Supplementary File for "Multi-Surrogate-Assisted Ant Colony Optimization for Expensive Optimization Problems with Continuous and Categorical Variables"

S-I. Proof of Propositions

A. Proof of Proposition 1

The upper bound of RBF's predicted error can be estimated as:

$$
\begin{aligned}
&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})| \\
=&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-f(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)+\hat{f}_{RBF}(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})| \\
\leq&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-f(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)|+|\hat{f}_{RBF}(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})| \\
\leq&(L_f+L_r)\cdot dis((\mathbf{x}^{cn},\mathbf{x}^{ca}),(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*))
\end{aligned}
$$

where the first equality holds by Assumption 2, the second inequality is the triangle inequality, and the third inequality holds by Assumption 1.

Similarly, the lower bound of RBF's predicted error can be estimated as:

$$
\begin{aligned}
&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})| \\
=&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-f(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)+\hat{f}_{RBF}(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})| \\
\geq&\big||f(\mathbf{x}^{cn},\mathbf{x}^{ca})-f(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)|-|\hat{f}_{RBF}(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})|\big|
\end{aligned}
$$

Let $F=\big||f(\mathbf{x}^{cn},\mathbf{x}^{ca})-f(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)|-|\hat{f}_{RBF}(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})|\big|$. According to Assumption 1, for F, we have:

$$
0\leq F\leq \max(L_f-\frac{1}{L_r},L_r-\frac{1}{L_f})\cdot dis((\mathbf{x}^{cn},\mathbf{x}^{ca}),(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*))
$$

thus

$$
|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})|\geq 0
$$

B. Proof of Proposition 2

The upper bound of LSBT's predicted error can be estimated as:

$$
\begin{aligned}
&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-\hat{f}_{LSBT}(\mathbf{x}^{cn},\mathbf{x}^{ca})| \\
=&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-f(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)+\hat{f}_{LSBT}(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)-\hat{f}_{LSBT}(\mathbf{x}^{cn},\mathbf{x}^{ca})| \\
=&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-f(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)| \\
\leq&L_f\cdot dis((\mathbf{x}^{cn},\mathbf{x}^{ca}),(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*))
\end{aligned}
$$

where the first equality holds by Assumption 2, the second equality holds by Assumption 3, and the third inequality holds by Assumption 1.

Similarly, the lower bound of LSBT's predicted error can be estimated as:

$$
|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-\hat{f}_{LSBT}(\mathbf{x}^{cn},\mathbf{x}^{ca})|\geq \frac{1}{L_f}\cdot dis((\mathbf{x}^{cn},\mathbf{x}^{ca}),(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*))
$$

S-II. Discussions

A. Effectiveness of the Three Selection Operators

To investigate the effectiveness of the three selection operators (i.e., the RBF-based selection, the LSBT-based selection, and the random selection), we designed three variants of MiSACO, called MiSACO-LSBT-Rand, MiSACO-RBF-Rand, and MiSACO-LSBT-RBF. In MiSACO-LSBT-Rand, the RBF-based selection was not used; in MiSACO-RBF-Rand, the LSBT-based selection was removed; and in MiSACO-LSBT-RBF, the random selection was not employed. The *AOFV* and *ASFEs* values are summarized in Table S-VII and Table S-VIII of the supplementary file, respectively. The results are discussed in the following:

- MiSACO-LSBT-Rand cannot give any better *AOFV* or *ASFEs* value on the ten type-1 and ten type-3 artificial test problems. Thus, we can conclude that MiSACO-LSBT-Rand is not good at dealing with type-1 and type-3 artificial test problems. The reason may be that without the RBF-based selection, MiSACO-LSBT-Rand cannot solve EOPCCVs with many continuous variables well.
- MiSACO-RBF-Rand cannot get any better *AOFV* or *ASFEs* value on the ten type-2 and ten type-3 artificial test problems. Therefore, MiSACO-RBF-Rand performs poor on solving type-2 and type-3 artificial test problems. This is because only using the RBF-based selection and the random selection cannot help MiSACO-RBF-Rand to solve EOPCCVs with many categorical variables effectively.
- For eight out of the ten type-1 artificial test problems (F2, F3, and F5-F10), nine out of the ten type-2 artificial test problems (F11-F13 and F15-F20), and all of the ten type-3 artificial test problems, MiSACO can obtain better *AOFV* values than MiSACO-LSBT-RBF. Meanwhile, for four type-1 artificial test problems (F3, F5, F6, and F10), seven type-2 artificial test problems (F11, F13-F16, F19, and F20), and six type-3 artificial test problems (F21, F24, F26, and F28-F30), MiSACO can provide better *ASFEs* values. Obviously, for all of these three types of artificial test problems, the random selection is able to enhance the performance of the algorithm. This is because when the constructed surrogate models are inaccurate, the random selection has the potential to improve their accuracies.

From the above analysis, we can conclude that all of the three selection operators are indispensable.

B. Effectiveness of the Surrogate-Assisted Local Search

We would like to ascertain whether the surrogate-assisted local search can improve the convergence performance of MiSACO. To this end, additional experiments were conducted. A variant of MiSACO, called MiSACO-WoLocal, was devised. In MiSACO-WoLocal, the surrogate-assisted local search was removed. The *AOFV* and *ASFEs* values provided by MiSACO-WoLocal and MiSACO are given in Table S-IX of the supplementary file.

For *AOFV*, MiSACO can obtain better values on all the 30 test problems except F25 and F29. For *ASFEs*, MiSACO can provide better values on 23 test problems (F1, F3-F6, F8-F11, F13-F15, F18-F21, F23-F25, and F28-F30). According to the Wilcoxon's rank-sum test, MiSACO performs better than MiSACO-WoLocal on 20 test problems in terms of *AOFV* and nine test problems in terms of *ASFEs*, respectively. Based on the above results, it can be concluded that the surrogate-assisted local search is capable of enhancing the convergence performance of MiSACO.

C. Effectiveness of RBF in MiSACO

In MiSACO, RBF was used as the surrogate model for continuous functions. However, other popular techniques, such as Kriging, can also be employed. One may be interested in the influence of RBF and Kriging on the performance of MiSACO. To this end, a variant of MiSACO, called MiSACO-Kriging, was designed. The *AOFV* and *ASFEs* values and runtime provided by MiSACO-Kriging and MiSACO are recorded in Table S-X and Table S-XI of the supplementary file, respectively.

From Table S-X and Table S-XI, MiSACO-Kriging and MiSACO show similar performance in terms of both *AOFV* and *ASFEs*. However, the runtime consumed by MiSACO-Kriging is significantly longer than that of MiSACO. This is because the computational time complexity of Kriging is much higher than that of RBF. Therefore, we employed RBF as the surrogate model for continuous functions in this paper.

D. Effectiveness of LSBT in MiSACO

In MiSACO, LSBT was employed as the surrogate model with a tree structure. As RF is also a famous surrogate model with a tree structure, one may be interested in whether RF can be incorporated into MiSACO. To answer this question, a variant of MiSACO, called MiSACO-RF, was developed by replacing LSBT with RF. The *AOFV* and *ASFEs* values provided by MiSACO-RF and MiSACO are recorded in Table S-XII of the supplementary file.

From Table S-XII, in terms of *AOFV*, MiSACO is better than MiSACO-RF on 27 test problems (i.e., F1-F4, F6-F21, F23-F26, and F28-F30). As far as *ASFEs* is concerned, MiSACO obtains better values on 24 test problems (i.e., F1, F3-F6, F10, F11, F13-16, F19-F21, F23-F26, and F28-F30). According to the Wilcoxon's rank-sum test, MiSACO performs better than MiSACO-RF on 23 test problems in terms of *AOFV* and 20 test problems in terms of *ASFEs*, respectively. Therefore, in this paper, we employed LSBT as the surrogate model with a tree structure.

The superiority of MiSACO against MiSACO-RF may be attributed to the following reasons. When training each binary regression tree in a RF, only a certain numbers of solutions in $\mathbb{DB}$ are used. If the number of solutions in $\mathbb{DB}$ is small, fewer solutions will be used to train each binary regression tree in a RF. At this time, it is difficult to guarantee the accuracy of each binary regression tree in a RF. As a result, the trained RF is also difficult to provide an accurate prediction. In contrast, when training each binary regression tree in a LSBT, all of the solutions in $\mathbb{DB}$ are used. This makes it possible for LSBT to provide better predicted values than RF. Since the number of solutions in $\mathbb{DB}$ is always a small value, we prefer to employ LSBT in our algorithm.

E. Influence of Different Distances in RBF

In Section II, we have mentioned that, to handle EOPCCVs, the distance used in RBF should be redefined. However, we have also mentioned another distance in Section V, i.e., Gower distance. One may be interested in the performance of MiSACO if Gower distance is used in RBF. To investigate this, we designed a variant of MiSACO, called MiSACO-Gower. In MiSACO-Gower, Gower distance was employed in RBF. The *AOFV* and *ASFEs* values provided by MiSACO and MiSACO-Gower are provided in Table S-XIII of the supplementary file.

From Table S-XIII, MiSACO obtains better values than MiSACO-Gower on 22 (i.e., F1, F2, F4-F7, F10-F12, F14, F15, F17, F20, F21-F27, F29, and F30) and 21 (i.e., F1, F3-F6, F10, F11, F13-16, F19-F21, F23-F26, and F28-F30) test problems in terms of *AOFV* and in terms of *ASFEs*, respectively. According to the Wilcoxon's rank-sum test, MiSACO performs better than MiSACO-Gower on 12 test problems in terms of *AOFV* and 15 test problems in terms of *ASFEs*, respectively. Thus, it can be concluded that Gower distance may not be a good choice for MiSACO.

F. Influence of N_{min}

In MiSACO, N_{min} was used to decide whether the surrogate-assisted local search is implemented or not. The influence of N_{min} was investigated by experiments. In the investigation, six test problems (i.e., F1, F6, F13, F18, F24, and F29) were selected, and N_{min} was set to four different values: $1*n_1$, $5*n_1$, $10*n_1$, and $20*n_1$. The *AOFV* and *ASFEs* values are summarized in Table S-XIV. From Table S-XIV, when N_{min} is equal to $5*n_1$, MiSACO achieves the best performance.

S-III. Results

TABLE S-I

Results of ACO$_{MV}$ and MiSACO over 20 Independent Runs on the Capacitated Facility Location Problems. The Wilcoxon's Rank-sum Test at a 0.05 Significance Level Was Performed Between ACO$_{MV}$ and MiSACO.

	ACO$_{MV}$		MiSACO
CFLP1	3192.08±2.30	$+$	**3185.42±2.21**
CFLP2	5803.43±16.93	$+$	**5776.52±5.39**
CFLP3	1404.17±30.73	$+$	**1381.19±2.93**
CFLP4	3442.87±163.90	$+$	**3378.06±127.74**
CFLP5	1080.75±33.51	$+$	**1050.35±12.54**
CFLP6	3356.49±646.10	$+$	**2550.43±117.73**
$+/-/\approx$	6/0/0		

TABLE S-II

Results of ACO$_{MV}$ and MiSACO over 20 Independent Runs on the Dubins Traveling Salesman Problems. The Wilcoxon's Rank-sum Test at a 0.05 Significance Level Was Performed Between ACO$_{MV}$ and MiSACO.

	ACO$_{MV}$		MiSACO
DTSP1	492.97±31.51	$+$	**465.88±28.90**
DTSP2	696.08±60.62	$+$	**607.59±38.60**
DTSP3	914.11±25.17	$+$	**775.79±44.85**
DTSP4	635.81±25.87	$+$	**585.83±34.84**
DTSP5	960.07±36.03	$+$	**830.08±24.51**
DTSP6	1229.80±62.92	$+$	**1143.03±63.28**
$+/-/\approx$	6/0/0		

TABLE S-III

AOFV Values Provided by CAL-SAPSO, EGO-Hamming, EGO-Gower, BOA-RF, and MiSACO on the 30 Artificial Test Problems. The Wilcoxon's Rank-sum Test at a 0.05 Significance Level Was Performed Between MiSACO and Each of CAL-SAPSO, EGO-Hamming, EGO-Gower, and BOA-RF.

