Advanced Foundation Engineering Final Review

Sang Yang

December 5, 2022

1 Rankine active and passive soil pressure

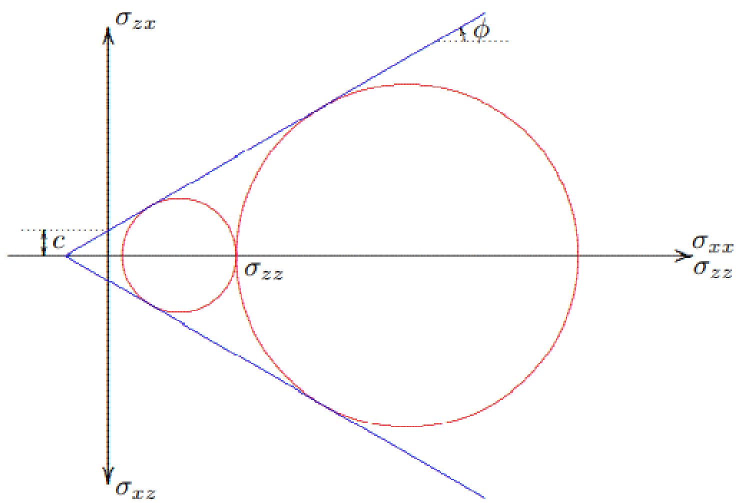

Figure 1: τ -σ and Mohr-Coulomb.

Fig.1 shows the relationship between Mohr-Coulomb and line $\tau = \sigma \tan\varphi + c$, when the Mohr-Coulomb circle tangent to the line, the soil reach the shear limit state.

σ_{zz} represents the self-weigh of soil ($\sigma_{zz} = \Sigma\gamma_i h_i$), there are two kinds of soil pressure: passive and active. As shown in Fig.1, active soil pressure represents the left circle (i.e., known $\sigma_1 = \sigma_{zz}$ to figure out σ_3), passive soil pressure represent the right circle (i.e., known $\sigma_3 = \sigma_{zz}$ to figure out σ_1).

Understanding above knowledge can help us to derive the following formula.

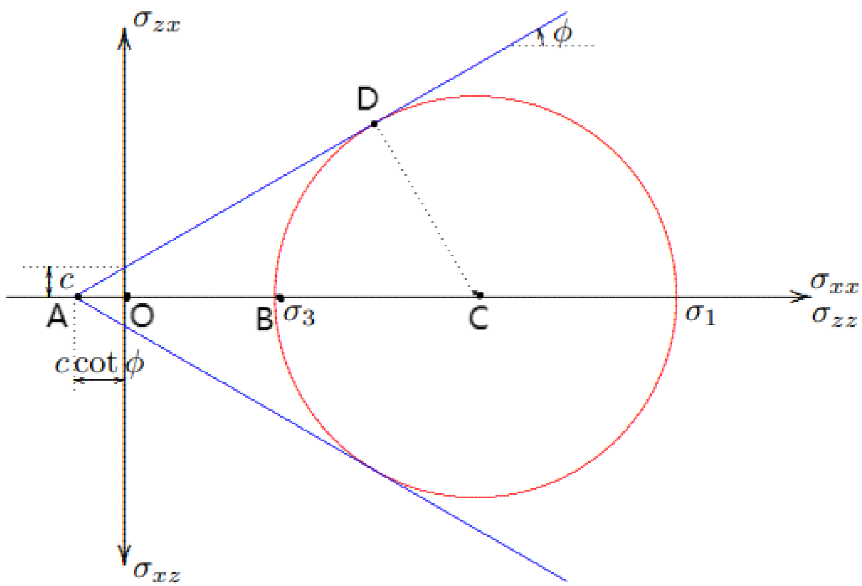

Figure 2: Active soil pressure.

Fig. 2 shows the active soil pressure derivation of Mohr-Coulomb circle. Knowing $\sigma_{zz} = \Sigma\gamma_i h_i = \sigma_1$, to figure out σ_3.

$\angle CAD = \phi$, $\angle ADC = 90°$, $\angle ACD = 90° - \phi$.

