Application of the Polynomial Maximization Method for Estimation Parameters of Autoregressive Models with Asymmetric Innovations

Serhii Zabolotnii[1], Oleksandr Tkachenko[2], and Zygmunt L. Warsza[3](✉)

[1] Cherkasy State Business College, Cherkasy, Ukraine
[2] Ivan Franko National University of Lviv, Lviv, Ukraine
[3] Łukasiewicz Research Network – Industrial Research Institute for Automation and Measurements PIAP, Al. Jerozolimskie 202, 02-486 Warszawa, Poland
zlw1936@gmail.com

Abstract. This paper considers the application of the Polynomial Maximization Method to find estimates of the parameters of autoregressive model with non-Gaussian innovation. This approach is adaptive and is based on the analysis of higher-order statistics. Analytical expressions that allow finding estimates and analyzing their uncertainty are obtained. Case of asymmetry of the distribution of autoregressive innovations is considered. It is shown that the variance of estimates of the Polynomial Maximization Method can be significantly less than the variance of the estimates of the linear approach (based on Yule-Walker equation or Ordinary Least Squares). The increase in accuracy depends on the values of the cumulant coefficients of higher orders of innovation residuals. The results of statistical modeling by the Monte Carlo method confirm the effectiveness of the proposed approach.

Keywords: Polynomial maximization method · Autoregressive models · Non-Gaussian distributions · Cumulant coefficients · Variance · Higher order statistics

1 Introduction

Autoregressive (AR) models are a way of describing time series, widely used to predict data of geophysical, economic, biomedical, sociological, and other origins. Traditionally, such models are synthesized and analyzed based on the assumption of the normality (Gaussian distribution) of the random component of the AR model, which is often called «innovation». However, a large number of researchers note the idealization of such an assumption, which greatly simplifies the solution of the problem of estimating informative parameters, but often does not correspond to the realities of practical tasks [1–6]. In such situations, to improve the estimation accuracy, a transition is made from simple nonparametric estimates based on the method of moments (Yule-Walker equations) or

R. Szewczyk et al. (Eds.): AUTOMATION 2022, AISC 1427, pp. 380–390, 2022.
https://doi.org/10.1007/978-3-031-03502-9_37

the least squares method (OLS) to a more implementation-complex parametric approach based on the maximum likelihood method (MLM). It is known that one of the key elements of this approach is the need to solve an additional type identification problem and estimate the parameters of the probability distribution to describe the random component of the AR model.

Currently, a large number of different types of distributions are used: exponential and gamma distribution [1], Poisson [1, 2], a family of exponential power distributions, including asymmetric modification [3], inverse Gaussian and Student's t distribution [4], mixes based on Gaussian [2] or Laplace distributions [5], etc. Such a large variety can make a difficult to choose of the right type of distributions, and in addition, the problem of analyzing the performance of resulting estimates is an important one [6].

An alternative approach to probabilistic description is the use of higher-order statistics (moment or cumulant functions) or their poly-spectra. A practical example of using this approach for constructing and estimate parameters of various predictive models (including AR) is the library of functions (HOSA toolbox) for the MATLAB package [7]. Based on the apparatus of cumulative description, a generalized higher ranks AR model for linear prediction of non-Gaussian processes [8] has been theoretically substantiated, and original methods for estimating parameters are being developed [9].

In this paper, for the first time, it is proposed to use the polynomial maximization method (PMM) [10] to finding the parameter estimates of AR-models. This relatively new estimation method was originally developed for solving problems of statistical radio-physics when processing signals with continuous time but was later adapted for the discrete case. It is based on the representation of the logarithm of the probability distribution density in the form of functional series. Conceptually, PMM is similar to MLM, as it also uses the principle of maximizing the functional from sample data in the vicinity of the true value of the estimated parameter. However, to form such a functional, not the probability distribution density is used, but the description of random variables in the form of a finite number of averaged statistics (moments, cumulants, etc.), which are much easier to obtain. Let us note the functionality of this method, which can be applied not only to find estimates of the scalar parameter [11], but also to find the moments of change point in the properties of a random sequence in a posteriori formulation of the problem [12], as well as to estimate the vector parameter linear [13] and polynomial [14] regression.