	Problem	CAL-SAPSO		EGO-Hamming		EGO-Gower		BOA-RF		MiSACO
		AOFV ± *Std Dev*		*AOFV* ± *Std Dev*		*AOFV* ± *Std Dev*		*AOFV* ± *Std Dev*		*AOFV* ± *Std Dev*
Type 1	F1	8.68E+01 ± 9.85E+02	+	1.99E+01 ± 1.69E+01	+	4.69E+01 ± 6.07E+01	+	2.03E+03 ± 8.38E+02	+	**6.21E−08 ± 2.24E−08**
	F2	3.19E+01 ± 1.13E+01	+	5.35E+01 ± 1.38E+01	+	5.41E+01 ± 7.58E+00	+	8.24E+01 ± 8.04E+00	+	**2.59E+01 ± 1.19E+01**
	F3	1.62E+01 ± 1.17E−04	+	3.82E+00 ± 1.99E+00	+	3.85E+00 ± 4.63E+00	+	1.12E+01 ± 9.33E−01	+	**3.04E−01 ± 7.50E−01**
	F4	2.95E+00 ± 1.55E+01	+	9.94E−01 ± 6.50E−01	+	3.92E−01 ± 2.08E−01	+	4.13E+01 ± 1.45E+01	+	**4.51E−09 ± 2.25E−09**
	F5	1.54E+01 ± 1.10E+01	+	1.43E+00 ± 2.96E−01	+	1.10E+00 ± 1.98E−01	+	1.93E+01 ± 7.23E+00	+	**2.39E−01 ± 2.30E−01**
	F6	4.86E+02 ± 1.16E+03	+	2.80E+01 ± 1.75E+01	+	2.11E+02 ± 3.31E+02	+	2.31E+03 ± 6.37E+02	+	**5.74E−08 ± 2.50E−08**
	F7	3.23E+01 ± 1.34E+01	+	5.29E+01 ± 9.38E+00	+	4.84E+01 ± 7.35E+00	+	7.37E+01 ± 1.16E+01	+	**2.40E+01 ± 1.22E+01**
	F8	1.61E+01 ± 1.40E+00	+	3.64E+00 ± 3.40E+00	+	2.68E+00 ± 2.18E+00	+	1.21E+01 ± 1.18E+00	+	**7.27E−02 ± 2.83E−01**
	F9	6.48E+00 ± 1.92E+01	+	7.01E−01 ± 4.74E−01	+	2.34E+00 ± 3.69E+00	+	4.33E+01 ± 1.26E+01	+	**1.41E−06 ± 6.15E−06**
	F10	1.27E+01 ± 1.26E+01	+	1.35E+00 ± 2.38E−01	+	1.12E+00 ± 2.04E−01	+	2.73E+01 ± 5.34E+00	+	**2.75E−01 ± 3.01E−01**
Type 2	F11	3.68E+02 ± 5.08E+02	+	1.04E+01 ± 1.07E+01	+	8.29E−02 ± 1.26E−01	+	1.09E+02 ± 6.25E+01	+	**9.43E−08 ± 1.06E−07**
	F12	5.83E+01 ± 1.12E+01	+	4.60E+01 ± 1.74E+01	≈	**4.35E+01 ± 1.49E+01**	≈	6.49E+01 ± 6.79E+00	+	4.71E+01 ± 1.73E+01
	F13	8.54E+00 ± 1.21E+00	+	3.93E+00 ± 1.89E+00	+	5.71E−01 ± 2.31E−01	+	4.89E+00 ± 1.49E+00	+	**2.26E−04 ± 1.17E−04**
	F14	8.70E+00 ± 1.15E+01	+	3.34E−01 ± 5.37E−01	+	1.65E+00 ± 4.77E+00	+	8.53E−01 ± 4.34E−01	+	**2.04E−09 ± 2.03E−09**
	F15	8.92E+00 ± 8.34E+00	+	1.15E+00 ± 1.89E−01	+	3.49E−01 ± 1.63E−01	+	1.42E+00 ± 4.48E−01	+	**2.82E−07 ± 3.27E−07**
	F16	7.57E+02 ± 1.19E+03	+	1.19E+02 ± 1.72E+02	+	2.34E+01 ± 7.08E+01	+	1.07E+03 ± 2.45E+02	+	**1.27E+00 ± 5.66E+00**
	F17	6.98E+01 ± 1.37E+01	+	4.77E+01 ± 1.85E+01	≈	**4.11E+01 ± 7.67E+00**	≈	7.16E+01 ± 7.89E+00	+	4.50E+01 ± 1.61E+01
	F18	1.08E+01 ± 8.70E−01	+	3.30E+00 ± 2.52E+00	+	2.89E+00 ± 5.45E−01	≈	6.70E+00 ± 1.59E+00	+	**1.55E+00 ± 1.96E+00**
	F19	3.42E+01 ± 2.75E+01	+	7.46E+00 ± 1.03E+01	+	1.76E+00 ± 1.39E+00	+	6.88E+00 ± 3.21E+00	+	**2.71E−01 ± 7.16E−01**
	F20	1.40E+01 ± 1.98E+01	+	3.24E+00 ± 3.20E+00	+	3.01E+00 ± 2.74E+00	+	1.17E+01 ± 3.67E+00	+	**1.05E−01 ± 3.23E−01**
Type 3	F21	5.77E+02 ± 9.35E+02	+	7.82E+00 ± 4.26E+00	+	1.11E+01 ± 3.07E+01	+	1.01E+03 ± 5.16E+02	+	**7.60E−08 ± 5.64E−08**
	F22	5.37E+01 ± 1.41E+01	≈	4.51E+01 ± 8.61E+00	≈	**4.42E+01 ± 9.04E+00**	≈	7.34E+01 ± 1.37E+01	+	4.78E+01 ± 1.32E+01
	F23	1.26E+01 ± 1.19E+00	+	3.39E+00 ± 2.74E+00	+	3.97E+00 ± 5.04E+00	+	7.83E+00 ± 1.42E+00	+	**2.38E−04 ± 8.58E−05**
	F24	1.21E+01 ± 1.10E+01	+	1.87E−01 ± 1.53E−01	+	1.33E−01 ± 9.41E−02	+	1.79E+01 ± 5.85E+00	+	**1.18E−09 ± 4.54E−10**
	F25	2.26E+01 ± 9.22E+00	+	1.20E+00 ± 1.64E−01	+	9.86E−01 ± 4.11E−02	+	6.22E+00 ± 2.59E+00	+	**1.24E−01 ± 2.34E−01**
	F26	7.44E+02 ± 1.48E+03	+	1.20E+01 ± 7.74E+00	+	1.45E+02 ± 2.13E+02	+	1.73E+03 ± 5.08E+02	+	**9.70E−08 ± 8.28E−08**
	F27	5.64E+01 ± 1.13E+01	+	5.42E+01 ± 1.07E+01	+	5.07E+01 ± 8.96E+00	+	7.95E+01 ± 1.29E+01	+	**4.15E+01 ± 1.78E+01**
	F28	1.43E+01 ± 1.03E+00	+	2.65E+00 ± 7.17E−01	+	2.15E+00 ± 2.30E+00	+	9.70E+00 ± 1.12E+00	+	**9.76E−01 ± 1.46E+00**
	F29	1.52E+01 ± 2.40E+01	+	5.80E−01 ± 3.82E−01	+	1.80E+00 ± 1.59E+00	+	1.97E+01 ± 6.72E+00	+	**2.19E−05 ± 9.28E−05**
	F30	1.58E+01 ± 1.43E+01	+	1.19E+00 ± 1.17E−01	+	1.19E+00 ± 4.85E−01	+	1.64E+01 ± 5.16E+00	+	**4.92E−01 ± 6.18E−01**
$+/-/\approx$		29/0/1		27/0/3		26/0/4		30/0/0		

TABLE S-IV
ASFEs VALUES PROVIDED BY CAL-SAPSO, EGO-HAMMING, EGO-GOWER, BOA-RF, AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO AND EACH OF CAL-SAPSO, EGO-HAMMING, EGO-GOWER, AND BOA-RF.

	Problem	CAL-SAPSO		EGO-Hamming		EGO-Gower		BOA-RF		MiSACO
		ASFEs±Std Dev		*ASFEs±Std Dev*		*ASFEs±Std Dev*		*ASFEs±Std Dev*		*ASFEs±Std Dev*
Type 1	F1	596.10±12.33	+	600.00±0.00	+	600.00±0.00	+	600.00±0.00	+	**249.95±36.49**
	F2	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	+	**600.00±0.00**
	F3	600.00±0.00	+	600.00±0.00	+	600.00±0.00	+	600.00±0.00	+	**375.85±101.67**
	F4	562.20±55.38	+	599.50±0.53	+	585.80±9.38	+	600.00±0.00	+	**159.10±25.60**
	F5	600.00±0.00	+	600.00±0.00	+	599.80±0.63	+	600.00±0.00	+	**208.90±37.03**
	F6	600.00±0.00	+	600.00±0.00	+	600.00±0.00	+	600.00±0.00	+	**269.40±56.18**
	F7	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	+	**600.00±0.00**
	F8	600.00±0.00	+	600.00±0.00	+	599.80±0.42	+	600.00±0.00	+	**391.60±63.22**
	F9	599.60±0.84	+	598.70±0.95	+	595.20±4.98	+	600.00±0.00	+	**231.15±65.29**
	F10	600.00±0.00	+	600.00±0.00	+	599.90±0.32	+	600.00±0.00	+	**270.20±123.87**
Type 2	F11	600.00±0.00	+	599.90±0.32	+	587.60±3.53	+	600.00±0.00	+	**285.75±73.21**
	F12	600.00±0.00	≈	**599.90±0.32**	≈	**599.50±1.58**	≈	**600.00±0.00**	+	600.00±0.00
	F13	600.00±0.00	+	599.90±0.32	+	594.10±4.20	+	559.80±37.78	+	**307.25±120.94**
	F14	600.00±0.00	+	596.30±2.21	+	527.50±49.70	+	584.90±47.75	+	**195.45±39.24**
	F15	599.90±0.32	+	599.90±0.32	+	587.60±3.34	+	600.00±0.00	+	**244.50±44.50**
	F16	600.00±0.00	+	600.00±0.00	+	598.80±1.23	+	600.00±0.00	+	**409.65±100.36**
	F17	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	+	**600.00±0.00**
	F18	600.00±0.00	+	599.60±0.70	+	598.60±1.17	+	600.00±0.00	+	**538.25±93.69**
	F19	600.00±0.00	+	599.90±0.32	+	597.70±4.64	+	600.00±0.00	+	**356.45±122.27**
	F20	600.00±0.00	+	600.00±0.00	+	599.60±0.84	+	600.00±0.00	+	**401.70±109.00**
Type 3	F21	600.00±0.00	+	600.00±0.00	+	599.50±0.85	+	600.00±0.00	+	**279.55±36.60**
	F22	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	+	**600.00±0.00**
	F23	600.00±0.00	+	600.00±0.00	+	599.40±1.07	+	600.00±0.00	+	**363.15±76.32**
	F24	600.00±0.00	+	595.30±1.34	+	526.50±58.93	+	600.00±0.00	+	**185.15±37.79**
	F25	600.00±0.00	+	600.00±0.00	+	599.50±0.53	+	600.00±0.00	+	**229.45±32.75**
	F26	600.00±0.00	+	600.00±0.00	+	599.90±0.32	+	600.00±0.00	+	**330.80±62.07**
	F27	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	+	**600.00±0.00**
	F28	600.00±0.00	+	600.00±0.00	+	599.50±0.53	+	600.00±0.00	+	**475.15±117.83**
	F29	599.70±0.95	+	597.40±1.90	+	596.30±6.04	+	600.00±0.00	+	**273.10±83.76**
	F30	600.00±0.00	+	600.00±0.00	+	599.50±0.53	+	600.00±0.00	+	**353.50±118.11**
+/−/≈		24/0/6		24/0/6		24/0/6		24/0/6		

TABLE S-V

AOFV VALUES PROVIDED BY CAL-SAPSO, EGO-HAMMING, EGO-GOWER, BOA-RF, AND MISACO ON THE SIX CAPACITATED FACILITY LOCATION PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO AND EACH OF CAL-SAPSO, EGO-HAMMING, EGO-GOWER, AND BOA-RF.

Problem	CAL-SAPSO		EGO-Hamming		EGO-Gower		BOA-RF		MiSACO
	AOFV±Std Dev		*AOFV±Std Dev*		*AOFV±Std Dev*		*AOFV±Std Dev*		*AOFV±Std Dev*
CFLP1	3203.22±2.20	+	3195.73±2.51	+	3191.68±1.76	+	3192.56±1.64	+	**3185.42±2.21**
CFLP2	5808.89±11.56	+	5800.91±10.08	+	5797.36±12.44	+	5795.30±11.89	+	**5776.52±5.39**
CFLP3	1403.34±9.37	+	1393.46±7.28	+	1387.62±2.26	+	1388.76±2.97	+	**1381.19±2.93**
CFLP4	3620.66±133.98	+	3659.44±115.64	+	**3356.00±125.48**	≈	3500.90±117.52	≈	3378.06±127.74
CFLP5	1090.61±98.20	+	1110.07±70.00	+	1053.74±2.75	≈	1050.95±3.37	≈	**1050.35±12.54**
CFLP6	3124.20±277.01	+	3098.86±222.44	+	2553.93±37.60	≈	3279.56±447.19	+	**2550.43±117.73**
+/−/≈	6/0/0		6/0/0		3/0/3		5/0/1		

TABLE S-VI

AOFV VALUES PROVIDED BY CAL-SAPSO, EGO-HAMMING, EGO-GOWER, BOA-RF, AND MISACO ON THE SIX DUBINS TRAVELING SALESMAN PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO AND EACH OF CAL-SAPSO, EGO-HAMMING, EGO-GOWER, AND BOA-RF.

Problem	CAL-SAPSO		EGO-Hamming		EGO-Gower		BOA-RF		MiSACO
	AOFV±Std Dev		*AOFV±Std Dev*		*AOFV±Std Dev*		*AOFV±Std Dev*		*AOFV±Std Dev*
DTSP1	490.55±10.23	+	499.08±25.95	+	479.46±17.36	+	495.81±9.81	+	**465.88±28.90**
DTSP2	720.20±22.30	+	739.70±35.04	+	705.55±24.88	+	718.00±19.80	+	**607.59±38.60**
DTSP3	923.73±23.21	+	952.06±28.13	+	932.76±31.01	+	975.70±11.24	+	**775.79±44.85**
DTSP4	667.26±14.77	+	674.05±34.42	+	629.15±16.22	+	644.97±3.45	+	**585.83±34.84**
DTSP5	1001.56±35.76	+	1011.25±33.53	+	906.68±34.32	+	1011.17±36.56	+	**830.08±24.51**
DTSP6	1377.33±36.28	+	1322.72±26.60	+	1327.63±52.65	+	1336.04±32.67	+	**1143.03±63.28**
+/−/≈	6/0/0		6/0/0		6/0/0		6/0/0		

TABLE S-VII

AOFV VALUES PROVIDED BY MISACO-LSBT-RAND, MISACO-RBF-RAND, MISACO-LSBT-RBF, AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO AND EACH OF MISACO-LSBT-RAND, MISACO-RBF-RAND, AND MISACO-LSBT-RBF.

