Gaussian Weighting Reversion Strategy for Accurate Online Portfolio Selection

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Abstract

This paper design and implement a new on-line portfolio selection strategy based on reversion mechanism and weighted on-line learning. This strategy, called "Gaussian Weighting Reversion" (GWR), improves the reversion estimator to form optimal portfolios and effectively overcomes the shortcomings of existing on-line portfolio selection strategies. Firstly, GWR uses Gaussian function to weight data in a sliding window to exploit the "time validity" of historical market data. It means that the more recent data are more valuable for market prediction than the earlier. Secondly, double estimations are first proposed to be made at each time point, and the average of double estimations is obtained to alleviate the influence of noise and outliers. Finally, Extensive evaluation on six public datasets shows the advantages of this strategy compared with other competing strategies, including the state-of-the-art ones.

Keywords: On-line portfolio selection, Gaussian weighting reversion, double estimations, adaptive learning.

1 Introduction

Portfolio refers to the asset collection of stocks, bonds, and derivative financial products, which are possessed by financial institutions or investors. Portfolio selection (PS) is generally about managing a range of assets to realize some financial goals, for example, the maximization of cumulative return or the risk adjusted return. The pioneering work can be traced to the mean-variance framework and effective boundary model[1], which aims to balance between the expected return (mean) and risk (variance). It is suitable for single-period (batch) PS. Later, the "Capital Growth Theory" was presented by Kelly[2], which is devoted to maximizing the expected log return for multi-period sequential PS.

This paper propose a novel passive aggressive on-line PS strategy. It differs from existing strategies in these aspects: firstly, considering that the previous reversion based strategies apply equal weight to all historical periods and this would lose much market information, exploit the "time validity" of historical market data through Gaussian function to weight data in a sliding window for more accurate price prediction; secondly, proposing the method of double estimations at each time point. The first estimation is based on the real values (the closing prices of stocks) of the current period and other historical data in the weighted window. The second estimation is made by the previous estimation about the current time point and historical data in the weighted window. And then the average of double estimations is obtained to alleviate the influence of noise and outliers, which can improve the robustness of the prediction model. The new strategy is named "Gaussian Weighting Reversion" (GWR) to explore the reversal characteristic of financial market. In order to verify the proposed strategy, numerous experiments on various public datasets are conducted. The experimental results indicate that the performance of GWR is significantly better than other strategies.

2 Related works

Implementing a simple yet effective multi-period on-line PS strategy GWR to exploit the reversion phenomenon of financial market, which predicts the next price via weighting historical data in a sliding window and is more accurate than other competing estimators.

Implementing a novel prediction method that makes double estimations at each time point to deal with noise and outliers effectively.

Through extensive experiments and comparing with existing strategies, the proposed GWR is state-of-the-art.

2.1 Problem setting

Now let us clarify the on-line PS problem in math. The financial market with *d* assets for *n* trading periods to be invested is considered. When the *t*th period finishes, the prices of assets are represented as a closing price vector $\mathbf{p}_t \in R_+^d$, and each element p_t^j means the closing price of asset *j*. The price ratio vector $\mathbf{x}_t \in R_+^d$ represents the change of each asset price, where each element x_t^j indicates the ratio of current closing price (corresponding to the *t*th period) to the previous closing price (corresponding to the $(t-1)$th period) of asset *j*, i.e., $\mathrm{x}_t^j = p_t^j / p_{t-1}^j$.$\mathbf{x}_1^n$=$(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)$ is denoted as a sequence of price ratio vectors from the first to the *n*th period.