The general idea of using PMM for estimating the parameters of AR models is based on the well-known mathematical analogy between an autoregressive model and linear (in terms of parameters) one-factor regression. Since the errors are statistically independent of observations, the latter can be used as predictors of linear regression to find estimates of the parameters of which linear estimates (Yula-Walker or OLS) can be used. At the same time, the inadequacy of the Gaussian hypothesis is not critical from the point of view that the linear estimates remain unbiased and consistent (although they are no longer effective), and the probabilistic properties of the AR of the residuals do not differ significantly from the properties of non-Gaussian errors. This fact opens up the possibility of overcoming the condition of having the necessary information about the value of higher-order statistics to describe the random component of AR models, because instead of the a priori values of these parameters one can use a posteriori estimates of such residuals. Thus, this approach can be interpreted as a two-stage adaptive procedure for estimating the parameters of AR models.

Note that the idea of such an adaptive approach based on the use of higher-order statistics in the process of solving the problem of estimating the parameters of time series models is not new. In particular, in [15], the use of a posteriori values of the asymmetry coefficients and kurtosis of the AR residuals after the use of the Yule-Walker estimates is considered. However, the fundamental difference is that the obtained a posteriori information is used for a more complex procedure for approximating the random component of the AR model based on an asymmetric exponential power distribution followed by adaptive refinement of estimates of informative parameters using the MLM.

This study is a continuation of works [11, 13, 14], which considered the use of PMM to solve the problem of estimating the parameters, respectively, of the constant component (shift of the distribution center) and linear and polynomial regressions under conditions of asymmetrically distributed statistical data. Its main purpose is to generalize the results of these works for an autoregressive model, t compare the efficiency of the estimates obtained by Monte Carlo modelling, and also to test the proposed approach to real statistical data of economic origin.

2 Mathematical Formulation of the Problem

Let there be a vector $\boldsymbol{X} = \{x_1, x_2, ..., x_N\}$ containing the values of the time series described by the autoregressive observation model

$$x_v = a_0 + \sum_{p=1}^{P} a_p x_{v-p} + \xi_v, \, v = \overrightarrow{1, N}, \tag{1}$$

To generalize the entry, we use the vector $\boldsymbol{\theta} = \{a_0, a_1, ...a_P\}$ including a constant component a_0 and P parameters (autoregressive coefficients). Countdowns ξ_v the random component of the AR model (1) are a sequence of centered ($E\{\xi\} = 0$), independent and equally distributed random variables. In this case, the distribution of the random component ξ differs from the Gaussian (normal) law and has significant asymmetry. An additional restriction constraint is that the random variable ξ has finite central moments μ_r up to the 4th order.

The general task is to find estimates of the vector parameter $\boldsymbol{\theta}$ based on statistical analysis of a set of samples $\boldsymbol{X}$. In this case, the values of the probability characteristics of the random component of the AR model may be a priori unknown.

3 Theoretical Results

3.1 Theoretical Foundations of PPM

To solve this problem, you can use a modification of the PMM designed to find estimates of the vector parameter under unequally distributed statistical data. It is based on the property of maximizing a functional as a stochastic polynomial of a general form [10]:

$$L_{SN} = \sum_{v=1}^{N} \sum_{i=1}^{S} \phi_i(x_v) \int^{a} k_{iv}(a)dz - \sum_{i=1}^{S} \sum_{v=1}^{N} \int^{a} \Psi_{iv} k_{iv}(a)dz, \tag{2}$$

It is assumed that random variables x_v are described by a sequence of mathematical expectations [10]:

$$\Psi_{iv} = E\{\phi_i(x_v)\},\ i = \overrightarrow{1, S},\ v = \overrightarrow{1, N} \tag{2}$$

which are double differentiated by the estimated parameter a.