	Problem	MiSACO-LSBT-Rand *AOFV* ± *Std Dev*		MiSACO-RBF-Rand *AOFV* ± *Std Dev*		MiSACO-LSBT-RBF *AOFV* ± *Std Dev*		MiSACO *AOFV* ± *Std Dev*
Type 1	F1	6.31E−08 ± 3.55E−08	≈	**4.59E−08 ± 2.82E−08**	≈	5.30E−08 ± 1.68E−08	≈	6.21E−08 ± 2.24E−08
	F2	2.83E+01 ± 9.93E+00	≈	2.91E+01 ± 1.71E+01	≈	3.29E+01 ± 9.83E+00	≈	**2.59E+01 ± 1.19E+01**
	F3	4.53E−01 ± 1.12E+00	+	**2.38E−01 ± 7.61E−01**	≈	7.13E−01 ± 1.19E+00	≈	3.04E−01 ± 7.50E−01
	F4	7.10E−06 ± 3.17E−05	+	1.36E−08 ± 4.64E−08	≈	**2.49E−09 ± 1.68E−09**	≈	4.51E−09 ± 2.25E−09
	F5	3.63E−01 ± 2.82E−01	≈	**2.24E−01 ± 2.51E−01**	≈	2.49E−01 ± 2.52E−01	≈	2.39E−01 ± 2.30E−01
	F6	9.72E−08 ± 8.21E−08	+	3.95E+00 ± 1.64E+01	+	7.49E−08 ± 2.79E−08	≈	**5.74E−08 ± 2.50E−08**
	F7	2.82E+01 ± 1.13E+01	≈	2.84E+01± 1.33E+01	≈	2.77E+01 ± 1.17E+01	≈	**2.40E+01 ± 1.22E+01**
	F8	8.68E−01 ± 1.36E+00	+	1.68E−01 ± 5.09E−01	≈	3.21E−01 ± 8.36E−01	+	**7.27E−02 ± 2.83E−01**
	F9	2.85E−02 ± 8.02E−02	+	1.69E−01 ± 6.07E−01	+	1.34E−01 ± 6.00E−01	+	**1.41E−06 ± 6.15E−06**
	F10	2.99E−01 ± 3.33E−01	≈	3.00E−01 ± 3.20E−01	≈	5.84E−01 ± 1.68E+00	≈	**2.75E−01 ± 3.01E−01**
Type 2	F11	6.10E−01 ± 1.73E+00	≈	2.77E+00 ± 1.24E+01	+	6.95E+01 ± 1.64E+02	+	**9.43E−08 ± 1.06E−07**
	F12	4.82E+01 ± 1.12E+01	≈	5.48E+01 ± 2.11E+01	+	4.85E+01 ± 6.29E+00	≈	**4.71E+01 ± 1.73E+01**
	F13	**1.85E−04 ± 7.55E−05**	≈	1.14E+00 ± 1.88E+00	≈	2.28E−01 ± 1.01E+00	≈	2.26E−04 ± 1.17E−04
	F14	2.83E−03 ± 7.68E−03	≈	2.15E−01 ± 4.17E−01	+	**1.36E−09 ± 1.97E−09**	≈	2.04E−09 ± 2.03E−09
	F15	7.94E−02 ± 2.43E−01	+	2.81E−01 ± 7.08E−01	≈	5.73E−07 ± 6.90E−07	≈	**2.82E−07 ± 3.27E−07**
	F16	2.36E+01 ± 8.55E+01	≈	1.46E+02 ± 2.24E+02	+	1.84E+02 ± 5.86E+02	+	**1.27E+00 ± 5.66E+00**
	F17	5.35E+01 ± 1.23E+01	≈	5.15E+01 ± 1.40E+01	≈	5.01E+01 ± 9.29E+00	≈	**4.50E+01 ± 1.61E+01**
	F18	**8.43E−01 ± 1.42E+00**	≈	3.38E+00 ± 2.15E+00	+	3.17E+00 ± 3.48E+00	≈	1.55E+00 ± 1.96E+00
	F19	**1.86E−01 ± 3.10E−01**	≈	1.53E+00 ± 1.58E+00	+	1.36E+00 ± 3.36E+00	+	2.71E−01 ± 7.16E−01
	F20	3.27E−01 ± 4.03E−01	+	3.24E+00 ± 3.61E+00	+	3.81E+00 ± 1.22E+01	+	**1.05E−01 ± 3.23E−01**
Type 3	F21	2.86E−01 ± 1.26E+00	≈	3.95E+00 ± 1.64E+01	+	2.24E+01 ± 1.00E+02	+	**7.60E−08 ± 5.64E−08**
	F22	4.81E+01 ± 1.27E+01	≈	5.08E+01 ± 1.67E+01	≈	5.11E+01 ± 1.62E+01	≈	**4.78E+01 ± 1.32E+01**
	F23	1.65E−01 ± 6.52E−02	+	1.86E−01 ± 8.31E−01	+	5.23E−02 ± 2.33E−01	+	**2.38E−04 ± 8.58E−05**
	F24	5.01E−04 ± 2.24E−03	+	1.04E−01 ± 3.21E−01	+	1.39E−09 ± 7.64E−10	≈	**1.18E−09 ± 4.54E−10**
	F25	3.19E−01 ± 3.33E−01	+	3.43E−01 ± 3.77E−01	≈	1.51E−01 ± 2.75E−01	+	**1.24E−01 ± 2.34E−01**
	F26	9.85E−01 ± 4.40E+00	≈	2.72E+01 ± 7.85E+01	+	6.27E+01 ± 1.57E+02	+	**9.70E−08 ± 8.28E−08**
	F27	4.34E+01 ± 1.63E+01	≈	4.39E+01 ± 1.15E+01	≈	4.30E+01 ± 1.48E+01	≈	**4.15E+01 ± 1.78E+01**
	F28	1.96E+00 ± 1.93E+00	≈	1.49E+00 ± 1.67E+00	≈	1.54E+00 ± 1.56E+00	≈	**9.76E−01 ± 1.46E+00**
	F29	1.20E−01 ± 2.36E−01	+	4.98E−01 ± 9.01E−01	+	9.58E−01 ± 2.96E+00	≈	**2.19E−05± 9.28E−05**
	F30	6.69E−01 ± 3.49E−01	+	8.20E−01 ± 7.84E−01	≈	5.30E−01 ± 3.74E−01	≈	**4.92E−01 ± 6.18E−01**
+/−/≈		12/0/18		14/0/16		10/0/20		

TABLE S-VIII

ASFEs VALUES PROVIDED BY MISACO-LSBT-RAND, MISACO-RBF-RAND, MISACO-LSBT-RBF, AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO AND EACH OF MISACO-LSBT-RAND, MISACO-RBF-RAND, AND MISACO-LSBT-RBF.

	Problem	MiSACO-LSBT-Rand		MiSACO-RBF-Rand		MiSACO-LSBT-RBF		MiSACO
		ASFEs ± *Std Dev*		*ASFEs* ± *Std Dev*		*ASFEs* ± *Std Dev*		*ASFEs* ± *Std Dev*
Type 1	F1	289.45 ± 51.18	+	**217.30 ± 75.74**	≈	221.60 ± 30.33	≈	249.95 ± 36.49
	F2	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	600.00 ± 0.00	≈	**600.00 ± 0.00**
	F3	362.85 ± 107.46	≈	**350.25 ± 97.75**	≈	400.70 ± 136.77	+	355.85 ± 101.67
	F4	223.00 ± 65.80	+	168.50 ± 65.50	≈	**150.25 ± 24.40**	≈	159.10 ± 25.60
	F5	291.85 ± 84.61	+	227.45 ± 60.24	≈	215.40 ± 47.74	≈	**208.90 ± 37.03**
	F6	368.25 ± 75.77	+	303.00 ± 139.55	≈	273.65 ± 84.48	≈	**269.40 ± 56.18**
	F7	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F8	419.55 ± 133.02	≈	380.65 ± 89.38	≈	**368.35 ± 117.16**	≈	391.60 ± 63.22
	F9	260.55 ± 54.94	+	236.80 ± 146.13	≈	**212.25 ± 103.96**	≈	231.15 ± 65.29
	F10	342.05 ± 77.51	+	327.55 ± 128.06	≈	282.90 ± 114.52	≈	**270.20 ± 123.87**
Type 2	F11	467.35 ± 77.98	+	371.20 ± 121.88	+	352.80 ± 159.45	+	**285.75 ± 73.21**
	F12	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F13	**293.45 ± 84.69**	≈	438.20 ± 128.56	+	307.95 ± 113.14	≈	307.25 ± 120.94
	F14	304.75 ± 53.62	+	314.30 ± 141.19	+	201.00 ± 71.51	≈	**195.45 ± 39.24**
	F15	413.40 ± 56.67	+	380.00 ± 146.59	+	246.50 ± 57.68	≈	**244.50 ± 44.50**
	F16	510.05 ± 78.44	+	538.90 ± 96.12	+	464.55 ± 147.50	+	**409.65 ± 100.36**
	F17	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F18	**432.75 ± 137.72**	≈	586.90 ± 29.26	≈	522.60 ± 131.29	≈	538.25 ± 93.69
	F19	399.35 ± 65.53	+	509.40 ± 135.33	+	366.25 ± 156.84	≈	**356.45 ± 122.27**
	F20	484.45 ± 69.50	≈	556.15 ± 83.22	+	410.15 ± 142.85	≈	**401.70 ± 109.14**
Type 3	F21	439.95 ± 79.69	+	350.25 ± 127.82	≈	290.25 ± 121.50	≈	**279.55 ± 36.60**
	F22	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	600.00 ± 0.00	≈	**600.00 ± 0.00**
	F23	380.35 ± 42.08	+	373.00 ± 73.12	≈	379.65 ± 82.56	≈	**363.15 ± 76.32**
	F24	328.05 ± 54.58	+	265.80 ± 132.05	+	194.65 ± 46.44	≈	**185.15 ± 37.79**
	F25	373.80 ± 54.57	+	309.60 ± 75.25	+	**222.90 ± 74.75**	≈	229.45 ± 32.75
	F26	458.85 ± 80.85	+	452.85 ± 113.08	+	351.35 ± 153.04	≈	**330.80 ± 62.07**
	F27	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F28	518.25 ± 106.08	≈	520.15 ± 98.98	≈	518.60 ± 118.47	+	**475.15 ± 117.83**
	F29	405.20 ± 66.87	+	361.35 ± 148.74	≈	304.05 ± 153.11	≈	**273.10 ± 83.76**
	F30	479.40 ± 79.15	+	453.95 ± 121.65	+	375.05 ± 148.82	+	**353.50 ± 118.11**
+/−/≈		18/0/12		11/0/19		5/0/25		

TABLE S-IX
RESULTS PROVIDED BY MISACO-WOLOCAL AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO AND MISACO-WOLOCAL.

