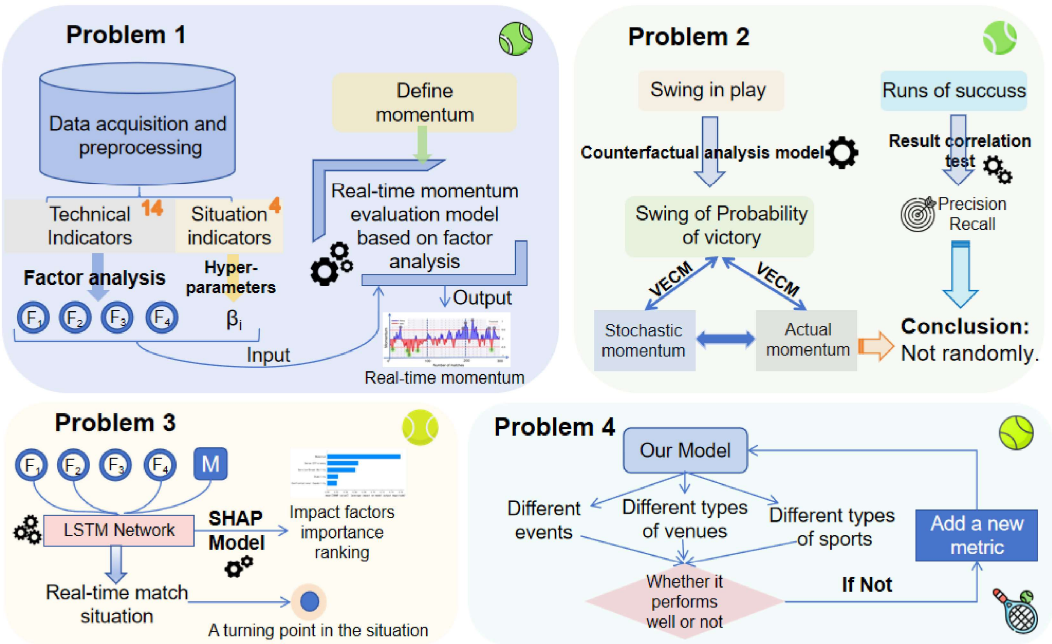


Figure 1: Our Approach

2 Assumption and Justification

For the rationality of problem research and the convenience of model development, we make the following assumptions in this paper to support our research findings:

- There is no significant difference in strength between players in the given dataset, and there are no major emergencies before or after the competition to eliminate the impact of extreme factors on the performance and momentum of players on the field.
- The data we collected for building the model is accurate and reliable.
- During the competition, the importance of each serve is the same except for the match point.
- There is a cointegration relationship between the situation on the field and the momentum of the players, which is a prerequisite for our VCEM analysis.

3 Notations

Table 1 shows the necessary notations and signs used in this paper. Other notations and signs will be declared or defined when using.

Table 1: Notions and Symbol Description

Symbol	Descripitions
X_t	the lever at moment t
Y_t	the observations of multiple variables at time point t
A_p	the time-lagged i period of the impact
ε_t	a k-dimensional error vector
λ_i	the estimated eigenvalue
T	the number of observed samples
λ_{r+1}	the r+1st eigenvalue of the estimate
β	short-run adjustment coefficient of momentum to the winning probability
ε_t	an error term denoting the unexplained part of the model
S	a subset of the features used in the mode

4 Model Preparation

4.1 Indicator Selection

This paper uses data from every match after the first two rounds of the Men's Singles at Wimbledon 2023 as the study object and uses data from the Women's Singles final of the Bad Homburg Open 2021 as the validation set. The technical statistics metrics of each match were obtained from all of them on the official website of the German Network or the attached dataset, and a total of 18 raw metrics were counted as follows:

We will construct our model based on the above raw metrics.

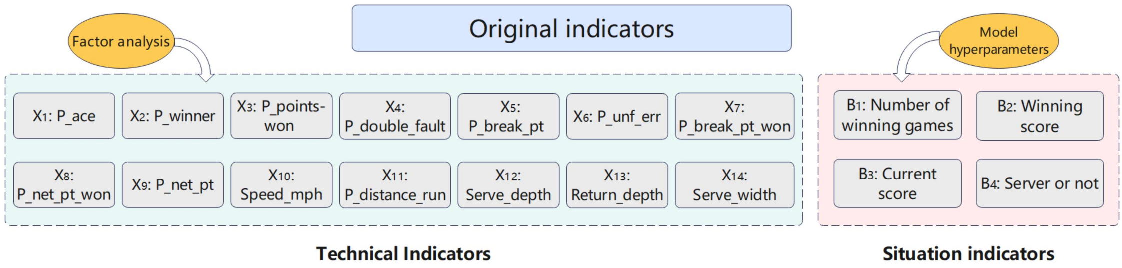


Figure 2: Raw metrics

4.2 Data Preprocessing

The completeness and quantifiability of the indicator data is a prerequisite for good modeling. We first used a python program to populate individual missing data with proximity values, despite the small number of events events. Subsequently, to facilitate statistical modeling, we quantified individual metrics in a reasonable manner. For example, we set "AD" to 50 for the current match score, and "Deep" and "Not Deep" to 1 and 0 for the depth of serve indicator.

5 A Real-time Momentum Assessment Model for Tennis Matches Based on Factor Analysis

5.1 Definition of Momentum

In the realm of tennis competitions, certain key metrics not only influence the shifts in a player's **Momentum** but also serve as indicators of the player's **Momentum**. Therefore, we will define and model **Momentum** through a selection of carefully chosen metrics, encompassing both players' technical performance and situational factors on the court. Given the distinct characteristics of technical and situational indicators, we apply different methodologies in our model construction. [3] For technical indicators, given their complexity and the potential for strong correlations among various factors, we employ factor analysis for dimensionality reduction. This approach allows us to capture the majority of the "**Momentum**" information from the original data using a limited number of composite factors. A player's recent performance in sets can act as a preliminary indicator of short-term competitive advantage. However, this measure falls short of fully representing "**Momentum**" as it overlooks the contextual nuances specific to matches and players. [4]Thus, we incorporate situational indicators on the court as supplementary measures. Recognizing that certain tennis-specific elements (such as the serving side) significantly affect a player's **Momentum** in ways that are challenging to quantify directly through machine learning techniques, we designate the court situation indicators as hyperparameters following specified rules:

β_1:Number of wins $\times$ 0.15 β_2:Average winning score $\times$ 0.10 β_3:current score $\times$ 0.01

β_4:If the receiver scores: The receiver's β_4= 1.35, The server's β_4=0.85;

If the server scores: The receiver's β_4= 0.95, The server's β_4=1.05;

In the model we define the concepts of "**lever**" and "**Momentum**", respectively. **Lever** is used as a measure of the effectiveness of a team player in a single round, weighted by the composite factor and hyperparameters of the round:

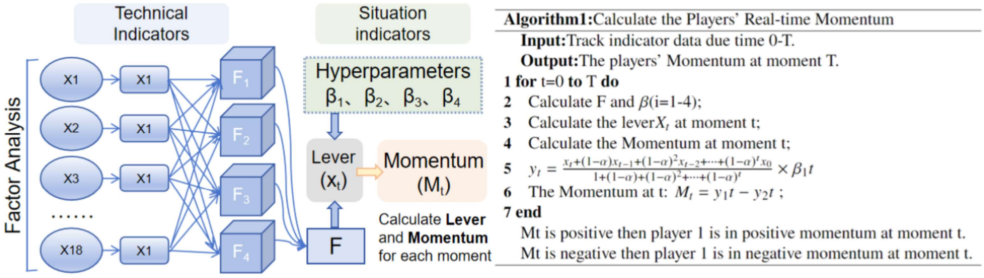


Figure 3: Schematic diagram of the principle of the model

$$x_t = a_1 F_{1t} + a_2 F_{2t} + a_3 F_{3t} + \cdots + \beta_{2t} + \beta_{3t} + \beta_{4t}$$

Momentum aims to describe which player is in control at any point of the match – who is currently winning more points? Who hit the ball of higher quality? And who is winning the important (high-leverage) points? It is defined as "an exponentially weighted moving average of the leverage gained by a player."

$$y_t = \frac{x_t+(1-\alpha)x_{t-1}+(1-\alpha)^2x_{t-2}+\cdots+(1-\alpha)^t x_0}{1+(1-\alpha)+(1-\alpha)^2+\cdots+(1-\alpha)^t} \times \beta_1 t$$

$$M_t = y_{1t} - y_{2t}$$

Where x_t is the **lever** at moment t. At each point in a match, there will be a single momentum value favouring one player. This better allows us to quantify periods of dominance beyond the standard "Player A has won x of the last y points," while also easily highlighting the specific points in a match that constitute true momentum swings.