At the start of the *t*th period, the capital is assigned among *d* assets according to a portfolio vector $\mathbf{b}_t$, and each element of $\mathbf{b}_t$ means the proportion of wealth invested in each asset. In general, suppose portfolio is self-financed and no margin/short is inadmissibility, which implies each element of portfolio vector is non-negative and the sum of a portfolio is one, that is, $\mathbf{b}_t \in \Delta d$, where $\Delta d = \left\{\mathbf{b}_t : \mathbf{b}_t \in R_+^d, \sum_{j=1}^d b_t^j = 1\right\}$. The investment procedure is represented as a portfolio strategy, which means $\mathbf{b}_1 = \frac{1}{d}\overrightarrow{\mathbf{1}}$ following a series of mappings $\mathbf{b}_t : \left(R_+^d\right)^{t-1} \rightarrow \Delta d, t = 2, 3, \ldots$, where $\mathbf{b}_t = \mathbf{b}_t\left(\mathbf{x}_1^{t-1}\right)$ is the portfolio adopted for the *t*th period, given the past market sequence $\mathbf{x}_1^{t-1} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{t-1})$. Denote $\mathbf{b}_1^n = (\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n)$ as the strategy from the first to the *n*th period.

At the end of the *t*th period, the portfolio $\mathbf{b}_t$ brings portfolio's period return s_t, which means the wealth grows by a factor of $s_t = \mathbf{b}_t \cdot \mathbf{x}_t = \sum_{j=1}^d b_t^j x_t^j$. As we reinvest, the portfolio wealth will get multiplicative increase. Therefore, after *n* trading periods, the portfolio strategy $\mathbf{b}_1^n$ generates the portfolio's cumulative wealth S_n, which achieves $\Pi_{t=1}^n \mathbf{b}_t \cdot \mathbf{x}_t$ times the initial wealth, namely, $S_n\left(\mathbf{b}_1^n, \mathbf{x}_1^n\right) = S_0 \Pi_{t=1}^n \left(\mathbf{b}_t \cdot \mathbf{x}_t\right)$, where S_0 is the initial wealth and set to one in this paper.

Finally, on-line PS problem can be formulated as a sequential decision task, and the algorithmic framework is outlined in Algorithm 1. The goal of portfolio manager is to design a strategy $\mathbf{b}_1^n$ to get the maximization of portfolio's cumulative wealth S_n. The PS is in a sequential mode. At the beginning of each trading period *t*, according to the historical information, the manager learns to select a new portfolio vector $\mathbf{b}_t$ for next price ratio vector $\mathbf{x}_t$, where the decision criterion varies among different managers. The resulting portfolio $\mathbf{b}_t$ can be scored by the portfolio's period return s_t. Such procedure continues until the end of the whole trading period, and the portfolio strategy is finally evaluated by final cumulative wealth S_n. In the above model, we ideally suppose no transaction costs, perfect market liquidity, and zero market impact.

Algorithm 1 On-Line PS Framework.

Input: Historical market price ratio sequence $\mathbf{x}_1^{t-1}$

Output: Final cumulative wealthy S_n

1: **Procedure:**
2: Initialization: $\mathbf{b}_1 = \frac{1}{d}\overrightarrow{\mathbf{1}}, S_0 = 1$.
3: **for** $t = 1 \to$ n **do**
4: Portfolio manager learns the portfolio: $\mathbf{b}_t$
5: Market releases the price ratio: $\mathbf{x}_t$
6: Update cumulative return: $s_t = \mathbf{b}_t \cdot \mathbf{x}_t$ and $S_t = S_{t-1} * s_t$
7: Portfolio manager updates on-line PS strategy
8: **end for**

2.2 Analysis of existing work

Kelly's formula is the foundation of numerous **PS** strategies:

$$\mathbf{b}_{t+1} = \underset{\mathbf{b}\in\Delta_d}{\arg\max} \sum_{i=1}^{k} q_i \log\left(\mathbf{b}\cdot\hat{\mathbf{x}}_{t+1}^{i}\right). \tag{1}$$

Generally, a portfolio manager predicts $\hat{\mathbf{x}}_{t+1}$ with k possible values $\hat{\mathbf{x}}_{t+1}^1, \ldots, \hat{\mathbf{x}}_{t+1}^k$, and their corresponding likelihood is $q_1, \ldots, q_k$. Here, $\hat{\mathbf{x}}_{t+1}^i$ means one possible combination vector of the prediction to next price ratio. Dissimilar prediction methods produce different $\hat{\mathbf{x}}_{t+1}^i$ and q_i, which causes different portfolios. Hence a precise and effective prediction method is essential to the success of a **PS** strategy.