	Problem	MiSACO-WoLocal *AOFV ± Std Dev*		MiSACO *AOFV ± Std Dev*	MiSACO-WoLocal *ASFEs ± Std Dev*		MiSACO *ASFEs ± Std Dev*
Type 1	F1	1.84E−02 ± 1.18E−02	+	**6.21E−08 ± 2.24E−08**	319.40 ± 21.71	+	**249.95 ± 36.49**
	F2	5.57E+01 ± 8.86E+00	+	**2.59E+01 ± 1.19E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F3	8.09E−01 ± 4.02E−01	+	**3.04E−01 ± 7.50E−01**	563.50 ± 41.66	+	**355.85 ± 101.67**
	F4	2.98E−04 ± 3.12E−04	+	**4.51E−09 ± 2.25E−09**	177.35 ± 23.13	≈	**159.10 ± 25.60**
	F5	3.57E−01 ± 1.40E−01	≈	**2.39E−01 ± 2.30E−01**	286.15 ± 26.90	+	**208.90 ± 37.03**
	F6	1.38E−02 ± 9.91E−03	+	**5.74E−08 ± 2.50E−08**	320.45 ± 21.41	+	**269.40 ± 56.18**
	F7	5.47E+01 ± 5.41E+00	+	**2.40E+01 ± 1.22E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F8	9.64E−01 ± 3.63E−01	+	**7.27E−02 ± 2.83E−01**	576.15 ± 28.62	+	**391.60 ± 63.22**
	F9	6.81E−04 ± 5.08E−04	+	**1.41E−06 ± 6.15E−06**	247.20 ± 29.33	≈	**231.15 ± 65.29**
	F10	3.00E−01 ± 1.79E−01	≈	**2.75E−01 ± 3.01E−01**	305.30 ± 40.98	≈	**270.20 ± 123.87**
Type 2	F11	2.93E−04 ± 3.80E−04	+	**9.43E−08 ± 1.06E−07**	287.50 ± 75.80	≈	**285.75 ± 73.21**
	F12	4.82E+01 ± 1.99E+01	≈	**4.71E+01 ± 1.73E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F13	1.23E−02 ± 9.72E−03	+	**2.26E−04 ± 1.17E−04**	316.55 ± 63.82	≈	**307.25 ± 120.94**
	F14	3.72E−02 ± 1.66E−01	+	**2.04E−09 ± 2.03E−09**	226.15 ± 58.14	≈	**195.45 ± 39.24**
	F15	2.39E−02 ± 1.95E−02	+	**2.82E−07 ± 3.27E−07**	261.10 ± 50.52	≈	**244.50 ± 44.50**
	F16	5.92E+00 ± 1.88E+01	+	**1.27E+00 ± 5.66E+00**	**396.30 ± 129.11**	≈	409.65 ± 100.36
	F17	5.02E+01 ± 1.73E+01	≈	**4.50E+01 ± 1.61E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F18	2.30E+00 ± 1.91E+00	≈	**1.55E+00 ± 1.96E+00**	554.20 ± 93.19	+	**538.25 ± 93.69**
	F19	2.93E−01 ± 5.50E−01	+	**2.71E−01 ± 7.16E−01**	366.65 ± 135.86	≈	**356.45 ± 122.27**
	F20	1.95E−01 ± 4.06E−01	+	**1.05E−01 ± 3.23E−01**	405.70 ± 109.40	≈	**401.70 ± 109.14**
Type 3	F21	2.53E−03 ± 2.49E−03	+	**7.60E−08 ± 5.64E−08**	294.40 ± 40.80	≈	**279.55** ± 36.60
	F22	4.99E+01 ± 1.20E+01	+	**4.78E+01 ± 1.32E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F23	2.77E−01 ± 1.63E−01	≈	**2.38E−04 ± 8.58E−05**	459.10 ± 49.46	+	**363.15 ± 76.32**
	F24	1.78E−04 ± 1.46E−04	+	**1.18E−09 ± 4.54E−10**	199.80 ± 35.88	≈	**185.15 ± 37.79**
	F25	**6.21E−02 ± 5.41E−02**	≈	1.24E−01 ± 2.34E−01	254.00 ± 22.53	+	**229.45 ± 32.75**
	F26	2.35E−03 ± 2.34E−03	+	**9.70E−08 ± 8.28E−08**	333.25 ± 64.27	≈	**330.80** ± 62.07
	F27	4.81E+01 ± 1.46E+01	≈	**4.15E+01 ± 1.78E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F28	9.89E−01 ± 9.57E−01	≈	**9.76E−01 ± 1.46E+00**	528.65 ± 63.43	+	**475.15 ± 117.83**
	F29	1.12E−03 ± 1.09E−03	+	**2.19E−05 ± 9.28E−05**	278.40 ± 56.19	≈	**273.10 ± 83.76**
	F30	**1.62E−01 ± 1.49E−01**	≈	4.92E−01 ± 6.18E−01	369.75 ± 80.04	≈	**353.50 ± 118.11**
+/−/≈		20/0/10			9/0/21		

TABLE S-X
RESULTS PROVIDED BY MISACO-KRIGING AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO-KRIGING AND MISACO.

	Problem	MiSACO-Kriging *AOFV ± Std Dev*		MiSACO *AOFV ± Std Dev*	MiSACO-Kriging *ASFEs ± Std Dev*		MiSACO *ASFEs ± Std Dev*
Type 1	F1	6.72E-08±2.21E-08	≈	**6.21E-08±2.24E-08**	288.40 ±58.96	≈	**249.95 ±36.49**
	F2	**2.44E+01±7.32E+00**	≈	2.59E+01±1.19E+01	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F3	4.38E-01±3.09E-01	≈	**3.04E-01±7.50E-01**	**313.40 ±145.04**	−	375.85 ±101.67
	F4	1.53E-08±1.56E-08	≈	**4.51E-09±2.25E-09**	197.40 ±32.22	≈	**159.10 ±25.60**
	F5	2.62E-01±9.93E-02	≈	**2.39E-01±2.30E-01**	274.60 ±70.70	+	**208.90 ±37.03**
	F6	7.21E-08±1.27E-08	≈	**5.74E-08±2.50E-08**	305.60 ±51.49	≈	**269.40 ±56.18**
	F7	**2.35E+01±4.97E+00**	≈	2.40E+01±1.22E+01	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F8	9.54E-01±2.00E+00	≈	**7.27E-02±2.83E-01**	418.40 ±152.48	≈	**391.60 ±63.22**
	F9	**1.85E-03±2.22E-03**	≈	2.59E-03±8.51E-03	**201.20 ±53.13**	≈	231.15 ±65.29
	F10	**1.67E-01±1.24E-01**	≈	2.75E-01±3.01E-01	**262.20 ±53.45**	≈	270.20 ±123.87
Type 2	F11	**8.68E-08±5.66E-08**	≈	9.43E-08±1.06E-07	315.60 ±46.72	≈	**285.75 ±73.21**
	F12	4.86E+01±1.19E+01	≈	**4.71E+01±1.73E+01**	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F13	3.22E-01±2.83E-05	≈	**3.19E-01±1.29E+00**	**257.20 ±76.04**	−	307.25 ±120.94
	F14	**1.10E-09±9.15E-10**	≈	2.04E-09±2.03E-09	229.00 ±75.72	≈	**195.45 ±39.24**
	F15	1.26E-01±2.82E-01	≈	**2.82E-07±3.27E-07**	279.00 ±45.29	≈	**244.50 ±44.50**
	F16	**9.86E-01±8.69E-08**	≈	1.27E+00±5.66E+00	411.40 ±79.02	≈	**409.65 ±100.36**
	F17	6.01E+01±6.82E+00	≈	**4.50E+01±1.61E+01**	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F18	**1.41E+00±2.00E+00**	≈	1.55E+00±1.96E+00	**503.20 ±171.32**	≈	538.25 ±93.69
	F19	3.53E-01±6.79E-08	≈	**2.71E-01±7.16E-01**	399.00 ±80.64	≈	**356.45 ±122.27**
	F20	1.86E-01±4.05E-01	≈	**1.05E-01±3.23E-01**	**361.60 ±96.96**	≈	401.70 ±109.00
Type 3	F21	8.13E-08±5.73E-08	≈	**7.60E-08±5.64E-08**	346.00 ±39.24	+	**279.55 ±36.60**
	F22	**3.42E+01±6.33E+00**	≈	4.78E+01±1.32E+01	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F23	5.31E-01±1.19E+00	≈	**1.52E-01±6.77E-01**	367.60 ±152.87	≈	**363.15 ±76.32**
	F24	**1.33E-09±1.07E-09**	≈	9.90E-07±4.42E-06	276.40 ±28.97	+	**185.15 ±37.79**
	F25	1.90E-01±2.75E-01	≈	**1.24E-01±2.34E-01**	244.40 ±40.46	≈	**229.45 ±32.75**
	F26	**6.63E-08±3.52E-08**	≈	9.70E-08±8.28E-08	391.00 ±50.31	≈	**330.80 ±62.07**
	F27	4.33E+01±1.05E+01	≈	**4.15E+01±1.78E+01**	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F28	**1.75E-02±3.84E-02**	≈	9.76E-01±1.46E+00	**345.00 ±141.24**	−	475.15 ±117.83
	F29	**1.60E-03±3.58E-03**	≈	3.74E-02±1.44E-01	322.20 ±71.29	≈	**273.10 ±83.76**
	F30	5.84E-01±8.77E-02	≈	**4.92E-01±6.18E-01**	354.20 ±62.60	≈	**353.50 ±118.11**
$+/-/\approx$		0/0/30			3/3/24		

TABLE S-XI
RUNTIME OF MISACO-KRIGING AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS.

	Type 1			Type 2			Type 3	
	Runtime (second)							
	MiSACO-Kriging	MiSACO		MiSACO-Kriging	MiSACO		MiSACO-Kriging	MiSACO
F1	200.78	102.18	F11	357.74	111.55	F21	211.83	107.96
F2	183.81	99.67	F12	338.41	111.41	F22	193.26	113.60
F3	178.98	105.40	F13	343.88	112.21	F23	216.68	102.01
F4	189.00	101.29	F14	374.83	111.74	F24	329.88	113.24
F5	190.74	99.18	F15	385.36	112.60	F25	335.04	109.60
F6	199.48	99.86	F16	227.33	118.48	F26	204.69	116.34
F7	179.61	97.36	F17	217.48	118.14	F27	189.81	118.49
F8	181.95	101.80	F18	211.56	117.07	F28	208.51	110.04
F9	188.57	101.99	F19	224.43	119.36	F29	201.70	118.56
F10	189.65	102.20	F20	220.73	116.61	F30	200.65	112.46

TABLE S-XII
RESULTS PROVIDED BY MISACO-RF AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO-RF AND MISACO.

	Problem	MiSACO-RF		MiSACO	MiSACO-RF		MiSACO
		AOFV ± Std Dev		*AOFV ± Std Dev*	*ASFEs ± Std Dev*		*ASFEs ± Std Dev*
Type 1	F1	3.26E-06±1.34E-05	+	**6.21E-08±2.24E-08**	342.35 ±61.50	+	**249.95 ±36.49**
	F2	2.64E+01±9.70E+00	≈	**2.59E+01±1.19E+01**	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F3	4.02E-01±8.71E-01	+	**3.04E-01±7.50E-01**	403.80 ±105.87	≈	**375.85 ±101.67**
	F4	4.37E-05±1.30E-04	+	**4.51E-09±2.25E-09**	241.45 ±80.73	+	**159.10 ±25.60**
	F5	**2.28E-01±2.18E-01**	≈	2.39E-01±2.30E-01	309.45 ±83.45	+	**208.90 ±37.03**
	F6	7.03E-08±7.51E+01	+	**5.74E-08±2.50E-08**	310.95 ±53.85	+	**269.40 ±56.18**
	F7	2.55E+01±1.41E+01	≈	**2.40E+01±1.22E+01**	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F8	1.06E+00±1.92E+00	+	**7.27E-02±2.83E-01**	468.05 ±137.25	+	**391.60 ±63.22**
	F9	6.17E-02±8.55E-01	+	**2.59E-03±8.51E-03**	251.05 ±107.95	≈	**231.15 ±65.29**
	F10	3.18E-01±6.15E-01	+	**2.75E-01±3.01E-01**	320.55 ±84.50	+	**270.20 ±123.87**
Type 2	F11	3.74E+01±1.48E+02	+	**9.43E-08±1.06E-07**	454.20 ±139.78	+	**285.75 ±73.21**
	F12	4.82E+01±1.56E+01	≈	**4.71E+01±1.73E+01**	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F13	9.25E-01±1.47E+00	+	**3.19E-01±1.29E+00**	445.50 ±145.48	+	**307.25 ±120.94**
	F14	2.24E-01±6.44E-01	+	**2.04E-09±2.03E-09**	297.25 ±150.56	+	**195.45 ±39.24**
	F15	6.73E-01±1.12E+00	+	**2.82E-07±3.27E-07**	443.35 ±143.74	+	**244.50 ±44.50**
	F16	9.92E+01±3.05E-08	+	**1.27E+00±5.66E+00**	548.70 ±71.57	+	**409.65 ±100.36**
	F17	5.48E+01±1.07E+01	≈	**4.50E+01±1.61E+01**	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F18	1.99E+00±1.21E+00	+	**1.55E+00±1.96E+00**	546.60 ±83.82	≈	**538.25 ±93.69**
	F19	7.98E-01±2.63E-01	+	**2.71E-01±7.16E-01**	476.55 ±135.21	+	**356.45 ±122.27**
	F20	1.69E+00±2.36E-01	+	**1.05E-01±3.23E-01**	542.80 ±83.76	+	**401.70 ±109.00**
Type 3	F21	4.08E+00±1.53E+01	+	**7.60E-08±5.64E-08**	493.20 ±92.32	+	**279.55 ±36.60**
	F22	**3.92E+01±1.06E+01**	≈	4.78E+01±1.32E+01	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F23	1.06E+00±1.49E+00	+	**1.52E-01±6.77E-01**	451.60 ±118.53	+	**363.15 ±76.32**
	F24	1.97E-01±3.74E-01	+	**9.90E-07±4.42E-06**	363.00 ±118.42	+	**185.15 ±37.79**
	F25	5.62E-01±3.73E-01	+	**1.24E-01±2.34E-01**	376.75 ±121.34	+	**229.45 ±32.75**
	F26	2.23E+01±1.56E+02	+	**9.70E-08±8.28E-08**	432.85 ±136.29	+	**330.80 ±62.07**
	F27	**3.40E+01±1.25E+01**	≈	4.15E+01±1.78E+01	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F28	1.68E+00±2.02E+00	+	**9.76E-01±1.46E+00**	475.85 ±125.74	≈	**475.15 ±117.83**
	F29	5.49E-01±9.06E-01	+	**3.74E-02±1.44E-01**	428.45 ±134.31	+	**273.10 ±83.76**
	F30	7.94E-01±2.57E+00	+	**4.92E-01±6.18E-01**	459.15 ±109.33	+	**353.50 ±118.11**
$+/-/\approx$		23/0/7			20/0/10		

TABLE S-XIII
RESULTS PROVIDED BY MISACO-GOWER AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO-GOWER AND MISACO.