$AO = \frac{c}{\tan\phi}$, $CD = R = \frac{\sigma_1 - \sigma_3}{2}$, $OC = \frac{\sigma_1+\sigma_3}{2}$.

$AO + OC = \frac{CD}{\cos(90° - \phi)}$

i.e :

$$\frac{c}{\tan\phi} + \frac{\sigma_1+\sigma_3}{2} = \frac{\sigma_1-\sigma_3}{2\cos(90°-\phi)} \tag{1}$$

Sort out the above formula:

$$\sigma_3 = \frac{1-\sin\phi}{1+\sin\phi}\sigma_1 - 2c\frac{\cos\phi}{\sin\phi+1} \tag{2}$$

$$\frac{1-\sin\phi}{1+\sin\phi} = \frac{1-2\sin\frac{\phi}{2}\cos\frac{\phi}{2}}{1+2\sin\frac{\phi}{2}\cos\frac{\phi}{2}} = \frac{(\sin\frac{\phi}{2}-\cos\frac{\phi}{2})^2}{(\sin\frac{\phi}{2}-\cos\frac{\phi}{2})^2} = (\frac{\tan\frac{\phi}{2}-1}{\tan\frac{\phi}{2}+1})^2 = (\tan(45°-\frac{\phi}{2}))^2 \tag{3}$$

$$\frac{\cos\phi}{\sin\phi+1} = \frac{1-\tan\frac{\phi}{2}}{1+\tan\frac{\phi}{2}} = \tan(45° - \frac{\phi}{2}) \tag{4}$$

so

$$\sigma_3 = \sigma_1(\tan(45^\circ - \frac{\phi}{2}))^2 - 2c\tan(45^\circ - \frac{\phi}{2}) = \sigma_1 K_a - 2c\sqrt{K_a} \tag{5}$$

Passive soil pressure can be obtained by the same method.

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December 6, 2022

1 Shallow Foundation Design

1.1 Characteristic value of foundation bearing capacity

$$f_a = f_{ak} + \eta_d \gamma_m (d - 0.5) + \eta_b \gamma (b - 3) \tag{1}$$

As shown in Eq. 1, modified characteristic value of foundation bearing capacity consists of two parts: width correction (b >3 m) and depth correction (d >0.5 m).

γ_m represents the weighted average weight of soil above the underside of the foundation, γ represents the weight of soil below the underside of the foundation, η_d, η_b can be obtained from table list based on the soil characteristics (i.e., e, I_L, I_p).

1.2 Shallow foundation design

No eccentric load:

$$\frac{F_k + G_k}{A} \leq f_a \tag{2}$$

F_k represents the sum of upper vertical load. G_k represents the weight of backfill soil (i.e., $G_k = \gamma_G V$, $\gamma_G \approx 20kN/m^3$, $V = dA$)

Eccentric load:

$$\frac{F_k + G_k}{A} \leq (1.2 \sim 1.3) f_a \tag{3}$$

$$f_{max/min} = \frac{F_k + G_k}{A}(1 \pm \frac{6e}{l}) \tag{4}$$

$$e = \frac{\Sigma M}{\Sigma N} \tag{5}$$

Eq. 2, 3 can also be expressed as follows:

$$A \geq \frac{F_k}{f_a - \gamma_G d} \tag{6}$$

$$A \geq (1.2 \sim 1.3)\frac{F_k}{f_a - \gamma_G d} \tag{7}$$

By Eq.2, 3, 6, 7 we can approximately calculate the bottom area of foundation. For shallow foundation, $\frac{l}{b} \approx 1 \sim 2$, thus we can further select the size of A (1.5 or 2 are most used.).

