Problem 1

a). $P(A|B) = \frac{P(A \cap B)}{P(B)}$ — ①

$P(A \cap B) = P(B|A) \cdot P(A)$ — ②

From ① and ②: $P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$

b). $P(A, B, C) = P(A \cap B \cap C)$

$= P(A \cap (B \cap C))$ — ③

Using Conditional Probability.

$P(A|B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)}$

So, ③ is equal to:

$P(A|B \cap C) \cdot P(B \cap C)$ — ④

Using Conditional Probability Again,

④ is equal to:

$P(A|B \cap C) \cdot P(B|C) \cdot P(C)$

Therefore, $P(A, B, C) = P(A|B \cap C) \cdot P(B|C) \cdot P(C)$.

c). Using the definition of expectation.

$E[X] = \sum_i a_i P(X = a_i)$

So, $E[X] = 1 \cdot P(A \text{ occur}) + 0 \cdot P(A \text{ not occur})$

$= P(A)$.

Proved.

d). 1). If X is independent of Y,

$$P(X) \cdot P(Y) = P(X \cap Y)$$

Using the information when $X=0$, $Y=0$.

$$P(X=0) = \frac{1}{15} + \frac{4}{15} + \frac{1}{10} + \frac{8}{45} = \frac{55}{90} = \frac{11}{18}$$

$$P(Y=0) = \frac{1}{15} + \frac{1}{15} + \frac{4}{15} + \frac{2}{15} = \frac{8}{15}$$

$$P(X=0, Y=0) = \frac{1}{15}.$$

Obviously, $P(X=0) \cdot P(Y=0) = \frac{11}{18} \cdot \frac{8}{15} \neq \frac{1}{15}$

Therefore, X and Y are not independent.

2). If X is conditionally independent of Y given Z.

$$P(X \cap Y \mid Z) = P(X \mid Z) \cdot P(Y \mid Z)$$

From the information.

$$P(X=0, Y=0 \mid Z=0) = \frac{1}{15}.$$

$$P(X=0 \mid Z=0) = \frac{1}{15} + \frac{1}{10} = \frac{5}{30} = \frac{1}{6}$$

$$P(Y=0 \mid Z=0) = \frac{2}{15}.$$

So, $P(X=0 \mid Z=0) \cdot P(Y=0 \mid Z=0) = \frac{1}{6} \cdot \frac{2}{15} = \frac{1}{45} \neq \frac{1}{15}$

Therefore, X and Y are not conditionally independent given Z.

3). $P(X=0 \mid X+Y>0) = \dfrac{P(X=0,\ X+Y>0)}{P(X+Y>0)}$

$$P(X=0,\ X+Y>0) = P(X=0,\ Y=1)$$
$$= \frac{1}{10} + \frac{8}{45} = \frac{5}{18}$$

$$P(X+Y>0) = P(X=0,\ Y=1) + P(X=1,\ Y=0) + P(X=1,\ Y=1)$$

$$\begin{cases} P(X=0,\ Y=1) = \frac{1}{10} + \frac{8}{45} = \frac{5}{18} \\ P(X=1,\ Y=0) = \frac{1}{15} + \frac{2}{15} = \frac{1}{5} \\ P(X=1,\ Y=1) = \frac{1}{10} + \frac{4}{45} = \frac{17}{90} \end{cases}$$

Therefore, $P(X+Y>0) = \frac{2}{3}$.

So, $P(X=0 \mid X+Y>0) = \frac{5}{18} \Big/ \frac{2}{3} = \frac{5}{12}$

Problem 2.

To run the code, I have a problem2.pynb file, simple run each section to see the result.

a). Answer: setosa: 50
versicolor: 50
virginica: 50.

b). Accuracy: 1.0

c). optimal value of k is 9,
the accuracy rate is 0.9733...

d). prediction: setosa.

Problem 3.

To run the code, I have a problem3.pynb file, simple run each section to see the result.

a). The elbow point occurs at k=4,
Therefore, we should have four clusters

b). there are 25 observations in each cluster.
The value of inertia is 4844.9....

c) - I cannot determine.

We have datasets of 14 dimensions.
but to plot the scatter plot, we only plot them taking the first two feature variables.
Therefore, there are lots of information missing here.