	Problem	MiSACO-Gower		MiSACO	MiSACO-Gower		MiSACO
		AOFV ± Std Dev		*AOFV ± Std Dev*	*ASFEs ± Std Dev*		*ASFEs ± Std Dev*
Type 1	F1	1.16E-04±2.99E-04	+	**6.21E-08±2.24E-08**	382.90±110.05	+	**249.95±36.49**
	F2	3.16E+01±7.91E+00	≈	**2.59E+01±1.19E+01**	**600.00±0.00**	≈	**600.00±0.00**
	F3	**2.35E-01±4.92E-01**	≈	3.04E-01±7.50E-01	448.40±92.61	≈	**355.85±101.67**
	F4	5.97E-07±1.25E-06	+	**4.51E-09±2.25E-09**	233.70±46.96	+	**159.10±25.60**
	F5	3.40E-01±2.08E-01	≈	**2.39E-01±2.30E-01**	319.10±57.36	+	**208.90±37.03**
	F6	2.98E-07±6.00E-07	+	**5.74E-08±2.50E-08**	340.20±69.41	≈	**269.40±56.18**
	F7	2.42E+01±6.68E+00	≈	**2.40E+01±1.22E+01**	**600.00±0.00**	≈	**600.00±0.00**
	F8	**3.55E-03±3.97E-03**	−	7.27E-02±2.83E-01	**378.70±38.99**	≈	391.60±63.22
	F9	**2.00E-06±2.88E-06**	−	2.59E-03±8.51E-03	**227.20±38.02**	≈	231.15±65.29
	F10	4.22E-01±2.17E-01	≈	**2.75E-01±3.01E-01**	289.50±79.86	≈	**270.20±123.87**
Type 2	F11	1.19E+00±1.54E+00	+	**9.43E-08±1.06E-07**	504.30±107.42	+	**285.75±73.21**
	F12	5.03E+01±1.04E+01	≈	**4.71E+01±1.73E+01**	**600.00±0.00**	≈	**600.00±0.00**
	F13	**4.32E-02±1.36E-01**	−	4.19E-01±1.29E+00	316.10±64.65	≈	**307.25±120.94**
	F14	4.05E-02±5.60E-02	+	**2.04E-09±2.03E-09**	256.20±59.73	+	**195.45±39.24**
	F15	5.47E-01±4.35E-01	+	**2.82E-07±3.27E-07**	451.80±115.21	+	**244.50±44.50**
	F16	**1.06E+01±2.15E+01**	≈	1.27E+00±5.66E+00	575.50±28.21	+	**409.65±100.36**
	F17	5.39E+01±8.76E+00	≈	**4.50E+01±1.61E+01**	**600.00±0.00**	≈	**600.00±0.00**
	F18	**7.82E-01±1.81E+00**	−	1.55E+00±1.96E+00	**455.70±112.13**	−	538.25±93.69
	F19	**2.31E-01±3.31E-01**	≈	2.71E-01±7.16E-01	415.40±61.19	≈	**356.45±122.27**
	F20	1.08E+00±3.42E-01	+	**1.05E-01±3.23E-01**	570.80±46.20	+	**401.70±109.14**
Type 3	F21	5.65E-01±1.78E+00	+	**7.60E-08±5.64E-08**	536.50±41.89	+	**279.55±36.60**
	F22	5.24E+01±1.22E+01	≈	**4.78E+01±1.32E+01**	**600.00±0.00**	≈	**600.00±0.00**
	F23	6.39E-04±5.32E-04	≈	**2.38E-04±8.58E-05**	429.80±36.47	+	**363.15±76.32**
	F24	2.07E-02±3.27E-02	+	**1.18E-09±4.54E-10**	381.60±91.09	+	**185.15±37.79**
	F25	3.36E-01±2.59E-01	+	**1.24E-01±2.34E-01**	471.30±79.99	+	**229.45±32.75**
	F26	2.83E+00±7.16E+00	+	**9.70E-08±8.28E-08**	511.20±80.14	+	**330.80±62.07**
	F27	4.67E+01±9.75E+00	≈	**4.15E+01±1.78E+01**	**600.00±0.00**	≈	**600.00±0.00**
	F28	**9.55E-01±1.38E+00**	≈	9.76E-01±1.46E+00	526.70±76.02	≈	**475.15±117.83**
	F29	1.83E-01±1.46E-01	+	**3.74E-02±1.44E-01**	449.90±67.93	+	**273.10±83.76**
	F30	5.48E-01±3.94E-01	≈	**4.92E-01±6.18E-01**	491.60±77.34	+	**353.50±118.11**
+/ − / ≈		12/4/14			15/1/14		

TABLE S-XIV
RESULTS OF MISACO WITH VARYING N_{min} ON THE SIX SELECTED ARTIFICIAL TEST PROBLEMS.

N_{min}	$1*n_1$	$5*n_1$	$10*n_1$	$20*n_1$
	AOFV ± Std Dev			
F1	1.06E-07±4.49E-08	**6.21E-08±2.24E-08**	3.26E+00±5.38E+00	3.79E+00±2.89E+00
F6	1.49E-07±1.71E-07	**5.74E-08±2.50E-08**	3.79E+00±4.60E+00	8.05E+00±4.48E+00
F13	7.77E-01±1.52E-04	3.19E-01±1.29E+00	**2.48E-01±7.63E-01**	4.09E-02±3.71E-02
F18	2.65E+00±8.64E-01	1.55E+00±1.96E+00	**4.06E-01±3.28E-01**	7.61E-01±3.81E-01
F24	1.32E-06±1.92E-06	**9.90E-07±4.42E-06**	1.07E-02±2.44E-02	2.11E-02±3.19E-02
F29	9.30E-02±1.88E-01	**3.74E-02±1.44E-01**	2.25E-01±3.62E-01	2.94E-01±2.49E-01
	ASFEs ± Std Dev			
F1	311.40 ±52.30	**249.95 ±36.49**	567.40 ±33.05	588.10 ±35.91
F6	371.90 ±64.31	**269.40 ±56.18**	592.20 ±9.82	599.10 ±2.85
F13	305.90 ±50.10	**307.25 ±120.94**	465.80 ±59.35	510.90 ±39.68
F18	536.25 ±124.58	538.25 ±93.69	**483.50 ±61.50**	567.00 ±40.36
F24	272.10 ±80.85	**185.15 ±37.79**	268.20 ±58.16	288.50 ±53.52
F29	347.90 ±83.13	**273.10 ±83.76**	371.90 ±132.75	434.20 ±59.75

S-IV. Test Problems

A. The Constructed Artificial Test Problems

The characteristics of the constructed artificial test problems are summarized in the Table S-XV.

TABLE S-XV
CHARACTERISTICS OF THE 30 ARTIFICIAL TEST PROBLEMS

	Problem	n_1	n_2	l_j	L_i^{cn}	U_i^{cn}	Basic Function
Type 1	F1	8	2	5	-100	100	Sphere Function
	F2	8	2	5	-100	100	Rastrigin Function
	F3	8	2	5	-100	100	Alckey Function
	F4	8	2	5	-100	100	Elliposoid Function
	F5	8	2	5	-100	100	Griewank Function
	F6	8	2	10	-100	100	Sphere Function
	F7	8	2	10	-100	100	Rastrigin Function
	F8	8	2	10	-100	100	Alckey Function
	F9	8	2	10	-100	100	Elliposoid Function
	F10	8	2	10	-100	100	Griewank Function
Type 2	F11	2	8	5	-100	100	Sphere Function
	F12	2	8	5	-100	100	Rastrigin Function
	F13	2	8	5	-100	100	Alckey Function
	F14	2	8	5	-100	100	Elliposoid Function
	F15	2	8	5	-100	100	Griewank Function
	F16	2	8	10	-100	100	Sphere Function
	F17	2	8	10	-100	100	Rastrigin Function
	F18	2	8	10	-100	100	Alckey Function
	F19	2	8	10	-100	100	Elliposoid Function
	F20	2	8	10	-100	100	Griewank Function
Type 3	F21	5	5	5	-100	100	Sphere Function
	F22	5	5	5	-100	100	Rastrigin Function
	F23	5	5	5	-100	100	Alckey Function
	F24	5	5	5	-100	100	Elliposoid Function
	F25	5	5	5	-100	100	Griewank Function
	F26	5	5	10	-100	100	Sphere Function
	F27	5	5	10	-100	100	Rastrigin Function
	F28	5	5	10	-100	100	Alckey Function
	F29	5	5	10	-100	100	Elliposoid Function
	F30	5	5	10	-100	100	Griewank Function

The constructed 30 artificial test problems are as follows:

$$
\begin{aligned}
\mathbf{F1}:\ min:\ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} z_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = \mathbf{z}'A \\
& z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 8\} \\ x_{i-8}^{ca} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734, 44.3837, 99.8131, -12.1793) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{99.8131, 38.7794, 97.4385, 66.3214, 83.6572\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{-12.1793, -81.4490, 94.5925, -20.7460, -23.4447\} \\
& x_1^{cn}, \ldots, x_8^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}_{best}^{cn} = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734,$

$44.3837)$ and $\mathbf{x}^{ca}_{best} = (99.8131, -12.1793)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F2}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} (z_i^2 - 10cos(2\pi z_i) + 10) \\
& \mathbf{z} = (z_1, \dots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \dots, 8\} \\ x^{ca}_{i-8} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \dots, o_{10}) = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346, 50.6776, 14.4813) \\
& \mathbf{v}_1 = \{v_1^1, \dots, v_1^5\} = \{50.6776, -39.6234, -61.7100, 97.7223, 63.1775\} \\
& \mathbf{v}_2 = \{v_2^1, \dots, v_2^5\} = \{14.4813, -97.4609, 92.2885, -3.8172, 83.2134\} \\
& x^{cn}_1, \dots, x^{cn}_8 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346)$ and $\mathbf{x}^{ca}_{best} = (50.6776, 14.4813)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F3}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 20 + e - 20\exp(-0.2\sqrt{\frac{1}{10}\sum_{i=1}^{10} z_i^2}) - \exp(\frac{1}{10}\sum_{i=1}^{10}\cos(2\pi z_i)) \\
& \mathbf{z} = (z_1, \dots, z_{10}) = 0.32 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \dots, 8\} \\ x^{ca}_{i-8} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \dots, o_{10}) = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349, 1.9141, -36.7252) \\
& \mathbf{v}_1 = \{v_1^1, \dots, v_1^5\} = \{1.9141, -12.6618, -3.5678, -18.1508, -7.8900\} \\
& \mathbf{v}_2 = \{v_2^1, \dots, v_2^5\} = \{-36.7252, 27.8390, -53.1350, -21.6579, -27.2089\} \\
& x^{cn}_1, \dots, x^{cn}_8 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349)$ and $\mathbf{x}^{ca}_{best} = (1.9141, -36.7252)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F4}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} i x_i^2 \\
& \mathbf{z} = (z_1, \dots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \dots, 8\} \\ x^{ca}_{i-8} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \dots, o_{10}) = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035, 63.0211, -96.1546) \\
& \mathbf{v}_1 = \{v_1^1, \dots, v_1^5\} = \{63.0211, 15.4585, -0.9510, 90.1246, 18.4635\} \\
& \mathbf{v}_2 = \{v_2^1, \dots, v_2^5\} = \{-96.1546, -3.5673, -59.5396, 13.2944, -21.4157\} \\
& x^{cn}_1, \dots, x^{cn}_8 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227,$

$, -22.2035)$ and $\mathbf{x}^{ca}_{best} = (63.0211, -96.1546)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F5}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 1 + \sum_{i=1}^{10} \frac{z_i^2}{4000} - \prod_{i=1}^{10} \cos(\frac{z_i}{\sqrt{i}}) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 6 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \ldots, 8\} \\ x^{ca}_{i-8} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6663, -39.7689, 39.6034, 52.0954) \\
& \mathbf{v}_1 = \{v^1_1, \ldots, v^5_1\} = \{39.6034, -98.0900, 66.9081, 5.6974, -52.6892\} \\
& \mathbf{v}_2 = \{v^1_2, \ldots, v^5_2\} = \{52.0954, -95.0943, 60.2757, -59.7487, -89.1344\} \\
& x^{cn}_1, \ldots, x^{cn}_5 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6663, -39.7689)$ and $\mathbf{x}^{ca}_{best} = (39.6034, 52.0954)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F6}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} z_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \ldots, 8\} \\ x^{ca}_{i-8} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734, 44.3837, 99.8131, -12.1793) \\
& \mathbf{v}_1 = \{v^1_1, \ldots, v^{10}_1\} = \{99.8131, 38.7794, 97.4385, 66.3214, 83.6572, 64.3900, 6.2714, 4.7893, 42.9978, -46.2021\} \\
& \mathbf{v}_2 = \{v^1_2, \ldots, v^{10}_2\} = \{-12.1793, -81.4490, 94.5925, -20.7460, -23.4447, -37.0443, -33.7724, -78.8025, 69.8750, \\
& \qquad 70.8563\} \\
& x^{cn}_1, \ldots, x^{cn}_8 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734, 44.3837)$ and $\mathbf{x}^{ca}_{best} = (99.8131, -12.1793)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F7}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} (z_i^2 - 10cos(2\pi z_i) + 10) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \ldots, 8\} \\ x^{ca}_{i-8} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346, 50.6776, 14.4813) \\
& \mathbf{v}_1 = \{v^1_1, \ldots, v^{10}_1\} = \{50.6776, -39.6234, -61.7100, 97.7223, 63.1775, -12.0348, -21.6271, -12.3744, 67.1491, \\
& \qquad -19.0775\} \\
& \mathbf{v}_2 = \{v^1_2, \ldots, v^{10}_2\} = \{14.4813, -97.4609, 92.2885, -3.8172, 83.2134, -89.4358, 10.1637, -86.6364, -64.1289, 6.0189\} \\
& x^{cn}_1, \ldots, x^{cn}_8 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558,$