To be mentioned, for existence of eccentric loading, after we select the size (l and b), we have to use Eq. 4, 5 to check whether the bearing capacity meets the following conditions:

$$\begin{cases} f_{max} = \dfrac{F_k + G_k}{A}(1 + \dfrac{6e}{l}) \leq (1.2 \sim 1.3)f_a \\ f_{min} = \dfrac{F_k + G_k}{A}(1 - \dfrac{6e}{l}) \geq 0 \end{cases} \tag{8}$$

1.3 Exercise

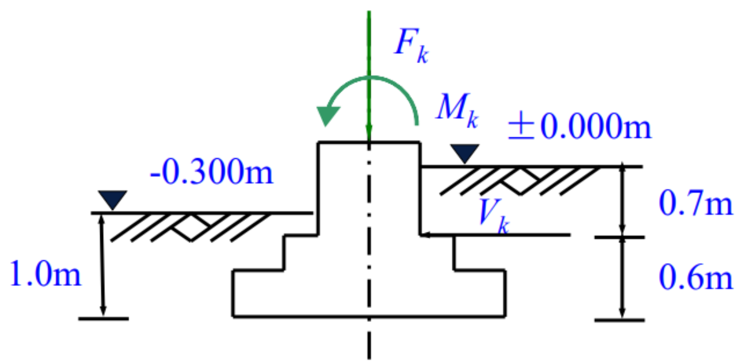

Figure 1: Exercise

Weight of a cohesive soil $\gamma = 18.2kN/m^3$, void ratio $e = 0.7$, liquidity index $I_L = 0.75$, Characteristic value of foundation bearing capacity $f_{ak} = 220kPa$. Now an external column foundation is built to act on the top surface of the foundation. Axial load $F_k = 830kN$, $M_k = 200kNm$, $V_k = 20kN$, foundation buried depth (calculated from the ground) is 1.0 m, and the indoor ground is 0.3 m higher than the outdoor ground. Try to determine the bottom of the rectangular foundation width.

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1 One Dimension Consolidation

1.1 Assumption and observation

We now return to derive the theory for time rate of settlement using an element of the soil sample of thickness dz and cross-sectional area $dA = dxdy$. We will assume the following:

- The soil is saturated, isotropic, and homogeneous.
- Darcy law is valid.
- Flow only occurs vertically.
- The strains are small.

We will use the following observations:

- The change in volume of the soil (ΔV) is equal to the change in volume of porewater expelled (ΔV_w), which is equal to the change in the volume of the voids (ΔV_v). Since the area of the soil is constant (the soil is laterally constrained), the change in volume is directly proportional to the change in height.
- At any depth, the change in vertical effective stress is equal to the change in excess porewater pressure at that depth. That is, $\partial\sigma'_z = \partial u$.

1.2 Derivation

For our soil element in Figure 9.6, the inflow of water is νdA and the outflow over the elemental thickness dz is $[\nu + \frac{\partial \nu}{\partial z}dz]dA$. The flow rate is the product of the velocity and the cross-sectional area normal to its (velocity) direction. The change in flow is then $\frac{\partial \nu}{\partial z}dzdA$. The rate of change in volume of water expelled, which is equal to the rate of change of volume of the soil, must equal the change in flow.

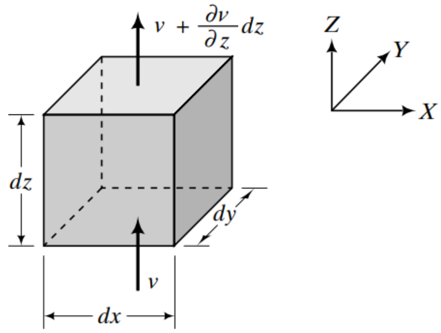

Figure 1: One-dimensional flow through a three-dimensional soil element.

That is,

$$\frac{\partial V}{\partial t} = \frac{\partial \nu}{\partial z} dz dA \tag{1}$$

Recall that the volumetric strain $\varepsilon_p = \frac{\partial V}{V} = \frac{\partial e}{1+e_0}$ and therefore

$$\partial V = \frac{\partial e}{1+e_o} dz dA = m_v \partial \sigma_z' dz dA = m_v \partial u dz dA \tag{2}$$

Substituting Eq. 2 into Eq. 1 and simplifying, we obtain

$$\frac{\partial \nu}{\partial z} = m_v \frac{\partial u}{\partial t} \tag{3}$$

The one-dimensional fl ow of water from Darcy's law is

$$\nu = k_z i = k_z \frac{\partial h}{\partial z} \tag{4}$$

where k_z is the hydraulic conductivity in the vertical direction.

