We need to apply the version of Bayes' rule for a discrete random variable conditioned on a continuous random variable:

$$p_{X|Z}(x \mid z) \;=\; \frac{p_X(x) f_{Z|X}(z \mid x)}{f_Z(z)} \;=\; \frac{p_X(x) f_{Z|X}(z \mid x)}{\sum_{k=0}^{1} p_X(k) f_{Z|X}(z \mid k)}.$$

Specifically,

$$\begin{aligned}
\mathbf{P}(X = 1 \mid Z = z) \;&=\; p_{X|Z}(1 \mid z) \;=\; \frac{p_X(1) f_{Z|X}(z \mid 1)}{\sum_{k=0}^{1} p_X(k) f_{Z|X}(z \mid k)} \\
&=\; \frac{p \frac{1}{2} \lambda e^{-\lambda|z-1|}}{(1-p)\frac{1}{2}\lambda e^{-\lambda|z+1|} + p\frac{1}{2}\lambda e^{-\lambda|z-1|}} \\
&=\; \frac{p e^{-\lambda|z-1|}}{(1-p) e^{-\lambda|z+1|} + p e^{-\lambda|z-1|}} \\
&=\; \frac{p e^{-\lambda|z-1|}}{(1-p) e^{-\lambda|z+1|} + p e^{-\lambda|z-1|}} \cdot \frac{e^{\lambda|z-1|}}{e^{\lambda|z-1|}} \\
&=\; \frac{p}{(1-p) e^{-\lambda(|z+1|-|z-1|)} + p}
\end{aligned}$$

The final manipulations are to ease interpretations for $p \to 0^+$, $p \to 1^-$, $\lambda \to 0^+$, and $\lambda \to \infty$. We observe that

$$\lim_{p \to 0^+} \mathbf{P}(X = 1 \mid Z = z) \;=\; 0 \qquad \text{and} \qquad \lim_{p \to 1^-} \mathbf{P}(X = 1 \mid Z = z) \;=\; 1;$$

these make sense: if the prior information gives us certainty about the value of X, the observation can be ignored. Next,

$$\lim_{\lambda \to 0^+} \mathbf{P}(X = 1 \mid Z = z) \;=\; p,$$

which makes sense because the distribution of Y becomes very flat as $\lambda \to 0^+$, making the observation uninformative. Finally,

$$\lim_{\lambda \to \infty} \mathbf{P}(X = 1 \mid Z = z) \;=\; \begin{cases} 1, & \text{if } |z+1| > |z-1|, \\ 0, & \text{if } |z+1| < |z-1|, \end{cases} \;=\; \begin{cases} 1, & \text{if } z > 0, \\ 0, & \text{if } z < 0; \end{cases}$$

this makes sense because if $\lambda \to \infty$, then Y will be very close to zero and so the sign of Z will be the same as the sign of X with high probability.