5.2 Model Construction

5.2.1 Model Fundamentals

Factor analysis is an effective algorithm for the comprehensive evaluation of multiple variables with high correlation, and its evaluation results can better reflect the momentum of tennis players. [5]The basic principle is: firstly, the correlation matrix is calculated for the normalized technical index data, and then the correlation matrix is calculated through the Correlation coefficient matrix R's eigenvalue $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p \geq 0$ and the corresponding eigenvectors $u_1, u_2, \cdots, u_p$, among $u_j = (u_{1j}, u_{2j}, \cdots, u_{nj})^T$. Calculate the tennis TEI primary load matrix:

$$A = \left[\sqrt{\lambda_1}u_1 \quad \sqrt{\lambda_2}u_2 \quad \cdots \quad \sqrt{\lambda_p}u_p\right]$$

Then, based on the primary loading matrix, the contribution of each common factor is calculated and the optimal number of principal factors is selected by the fragmentation diagram. The extracted factor loading matrix is rotated to obtain the matrix $B = \hat{A}T$, among $\hat{A}$ is the first m columns of A and T is an orthogonal matrix.Constitutive factor model:

$$\begin{cases} \tilde{x_1} = b_{11}F_1 + \cdots + b_{1m}F_m \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \tilde{x_1} = b_{p1}F_1 + \cdots + b_{pm}F_m \end{cases}$$

The regression method was then used to find the individual factor score function:

$$\hat{F}_j = b_{j1}\tilde{x_1} + \cdots + b_{jp}\tilde{x_p}, j = 1, 2, \ldots, m$$

$$\text{Among,}\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mp} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}\text{,then } \begin{bmatrix} b_1^T & b_2^T & \cdots & b_m^T \end{bmatrix} = R^{-1}A$$

Finally, the composite factor score formula was utilized to calculate the tennis player's composite score for each round.

5.2.2 Indicator Correlation Test

The precondition of factor analysis is that there is a strong correlation between the observed variables, otherwise it is difficult to have a shared factor. In this paper, KMO and Bartlett's test of sphericity were used to analyze the correlation of indicators, and the value of KMO was measured to be 0.856, which is greater than 0.5, and at the same time, P = 0.00 ¡ 0.05, so it indicates that this data is suitable for factor analysis.

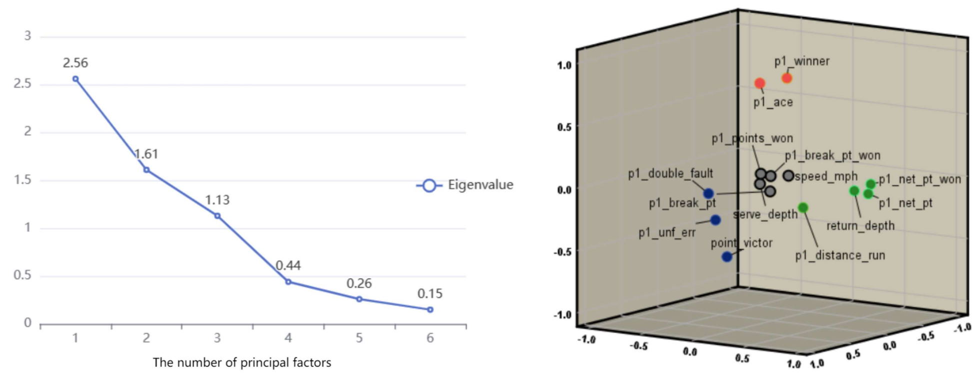
Table 2: KMO and Bartlett sphericity test

KMO Quantity	Bartlett's test of sphericity
0.856	Approximate chi-square=474.193,Degrees of freedom=91,P=0.00

5.2.3 Factor Analysis Results

The optimal count of common factors is identified through a scree plot, revealing significant eigenvalue shifts for up to 4 factors before stabilizing, thus determining 4 as the optimal number of common factors. The rotated variance contribution rates of the common factors from 1 to 4 are 20.000%, 18.072%, 14.683%, and 13.695%, and the four eigenvalues of the common factors are 2.562, 1.614, 1.113, and 1.006, respectively. Using the maximum variance method to rotate the initial matrix orthogonally, the practical significance of the four common factors can be better understood and interpreted: in the common factor 1, according to the index loading coefficients of the distance run, unforced errors, and the winning score in descending order, respectively. These indicators reflect the holding ability of the players, so we named Common Factor 1 as the holding ability factor. The loadings of the indicators in the Common Factor 2 in descending order are serve score, Aces, and depth of serve, which measure the actual score of the player's serve, so we named the Common Factor 2 as the efficiency of serve score factor. The load factor of each variable indicator of the public factor 3 in descending order are unforced errors, two service errors lost points, net points, named stability factor. Common factor 4 The first two factors are the break score, the reception score, the reception score rate is closely related to the success rate of the break, so the two technical and tactical indicators are named as the break ability factor.

Finally we get the factor scoring model as:



(a) Gravel chart (b) Rotating the spatial factor distribution

Figure 4: Factor analysis results

$$F_1 = 0.084X_1 + 0.035X_2 + 0.149X_3 + \cdots + 0.441X_{14} \; F_2 = 0.360X_1 - 0.006X_2 - 0.062X_3 + \cdots - 0.061X_{14}$$
$$F_3 = -0.149X_1 - 0.459X_2 + 0.518X_3 + \cdots + 0.194X_{14} \; F_4 = -0.141X_1 + 0.177X_2 + 0.023X_3 + \cdots + 0.018X_{14}$$

$$F_{total} = \frac{0.2F_1 + 0.18072F_2 + 0.14683F_3 + 0.13695F_4}{0.6645}$$

Index	Factor 1	Factor 2	Factor 3	Factor 4	Index	Factor 1	Factor 2	Factor 3	Factor 4
aces	.084	.036	-.149	-.141	p_net_pt_won	.006	.147	.351	.224
p_winner	.035	.006	.459	.177	p_net_pt	.103	-.126	.413	.004
winner_shot_type	.149	.062	.518	.023	speed_mph	.268	.005	.007	-.153
p_double_fault	.056	.521	.124	-.044	p_distance_run	.132	.008	.213	.211
p_break_pt	.012	.144	-.175	.354	serve_depth	.255	.617	.267	.016
p_unf_err	-.536	.167	-.002	.000	return_depth	.154	.348	.366	-.287
p_break_pt_won	.382	.045	.241	.221	serve_width	.441	-.106	.194	. 018

Figure 5: Factor analysis results

5.2.4 Real-time momentum calculation

After the construction of the factor analysis model, we use the data of players' indexes at the current moment (moment t) to calculate the "leverage" of the game in this bureau, and combine the "leverage" from moment 0 to moment t to calculate the "momentum" of the players at the current moment.

To test the generalization performance of the model, this paper uses the constructed model to evaluate the match between Yafan Wang vs Donna Vekic from the semi-final of the 2019 WTA 250 Acapulco Tournament, and plotted the real-time momentum graphs of the duo (as shown in the figure below), with in-game data from the official website of oddspedia (Yafan Wang vs Donna Vekic " Predictions, Odds, Live Scores & Stats (oddspedia.com)).