Partial differentiation of Eq. 4 with respect to z gives

$$\frac{\partial \nu}{\partial z} = k_z \frac{\partial^2 h}{\partial z^2} \tag{5}$$

The porewater pressure at any time is

$$u = h \gamma_w \tag{6}$$

where h is the height of water in the burette.

Partial differentiation of Eq. 6 with respect to z gives

$$\frac{\partial^2 h}{\partial z^2} = \frac{\partial^2 u}{\partial z^2} \frac{1}{\gamma_w} \tag{7}$$

By substitution of Eq. 7 into Eq. 5, we get

$$\frac{\partial \nu}{\partial z} = \frac{k_z}{\gamma_w} \frac{\partial^2 u}{\partial z^2} \tag{8}$$

Equating Eq. 8 and Eq. 3, we obtain

$$\frac{\partial u}{\partial t} = \frac{k_z}{m_v \gamma_w} \frac{\partial^2 u}{\partial z^2} \tag{9}$$

We can replace $\frac{k_z}{m_v \gamma_w}$ by a coefficient C_v called the coefficient of consolidation

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December 6, 2022

1 Bearing capacity of shallow foundation

1.1 Assumption of Prandtl. L - Reissner. H

When using the limit equilibrium theory to solve the ultimate bearing capacity of foundation, if the following three simplified assumptions are made for the problem, the complex problem can be greatly simplified. The three assumptions are as follows:

- The foundation soil below the foundation base is regarded as a weightless medium, that is, it is assumed that the gravity of the soil below the foundation bottom is $\gamma = 0$.
- The bottom surface of the foundation is completely smooth. Because there is no friction, the compressive stress of the base is perpendicular to the ground and becomes a principal stress.
- For shallow foundation with embedded depth d less than foundation width b, the base plane can be regarded as the foundation surface, and the sliding surface only extends to this assumed foundation surface. The dead weight stress γd of soil on both sides of the foundation above this plane is taken as the uniformly distributed load $q = \gamma d$ acting on both sides of the foundation, and d is the embedded depth of the foundation.

After such simplification, the load on the foundation surface is shown in the figure.

1.2 Solve of Prandtl. L - Reissner. H

When the load reaches the ultimate load force A continuous sliding surface appears in the foundation. The sliding soil mass can be divided into three areas, as shown in Fig. 1 and Fig. 2. Zone 1 is the Rankine active zone, Zone III is the Rankine passive zone, and Zone II is the transition zone. The included angle

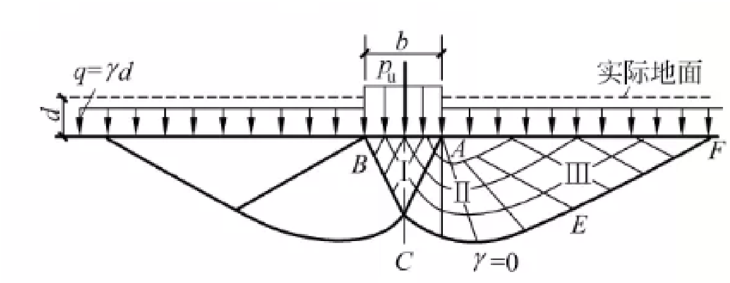

Figure 1: Slip line network without heavy medium foundation.

between the sliding line of the Rankine active zone and the horizontal plane is $(45° + \frac{\phi}{2})$, and the included angle between the sliding line of the Rankine passive zone and the horizontal plane is $(45° - \frac{\phi}{2})$. There are two groups of sliding lines in the transition zone II, one group is the ray from point A and point at the edge of the load; the other group is the logarithmic spiral connecting the sliding lines of zone I and III, which can be expressed as

$$r = r_0 e^{\psi \tan \varphi} \tag{1}$$

Where, φ is the internal friction angle of soil; R_0 is the initial radius of Zone II, and its value is equal to the boundary length AC of Zone I; ψ is the angle between ray r and r_0,