Now we concentrate on the related strategies PAMR, OLMAR, and RMR, which all exploit the reversion property of financial markets. PAMR implicitly assumes $\hat{\mathbf{x}}_{t+1} = \frac{1}{\mathbf{x}_t}$, namely, estimating the next price ratio by the inverse of last price ratio, which is a single-period strategy. OLMAR improves PAMR through predicting the next price with moving average, i.e., $\hat{\mathbf{p}}_{t+1} = avg_t(l) = \frac{1}{l}\sum_{i=t}^{i=t-l+1}\mathbf{p}_i$, where l denotes the size of moving window. Afterward, L1-median is adopted by RMR as the estimator, which is the point with minimal sum of Euclidean distances to l given points of price data. Officially, $\hat{\mathbf{p}}_{t+1} = arg\min_{\mathbf{p}} \sum_{i=0}^{l-1}\|\mathbf{p}_{t-i} - \mathbf{p}\|$, where $\|\cdot\|$ d enotes the Euclidean norm. It is demonstrated that RMR is more robust to noise and outliers than the existing strategies.

In conclusion, PAMR, as a single-period strategy of PS, does not make use of enough market information to make the prediction. Although OLMAR and RMR are developed as multi-period strategy of PS and utilize multi-period historical data, they treat them equally. These approaches ignore the time effect of market data. Furthermore, PAMR and OLMAR are susceptible to noise and outliers. RMR is claimed to be robust to noise and outliers to some extent due to the employment of L1-median, but it still performs poorly in some real markets. For example, on SP500 dataset, RMR even performs worse than OLMAR. The shortcomings of existing strategies prompt us to develop a new on-line PS strategy.

3 Method

3.1 Overview

The proposed strategy exploits Gaussian weighting reversion and passive aggressive on-line learning. The fundamental operation is to obtain the next price ratio $\hat{x}_{t+1}$ via two estimations based on Gaussian weighting, and then maximize the expected return $\mathbf{b}_t \cdot \hat{\mathbf{x}}_{t+1}$ while keeping the last portfolio information to construct optimal portfolio.

3.2 Gaussian weighting

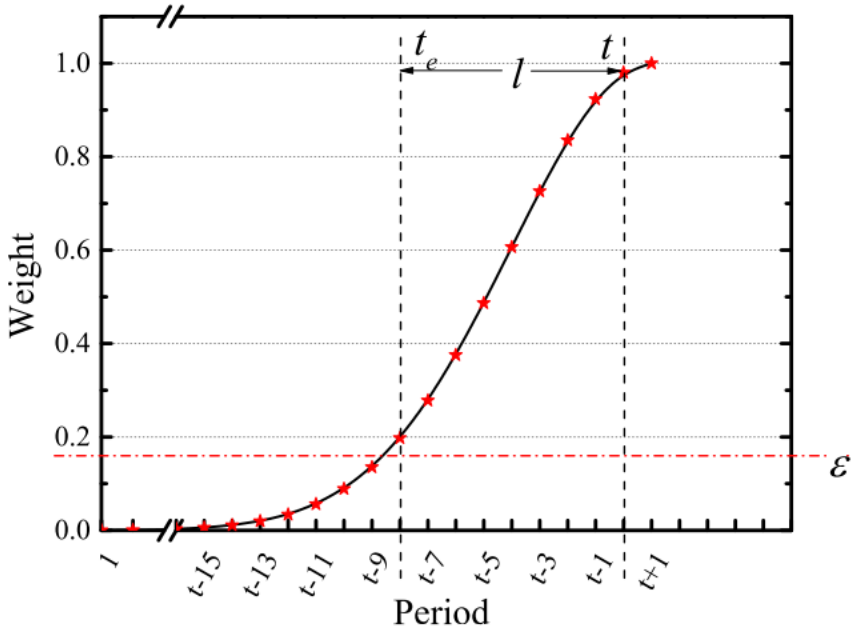

Figure 1: The diagram of Gaussian weighting.