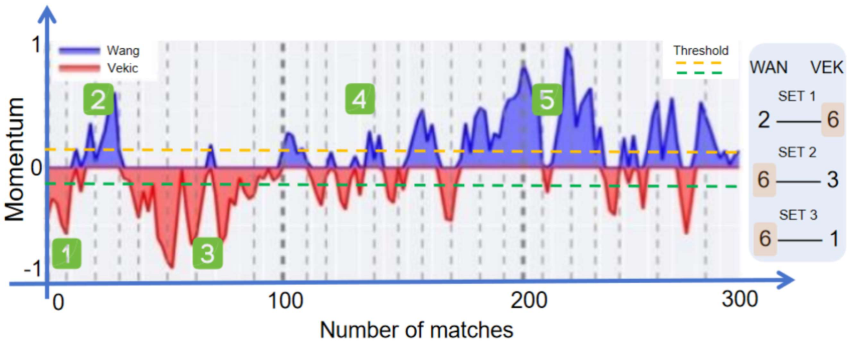


Figure 6: Visualization of real-time momentum

In the match Wang ultimately won in three sets (2-6, 6-3, 6-1), the outcomes were closely aligned with our real-time momentum assessments. Overall, despite some fluctuations in momentum during each set, the side with the greater overall momentum consistently emerged victorious.

Upon detailed analysis, we segmented the match into five phases based on changes in player's momentum. During Phase 1 (Games 1-2 of Set 1), Vekic held the momentum, indicating her dominance and superior performance, leading 2-0 at the outset; in Phase 2 (Games 3-4 of Set 1), Wang gained momentum, correlating with her winning two consecutive games; in Phase 3 (Games 4-8 of Set 1), Vekic recaptured momentum, winning four straight games to secure the first set; in Phase 4 (Games 1-6 of Set 2), momentum between the two was fluctuating, with neither player having a clear advantage, resulting in a 3-3 scoreline; and in Phase 5 (Games 7-9 of Set 2 and all of Set 3), Wang successfully seized and maintained momentum, winning three games to take Set 2 and dominating Set 3 with a 6-1 victory.

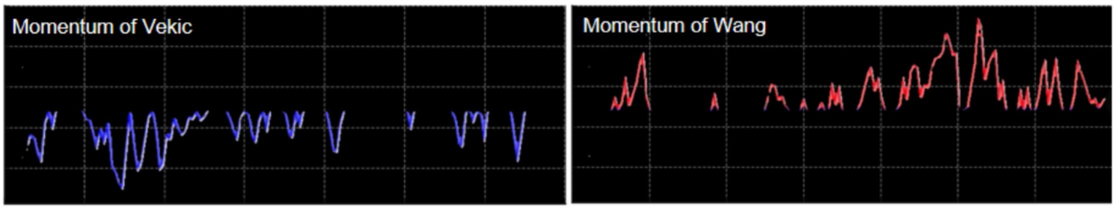


Figure 7: Momentum of the two players

In order to accurately measure the strengths and weaknesses of the players' performances over time, we used a python program to separate the momentum of each player's moments of better performance (as Figure 7 above) and quantify "how much better" using momentum scores. We defined the notion of superiority z as follows: the superiority of the dominant side in a phase is the average of the ratio of the absolute value of the difference in momentum scores to the ratio of the smaller momentum scores for each game in that phase.

$$z = \frac{1}{n}\sum_i^n \frac{|M_i|}{min\{y_{1i}, y_{2i}\}} = \frac{1}{n}\sum_i^n \frac{|y_{1i} - y_{2i}|}{min\{y_{1i}, y_{2i}\}}$$

We calculated the superiority rates for time periods 1-5 separately: in time periods 1 and 3, Vekic outperformed Wang by 27.145% and 34.584%, respectively; in time periods 2, 4, and 5, Wang outperformed Vekic by 24.518%, 9.002%, and 36.442%, respectively. In the whole match, the better performer Wang was 10.292% better than Vekic. This is highly consistent with the actual match data.

Table 3: Performance Winners and Winning Percentage over Time Periods

	Time period 1	Time period 2	Time period 3	Time period 4	Time period 5
Better performer	Vekic	Wang	Vekic	Wang	Wang
Winning rate	27.145%	24.518%	34.584%	9.002%	36.442%

6 Randomized Assessment of Field Situation Fluctuations and Continuous Scoring

6.1 Quantitative and Stochastic Assessment of Fluctuations in Match Situations

While momentum is typically associated with the match situation, it is important to note that it cannot be solely linked to it. For instance, in tennis matches, there are instances where players win several matches in a row, leading to a temporary burst of momentum, but without significantly changing the overall match situation. To measure the fluctuations in the match situation, we utilize the real-time victory rate of players during a match, considering momentum as a crucial influence.

To analyze the randomness of match situation fluctuations, we employ a specific approach. First, we use a counterfactual analytical prediction framework to derive real-time win probabilities of players, quantifying the match situation. Then, a Vector Error Correction Model is constructed to examine the correlation between real match momentum, random momentum, and the fluctuation of winning percentage. The results reveal a strong correlation between calculated momentum and winning percentage in both short and long terms, while no correlation exists between random momentum and winning percentage. These findings provide robust evidence that tournament fluctuations are not random but influenced by factors like momentum. This further validates the effectiveness of our modeling approach discussed in the previous paper.

6.1.1 Win Rate Assessment Based on a Counterfactual Analytical Modeling Framework

Utilizing counterfactual analysis, we can discern the influence of individual events within a match on the overall outcome. In this context, ”events” refer to predicted outcomes of instances, while ”causes” are specific feature values of these instances that are input into the model and lead to a certain prediction. We construct a Backpropagation (BP) neural network model based on the dynamic features of the match. During training, we employ counterfactual analysis to simulate changes in win probability at different stages of the match. For instance, by hypothesizing a player's loss of a point or a service fault at a critical moment, we adjust the outcome prediction to understand how win probability alters under the opposite scenario. Additionally, we incorporate a four-objective loss function, aiming to minimize loss to optimize our model's adaptability to various weighted match features for improved prediction accuracy. (The loss function is as follows)

$$L(x, x', y', X^{obs}) = (o_1(\hat{f}(x'), y'), o_2(x, x'), o_3(x, x'), o_4(x', X^{obs}))$$

The first objective o_1 reflects that the prediction of our counterfactual x' should be as close as possible to our desired prediction y'.The second objective o_2 reflects that our counterfactual should be as similar as possible to our instance x. By minimizing o_3 we aim for our third criterion – sparse feature changes. The fourth objective o_4 reflects that our counterfactuals should have likely feature values/combinations.

By appropriately adjusting and validating the model by comparing the actual observations with the simulation results, it is possible to assess the change in the win rate at different stages.