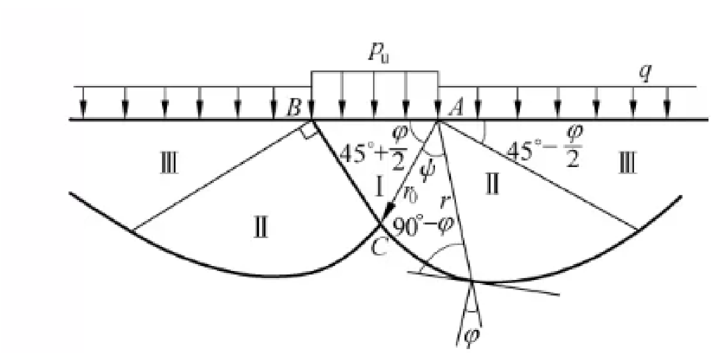

Figure 2: Transition zone of sliding body.

1.3 Rigid body balance method

Cut the sliding soil mass in the foundation as shown in Fig. 2 along the centerline of Zone I and Zone III. The soil OCEGAO is taken as the isolator, as shown in Fig. 2. Regardless of the dead weight of the soil, the forces around the isolator are as follows.

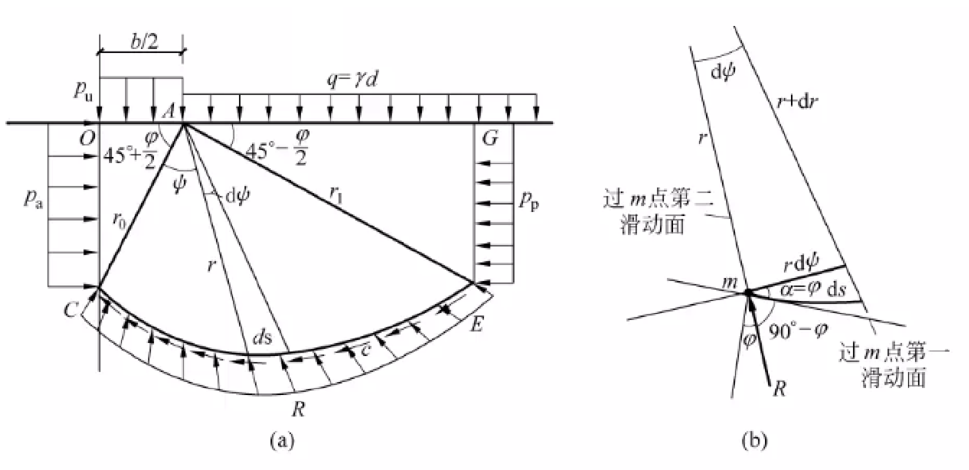

Figure 3: Calculation of ultimate bearing capacity by force balance method.

$\overline{OA}$: P_u to be solved.
$\overline{AG}$: $q = \gamma g$
$\overline{OC}$: Rankine active soil pressure, $P_a = P_u K_a - 2c\sqrt{k_a}$
$\overline{GE}$: Rankine passive soil pressure, $P_p = \gamma g K_p + 2c\sqrt{K_p}$

$\overset{\frown}{CE}$: One is cohesion C, which is uniformly distributed along the tangent line of CE surface, and the resultant force R of normal pressure and friction, which points to point A.

Since the isolator is in equilibrium, the force on each boundary surface takes the moment of point A, there should be $\Sigma A = 0$

$$P_u \frac{b^2}{8} + P_a \frac{\overline{OC^2}}{2} = q\frac{\overline{AG^2}}{2} + P_p \frac{\overline{GE^2}}{2} + M_c \tag{2}$$

$$M_c = \int c \cdot ds \cdot \cos\varphi \cdot dr \tag{3}$$

In the function, $ds = \frac{rd\psi}{\cos\varphi}$, take it into Eq. 3

$$M_c = \int_0^{\frac{\pi}{2}} cr^2\, d\psi = c\int_0^{\frac{\pi}{2}} (r_0 e^{\psi\tan\varphi})^2\, d\psi = cr_0^2 \frac{1}{2\tan\varphi}(e^{\pi\tan\varphi} - 1) \tag{4}$$