We only use the whole left-half of Gaussian function to exploit the historical data that the recent information is more important than the early. For the prediction of the $(t+1)$th period, this function is centered at the $(t+1)$th period, and τ is a parameter of Gaussian function as follows:

$$gf(x, t+1) = e^{-\dfrac{(t+1-x)^2}{2\tau^2}}. \tag{2}$$

Since the function extends to endless range, another parameter ε is used to control the weighted range. It means only the range that satisfies $gf(x, t+1) \geqslant \varepsilon$ in the left-half of function in (2) is considered, which is named the weighted window. Due to the constraint of the parameter ε, the earliest period t_e should has the smallest $gf(t_e, t+1)$ in the window and $gf(t_e, t+1) \geqslant \varepsilon$. In Figure 1, $t_e = \mathrm{t} - 8$. The range between t_e and t forms the weighted window with width l, which can be calculated by (2) and the above constrain:

$$l = \left\lfloor \sqrt{-2\tau^2 \ln \varepsilon} \right\rfloor. \tag{3}$$

3.3 Double estimations

The first estimation is obtained by Gaussian weighting on the historical data up to the tth period which belongs to the weighted window, i.e., the prices between t_e and t: $\{\mathbf{p}_{t-l+1}, \mathbf{p}_{t-l+2}, \ldots, \mathbf{p}_{t-1}, \mathbf{p}_t\}$. Denote the first estimation by $\mathbf{pp1}(t, t+1)$, which is expressed as:

$$\mathbf{pp1}(t, t+1) = \frac{\sum_{i=t-l+1}^{i=t} e^{-\dfrac{(t+1-i)^2}{2\tau^2}} \mathbf{p}_i}{\sum_{i=t-l+1}^{i=t} e^{-\dfrac{(t+1-i)^2}{2\tau^2}}}. \tag{4}$$

where $\mathbf{p}_t$ is the actual closing price of market in the *t*th period, which may contain noise or even is an outlier.

In order to reduce the impact of noise and outliers, the second estimation is made by replacing $\mathbf{p}_t$ with the estimation of the *t*th period, i,e., $\mathbf{pp1}(t-1, t)$, which is got at the end of the $t-1$th period. At this time (the end of *t*th period), the closing price of the *t*th period has been released, but we still use the primary estimation $\mathbf{pp1}(t-1, t)$ in the $t-1$th period. This part plays an important role of improving the model robustness. Denote the second estimation by $\mathbf{pp2}(t, t+1)$.And it is obtained by Gaussian weighting over the data sequence:$\{\mathbf{p}_{t-l+1}, \mathbf{p}_{t-l+2}, \ldots, \mathbf{p}_{t-1}, \mathbf{pp1}(t-1, t)\}$, that is,

$$\mathbf{pp2}(t, t+1) = \frac{\sum_{i=t-l+1}^{i=t-1} e^{-\dfrac{(t+1-i)^2}{2\tau^2}} \mathbf{p}_i + e^{-\dfrac{1}{2\tau^2}} \mathbf{pp1}(t-1, t)}{\sum_{i=t-l+1}^{i=t} e^{-\dfrac{(t+1-i)^2}{2\tau^2}}}. \tag{5}$$

The final prediction of $\hat{\mathbf{p}}_{t+1}$ is obtained by averaging the above double estimations as follows:

$$\hat{\mathbf{p}}_{t+1} = (\mathbf{pp1}(t, t+1) + \mathbf{pp2}(t, t+1))/2. \tag{6}$$

Figure 2 shows an example of predicting the price of the (*t* +1)th period. In this example, comparing with the prices before the *t*th period, the price in the *t*th period has a sharp down, which is an outlier. However, the first estimation $pp1(t-1, t)$ for the *t*th period is larger than the actual price of the *t*th period, which makes the second estimation $pp2(t, t+1)$ for the (*t* +1)th period larger than $\mathbf{pp1}(t, t+1)$. In the traditional way, $pp1(t, t+1)$ is the final prediction for the (*t* +1)th period. However, for our method of double estimations, the final prediction for the (*t* +1)th period is obtained by averaging $pp1(t, t+1)$ and $pp2(t, t+1)$, and it is much larger than the price of the *t*th period, so that the outlier in the *t*th period does not have much influence on next price prediction.