$-77.1346)$ and $\mathbf{x}^{ca}_{best} = (50.6776, 14.4813)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F8}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 20 + e - 20\exp(-0.2\sqrt{\frac{1}{10}\sum_{i=1}^{10} z_i^2}) - \exp(\frac{1}{10}\sum_{i=1}^{10}\cos(2\pi z_i)) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.32 * \mathbf{z}'A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \ldots, 8\} \\ x^{ca}_{i-8} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349, 1.9141, -36.7252) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{1.9141, -12.6618, -3.5678, -18.1508, -7.8900, 1.7955, -45.1022, -14.7915, -47.5095, \\
& \qquad 57.2121\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-36.7252, 27.8390, -53.1350, -21.6579, -27.2089, -58.0376, 19.1243, 2.8412, -17.4512, \\
& \qquad -58.3012\} \\
& x^{cn}_1, \ldots, x^{cn}_8 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349)$ and $\mathbf{x}^{ca}_{best} = (1.9141, -36.7252)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F9}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} i x_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}'A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \ldots, 8\} \\ x^{ca}_{i-8} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035, 63.0211, -96.1546) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{63.0211, 15.4585, -0.9510, 90.1246, 18.4635, -17.3490, 96.5306, -14.2523, -37.9036, 58.6272\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-96.1546, -3.5673, -59.5396, 13.2944, -21.4157, 25.7777, 0.5632, 73.3501, -29.0539, -79.8143\} \\
& x^{cn}_1, \ldots, x^{cn}_8 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035)$ and $\mathbf{x}^{ca}_{best} = (63.0211, -96.1546)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F10}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 1 + \sum_{i=1}^{10} \frac{z_i^2}{4000} - \prod_{i=1}^{10} \cos(\frac{z_i}{\sqrt{i}}) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 6 * \mathbf{z}'A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \ldots, 8\} \\ x^{ca}_{i-8} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6663, -39.7689, 39.6034, 52.0954) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{39.6034, -98.0900, 66.9081, 5.6974, -52.6892, -22.8678, -21.4237, -70.5337, 8.6142, -89.3348\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{52.0954, -95.0943, 60.2757, -59.7487, -89.1344, 70.3986, -55.3637, 17.6185, -72.3865, \\
& \qquad -10.6520\} \\
& x^{cn}_1, \ldots, x^{cn}_8 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6663,$

$-39.7689)$ and $\mathbf{x}^{ca}_{best} = (39.6034, 52.0954)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$\mathbf{F11}: \ min: \ f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} z_i^2$$
$$\mathbf{z} = (z_1, \ldots, z_{10}) = \mathbf{z}'A$$
$$z'_i = \begin{cases} x_i^{cn} - o_i, & i \in \{1,2\} \\ x_{i-2}^{ca} - o_i, & i \in \{3, \ldots, 10\} \end{cases}$$
$$\mathbf{o} = (o_1, \ldots, o_{10}) = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734, 44.3837, 99.8131, -12.1793)$$
$$\mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{-95.5110, 10.9166, -86.3500, 6.3552, -52.8390\}$$
$$\mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{-68.7425, 2.4009, -26.8628, 52.9171, -94.4758\}$$
$$\mathbf{v}_3 = \{v_3^1, \ldots, v_3^5\} = \{8.7344, 2.0220, 1.2974, -37.0691, -79.2651\}$$
$$\mathbf{v}_4 = \{v_4^1, \ldots, v_4^5\} = \{0.0577, -66.8891, -24.5506, -96.2061, 45.4579\}$$
$$\mathbf{v}_5 = \{v_5^1, \ldots, v_5^5\} = \{-36.7734, 11.1348, 40.9187, -32.3377, 62.3757\}$$
$$\mathbf{v}_6 = \{v_6^1, \ldots, v_6^5\} = \{44.3837, -84.2635, -31.8857, -99.0299, 23.2041\}$$
$$\mathbf{v}_7 = \{v_7^1, \ldots, v_7^5\} = \{99.8131, 38.7794, 97.4385, 66.3214, 83.6572\}$$
$$\mathbf{v}_8 = \{v_8^1, \ldots, v_8^5\} = \{-12.1793, -81.4490, 94.5925, -20.7460, -23.4447\}$$
$$x_1^{cn}, x_2^{cn} \in [-100, 100]$$
$$x_j^{ca} \in \mathbf{v}_j, j \in \{1,2,3,4,5,6,7,8\}$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (7.7624, -51.0984)$ and $\mathbf{x}^{ca}_{best} = (-95.5110, -68.7425, 8.7344, 0.0577, -36.7734, 44.3837, 99.8131, -12.1793)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$\mathbf{F12}: \ min: \ f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} (z_i^2 - 10cos(2\pi z_i) + 10)$$
$$\mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}'A$$
$$z'_i = \begin{cases} x_i^{cn} - o_i, & i \in \{1,2\} \\ x_{i-2}^{ca} - o_i, & i \in \{3, \ldots, 10\} \end{cases}$$
$$\mathbf{o} = (o_1, \ldots, o_{10}) = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346, 50.6776, 14.4813)$$
$$\mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{-97.8543, -98.8581, -93.8085, -78.8370, 81.9188\}$$
$$\mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{39.7223, -99.8416, -4.6454, 74.7200, -80.8983\}$$
$$\mathbf{v}_3 = \{v_3^1, \ldots, v_3^5\} = \{28.3686, -19.5009, -96.7178, 9.0181, 58.1322\}$$
$$\mathbf{v}_4 = \{v_4^1, \ldots, v_4^5\} = \{61.1286, 4.1286, -3.9445, -15.1705, -7.9428\}$$
$$\mathbf{v}_5 = \{v_5^1, \ldots, v_5^5\} = \{38.9558, 37.3101, 83.0975, -98.6905, -75.3426\}$$
$$\mathbf{v}_6 = \{v_6^1, \ldots, v_6^5\} = \{-77.1346, -2.0340, 70.8802, 51.3176, -24.3460\}$$
$$\mathbf{v}_7 = \{v_7^1, \ldots, v_7^5\} = \{50.6776, -39.6234, -61.7100, 97.7223, 63.1775\}$$
$$\mathbf{v}_8 = \{v_8^1, \ldots, v_8^5\} = \{14.4813, -97.4609, 92.2885, -3.8172, 83.2134\}$$
$$x_1^{cn}, x_2^{cn} \in [-100, 100]$$
$$x_j^{ca} \in \mathbf{v}_j, j \in \{1,2,3,4,5,6,7,8\}$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (0.5876, -84.9703)$ and $\mathbf{x}^{ca}_{best} = (-97.8543, 39.7223, 28.3686,$

$61.1286, 38.9558, -77.1346, 50.6776, 14.4813)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F13}: \; min: \;\; & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 20 + e - 20\exp(-0.2\sqrt{\frac{1}{10}\sum_{i=1}^{10} z_i^2}) - \exp(\frac{1}{10}\sum_{i=1}^{10}\cos(2\pi z_i)) \\
& \mathbf{z} = (z_1, \dots, z_{10}) = 0.32 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1,2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3, \dots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \dots, o_{10}) = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349, 1.9141, -36.7252) \\
& \mathbf{v}_1 = \{v_1^1, \dots, v_1^5\} = \{47.8659, -50.2789, -51.4218, -5.7807, 59.7290\} \\
& \mathbf{v}_2 = \{v_2^1, \dots, v_2^5\} = \{-17.0994, 17.6311, -52.2380, -37.5205, -50.8626\} \\
& \mathbf{v}_3 = \{v_3^1, \dots, v_3^5\} = \{-32.5756, 27.7492, -33.1525, 55.2097, -38.2367\} \\
& \mathbf{v}_4 = \{v_4^1, \dots, v_4^5\} = \{-29.2208, -14.6371, 31.1216, -14.0404, 23.6011\} \\
& \mathbf{v}_5 = \{v_5^1, \dots, v_5^5\} = \{-32.7262, 53.3853, -56.2406, 51.0570, 21.1575\} \\
& \mathbf{v}_6 = \{v_6^1, \dots, v_6^5\} = \{-43.5349, -59.4140, -49.4245, -9.3890, -56.9489\} \\
& \mathbf{v}_7 = \{v_7^1, \dots, v_7^5\} = \{1.9141, -12.6618, -3.5678, -18.1508, -7.8900\} \\
& \mathbf{v}_8 = \{v_8^1, \dots, v_8^5\} = \{-36.7252, 27.8390, -53.1350, -21.6579, -27.2089\} \\
& x^{cn}_1, x^{cn}_2 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1,2,3,4,5,6,7,8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (5.3830, 0.1560)$ and $\mathbf{x}^{ca}_{best} = (47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349, 1.9141, -36.7252)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F14}: \; min: \;\; & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} i x_i^2 \\
& \mathbf{z} = (z_1, \dots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1,2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3, \dots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \dots, o_{10}) = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035, 63.0211, -96.1546) \\
& \mathbf{v}_1 = \{v_1^1, \dots, v_1^5\} = \{99.8101, -31.6099, 89.6818, 56.8515, -88.4346\} \\
& \mathbf{v}_2 = \{v_2^1, \dots, v_2^5\} = \{-69.8675, 40.6365, -76.1113, 65.0137, -93.6473\} \\
& \mathbf{v}_3 = \{v_3^1, \dots, v_3^5\} = \{46.3398, 57.3869, -59.6658, 54.2175, -66.4293\} \\
& \mathbf{v}_4 = \{v_4^1, \dots, v_4^5\} = \{-94.7804, 90.0091, -12.3052, 27.1707, 89.5608\} \\
& \mathbf{v}_5 = \{v_5^1, \dots, v_5^5\} = \{-14.1227, 70.3685, -3.5821, -57.3306, -21.0553\} \\
& \mathbf{v}_6 = \{v_6^1, \dots, v_6^5\} = \{-22.2035, -94.6947, -28.3114, 79.0958, -32.9711\} \\
& \mathbf{v}_7 = \{v_7^1, \dots, v_7^5\} = \{63.0211, 15.4585, -0.9510, 90.1246, 18.4635\} \\
& \mathbf{v}_8 = \{v_8^1, \dots, v_8^5\} = \{-96.1546, -3.5673, -59.5396, 13.2944, -21.4157\} \\
& x^{cn}_1, x^{cn}_2 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1,2,3,4,5,6,7,8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (6.5706, -4.7415)$ and $\mathbf{x}^{ca}_{best} = (99.8101, -69.8675, 46.3398, -94.7804,$

$-14.1227, -22.2035, 63.0211, -96.1546)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F15}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 1 + \sum_{i=1}^{10} \frac{z_i^2}{4000} - \prod_{i=1}^{10} \cos(\frac{z_i}{\sqrt{i}}) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 6 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1,2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6664, -39.7689, 39.6034, 52.0954) \\
& \mathbf{v}_1 = \{v^1_1, \ldots, v^5_1\} = \{-12.0721, 2.9979, -19.0989, -67.9707, -5.3256\} \\
& \mathbf{v}_2 = \{v^1_2, \ldots, v^5_2\} = \{-10.4880, 49.2424, 64.1867, 5.7290, -95.5992\} \\
& \mathbf{v}_3 = \{v^1_3, \ldots, v^5_3\} = \{2.4412, 63.1647, -11.6669, -31.9233, -87.5237\} \\
& \mathbf{v}_4 = \{v^1_4, \ldots, v^5_4\} = \{-86.8010, 58.9903, -71.2887, 55.7150, 59.1220\} \\
& \mathbf{v}_5 = \{v^1_5, \ldots, v^5_5\} = \{-32.6664, 14.3942, 19.6051, 23.8658, 92.7302\} \\
& \mathbf{v}_6 = \{v^1_6, \ldots, v^5_6\} = \{-39.7689, 51.2260, 95.1572, -52.9366, 81.0941\} \\
& \mathbf{v}_7 = \{v^1_7, \ldots, v^5_7\} = \{39.6034, -98.0900, 66.9081, 5.6974, -52.6892\} \\
& \mathbf{v}_8 = \{v^1_8, \ldots, v^5_8\} = \{52.0954, -95.0943, 60.2757, -59.7487, -89.1344\} \\
& x^{cn}_1, x^{cn}_2 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1,2,3,4,5,6,7,8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (84.9982, 65.1519)$ and $\mathbf{x}^{ca}_{best} = (-12.0721, -10.4880, 2.4412, -86.8010, -32.6664, -39.7689, 39.6034, 52.0954)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F16}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} z_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1,2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734, 44.3837, 99.8131, -12.1793) \\
& \mathbf{v}_1 = \{v^1_1, \ldots, v^{10}_1\} = \{-95.5110, 10.9166, -86.3500, 6.3552, -52.8390, 30.5276, 77.9978, -14.5499, -53.7453, 93.4961\} \\
& \mathbf{v}_2 = \{v^1_2, \ldots, v^{10}_2\} = \{-68.7425, 2.4009, -26.8628, 52.9171, -94.4758, -19.8521, 18.5924, 16.7370, -83.7091, -33.1471\} \\
& \mathbf{v}_3 = \{v^1_3, \ldots, v^{10}_3\} = \{8.7344, 2.0220, 1.2974, -37.0691, -79.2651, -13.4857, 91.8744, 41.5887, 54.6449, -53.6206\} \\
& \mathbf{v}_4 = \{v^1_4, \ldots, v^{10}_4\} = \{0.0577, -66.8891, -24.5506, -96.2061, 45.4579, 18.1319, 58.3241, 71.9285, -75.8105, -22.9862\} \\
& \mathbf{v}_5 = \{v^1_5, \ldots, v^{10}_5\} = \{-36.7734, 11.1348, 40.9187, -32.3377, 62.3757, 24.4644, 86.2232, 90.6468, -99.4558, 89.3221\} \\
& \mathbf{v}_6 = \{v^1_6, \ldots, v^{10}_6\} = \{44.3837, -84.2635, -31.8857, -99.0299, 23.2041, 13.1996, -43.5760, 26.1334, -6.6750, -22.8134\} \\
& \mathbf{v}_7 = \{v^1_7, \ldots, v^{10}_7\} = \{99.8131, 38.7794, 97.4385, 66.3214, 83.6572, 64.3900, 6.2714, 4.7893, 42.9978, -46.2021\} \\
& \mathbf{v}_8 = \{v^1_8, \ldots, v^{10}_8\} = \{-12.1793, -81.4490, 94.5925, -20.7460, -23.4447, -37.0443, -33.7724, -78.8025, 69.8750, \\
& \qquad 70.8563\} \\
& x^{cn}_1, \ldots, x^{cn}_2 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1,2,3,4,5,6,7,8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (7.7624, -51.0984)$ and $\mathbf{x}^{ca}_{best} = (-95.5110, -68.7425, 8.7344,$

