Southeast University Exam Paper

Modular Name Numerical Analysis Semester 2020-2021 Semester 1 Score ____________

Program Civil Engineering (SEU-Monash University Joint Programe)

Exam Form Closed-book Exam Duration 150 minutes

（Calculator is permitted）

No.	1	2	3	4	5	6	7	8	9
Marks									
Examiner									

自觉遵守考场纪律 如考试作弊 此答卷无效

Student ID ________ Name ________

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1. (10 Marks)

(1) Use Horner's method to find $p(4)$, where

$$p(x) = 3x^4 - 5x^3 - 8x + 2;$$

(2) Suppose $x_1 = 0.356$, $x_2 = 1.25$ are approximations to x_1^*, x_2^* of three significant figures. Derive the bound of $e_r(x_1 x_2)$.

2. (10 Marks) Find all the roots accurate within 10^{-3} for the following equation

$$e^x + x - \sin x = 0$$

by the iterative method.

3. (10 Marks) Solve the linear system

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 4 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ \frac{9}{2} \end{bmatrix}$$

by Gaussian elimination with partial pivoting.

4. (10 Marks) Determine the domain of real number α such that the following iterative method converges.

$$\begin{cases} x_1^{(k+1)} = (b_1 - \alpha x_2^{(k)})/2, \\ x_2^{(k+1)} = (b_2 - x_1^{(k+1)} - \alpha x_3^{(k)})/2, \\ x_3^{(k+1)} = b_3 - x_2^{(k+1)}. \end{cases}$$

5. (12 Marks) Find an interpolating polynomial $H(x)$ of degree at most 3 such that

$$H(a) = f(a),\ H'(a) = f'(a),\ H'(b) = f'(b), H''(b) = f''(b).$$

6. (10 Marks) Find the least squares polynomial approximation of degree 2 on the interval [-1,1] for the function $f(x) = e^x$.

7. (14 Marks) Suppose $f \in \mathcal{C}^4[a,b]$ and $I(f) = \int_a^b f(x)\mathrm{d}x$. Let

$$M(f) = (b-a)f(\frac{a+b}{2}) + \frac{(b-a)^3}{24}f''(\frac{a+b}{2}).$$

(1) Determine the degree of precision of $M(f)$;

(2) Show that a real number $\xi \in (a,b)$ exists such that $I(f) - M(f) = \frac{b-a}{120}\left(\frac{b-a}{2}\right)^4 f^{(4)}(\xi)$;

(3) Choose an integer n and let $h = (b-a)/n,\ x_i = a + ih,\ i = 0, 1, \cdots, n$. Construct a composite quadrature $M_n(f)$ for n subintervals based on $M(f)$ and compute $\lim_{h \to 0} \dfrac{I(f) - M_n(f)}{h^4}$.

8. (12 Marks) Given

$$\begin{cases} y' = f(x, y), \ a < x \le b, \\ y(a) = \eta, \end{cases}$$

choose a positive integer n and let $h = (b-a)/n$, $x_i = a + ih$, $0 \le i \le n$; $y_i \approx y(x_i)$, $0 \le i \le n$, $y_0 = \eta$. Determine the constant λ such that the following difference equation

$$y_{i+1} = \lambda(y_i - y_{i-1}) + y_{i-2} + \frac{3+\lambda}{2} h \left[f(x_i, y_i) + f(x_{i-1}, y_{i-1}) \right],$$

has the highest order and derive the local truncation error.

9. (12 Marks) Suppose

$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial u}{\partial x} + u = 0, \ 0 < x < 1, \ 0 < t \le 1, \\ u(x,0) = x\sin(1-x), \ 0 \le x \le 1, \\ u(0,t) = 0, \ u(1,t) = 0, \ 0 < t \le 1. \end{cases}$$

has smooth solution $u(x,t)$. Choose positive integers M and N. Let $h = 1/M, \ \tau = 1/N, \ x_i = ih, \ t_k = k\tau, \ u(x_i, t_k) \approx u_i^k \ (0 \le i \le M, \ 0 \le k \le N)$.

(1) Derive the following difference scheme:

$$\begin{cases} \dfrac{1}{\tau}(u_i^k - u_i^{k-1}) - \dfrac{1}{h^2}(u_{i+1}^k - 2u_i^k + u_{i-1}^k) + \dfrac{1}{2h}(u_{i+1}^k - u_{i-1}^k) + u_i^k = 0, \\ \hfill 1 \le i \le M-1, \ 1 \le k \le N, \\ u_i^0 = x_i \sin(1 - x_i), \ 0 \le i \le M, \\ u_0^k = 0, \ u_M^k = 0, \ 1 \le k \le N, \end{cases}$$

and give the truncation error;

(2) Let $r = \tau/h^2$. Show that $\|u^k\|_\infty \le \|u^0\|_\infty (1 \le k \le N)$ holds for any r.