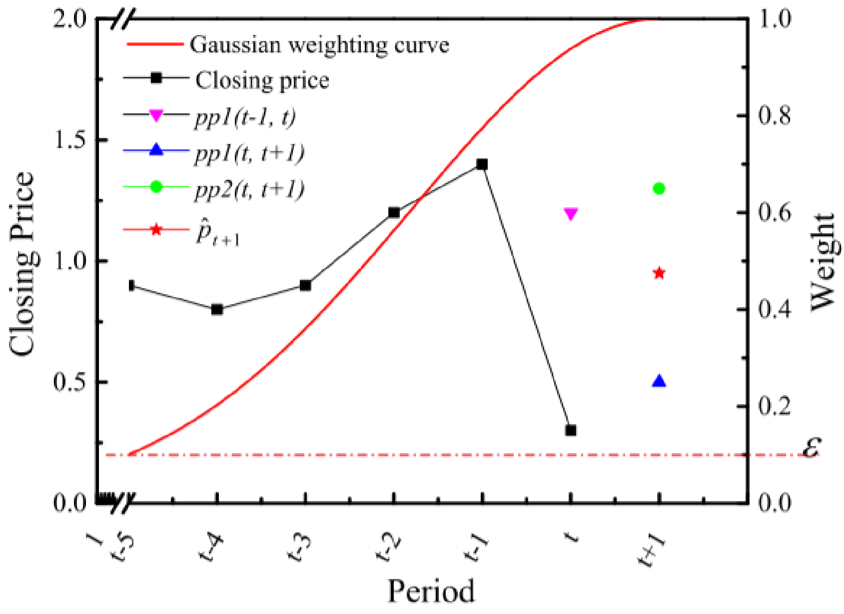

Figure 2: The illustration of the prediction for the (t+1)th period.

Finally, The predicted price ratio at the (t +1)th period is

$$\hat{\mathbf{x}}_{t+1} = \frac{\hat{\mathbf{p}}_{t+1}}{\mathbf{p}_t} = \frac{\mathbf{pp1}(t, t+1) + \mathbf{pp2}(t, t+1)}{2\mathbf{p}_t}. \tag{7}$$

The step is outlined in Algorithm 2, where ε is the parameter to determine the size of the weighted window and τ is the parameter in Gaussian function to determine the weight and size of the weighted window.

Algorithm 2 Relative Price Prediction($\mathbf{p}$, ε, τ).

Input: data $\mathbf{p}_1, \ldots, \mathbf{p}_t$, parameters ε, and τ
Output: predicted $\hat{\mathbf{x}}_{t+1}$

1: **Procedure:**
2: calculate $\mathbf{pp1}(t, t+1)$ according to (4)
3: calculate $\mathbf{pp2}(t, t+1)$ according to (5)
4: $\hat{\mathbf{x}}_{t+1} = \dfrac{\mathbf{pp1}(t, t+1) + \mathbf{pp2}(t, t+1)}{2\mathbf{p}_t}$
5: **return** $\hat{\mathbf{x}}_{t+1}$

3.4 Passive aggressive

After getting the final prediction $\hat{\mathbf{p}}_{t+1}$, the portfolio $\hat{\mathbf{p}}_{t+1}$ is obtained according to passive aggressive on-line learning algorithm[3], which can exploit the reversal phenomenon in financial markets. This on-line learning algorithm is generally used for classification. When the classification is correct, this algorithm passively keeps the previous solution and aggressively obtains a new solution when the classification is wrong. For the presented formulation of our strategy, optimize the following problem to form the next portfolio:

$$\mathbf{b}_{t+1} = \arg\min_{\mathbf{b}\in\Delta d} \frac{1}{2} \|\mathbf{b} - \mathbf{b}_t\|^2 \text{ s.t. } \mathbf{b} \cdot \hat{\mathbf{x}}_{t+1} \geqslant \delta. \tag{8}$$