$0.0577, -36.7734, 44.3837, 99.8131, -12.1793)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F17}:\ min:\ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} (z_i^2 - 10cos(2\pi z_i) + 10) \\
& \mathbf{z} = (z_1, \dots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \dots, 2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3, \dots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \dots, o_{10}) = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346, 50.6776, 14.4813) \\
& \mathbf{v}_1 = \{v_1^1, \dots, v_1^{10}\} = \{-97.8543, -98.8581, -93.8085, -78.8370, 81.9188, -2.9181, -50.9363, 31.9732, 69.0271, 85.5965\} \\
& \mathbf{v}_2 = \{v_2^1, \dots, v_2^{10}\} = \{39.7223, -99.8416, -4.6454, 74.7200, -80.8983, -54.2256, 73.2056, -29.1147, -11.7113, 16.2659\} \\
& \mathbf{v}_3 = \{v_3^1, \dots, v_3^{10}\} = \{28.3686, -19.5009, -96.7178, 9.0181, 58.1322, -8.9950, 99.3704, 28.2749, 60.8476, -53.6360\} \\
& \mathbf{v}_4 = \{v_4^1, \dots, v_4^{10}\} = \{61.1286, 4.1286, -3.9445, -15.1705, -7.9428, -51.8438, 76.1710, -37.0110, -34.0411, -61.8862\} \\
& \mathbf{v}_5 = \{v_5^1, \dots, v_5^{10}\} = \{38.9558, 37.3101, 83.0975, -98.6905, -75.3426, 93.3490, 52.6871, -77.6949, 3.4488, -14.2671\} \\
& \mathbf{v}_6 = \{v_6^1, \dots, v_6^{10}\} = \{-77.1346, -2.0340, 70.8802, 51.3176, -24.3460, 63.8007, 93.8858, 11.8351, 57.5630, -0.9117\} \\
& \mathbf{v}_7 = \{v_7^1, \dots, v_7^{10}\} = \{50.6776, -39.6234, -61.7100, 97.7223, 63.1775, -12.0348, -21.6271, -12.3744, 67.1491, \\
& \qquad -19.0775\} \\
& \mathbf{v}_8 = \{v_8^1, \dots, v_8^{10}\} = (14.4813, -97.4609, 92.2885, -3.8172, 83.2134, -89.4358, 10.1637, -86.6364, -64.1289, 6.0189\} \\
& x_1^{cn}, \dots, x_2^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5, 6, 7, 8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (0.5876, -84.9703)$ and $\mathbf{x}^{ca}_{best} = (-97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346, 50.6776, 14.4813)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F18}:\ min:\ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 20 + e - 20\exp(-0.2\sqrt{\frac{1}{10}\sum_{i=1}^{10} z_i^2}) - \exp(\frac{1}{10}\sum_{i=1}^{10}\cos(2\pi z_i)) \\
& \mathbf{z} = (z_1, \dots, z_{10}) = 0.32 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \dots, 2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3, \dots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \dots, o_{10}) = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349, 1.9141, -36.7252) \\
& \mathbf{v}_1 = \{v_1^1, \dots, v_1^{10}\} = \{47.8659, -50.2789, -51.4218, -5.7807, 59.7290, -16.4921, 4.6304, -51.4052, -56.5858, -1.4768\} \\
& \mathbf{v}_2 = \{v_2^1, \dots, v_2^{10}\} = \{-17.0994, 17.6311, -52.2380, -37.5205, -50.8626, -18.6013, -3.3562, -46.1564, 23.0041, \\
& \qquad -24.2366\} \\
& \mathbf{v}_3 = \{v_3^1, \dots, v_3^{10}\} = \{-32.5756, 27.7492, -33.1525, 55.2097, -38.2367, -34.8929, 54.6377, -45.0192, 26.5879, \\
& \qquad -53.1159\} \\
& \mathbf{v}_4 = \{v_4^1, \dots, v_4^{10}\} = \{-29.2208, -14.6371, 31.1216, -14.0404, 23.6011, -27.5998, 50.6179, 26.2418, 43.5885, 15.1259\} \\
& \mathbf{v}_5 = \{v_5^1, \dots, v_5^{10}\} = \{-32.7262, 53.3853, -56.2406, 51.0570, 21.1575, 20.4656, -28.2398, -13.7042, -18.0062, 10.0906\} \\
& \mathbf{v}_6 = \{v_6^1, \dots, v_6^{10}\} = \{-43.5349, -59.4140, -49.4245, -9.3890, -56.9489, -50.0486, -53.9074, -58.2566, 17.3949, \\
& \qquad -27.1704\} \\
& \mathbf{v}_7 = \{v_7^1, \dots, v_7^{10}\} = \{1.9141, -12.6618, -3.5678, -18.1508, -7.8900, 1.7955, -45.1022, -14.7915, -47.5095, -57.2121\} \\
& \mathbf{v}_8 = \{v_8^1, \dots, v_8^{10}\} = \{-36.7252, 27.8390, -53.1350, -21.6579, -27.2089, -58.0376, 19.1243, 2.8412, -17.4512, \\
& \qquad -58.3012\} \\
& x_1^{cn}, \dots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5, 6, 7, 8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (5.3830, 0.1560)$ and $\mathbf{x}^{ca}_{best} = (47.8659, -17.0994, -32.5756, -29.2208,$

$-32.7262, -43.5349, 1.9141, -36.7252)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F19}: \ min: \ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} i x_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \ldots, 2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035, 63.0211, -96.1546) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{99.8101, -31.6099, 89.6818, 56.8515, -88.4346, -57.0659, -31.5381, -49.4641, 25.9274, 9.5564\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-69.8675, 40.6365, -76.1113, 65.0137, -93.6473, 92.6166, 80.1439, 76.2718, 26.7529, 37.0511\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{46.3398, 57.3869, -59.6658, 54.2175, -66.4293, 23.2850, 8.9172, 5.8984, -81.7778, -29.9939\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{-94.7804, 90.0091, -12.3052, 27.1707, 89.5608, 23.1004, -82.2399, 22.3617, 87.6216, -60.2491\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{-14.1227, 70.3685, -3.5821, -57.3306, -21.0553, 22.6570, -8.0322, 66.9846, 22.4542, -21.2894\} \\
& \mathbf{v}_6 = \{v_6^1, \ldots, v_6^{10}\} = \{-22.2035, -94.6947, -28.3114, 79.0958, -32.9711, -68.5177, -37.1964, 49.1284, -50.3006, \\
& \qquad 31.9762\} \\
& \mathbf{v}_7 = \{v_7^1, \ldots, v_7^{10}\} = \{63.0211, 15.4585, -0.9510, 90.1246, 18.4635, -17.3490, 96.5306, -14.2523, -37.9036, 58.6272\} \\
& \mathbf{v}_8 = \{v_8^1, \ldots, v_8^{10}\} = \{-96.1546, -3.5673, -59.5396, 13.2944, -21.4157, 25.7777, 0.5632, 73.3501, -29.0539, -79.8143\} \\
& x^{cn}_1, \ldots, x^{cn}_2 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5, 6, 7, 8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (6.5706, -4.7415)$ and $\mathbf{x}^{ca}_{best} = (99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035, 63.0211, -96.1546)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F20}: \ min: \ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 1 + \sum_{i=1}^{10} \frac{z_i^2}{4000} - \prod_{i=1}^{10} \cos(\frac{z_i}{\sqrt{i}}) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 6 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \ldots, 2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6664, -39.7689, 39.6034, 52.0954) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{-12.0721, 2.9979, -19.0989, -67.9707, -5.3256, 41.2393, 88.4943, -89.4662, 12.4275, 53.2186\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-10.4880, 49.2424, 64.1867, 5.7290, -95.5992, 66.8645, 56.9595, -27.3189, 77.8113, -53.0569\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{2.4412, 63.1647, -11.6669, -31.9233, -87.5237, 48.4239, -75.0023, 49.5995, -83.9520, \\
& \qquad -81.1888\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{-86.8010, 58.9903, -71.2887, 55.7150, 59.1220, 37.2548, 75.7530, -10.7222, -1.7561, 97.9166\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{-32.6664, 14.3942, 19.6051, 23.8658, 92.7302, 0.4261, 19.6791, 60.4852, -39.3893, -35.6968\} \\
& \mathbf{v}_6 = \{v_6^1, \ldots, v_6^{10}\} = \{-39.7689, 51.2260, 95.1572, -52.9366, 81.0941, -67.8047, -13.8984, 74.2628, 41.1879, 53.5652\} \\
& \mathbf{v}_7 = \{v_7^1, \ldots, v_7^{10}\} = \{39.6034, -98.0900, 66.9081, 5.6974, -52.6892, -22.8678, -21.4237, -70.5337, 8.6142, -89.3348\} \\
& \mathbf{v}_8 = \{v_8^1, \ldots, v_8^{10}\} = \{52.0954, -95.0943, 60.2757, -59.7487, -89.1344, 70.3986, -55.3637, 17.6185, -72.3865, \\
& \qquad -10.6520\} \\
& x^{cn}_1, \ldots, x^{cn}_2 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5, 6, 7, 8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (84.9982, 65.1519)$ and $\mathbf{x}^{ca}_{best} = (-12.0721, -10.4880, 2.4412,$

$-86.8010, -32.6664, -39.7689, 39.6034, 52.0954)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F21}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} z_i^2 \\
& \mathbf{z} = (z_1, \dots, z_{10}) = \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \dots, 5\} \\ x^{ca}_{i-5} - o_i, & i \in \{6, \dots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \dots, o_{10}) = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734, 44.3837, 99.8131, -12.1793) \\
& \mathbf{v}_1 = \{v^1_1, \dots, v^5_1\} = \{0.0577, -66.8891, -24.5506, -96.2061, 45.4579\} \\
& \mathbf{v}_2 = \{v^1_2, \dots, v^5_2\} = \{-36.7734, 11.1348, 40.9187, -32.3377, 62.3757\} \\
& \mathbf{v}_3 = \{v^1_3, \dots, v^5_3\} = \{44.3837, -84.2635, -31.8857, -99.0299, 23.2041\} \\
& \mathbf{v}_4 = \{v^1_4, \dots, v^5_4\} = \{99.8131, 38.7794, 97.4385, 66.3214, 83.6572\} \\
& \mathbf{v}_5 = \{v^1_5, \dots, v^5_5\} = \{-12.1793, -81.4490, 94.5925, -20.7460, -23.4447\} \\
& x^{cn}_1, \dots, x^{cn}_5 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344)$ and $\mathbf{x}^{ca}_{best} = (0.0577, -36.7734, 44.3837, 99.8131, -12.1793)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F22}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} (z_i^2 - 10cos(2\pi z_i) + 10) \\
& \mathbf{z} = (z_1, \dots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \dots, 5\} \\ x^{ca}_{i-5} - o_i, & i \in \{6, \dots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \dots, o_{10}) = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346, 50.6776, 14.4813) \\
& \mathbf{v}_1 = \{v^1_1, \dots, v^5_1\} = \{61.1286, 4.1286, -3.9445, -15.1705, -7.9428\} \\
& \mathbf{v}_2 = \{v^1_2, \dots, v^5_2\} = \{38.9558, 37.3101, 83.0975, -98.6905, -75.3426\} \\
& \mathbf{v}_3 = \{v^1_3, \dots, v^5_3\} = \{-77.1346, -2.0340, 70.8802, 51.3176, -24.3460\} \\
& \mathbf{v}_4 = \{v^1_4, \dots, v^5_4\} = \{50.6776, -39.6234, -61.7100, 97.7223, 63.1775\} \\
& \mathbf{v}_5 = \{v^1_5, \dots, v^5_5\} = (14.4813, -97.4609, 92.2885, -3.8172, 83.2134\} \\
& x^{cn}_1, \dots, x^{cn}_5 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686)$ and $\mathbf{x}^{ca}_{best} =$