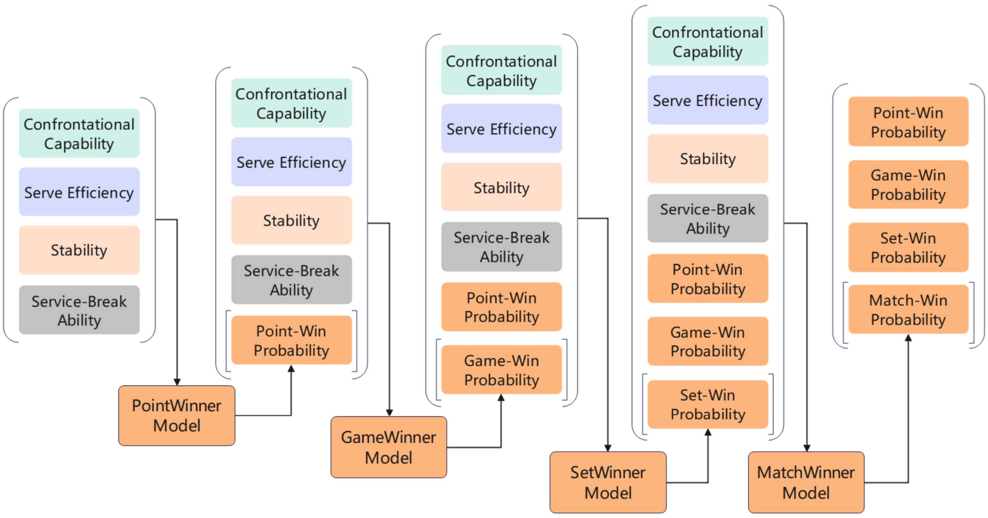


Figure 8: Framework structure based on a counterfactual analytical model

Based on the above modeling approach combined with counterfactual analysis, we constructed the following framework to achieve interpretable predictions for different scales of victory probabilities at each stage. As shown in the figure, we constructed a chained framework, where each model is connected in the order specified by the chain, and the next prediction is made using all the features plus the prediction result of the previous model in the chain, so that we can predict the winning probability at different scales (innings, boards, and matches), taking into account the influence of the past match progress on the next match.

6.1.2 Correlation Analysis Based on VECM

After obtaining the real-time win rate of the field through the counterfactual analysis model, we construct the Vector Error Correction Model (VECM) to perform the correlation analysis between the real-time momentum and the real-time win rate.The VECM applies to non-stationary multiple time series data, and determines whether the linear combination of multiple series data has a stable equilibrium through the methods of covariance verification and error correction. relationship. We designed comparative experiments using actual momentum data and momentum data generated from randomized sequences to determine whether the fluctuations of the game situation are random or influenced by momentum.

1) Long-term correlation test

We use the Johansen cointegration test to indirectly verify the long-term stable correlation between

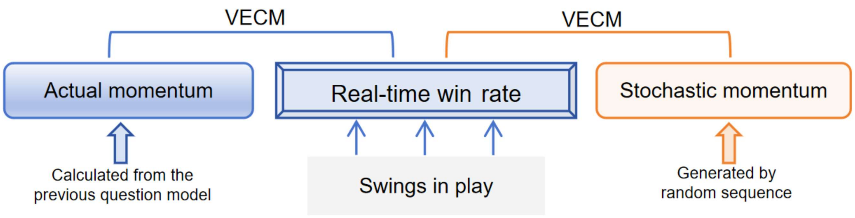


Figure 9: Comparative experiment process

momentum and win rate.The Johansen cointegration test is based on the VAR model, and the cointegration relationship is judged by hypothesis testing.The expression formula of the VAR model is as follows: where Y_t is a k-dimensional non-stationary vector representing the observations of multiple variables at time point t; A_p denotes the time-lagged i period of the impact; p is the lag order of the model, indicating the number of periods of the time lag considered; and ε_t is a k-dimensional error vector, indicating the random perturbation term of the model.

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + BX_t + \varepsilon_t$$

After modeling, the residual series autocorrelation test was performed to validate and how well the residual plot represented the data.

Table 4: Autocorrelation test for residuals

lag	chi2	df	Probability >shi2
1	6.3954	9	0.66237
2	27.145‰	9	0.62139

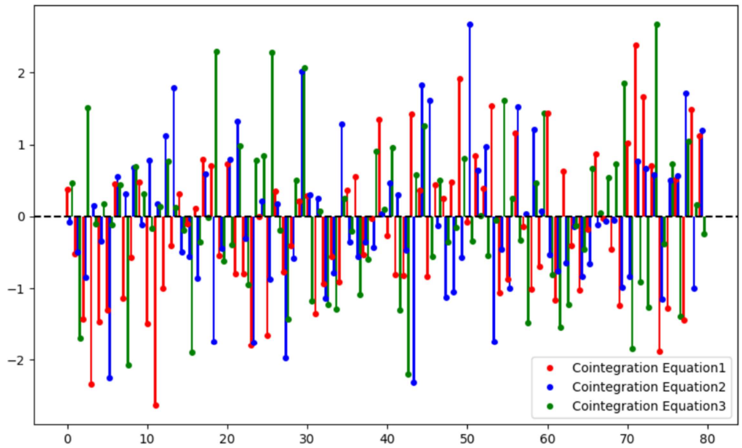


Figure 10: Residual data for the VECM cointegration equation

From the autocorrelation test results as well as the residual plots, we can get that there is no significant autocorrelation in the residuals, indicating that the model can express the dynamics and relationships of the time series data better.

We assume that there is no cointegration relationship between momentum and win rate, and test the hypothesis by calculating the eigenroot statistic and the maximum eigenroot statistic. The trace statistic is the sum of the sorted values of the characteristic root statistic. If the characteristic root statistic is greater than the critical value of 5%, the original hypothesis can be rejected, indicating the existence of a cointegration relationship. By testing the existence of a cointegration relationship, we can infer that there is a long-term and stable relationship between the momentum and the winning percentage, thus indirectly indicating the correlation between them.

The following are the formulas for the calculation of the characteristic root statistic and the maximum characteristic root statistic, and the critical values can be obtained through the Johansen test table.

The formula for the characteristic root statistic is as follows, among r is the number of cointegrating vectors, where r ranges from 0 to k-1 (k is the number of variables in the time series)

$$\Lambda_{trace} = -T \sum_{i=r+1}^{k} ln(1 - \lambda_i)$$

where λ_i is the estimated eigenvalue and T is the number of observed samples.The formula for the maximum eigenroot statistic is given below:

$$\Lambda_{max} = -Tln(1 - \lambda_{r+1})$$

Where λ_{r+1} is the r+1st eigenvalue of the estimate.

2) Short-term correlation test

The model includes an error correction term to measure the short-term dynamic adjustment between momentum and the probability of winning. By analyzing the sign and significance of this term, we can determine the short-term relationship between momentum (M) and winning probability (W), which can be expressed as:

$$\triangle M_t = \alpha + \beta \triangle W_{t-1} + \sum_{i=1}^{p-1} \gamma_i \triangle M_{t-i} + \sum_{j=1}^{q} \delta_j \triangle W_{t-j} + \varepsilon_t$$

Where $\triangle M_t$ and $\triangle W_t$ denote the change in momentum and winning probability, respectively, at time point t. α is a constant term, β is the short-run adjustment coefficient of momentum to the winning probability, y_i and δ_j are the coefficients of the lag terms for both, and p and q denote the lag order for each, respectively.ε_t is an error term denoting the unexplained part of the model.

By estimating and analyzing the coefficients and error terms in the model, we can understand the process of short-term dynamic adjustment between momentum and the probability of winning and determine whether a short-term relationship exists.The coefficient β indicates the extent to which changes in the probability of winning adjust to changes in momentum in the short run. If β is significantly non-zero, i.e., the estimate of β is large compared to its standard error, we can conclude that changes in the probability of winning have a significant effect on momentum in the short term. The error term ε_t can be considered as a measure of the degree of short-term adjustment between momentum and winning percentage. If the error term ε_t is significantly non-zero, i.e., the estimate of ε_t is large compared to its standard error, we can conclude that there is a short-term adjustment mechanism between momentum and winning percentage, i.e., there is a significant short-term relationship between momentum and game fluctuations.Below are the test results:

The experimental results show that the actual momentum of each stage for the change of the game winning rate in the long-term equilibrium relationship with the short-term dynamic relationship, the randomness of the momentum and the winning rate in the long term and the short term have

Table 5: Cointegration test results

params	LL	eigenvalue	trace statistics	5% critical
26	-549.58	0.662	44.7931	29.68
29	-541.61	0.531	18.8135	15.41

no significant relationship, indicating that there is a strong correlation between momentum and the fluctuation of the situation of the game, the fluctuation of the game is not random, on the contrary, the momentum in the game plays a very important role. The experiment also verifies the validity of our momentum calculation model.