Bring various boundary conditions and forces into Eq. 2, we can obtain

$$\begin{aligned} P_u =& q\tan^2(45° + \frac{\varphi}{2})e^{\pi\tan\varphi} + c\cot\varphi(\tan^2(45° + \frac{\varphi}{2}) - 1) \\ =& qN_q + cN_c \end{aligned} \tag{5}$$

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December 7, 2022

1 Pile foundation

1.1 Bearing capacity

Single one pile bearing capacity:

$$Q_{uk} = Q_{sk} + Q_{pk} = u\Sigma q_{sik} l_i + q_{pk} A_p \tag{1}$$

Where q_{sik} represents standard value of ultimate lateral resistance of layer i soil, q_{pk} represents the standard value of ultimate end resistance.

1.2 Negative friction

Negative friction of pile side will produce pull load, The negative friction resistance standard value of layer i soil in a single pile surrounding the neutral point can be calculated as follows:

$$q_{si}^n = \sigma_i' \xi_{ni} \tag{2}$$

When a large area of load is distributed on the ground:

$$\sigma_i' = p + \sigma_{\gamma i}' \tag{3}$$

$$\sigma_{\gamma i}' = \sum_{e=1}^{i-1} \gamma_e \Delta z_e + \frac{1}{2}\gamma_i \Delta z_i \tag{4}$$

The pull down load of foundation considering pile group effect can be calculated as follows

$$Q_g^n = \eta_n u \sum_{i=1}^{n} q_{si}^n l_i \tag{5}$$

$$\eta_n = \frac{s_{ax} s_{ay}}{\pi d(\frac{q_s^n}{\gamma_m} + \frac{d}{4})} \tag{6}$$

1.3 Group piles settlement

Pile settlement consist of three parts:

1. The elastic compression of the pile itself.
2. Soil compression under pile tip due to the downward transfer of side friction.
3. Compression of soil under pile tip due to pile tip load.

The settlement below the pile tip is calculated by taking the imaginary foundation as a rigid whole without considering the compression amount of pile and the relative displacement between pile and soil.

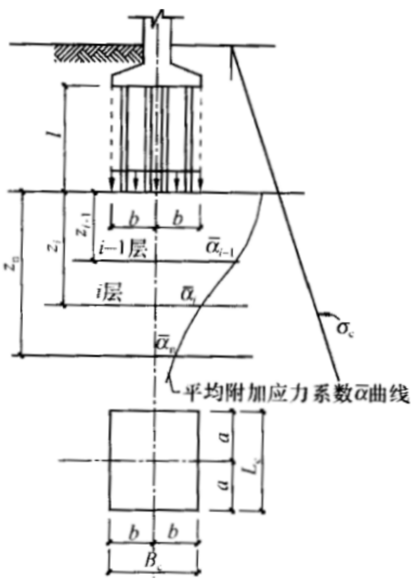

Figure 1: Group pile settlement.

Adopt equivalent layered summation method, the equivalent action surface is located at the pile end plane, the equivalent action area is the projected area of the pile cap. The equivalent additional pressure is approximately taken as the additional pressure on the bottom plane of the cushion cap.

The settlement can be calculated by the following equation:

$$s = \psi \cdot \psi_{\mathrm{c}} \cdot s' = \psi \cdot \psi_{\mathrm{e}} \cdot \sum_{j=1}^{m} p_{0j} \sum_{i=1}^{n} \frac{z_{ij}\bar{\alpha}_{ij} - z_{(i-1)j}\bar{\alpha}_{(i-1)j}}{E_{si}} \tag{7}$$

When calculating the center settlement of rectangular pile foundations, the above equation can be simplified to:

$$s = \psi \cdot \psi_{\mathrm{c}} \cdot s' = 4 \cdot \psi \cdot \psi_{\mathrm{c}} \cdot p_0 \sum_{i=1}^{n} \frac{z_i\bar{\alpha}_i - z_{i-1}\bar{\alpha}_{i-1}}{E_{si}} \tag{8}$$