$(61.1286, 38.9558, -77.1346, 50.6776, 14.4813)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F23}:\ min:\ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 20 + e - 20\exp(-0.2\sqrt{\frac{1}{10}\sum_{i=1}^{10} z_i^2}) - \exp(\frac{1}{10}\sum_{i=1}^{10}\cos(2\pi z_i)) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.32 * \mathbf{z}' A \\
& z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349, 1.9141, -36.7252) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{-29.2208, -14.6371, 31.1216, -14.0404, 23.6011\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{-32.7262, 53.3853, -56.2406, 51.0570, 21.1575\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^5\} = \{-43.5349, -59.4140, -49.4245, -9.3890, -56.9489\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^5\} = \{1.9141, -12.6618, -3.5678, -18.1508, -7.8900\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^5\} = \{-36.7252, 27.8390, -53.1350, -21.6579, -27.2089\} \\
& x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is $\mathbf{x}^{cn}_{best} = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756)$ and $\mathbf{x}^{ca}_{best} = (-29.2208, -32.7262, -43.5349, 1.9141, -36.7252)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F24}:\ min:\ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} i x_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035, 63.0211, -96.1546) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{-94.7804, 90.0091, -12.3052, 27.1707, 89.5608\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{-14.1227, 70.3685, -3.5821, -57.3306, -21.0553\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^5\} = \{-22.2035, -94.6947, -28.3114, 79.0958, -32.9711\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^5\} = \{63.0211, 15.4585, -0.9510, 90.1246, 18.4635\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^5\} = \{-96.1546, -3.5673, -59.5396, 13.2944, -21.4157\} \\
& x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398)$ and $\mathbf{x}^{ca}_{best} =$

$(-94.7804, -14.1227, -22.2035, 63.0211, -96.1546)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F25}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 1 + \sum_{i=1}^{10} \frac{z_i^2}{4000} - \prod_{i=1}^{10} \cos(\frac{z_i}{\sqrt{i}}) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 6 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \ldots, 5\} \\ x^{ca}_{i-5} - o_i, & i \in \{6, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6664, -39.7689, 39.6034, 52.0954) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{-86.8010, 58.9903, -71.2887, 55.7150, 59.1220\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{-32.6664, 14.3942, 19.6051, 23.8658, 92.7302\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^5\} = \{-39.7689, 51.2260, 95.1572, -52.9366, 81.0941\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^5\} = \{39.6034, -98.0900, 66.9081, 5.6974, -52.6892\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^5\} = \{52.0954, -95.0943, 60.2757, -59.7487, -89.1344\} \\
& x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412)$ and $\mathbf{x}^{ca}_{best} = (-86.8010, -32.6664, -39.7689, 39.6034, 52.0954)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F26}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} z_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \ldots, 5\} \\ x^{ca}_{i-5} - o_i, & i \in \{6, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734, 44.3837, 99.8131, -12.1793) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{0.0577, -66.8891, -24.5506, -96.2061, 45.4579, 18.1319, 58.3241, 71.9285, -75.8105, -22.9862\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-36.7734, 11.1348, 40.9187, -32.3377, 62.3757, 24.4644, 86.2232, 90.6468, -99.4558, 89.3221\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{44.3837, -84.2635, -31.8857, -99.0299, 23.2041, 13.1996, -43.5760, 26.1334, -6.6750, -22.8134\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{99.8131, 38.7794, 97.4385, 66.3214, 83.6572, 64.3900, 6.2714, 4.7893, 42.9978, -46.2021\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{-12.1793, -81.4490, 94.5925, -20.7460, -23.4447, -37.0443, -33.7724, -78.8025, 69.8750, \\
& \qquad 70.8563\} \\
& x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344)$ and $\mathbf{x}^{ca}_{best} =$

$(0.0577, -36.7734, 44.3837, 99.8131, -12.1793)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F27}:\ min:\ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10}(z_i^2 - 10cos(2\pi z_i) + 10) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}'A \\
& z'_i = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346, 50.6776, 14.4813) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{61.1286, 4.1286, -3.9445, -15.1705, -7.9428, -51.8438, 76.1710, -37.0110, -34.0411, \\
& \qquad -61.8862\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{38.9558, 37.3101, 83.0975, -98.6905, -75.3426, 93.3490, 52.6871, -77.6949, 3.4488, -14.2671\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{-77.1346, -2.0340, 70.8802, 51.3176, -24.3460, 63.8007, 93.8858, 11.8351, 57.5630, -0.9117\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{50.6776, -39.6234, -61.7100, 97.7223, 63.1775, -12.0348, -21.6271, -12.3744, 67.1491, \\
& \qquad -19.0775\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{14.4813, -97.4609, 92.2885, -3.8172, 83.2134, -89.4358, 10.1637, -86.6364, -64.1289, 6.0189\} \\
& x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686)$ and $\mathbf{x}^{ca}_{best} = (61.1286, 38.9558, -77.1346, 50.6776, 14.4813)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F28}:\ min:\ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 20 + e - 20\exp(-0.2\sqrt{\frac{1}{10}\sum_{i=1}^{10} z_i^2}) - \exp(\frac{1}{10}\sum_{i=1}^{10}\cos(2\pi z_i)) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.32 * \mathbf{z}'A \\
& z'_i = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349, 1.9141, -36.7252) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{-29.2208, -14.6371, 31.1216, -14.0404, 23.6011, -27.5998, 50.6179, 26.2418, 43.5885, 15.1259\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-32.7262, 53.3853, -56.2406, 51.0570, 21.1575, 20.4656, -28.2398, -13.7042, -18.0062, \\
& \qquad 10.0906\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{-43.5349, -59.4140, -49.4245, -9.3890, -56.9489, -50.0486, -53.9074, -58.2566, 17.3949, \\
& \qquad -27.1704\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{1.9141, -12.6618, -3.5678, -18.1508, -7.8900, 1.7955, -45.1022, -14.7915, -47.5095, \\
& \qquad -57.2121\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{-36.7252, 27.8390, -53.1350, -21.6579, -27.2089, -58.0376, 19.1243, 2.8412, -17.4512, \\
& \qquad 58.3012\} \\
& x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756)$ and $\mathbf{x}^{ca}_{best} =$

$(-29.2208, -32.7262, -43.5349, 1.9141, -36.7252)$, and $f(\mathbf{x}_{best}^{cn}, \mathbf{x}_{best}^{ca}) = 0$.

$$
\begin{aligned}
\textbf{F29}:\ min:\ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} i x_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035, 63.0211, -96.1546) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{-94.7804, 90.0091, -12.3052, 27.1707, 89.5608, 23.1004, -82.2399, 22.3617, 87.6216, -60.2491\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-14.1227, 70.3685, -3.5821, -57.3306, -21.0553, 22.6570, -8.0322, 66.9846, 22.4542, -21.2894\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{-22.2035, -94.6947, -28.3114, 79.0958, -32.9711, -68.5177, -37.1964, 49.1284, -50.3006, \\
& \qquad 31.9762\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{63.0211, 15.4585, -0.9510, 90.1246, 18.4635, -17.3490, 96.5306, -14.2523, -37.9036, 58.6272\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{-96.1546, -3.5673, -59.5396, 13.2944, -21.4157, 25.7777, 0.5632, 73.3501, -29.0539, -79.8143\} \\
& x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}_{best}^{cn} = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398)$ and $\mathbf{x}_{best}^{ca} = (-94.7804, -14.1227, -22.2035, 63.0211, -96.1546)$, and $f(\mathbf{x}_{best}^{cn}, \mathbf{x}_{best}^{ca}) = 0$.

$$
\begin{aligned}
\textbf{F30}:\ min:\ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 1 + \sum_{i=1}^{10} \frac{z_i^2}{4000} - \prod_{i=1}^{10} \cos(\frac{z_i}{\sqrt{i}}) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 6 * \mathbf{z}' A \\
& z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6664, -39.7689, 39.6034, 52.0954) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{-86.8010, 58.9903, -71.2887, 55.7150, 59.1220, 37.2548, 75.7530, -10.7222, -1.7561, 97.9166\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-32.6664, 14.3942, 19.6051, 23.8658, 92.7302, 0.4261, 19.6791, 60.4852, -39.3893, -35.6968\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{-39.7689, 51.2260, 95.1572, -52.9366, 81.0941, -67.8047, -13.8984, 74.2628, 41.1879, 53.5652\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{39.6034, -98.0900, 66.9081, 5.6974, -52.6892, -22.8678, -21.4237, -70.5337, 8.6142, \\
& \qquad -89.3348\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{52.0954, -95.0943, 60.2757, -59.7487, -89.1344, 70.3986, -55.3637, 17.6185, -72.3865, \\
& \qquad -10.6520\} \\
& x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}_{best}^{cn} = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412)$ and $\mathbf{x}_{best}^{ca} = (-86.8010, -32.6664, -39.7689, 39.6034, 52.0954)$, and $f(\mathbf{x}_{best}^{cn}, \mathbf{x}_{best}^{ca}) = 0$.

B. Capacitated Facility Location Problems

The parameters of the capacitated facility location problems used in this paper were generated as follows: D_j was generated from $U[5, 35]$ (U represents the uniform distribution), C_r was generated from $U[1000, 3000]$, $F_{i,r}$ is generated from $U[300, 1300]$, and $Q_{i,j}$ was generated from $U[0, 1]$. By setting m, n, and s, six capacitated facility location problems, i.e., CFLP1-CFLP6, were generated. The parameter settings of CFLP1-CFLP6 are listed in Table S-XVI.

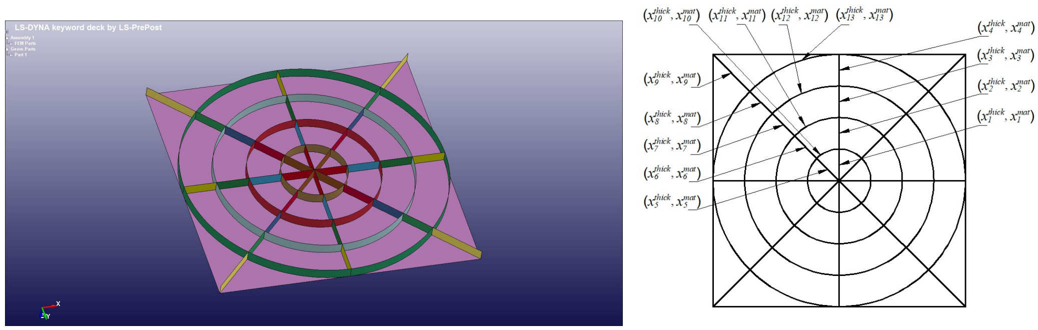
TABLE S-XVI
PARAMETERS SETTINGS OF THE SIX CAPACITATED FACILITY LOCATION PROBLEMS

	m	n	s
CFLP1	5	5	1
CFLP2	10	5	1
CFLP3	5	5	4
CFLP4	10	5	4
CFLP5	5	5	8
CFLP6	10	5	8

C. Dubins Traveling Salesperson Problems

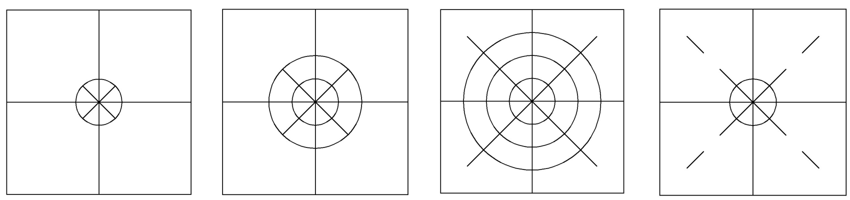
The parameters of the constructed Dubins traveling salesperson problems are set as follows. For DTSP1-DTSP3, the waypoints were randomly generated in the range of $[-50, 50]$, and the numbers of waypoints were set to 5, 10, and 15, respectively. For DTSP4-DTSP6, the waypoints were randomly generated in the range of $[-100, 100]$, and the numbers of waypoints were also set to 5, 10, and 15, respectively. For all these six problems, the minimal turning radius was set to 1.

S-V. FIGURES



(a) (b)

Fig. S-1. Structure of the stiffened plate. (a) the considered FEA model (b) the variable distribution of the structure of the stiffened plate.



(a) (b) (c) (d)

Fig. S-2. Topological structures of the stiffened plates reported in [1] and obtained by EGO-Gower, GA, and MiSACO. (a) the original design (b) EGO-Gower (c) GA (d) MiSACO

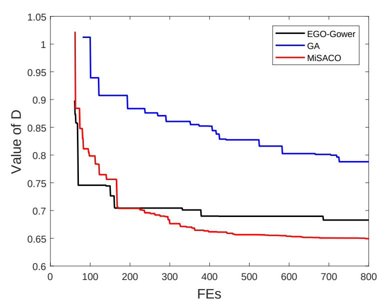


Fig. S-3. Convergence curves derived from EGO-Gower, GA, and MiSACO.

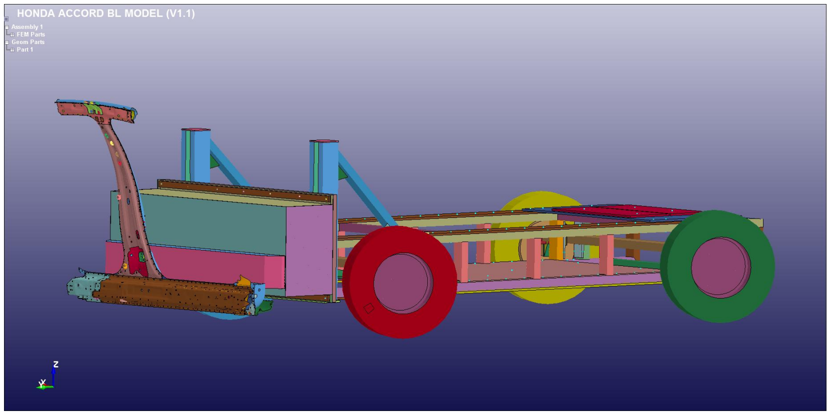


Fig. S-4. Simplified FEA model used in this paper.

REFERENCES

[1] T. Liu, G. Sun, J. Fang, J. Zhang, and Q. Li, “Topographical design of stiffener layout for plates against blast loading using a modified algorithm,” *Structural and Multidisciplinary Optimization*, vol. 59, no. 2, pp. 335–350, 2019.