6.2 Assessment of the Randomness of Continuous Scores Based on Outcome-related Correlation Testing

We aim to analyze whether a player's consecutive scoring is influenced by their momentum or occurs randomly. To minimize the impact of luck and coincidence, we define winning streaks as instances of three or more consecutive victories. Continuing with the example from question one, the match between Yafan Wang and Donna Vekic, we will employ the model developed in the previous question to calculate the player's overall momentum throughout the match. Winning streaks will be marked with green dots on the momentum chart.

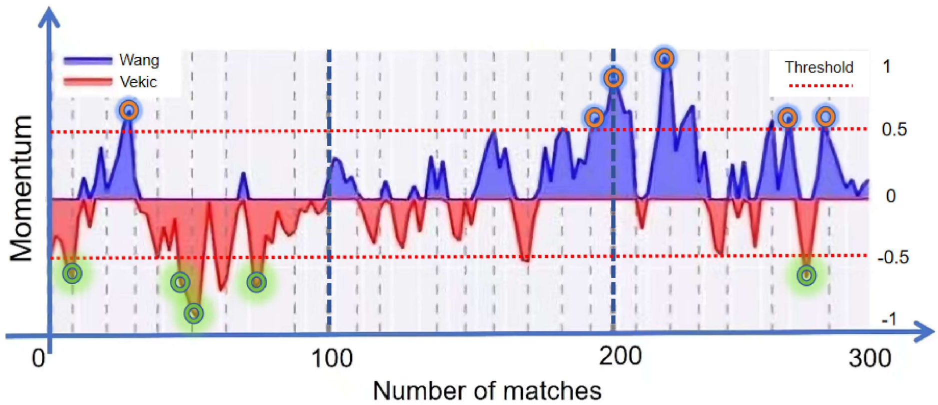


Figure 11: Visualization of the outcome-related correlation testing

We set the momentum threshold at 0.5, considering it as a significant momentum level that can impact the game situation. From the analysis of the provided images, we observed a total of 12 winning streaks in the matches. Among them, 11 streaks occurred when the player's momentum was above the threshold, indicating a higher likelihood of streaks occurring when the player's momentum is higher.

By examining all the matches in the dataset and identifying time points where streaks occurred, we compared the corresponding momentum values with the threshold. Based on our calculations, we can generate a table showing the accuracy recall rate, indicating the accuracy of the player's momentum being above the threshold during streaks.

We counted all the time points in all the matches in this dataset where streaks of goals occurred, found the corresponding momentum values, and compared them with the threshold value, and we believe that the player's momentum value should be above the threshold when streaks occur, and based on the calculations, we can draw a table of the accuracy recall rate.

Table 6: Accuracy and recall

Accuracy	Recall	Random Probability
91.67%	84.46%	0.08

According to the above analysis, if we assume that the player's "momentum" will have a certain impact on whether the player can score continuously, then the accuracy and recall of the continuous scoring time points detected by this method are higher, then we conclude that the player maintains a high momentum when scoring continuously, so we believe that whether the player can score continuously is closely related to his or her momentum, and is not randomly generated.

7 Prediction of Game Situation and Evaluation of Influencing Factors

7.1 Match Situation Prediction Model Based on LSTM

The fluctuation of a match is manifested as a player gaining or losing a winning advantage at a certain node, i.e., its winning probability changes from being lower than the opponent to being higher than the opponent or from being higher than the opponent to being lower than the opponent at a certain node. The fluctuation of a match is affected by the past and current match states, and whether a certain point of the match makes the situation turn around is not only related to the current momentum and performance of the player, but also related to the contextual context such as the match scores, so we use the LSTM model to combine the individual features of the match process for modeling, and efficiently capture the patterns and regularities in the sequences.

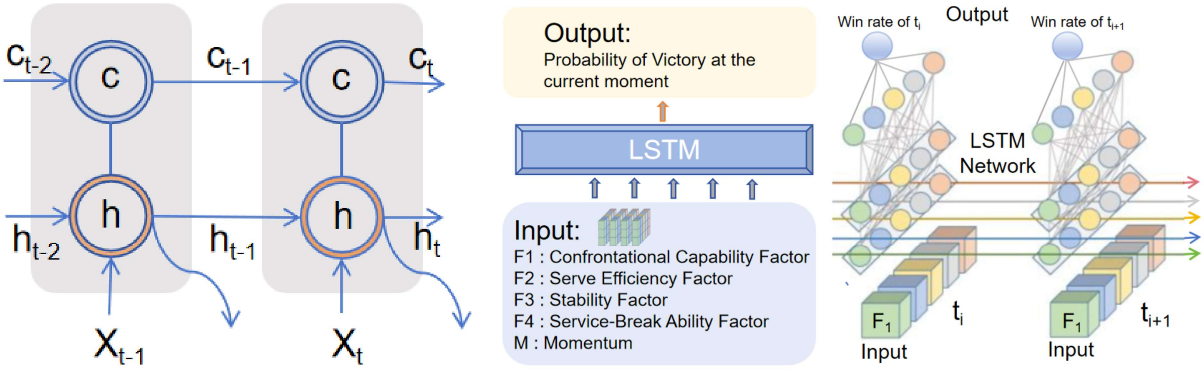
LSTM (Long Short-Term Memory) is a variant of Recurrent Neural Networks (RNNs) specifically designed to process sequential data and has a stronger memory capacity to capture long term dependencies compared to traditional RNNs.The basic structure of the LSTM is a recurrent unit that processes the inputs of each race node taking into account the prediction of the previous race node's The output is used as part of the input, and the flow of information and the update of the memory is controlled by introducing gating mechanisms, including input gates, forgetting gates and output gates.

The architecture of the model is shown in Figure 15. At time t, There are **three inputs** to the LSTM network :

- x_t:Input value of the network at the current moment
- h_{t-1}:The output value of the LSTM at the previous moment
- c_{t-1}:The state of the unit at the previous moment

And there are **two outputs** to the LSTM network:

- h_t:Current moment LSTM output value
- c_t:Unit status at the current moment



(a) Schematic diagram of the LSTM network (b) LSTM-based win rate prediction model

Figure 12: LSTM network

The hidden state (unitary state) of a model neuron is the recurrent neural network's "memory" of the input data, denoted by c_t, which is the neuron's "memory" after the time t. This vector covers the "summary" of all the inputs that the neural network has received up to the t+1 time.

The input x_t of the model consists of five components: the four common factor values (defined in Problem 1), and the current athlete momentum value (calculated from the Problem 1 model). In order to incorporate the effect of the strength gap between the two athletes and other relevant antecedent information, the initialized output value of the model, h_0, is the predicted value of the tennis industry's odds for the current match before the start of the match, and in order to improve the robustness of the model, the initialized unit state, c_0, is set to be a random sample of the training set. The model training data comes from 32 men's singles matches of Indian Wells Masters 2019 except for the first two rounds, where the real-time win rate data of the tournament comes from the official website of Bet365, a famous betting company. We used the trained model to predict the in-court fluctuations of the Indian Wells Masters 2019 semifinals in real time with a prediction accuracy of 98.155, marking two situation turning points (point 38 and point 57).

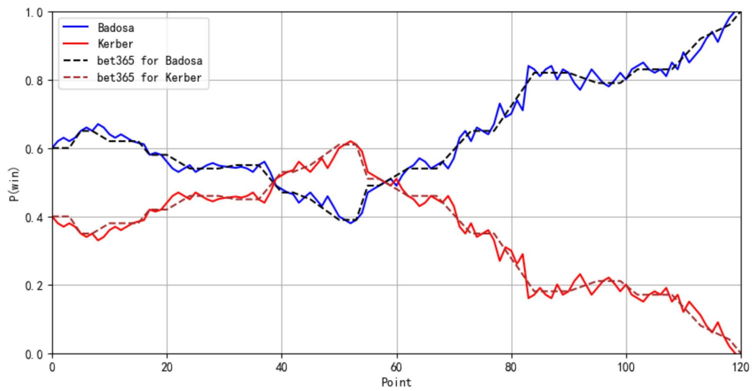


Figure 13: Comparison to marked odds:P.Bardosa vs. A.Kerber (6-4,7-5), Semifinal,Indian Wells 2021

We visualized the results of the model predictions, before the start of the race, bookmaker Beat365 estimated the winning margin between Badosa and Kerber based on the pre-fight rankings at 0.6:0.4.