The final solution of (8) without considering the non-negativity constraint is

$$\mathbf{b}_{t+1} = \mathbf{b}_t + \omega_{t+1}\left(\hat{\mathbf{x}}_{t+1} - \bar{x}_{t+1}\overrightarrow{\mathbf{1}}\right), \tag{9}$$

where $\overrightarrow{\mathbf{1}}$ is the vector with dimension d, that is $\overrightarrow{\mathbf{1}} \in R^d$, $\bar{x}_{t+1} = \left(\hat{\mathbf{x}}_{t+1} \cdot \overrightarrow{\mathbf{1}}\right)/d$ denotes the average of estimated price ratio, and ω_{t+1} is the Lagrangian multiplier calculated as

$$\omega_{t+1} = \max\left\{\frac{\delta - \mathbf{b}_t \cdot \hat{\mathbf{x}}_{t+1}}{\left\|\hat{\mathbf{x}}_{t+1} - \bar{\mathbf{x}}_{t+1}\overrightarrow{\mathbf{1}}\right\|^2}, 0\right\}. \tag{10}$$

Note that the resulting portfolio may go out of simplex domain. Thus, for ensuring that the portfolio is non-negative, the above portfolio is finally projected to simplex domain[4]. The step is outlined in Algorithm 3.

Algorithm 3 Passive Aggressive($\hat{\mathbf{x}}_{t+1}$, δ, $\mathbf{b}_t$).

Input: predicted next price ratio vector $\hat{\mathbf{x}}_{t+1}$; reversion threshold δ; current portfolio $\mathbf{b}_t$

Output: desired $\mathbf{b}_{\mathrm{t+1}}$

1: **Procedure:**
2: Calculate the following variable:
3: $\omega_{t+1} = \max\left\{\frac{\delta - \mathbf{b}_t \cdot \hat{\mathbf{x}}_{t+1}}{\left\|\hat{\mathbf{x}}_{t+1} - \bar{x}_{t+1}\overrightarrow{\mathbf{1}}\right\|^2}, 0\right\}$
4: Update the portfolio:
5: Project $\mathbf{b}_{t+1}$ to simplex domain
6: **return** the projected $\mathbf{b}_{t+1}$

3.5 Algorithm integration

The whole process of GWR strategy can be divided into two steps: the first step is to predict the next price ratio by double estimations which are based on Gaussian weighting over historical data up to the current time point and the preliminary estimation of the current time point; the second step is to determine the portfolio by passive aggressive on-line learning.The whole process of on-line PS following the problem setting for GWR strategy is outlined in Algorithm 4.

Algorithm 4 The GWR Algorithm.

Input: market sequence $\mathbf{x}_1^n$; parameters: δ, τ , and ε.

Output: cumulative wealthy after the nth period S_n

1: **Procedure:**
2: Initialization: $\mathbf{b}_1 = \frac{1}{d}\overrightarrow{\mathbf{1}}, S_0 = 1.$
3: **for** t = 1 → n **do**
4: Receive stock price ratio: $\mathbf{x}_t$
5: Update cumulative return: $s_t = \mathbf{b}_t \cdot \mathbf{x}_t$ and $S_t = S_{t-1} * s_t$
6: Predict the next price ratio vector:
7: $\hat{\mathbf{x}}_{t+1} =$ *Relative Price Prediction* $(\mathbf{p}, \varepsilon, \tau)$
8: Update the portfolio:
9: $\mathbf{b}_{t+1} =$ *Passive Aggressive* $(\hat{\mathbf{x}}_{t+1}, \delta, \mathbf{b}_t)$
10: **end for**

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4 Implementation details

4.1 Comparing with released source codes

No related source codes are available. My work mainly includes the following points: (1) Implemented the code that predict the next price ratio by double estimations which are based on Gaussian weighting over historical data up to the current time point and the preliminary estimation of the current time point. (2) Implemented the code that determine the portfolio by passive aggressive on-line learning. (3) Extensive experiments on six public datasets shows the advantages of our strategy compared with other six competing strategies(the other six competing algorithm code provided by our research group), including the state-of-the-art ones.

