University Physics A(1) 2014

Worksheet #6

Name (名字): Student number (学号):

New words: Write the Chinese next to these words as you learn them.

conservative force
thermal energy
dissipation
power
center of mass
translation
rotation
vibration
angular speed
moment of inertia

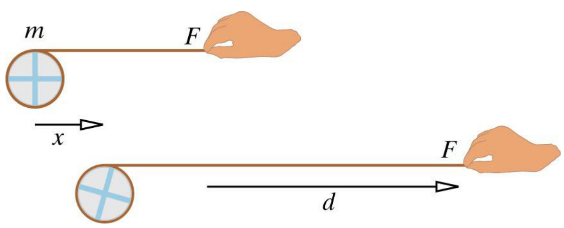
Problems Show all working.

(1) [9.P.29] Show (证明) that the moment of inertia of a disk (盘) of mass M and radius R is $\frac{1}{2}MR^2$. Some hints:

- Divide the disk into narrow rings(环), each ofradius r and width dr.
- The contribution to I by one of these rings is simply $r^2\,dm$, where dm is the amount of mass contained in that particular ring.
- The mass of any ring is the total mass times the fraction of the total area occupied by the area of the ring.
- The area of a ring is approximately $2\pi r\,dr$.

Use integral calculus(积分) to add up all the contributions.

(2) [9.P.33] A string (绳子) is wrapped around a disk (盘) of mass 2.1 kg (its density is not necessarily uniform – 它的密度不一定是均匀的). The disk is lying flat on a nearly frictionless surface(没有摩擦力的表面). Starting from rest, you pull the string with a constant force of 9 N. At the instant when the center of the disk has slid a distance 0.11 m, your hand has moved a distance of 0.28 m (see the figure below). You can neglect the mass of the string.



(a) At this instant, what is the speed of the center of mass of the disk?

(b) At this instant, how much rotational kinetic energy does the disk have relative to its center of mass?

(c) At this instant, the angular speed of the disk is 7.5 radians/s. What is the moment of inertia of the disk?

第六周作业

(1) $\because dI = r^2 dm = 2\pi\sigma r^3 dr$

$\therefore I = \int dI = 2\pi\sigma \int_0^R r^3 dr = 2\pi\sigma \cdot \frac{1}{4}R^4 = \frac{1}{2}\pi\left(\frac{M}{\pi R^2}\right)\cdot R^4 = \frac{1}{2}MR^2$

(2) (a) 把圆盘当作系统，由能量守恒定律得:

$\Delta K_{trans} = W_{ext}$

即 $K_{trans,f} = \vec{F}_{net} \cdot \Delta\vec{r}_{cm}$

$\frac{1}{2}M|\vec{V}_{cm}|^2 = |\vec{F}_{net}| \cdot |\Delta\vec{r}_{cm}|$ 则 $|\vec{V}_{cm}| = \sqrt{\frac{2|\vec{F}_{net}|\cdot|\Delta\vec{r}_{cm}|}{M}}$

$= \sqrt{\frac{2\times 9\times 0.11}{2.1}} \approx 0.971 m/s$

(b) 把圆盘和手看作一个系统，由能量守恒定律得:

$\Delta K_{trans} + \Delta K_{rot} = \vec{F}_{hand} \cdot \Delta\vec{r}_{hand}$

$K_{trans,f} + K_{rot,f} = |\vec{F}_{hand}| \cdot |\Delta\vec{r}_{hand}| \cos 0°$

即 $\vec{F}_{net} \cdot \Delta\vec{r}_{cm} + K_{rot,f} = |\vec{F}_{hand}| \cdot |\Delta\vec{r}_{hand}|$

$K_{rot,f} = |\vec{F}_{hand}| \cdot |\Delta\vec{r}_{hand}| - \vec{F}_{rot} \cdot \Delta\vec{r}_{cm}$

$= |\vec{F}_{hand}| \cdot (|\Delta\vec{r}_{hand}| - |\Delta\vec{F}_{cm}|)$

$= 9 \times (0.28 - 0.11) = 1.53 J$

(c) $K_{rot} = \frac{1}{2}I\omega^2$

则 $I = \frac{2K_{rot}}{\omega^2} = \frac{2\times 1.53}{7.5^2} kg\cdot m^2 = 0.0544 kg\cdot m^2$