University Physics A(1) 2014

Worksheet #4

Name: Student number:

New words: Write the Chinese next to these words as you learn them.

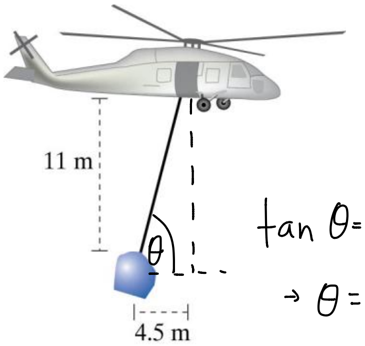
statics	radial
dynamics	centripetal
parallel	radius
perpendicular	diameter
tangential	circumference

Problems Show all working.

(1) [5.P.19] A helicopter flies to the right (in the +*x* direction) at a constant speed of 12 m/s, parallel to the surface of the ocean. A 900 kg package is suspended below the helicopter by a cable (缆绳) as shown in the figure below; the package is also traveling to the right in a straight line, at a constant speed of 12 m/s. The pilot is worried about whether the cable, whose breaking strength is 9300 N, is strong enough to support this package.

(a) Choose the package as the system. What is the rate of change of the momentum of the system?

Constant velocity, so $\boxed{\frac{d\vec{p}}{dt} = 0}$



$\tan\theta = \frac{11}{4.5}$

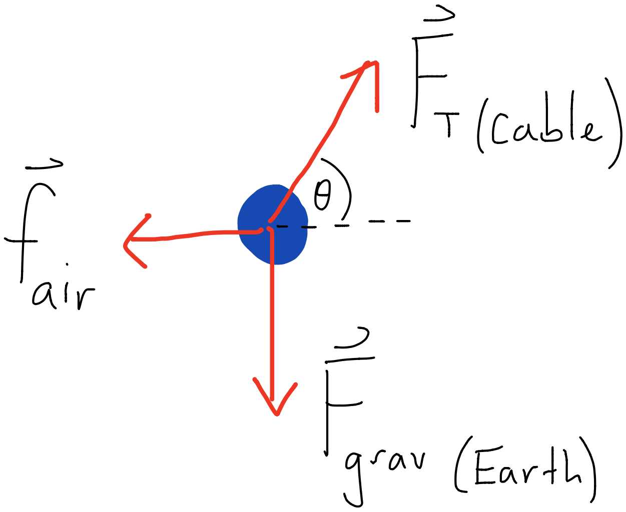
$\rightarrow \theta = \tan^{-1}\left(\frac{11}{4.5}\right)$

$= 67.8°$

(b) List all the objects that are exerting forces on the package.

Cable, Earth, air

(c) Draw a careful force diagram ("free-body diagram") showing all the forces acting on the package. Label each force with the name of the object exerting the force.



(d) What is the magnitude of the tension in the cable supporting the package? Carefully show all steps in your work (starting from the Momentum Principle).

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

$$\vec{0} = \vec{F}_{grav} + \vec{F}_T + \vec{f}_{air}$$

$\boxed{x}$ $\quad 0 = F_T \cos\theta - f_{air} \quad (1)$

$\boxed{y}$ $\quad 0 = F_T \underbrace{\cos(90^\circ - \theta)}_{\sin\theta} - mg \quad (2)$

$$F_T = \frac{mg}{\sin\theta} = \frac{(900\text{ kg})(9.8\text{ m/s}^2)}{\sin(67.8^\circ)}$$

$$= \boxed{9.5 \times 10^3\text{ N}}$$

(e) Write the force exerted on the package by the cable as a vector.

$$\vec{F}_T = F_T\,\hat{F}_T = (9.5\times10^3\text{ N})\langle \cos 67.8^\circ, \cos 22.2^\circ, 0\rangle = \boxed{\langle 3.6\times10^3, 8.8\times10^3, 0\rangle\text{ N}}$$

(f) What is the magnitude of the force exerted by the air on the package? Show all steps in your work

From Eq (1) in part (d):

$$f_{air} = F_T \cos\theta = \boxed{3.7\times10^3\text{ N}}$$

(g) Write the force on the package by the air as a vector.

$$\vec{f}_{air} = f_{air}\,\hat{f}_{air} = (3.7\times10^3\text{ N})\langle -1, 0, 0\rangle = \boxed{\langle -3.7\times10^3, 0, 0\rangle\text{ N}}$$

(h) Is the cable in danger of breaking?

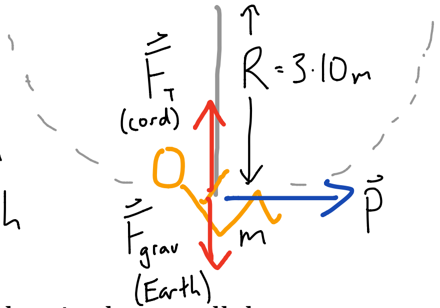
$\boxed{\text{Yes!}}$ The breaking strength is 9300 N, but the tension force in the cable has a magnitude of 9500 N.

(2) [5.P.48/49] A child of mass 26 kg swings (摇摆) at the end of an elastic cord (弹性绳). At the bottom of the swing, the child's velocity is horizontal, and the speed is 8 m/s. At this instant the cord is 3.10 m long. [You can neglect air resistance.]

First, draw a picture of the situation. Then answer the following questions.

