Poisson(λ) arrivals during $[0, T]$; T: independent exponential(μ)
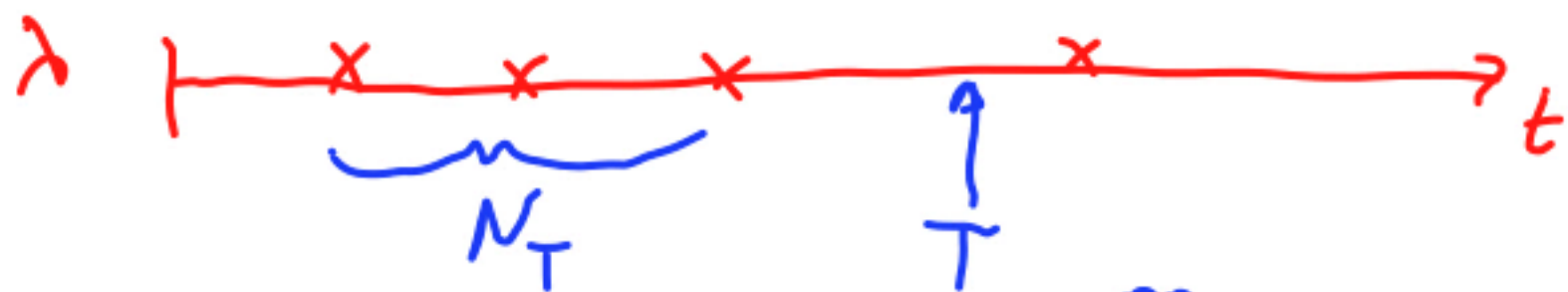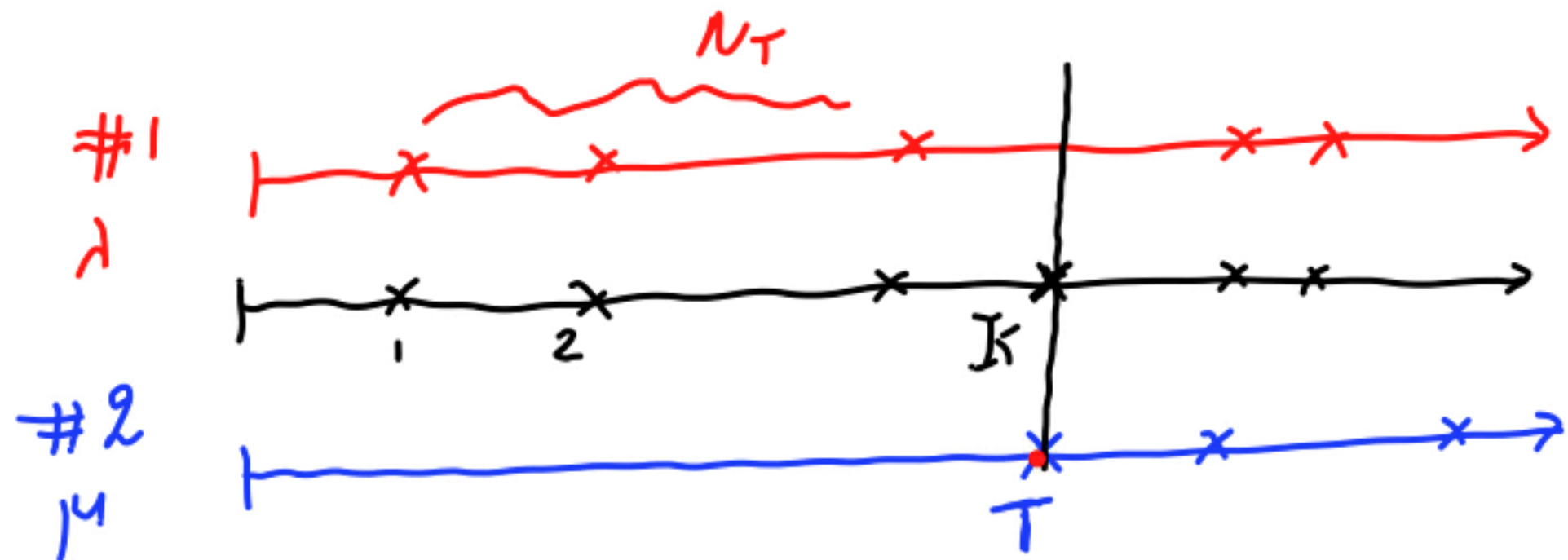

$$P(N_T = k) = \int_0^\infty P(N_T = k \mid T = t) \, f_T(t) \, dt$$

$$P(N_t = k \mid T = t)$$

$$= \int_0^\infty \frac{\lambda^k e^{-\lambda t}}{k!} \, \mu e^{-\mu t} \, dt \; .$$

Poisson(λ) arrivals during $[0, T]$; T: independent exponential(μ)

N_T

#1
λ

#2
μ

$1 \quad 2 \qquad K$

T

$K = N_T + 1$

$K =$ # arrivals in merged process till get one
that comes from blue

Arrival in merged process: trial

"success" if it comes from blue

$P(\text{success}) = \dfrac{\mu}{\lambda + \mu}$

independent

$K =$ # trials till success

Geometric with parameter $\mu/(\lambda + \mu)$