At point 38, Kerber turned the winning margin back on the back of a good start to the race, but the momentum didn't hold up very well, at point 57, the Badosa regained the field advantage and went on to win the rest of the match in a landslide. Using point 38 and point 57 as a dividing point the match can be divided into 3 phases: at point 1 - point 37 and point 58 - point 120 Badosa dominated, at point 39 - point 56 Kerber dominated.

7.2 A Model for Assessing the Influencing Factors of the Playing Field Situation Based on the SHAP Model

Although LSTM networks can perform the prediction task well through complex computational transformations, they are basically in a "black box" state internally, making it difficult to analyze the specific impact of each indicator on the dependent variable. Therefore, this paper introduces the SHapley Additive exPlanation (SHAP) model to quantitatively analyze the influence of each indicator through the method of ex-post interpretation.

The core idea of the SHAP model is to calculate the marginal contribution of features to the model output, and then interpret the "black-box model" from both global and local levels, which is calculated by the formula:$g(z') = \phi_0 + \sum_{j=1}^{M} \phi_j$,where g is the explanatory model, M is the number of input features,ϕ is the imputation value (Shapley value) for each feature, and ϕ_0 is a constant.It is easy to see from the formula that solving the explanatory model for each impact indicator focuses on calculating the Shapley value for each indicator. For a certain indicator j in LSTM model, the shapley value needs to be calculated for all possible combinations of features (including different orders), and then weighted and summed, i.e:

$$\phi_j(val) = \sum_{S \subseteq \{x_1,x_2,\ldots,x_p\}/\{x_j\}} \frac{|S|!(p-|S|-1)!}{p!}(val(S \cup \{x_j\}) - val(S))$$

Where S is a subset of the features used in the model, x is a vector of feature values to be interpreted, p is the number of features , and val(S) refers to the output value of the model under the feature combination S.We used the shap library in python to build an interpreter for the LSTM model, parsing the specific role of each factor in a single match, and the magnitude of the influence of each factor in all matches, respectively. Here we present the study using the 2023-wimbledon-1601 match player Carlos Alcaraz:

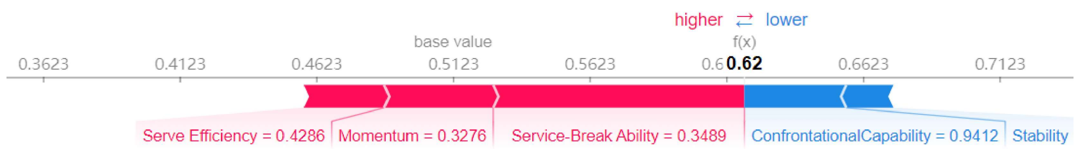
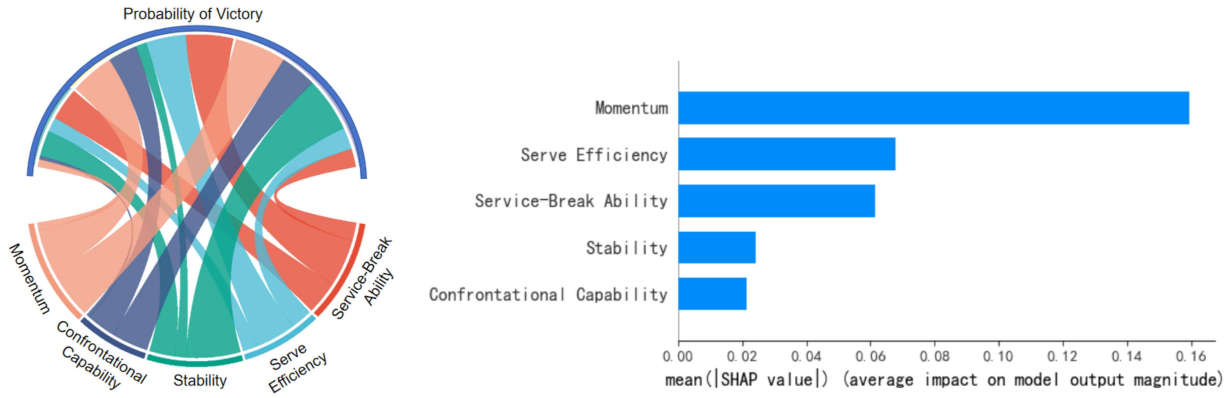


Figure 14: SHAP force plot for Carlos Alcaraz (2023-wimbledon-1601)

As shown by shap.force_plot, excellent ConfrontationalCapability, strong Momentum and good Serve Efficiency contributed to Carlos Alcaraz's win, while weaker Confrontational Capability and Stability had a negative impact on Carlos Alcaraz's game. From the results of the shap model, it is clear that Carlos Alcaraz is one of the more offensive players, while he is weak in Consistency and Stability.



(a) Chord diagram of the impact factors (b) Ranking of the importance of impact factors

Figure 15: Analysis of influencing factors

Meanwhile, we also calculated the SHAP_value of each influential factor based on the whole data, so as to rank the importance of the indicator, and visualized the interaction between the factors through the chord diagram. As can be seen from the figure, the most influential factor on the fluctuation of the race situation is Momentum, whose influential role is about two times that of the second place, followed by Service-Break Ability and Serve Efficiency, and Stability and Confrontational Capability have relatively less influence on the fluctuation of the race situation.

7.3 Match Strategy Recommendations Based on Momentum and Rankings

Based on the above model analysis, we found that among the factors affecting the fluctuation of the game situation, ”momentum” plays the most significant role. Therefore, according to the five different states of momentum that may appear during the match, we put forward targeted suggestions to coaches and players, which will enable players to fully adjust their strategies when facing new matches and new opponents, and to utilize their momentum to achieve incredible results in the matches.

Since Momentum is highly correlated with a player's technical strength, a tennis player's Momentum tends to be unfavorable when facing a stronger opponent, while Momentum tends to be favorable when facing a weaker opponent, so it is necessary to consider the ranking of both players' strengths in formulating a strategy. We make targeted recommendations for athletes in the manner shown in the figure below.

- Strategy 1: When a player's momentum is at a complete disadvantage at the current moment, the player should slow down the pace of the race and focus on the rituals of flow, focusing on his/her own pace in the race, and even pausing the race to adjust to the unfavorable situation at the necessary moment.

- Strategy 2: When the player is out of the low momentum, he should gradually increase his ”vigor” and become more aggressive in the game.

- Strategy 3: When the players' momentum is equal, then we should not only pay attention to fight for the score, but also pay attention to fight for the right to dominate the momentum, to play

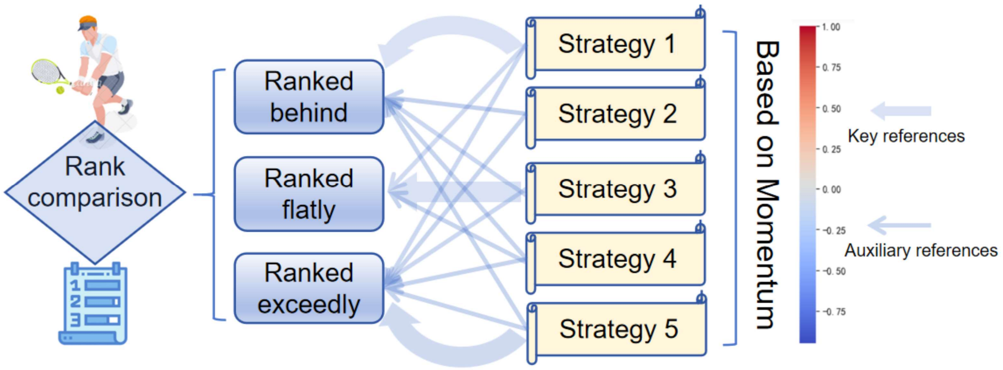


Figure 16: Match strategys based on Momentum and rankings

steadily and steadily, and in the appropriate actual strong offense in order to break through and lead the momentum.