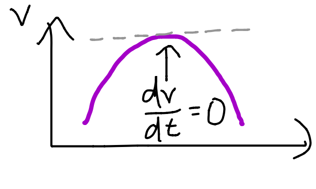
SYSTEM: child

SURROUNDINGS: cord
Earth



(a) At this instant, what is the parallel component of the rate of change of the child's momentum?

$$\left|\left(\frac{d\vec{p}}{dt}\right)_{\parallel}\right| \simeq m\frac{d|\vec{v}|}{dt} = \boxed{0}\text{, since speed is at a maximum}$$



(b) At this instant, what is the perpendicular component of the rate of change of the child's momentum?

$$\left|\left(\frac{d\vec{p}}{dt}\right)_{\perp}\right| = |\vec{p}|\frac{|\vec{v}|}{R} \simeq mv\frac{v}{R} = \frac{mv^2}{R} = \frac{(26\,kg)(8\,m/s)^2}{3.10\,m} = \boxed{537\,kg\,m/s^2\text{; direction is up.}}$$

(c) At this instant, what is the *net* force acting on the child? (Use the Momentum Principle.)

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

$\boxed{x}$: $\left|\left(\frac{d\vec{p}}{dt}\right)_{\parallel}\right| = |\vec{F}_{net,\parallel}|$

$0 = F_{net,x}$

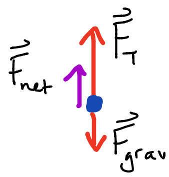
$\boxed{y}$: $\left|\left(\frac{d\vec{p}}{dt}\right)_{\perp}\right| = |\vec{F}_{net,\perp}|$

$537\,N = F_{net,y}$

$$\boxed{\vec{F}_{net} = \langle 0, 537, 0\rangle\,N}$$

(d) What is the magnitude of the force that the elastic cord exerts on the child? (It helps to draw a force diagram.)

$$\vec{F}_{net} = \vec{F}_T + \vec{F}_{grav}$$



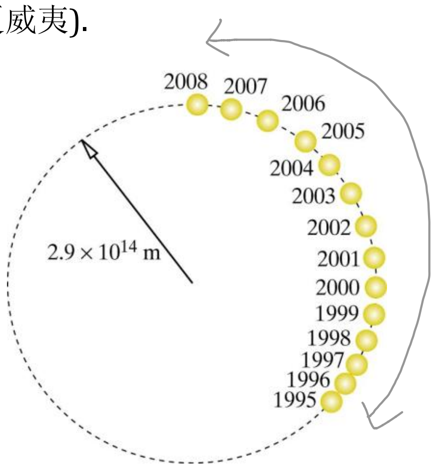
$$\rightarrow |\vec{F}_T| = |\vec{F}_{net} - \vec{F}_{grav}| = F_{net} + mg = 537\,N + (26\,kg)(9.8\,m/s^2) = \boxed{792\,N}$$

(e) The relaxed length of the elastic cord is 3.06 m. What is the stiffness of the cord?

$$F_T = k_s|s|$$

$$\rightarrow k_s = \frac{F_T}{|s|} = \frac{F_T}{L - L_0} = \frac{792\,N}{3.10\,m - 3.06\,m} = \frac{792\,N}{0.04\,m} = \boxed{1.98\times10^4\,N/m}$$

(3) Astronomers (天文学家) believe there is a massive black hole (黑洞) at the center of our Milky Way galaxy (银河系). They know this because they have observed the motion of stars orbiting around the center of the galaxy. The figure below shows the motion of one of these stars, called S0-20; the position of S0-20 was measured every year from 1995 until 2008, using the large Keck telescope (凯克望远镜) in Hawaii (夏威夷).



$\approx \frac{1}{3} \times \text{circumference}$

(a) Using the positions and times shown in the figure, what is the approximate speed of this star in m/s? Also express the speed as a fraction of the speed of light.

$$V = \frac{\text{distance}}{\text{time}} \approx \frac{\frac{1}{3} \times 2\pi R}{(2008-1995)\,\text{years}} = \frac{\frac{1}{3} \times 2\pi\,(2.9 \times 10^{14}\,\text{m})}{13\,\text{y} \times (\pi \times 10^{7}\,\text{s/y})}$$

Useful fact! 1 year $\simeq \pi \times 10^7$ s

$$= \boxed{1.5 \times 10^{6}\,\text{m/s} = 5 \times 10^{-3}\,c}$$

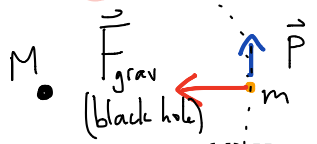
(b) This is an extraordinarily high speed for a macroscopic (宏观的) object. Is it reasonable to approximate the star's momentum as $m\vec{v}$?

Yes; $\gamma = \frac{1}{\sqrt{1-(5\times10^{-3})^2}} = 1.000013$, so $\gamma m\vec{v} \simeq m\vec{v}$.

(c) Based on these data (数据), estimate the mass of the massive black hole around which this star is orbiting. (Use the Momentum Principle.)

SYSTEM: star

SURROUNDINGS: black hole



$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

$$\hat{x}: \left|\left(\frac{d\vec{p}}{dt}\right)_{\perp}\right| = \left|\vec{F}_{net,\perp}\right|$$

$$\frac{\cancel{m}v^2}{\cancel{R}} = G\frac{M\cancel{m}}{R^{\cancel{2}}}$$

$$M = \frac{v^2 R}{G} \approx \frac{(1.5\times10^{6}\,\text{m/s})^2\,(2.9\times10^{14}\,\text{m})}{6.7\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}}$$

$$= \boxed{9.7 \times 10^{36}\,\text{kg}}$$

(d) How many of our Suns does this mass represent? (Look at the reference sheet on the class webpage for the mass of the Sun.)

$$\frac{M_{\text{black hole}}}{M_{\text{sun}}} = \frac{9.7\times10^{36}\,\text{kg}}{2\times10^{30}\,\text{kg}} = \boxed{5\times10^{6}\ \text{Suns!}}$$