4.2 Experimental

In this section, we will experimentally evaluate our strategies and competing strategies on six real public datasets.

4.2.1 Experimental datasets

The six public datasets have wide application in on-line PS strategy test[5][6][7], and the specific information of six public datasets is shown in Table 1.

Table 1: The Summary Of Six Real Datasets

Dataset	Region	Time frame	#Days	#Assets
DJIA	US	1/1/2001-14/1/2003	507	30
SP500	US	2/1/1998-31/1/2003	1276	25
NYSE_O	US	3/7/1962-31/12/1984	5651	36
NYSE_N	US	1/1/1985-30/6/2010	6431	23
TSE	CA	4/1/1994-31/12/1998	1259	88
MSCI	GLOBAL	1/4/2006-31/3/2010	1043	24

4.2.2 Experimental setup

In the below experiments, the proposed GWR strategy is realized. And it is compared with a set of benchmarks and ever state-of-the-art competitors including related reversion strategies. The specific parameter settings of competing strategies are consistent with the original papers. For the GWR strategy, there are three hyper-parameters δ, τ, and ε. With the goal of consistency in the comparison, this paper setup the parameters $\delta = 50$, $\tau = 2.8$, and $\varepsilon = 0.005$ in the experiments[8].

4.3 Main contributions

Firstly, exploit the “time validity” of historical market data through Gaussian function to weight data in a sliding window for more accurate price prediction.

Secondly, implement the method of double estimations at each time point. The first estimation is based on the real values (the closing prices of stocks) of the current period and other historical data in the weighted window. The second estimation is made by the previous estimation about the current time point and historical data in the weighted window. And then the average of double estimations is obtained to alleviate the influence of noise and outliers, which can improve the robustness of the prediction model.

In order to verify the proposed strategy, numerous experiments on various public datasets are conducted. The experimental results indicate that the performance of GWR is significantly better than other strategies.

5 Results and analysis

The contrast results of cumulative wealth achieved by different strategies on the six datasets in Table 1 are listed in Table 2. Here, the strategies are compared with other six competitors, including the existing reversion strategies PAMR, RMR and OLMAR.

As Table 2 shows, the GWR surpass all existing strategies on TSE, MSCI, NYSE_O and NYSE_N, and is ranked the 3nd on DJIA (only slightly inferior to OLMAR and RMR). and is ranked the 2nd on SP500 (only slightly inferior to OLMAR). In particular, the cumulative wealth achieved by GWR on NYSE_O is three times more than RMR which is at the second place. GWR has more outstanding performance than multi-period reversion strategies (OLMAR and RMR) on other datasets, which shows that the effectiveness of nonlinear weighting and double estimations mechanism adopted in this strategies.Table 3 shows the experimental data of the original paper. Due to the accuracy problem, my experimental results will not be exactly the same as those of the original paper, but they are also very close.

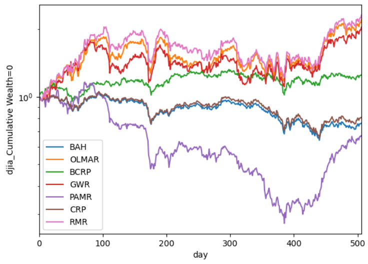
Then the accumulation process of how each strategy achieving the final cumulative wealth is presented in Figure 3. These strategies include the proposed GWR, relevant reversion competitors and two benchmarks. Experimental results suggest that our strategies GWR always perform better than others during the entire trading period except DJIA and SP500 datasets. This proves the effectiveness of our strategies again.

Table 2: My experiment results.