- Strategy 4: When a player's momentum is favorable, he or she needs to reflect on how he or she has gained a temporary advantage and seeks to maintain it, so as to rationalize the momentum's advantage and play it wisely, instead of getting lucky or being overconfident.

- Strategy 5: When a player's momentum is crushing the opponent, the opponent is most likely to change strategy, and the player needs to aggressively seek to mix up his play to keep the game at an absolute premium.

At the same time, it should be pointed out that, in addition to momentum, the turning point of the game also plays an important role in the adjustment of the game strategy. In the following, we will construct a real-time analysis model of the form of the game to realize the real-time prediction of the victory rate during the game and the prediction of the turning point, so as to allow the players to have a better judgment and adjustment of the game situation and their own strategy.

8 Model Evaluation and Generalizability Analysis

8.1 Model Generalizability Analysis

To test the generalization ability of the model, we use the model constructed in the previous question to predict the volatility of unknown matches. Outside the given dataset, we selected the match between Pegula.J vs Sakkari M. from Group A of 2023 WTA Tour Finals Cancun, the player data of the whole match from Sofascore official website, and real-time win rate data from bet365 official website. The results show that the model prediction accuracy is 92.26%. We visualize the model's prediction results and the fluctuation curve of the win rate analyzed by the bookmaker:

As can be seen from the Figure 17, the prediction result of our model is basically fitted with the actual winning percentage curve, and the model still has a relatively good prediction ability in a new and unknown environment of the game. The fluctuation of the winning percentage of this game is more complicated and there are two reversals of the game situation, the model captures the effective information keenly and predicts the appearance of the two reversals more accurately, so we can conclude

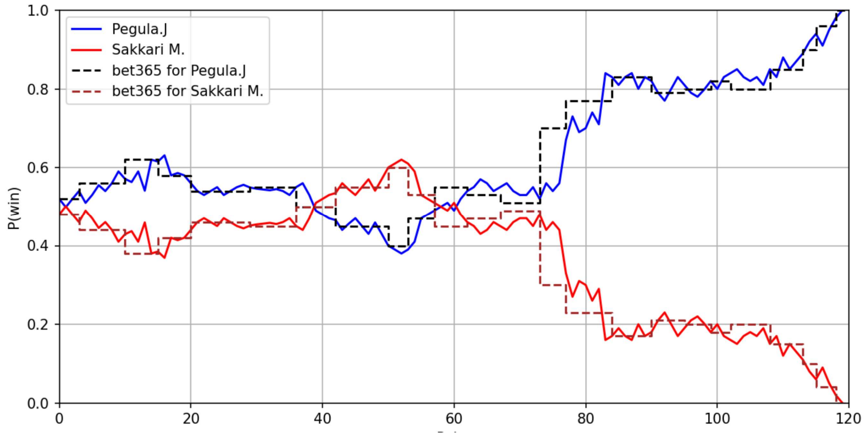
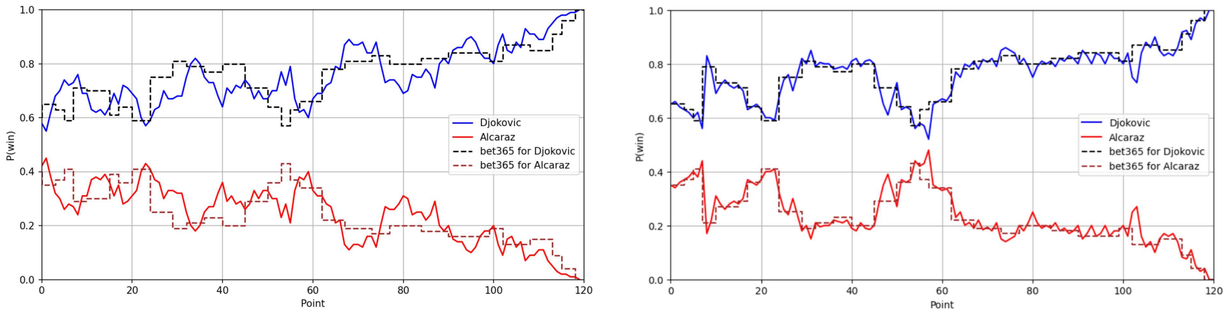


Figure 17: Winning percentage fluctuation prediction

that the game situation assessment model developed by us can predict the fluctuation of the game well in a wide range.

8.2 Factors for Possible Inclusion in Future Model

Although our model achieves a better performance in the prediction of hard court singles tennis matches, it sometimes does not perform as well in other special scenarios. The following graph shows the model's prediction for a tennis match played on red clay (unconventional terrain): the match between Djokovic vs Alcaraz from the Semi-Finals of the 2023 French Open, with a dataset taken from Eurosport's official website.



(a) (b)

Figure 18: Fitted images without site considerations

We observed a significant deviation between our model's predictions and the actual win rates in matches played on clay courts. This led us to analyze and conclude that the type of playing surface could variably affect a player's performance. The degree to which different metrics influence match outcomes on clay surfaces differs from their impact on hard courts, indicating the necessity to include the surface factor within our model's analytical parameters. Consequently, we reconstructed the input parameters of our model to incorporate the surface type, which allowed us to identify new common factors and update our model's calculation methodology. The comparison of model predictions before and after optimization is illustrated in Figure 18.

After considering the venue factor, we found that the fitting accuracy grew from the original 74.54% to 91.67%, which was significantly improved, so we believe that the venue factor may become a factor to be included in the future model. Based on this idea, we also analyzed the factors that may have an impact on the outcome of tennis matches, and finally came up with some of the factors that need to be included in the future model: the weather factor, pre-match physical condition, home and away factors, wind factor, past wins and losses of several matches, and so on.

8.3 Generalizability of the Model to Other Sports

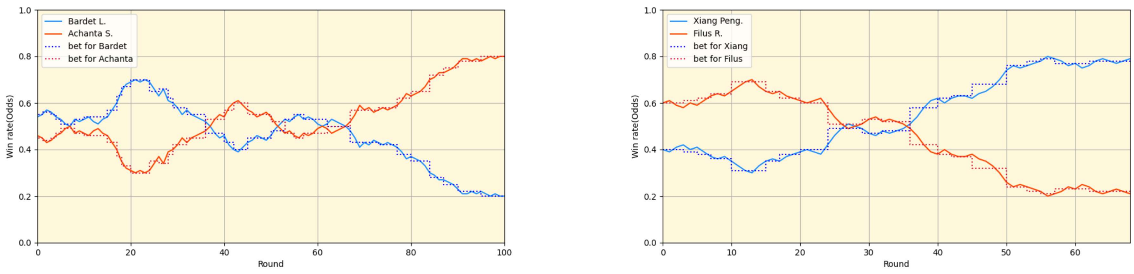
In the face of different types of sports events, we need to take into account the rules of the event, the environment of the venue, the degree of competition, the length of the game, the specifications of the athletes and even gender and other factors, so the generality of the model needs to be specifically analyzed and adjusted according to the actual application of the model. For example, there are differences between table tennis and tennis matches, such as shorter duration, less running distance, and faster pace of the game, so the model applied to tennis matches in the previous section can not be fully applied to the prediction, so we utilize the methodology and framework described in the previous section, add the factors related to table tennis matches, and construct the features of the model by adjusting the weights and regrouping the factors to build the model suitable for the prediction of table tennis matches.