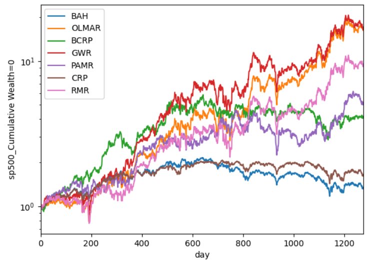
Strategies	DJIA	SP500	NYSE_O	NYSE_N	TSE	MSCI
BHA	0.76	1.33	14.21	18.23	1.59	0.89
OLMAR	**2.20**	**16.78**	7.65E+16	4.20E+08	59.00	14.56
BCRP	1.25	4.05	248.50	120.76	6.59	1.49
GWR	2.03	17.02	**5.68E+17**	**6.82E+08**	**450.47**	**22.31**
PAMR	0.67	5.02	5.08E+15	1.29E+06	257.86	14.99
CRP	0.81	1.63	26.67	31.82	1.57	0.91
RMR	**2.30**	9.46	1.77E+17	6.24E+08	245.17	15.26

Table 3: The experimental results of the paper.

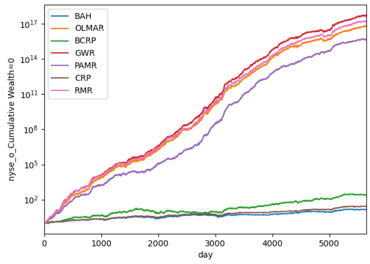
Strategies	DJIA	SP500	NYSE_O	NYSE_N	TSE	MSCI
BHA	0.79	1.35	14.30	18.32	1.57	0.89
OLMAR	2.33	16.53	7.73E+16	4.29E+08	56.55	14.39
BCRP	1.20	4.09	248.75	119.32	6.71	1.47
GWR	2.33	**17.97**	**5.69E+17**	**6.98E+08**	**453.55**	**22.35**
PAMR	0.70	5.05	4.87E+15	1.32E+06	257.78	14.63
CRP	0.80	1.54	22.57	26.09	1.55	0.88
RMR	**2.49**	8.93	1.67E+17	3.17E+08	168.49	16.18



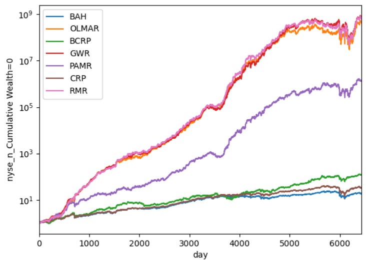
(a) DJIA.



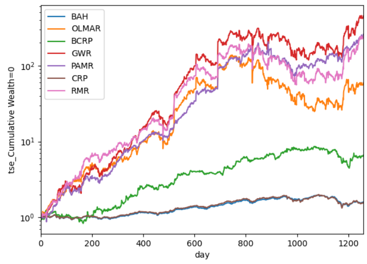
(b) SP500.



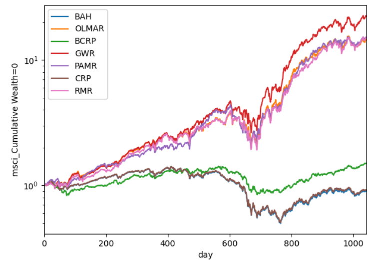
(c) NYSE_O.



(d) NYSE_N.



(e) TSE



(f) MSCI

Figure 3: The trend chart of cumulative wealth obtained by different strategies during the entire trading period on six public datasets.

6 Conclusion and future work

In this paper, a novel multi-period on-line portfolio selection strategy named GWR is proposed, which takes full advantage of the market information by weighting the historical data with Gaussian function, and decreases the impact of noise and outliers by averaging double estimations.

Vast experiments are conducted on six public datasets and the experimental results show that this strategies outperform seven existing competitors. What is more, this strategies have low computational complexity and are easy to implement. All those features and advantages support the practical applications of our strategies.

In the future, we will grope for more advanced models for market data mining to further improve performance, consider transaction costs into the optimization model, and devise new on-line learning strategies for portfolio selection.

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