Technical Indicators				
P_winner	P_points_win	P_counterdrive	P_flat	P_chop
P_topspin	P_backspin	Crossover_speed	P_ace	P_double_fault
Situation Indicators				
Winning_score	Winning game nums	Winning_score	Winning game nums	

Figure 19: Characterization factors of the table tennis model

Utilizing data from Blogabet and SofaScore, we gathered partial match statistics and corresponding historical odds for the WTT Contender Zagreb 2023 to train our model. Subsequently, we randomly selected two matches to demonstrate the predictive outcomes.

From the visualization results, we can see that the model prediction results fit well, and the prediction accuracies for the two matches are 93.33% and 88.65%, respectively.In addition, we fine-tuned the tennis prediction model in the paper to obtain a prediction system for tennis championships, which also achieved good prediction results. Thus, we can conclude that the systematic evaluation and prediction modeling method proposed in this paper has good generality. When used in different scenarios, the inputs of the model need to be constructed according to the specific characteristics of the tournament, and more stable and accurate results can be obtained.



(a) WTT Contender Zagreb 2023 Bardet L. vs Achanta S. Fitting Effectiveness

(b) WTT Contender Zagreb 2023 Xiang Peng L. vs Filus R. Fitting Effectiveness

Figure 20: Fitted images of winning percentage in table tennis matches

9 Sensitivity Analysis

Good sensitivity and certain stability are necessary conditions for an excellent model. In order to test the sensitivity of the real-time momentum evaluation model established in Problem 1, we slide the values of the four common factors in the interval of 95%-105%, and calculate the momentum evaluation values under different values respectively. We selected the momentum change at the 60th play of the 2023-wimbledon-1301 game for visualization:

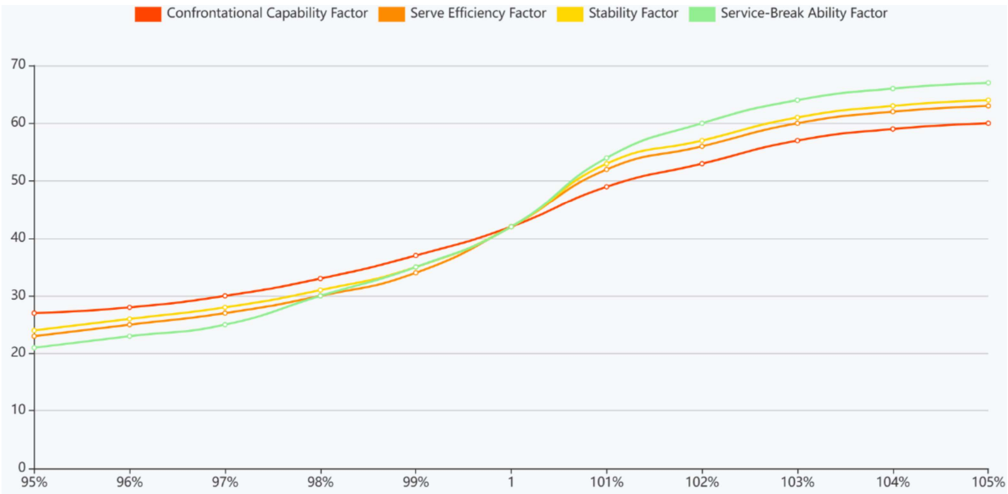


Figure 21: Momentum change curve

As can be seen from the figure, when the value of each factor is increased from 95% to 105%, the momentum value shows different degrees of growth, which indicates that these factors are sensitive to the change of the indicator. Among them, the overall slope of the curve corresponding to Service-Break Ability Factor is the largest, which reflects that the model is the most sensitive to Service-Break Ability Factor, and this conclusion is consistent with the results of our analysis in Problem 3.

10 Model Advantages and Disadvantages

10.1 Advantages

- This paper widely and effectively selects the data indicators affecting momentum to construct the model, and adopts different processing methods for the data according to the different characteristics of technical indicators and situation indicators. The redundant technical index data are refined and extracted using factor analysis; the role of the race situation on momentum is fully considered through the hyper-parameterized situation.
- In testing whether the fluctuation of the match situation is random, we construct the Vector Error Correction Model, which can not only determine the long-term stable relationship between momentum and the situation, but also explore the short-term interaction between the two. At the same time, by setting up the comparison test, we directly and powerfully prove the conclusion that the fluctuation of the game situation is not completely random.
- This paper uses the SHAP model to analyze the interpretability of LSTM network, and realizes the quantification of the importance of the influencing factors through the calculation of the shap_value of different influencing factors, so that the neural network is no longer a ”black box”.

10.2 Disadvantages

- In the construction process of the real-time momentum assessment model, the selection of hyperparameter values comes from experience and debugging, which is somewhat subjective.
- Although the model's considerations are already quite comprehensive, they are not sufficient to cover all game situations, and tasks in special environments require additional fine-tuning of the model.

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Momentum in Tennis Match Track, Evaluate and Use

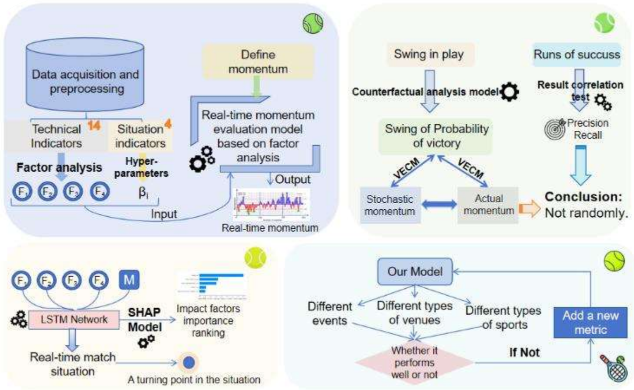
Background

Tennis is a multidimensional sport, and momentum during a match is a crucial factor that is believed to be not only influenced by the player's state of mind but can also reflect it. Moreover, it can have a profound impact on the remainder of the match. For instance, unexpected reversals are often attributed to shifts in momentum.

Description

What does Our Ongoing Study Do?

We have developed a model to analyze the real-time momentum of players based the match data. The momentum falls within a range of 0 to 1, which can be seen as a reflection of the athlete's state. Our research has revealed a strong correlation between the momentum and the overall trend of the game, as well as the occurrence of consecutive scoring situations. By conducting assessment of momentum and incorporating it into our modeling along with relevant factors, we have successfully predicted potential fluctuations in the game.

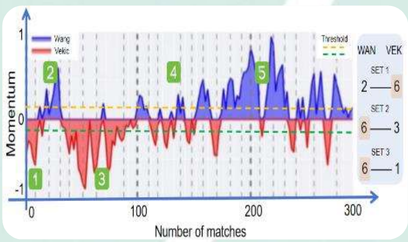


Suggestion

We firmly believe that momentum, in addition to the individual's strength, is the most influential factor in determining the outcome of a match. It encompasses both the player's performance and the fluctuations in their mindset, making it a crucial element that can dictate the course of a game.

To leverage this understanding, the coaching team can utilize our model to analyze real-time data during matches, providing them with accurate momentum assessments for each player. This enables coaches to offer timely feedback and make appropriate suggestions regarding the magnitude of momentum.

How to Incorporate Momentum into Coaching Strategies?



- ✓ When the player's momentum is lower than the opponent's, he should stabilize his game rhythm, avoid mistakes, and focus on winning steadily. Remaining patience and making timely adjustments to regain control can be helpful.
- ✓ When the player's momentum is equal to the opponent's, he should stabilize his own momentum and actively seek opportunities to shift the momentum in his favor. Proactive play and attentiveness are key.
- ✓ If the player has a momentum advantage, he should capitalize on it by scoring consecutive points. However, he should avoid letting scoring affect his own state

By incorporating our analysis into their coaching methods, coaches can go beyond solely focusing on improving their players' strength. They can also harness the power of momentum to gain remarkable advantages in matches. This highlights the utmost significance of our study and its potential to revolutionize coaching approaches in tennis.