By the well-known analogy with the MLM, the parameter estimate can be found from the solution of an equation of the form [10]:

$$\frac{d}{da} L_{SN} \bigg|_{a=\hat{a}} = \sum_{i=1}^{S} \sum_{v=0}^{N} k_{iv} [\phi_i(x_v) - \Psi_{iv}] \bigg|_{a=\hat{a}} = 0, \tag{4}$$

where the optimal coefficients k_{iv} that maximize functional (2) are found from solving a systems of linear algebraic equations [10]:

$$\sum_{j=1}^{S} k_{jv} F_{(i,j)v} = \frac{d}{da} \Psi_{iv},\ i = \overrightarrow{1, S},\ v = \overrightarrow{1, N}, \tag{5}$$

where $F_{(i,j)v} = \Psi_{(i,j)v} - \Psi_{iv}\Psi_{jv}$, $\Psi_{(i,j)v} = E\big\{\phi_i(x_v)\phi_j(x_v)\big\}$, $i, j = \overrightarrow{1, S}$.

This approach can be extended to the case of finding estimates of a vector parameter $\boldsymbol{\theta} = \{a_0, a_1, ...a_P\}$. In such a situation, it is necessary to use $P + 1$ polynomials general form (2) for each component of the vector parameter. Thus, the desired estimates can be found as solution to a systems of equations of the form [10]:

$$\sum_{i=1}^{S} \sum_{v=1}^{N} k_{iv}^{(p)} [\phi_i(x_v) - \Psi_{iv}] \bigg|_{a_p=\hat{a}_p} = 0,\ p = \overrightarrow{0, P} \ldots \tag{6}$$

To solve the problem in the formation of the stochastic polynomial (2) as basic functions, we can use power transformations of sample values, i.e. $\phi_i(x_v) = x_v^i$. In this case, the sequence of mathematical expectations (3) is a set of initial moments of the corresponding order.

3.2 PPM-Estimates of AR Parameters with Polynomial Degree S = 1

By using a stochastic polynomial of order $S = 1$ PMM-estimates of the vector parameter $\boldsymbol{\theta}$ can be found from the solution a system of equations of the form:

$$\sum_{v=1}^{N} \left\{ k_{1,v}^{(p)} \left[x_v - \left(a_0 + \sum_{p=1}^{P} a_p x_{v-p} \right) \right] \right\} = 0,\ p = \overrightarrow{0, P}. \tag{7}$$

where are the optimal coefficients, which are found as solution of system (5)

$$k_{1,v}^{(p)} = \frac{1}{\mu_2} \frac{\partial}{\partial a_p} \left[\sum_{p=0}^{P} a_p x_{v-p} \right] = \frac{x_{v-p}}{\mu_2},\ p = \overrightarrow{0, P}, \tag{8}$$

where μ_2 – central moment of the 2nd order of the random component ξ.

Note that after the substitution of (8) into (7), the resulting system for finding the PMM-estimates parameters at $S = 1$ becomes equivalent to the linear system of estimates of the least squares method. This means that the accuracy of both methods is the same [13, 14]. It is known that the efficiency of such OLS estimates is significantly reduced when the distribution of the random component of the AR model differs from the Gaussian distribution. Therefore, a new approach to non-linear estimation of AR parameters based on the use of quadratic power stochastic polynomials is discussed.

3.3 PPM-Estimates of AR Parameters with Polynomial Degree S = 2

Using stochastic polynomials of order $S = 2$, the PMM-estimates for the AR model (1) can be found by solving a system of equations of the form:

$$\sum_{v=1}^{N}\left\{k_{1,v}^{(p)}\left[x_v-\left(a_0+\sum_{p=1}^{P}a_p x_{v-p}\right)\right]+k_{2,v}^{(p)}\left[[x_v]^2-\left(a_0+\sum_{p=1}^{P}a_p x_{v-p}\right)^2-\mu_2\right]\right\}=0,\quad p=\overrightarrow{0,P},\tag{9}$$

where: the optimal coefficients $k_{i,v}^{(p)}$, $i=\overrightarrow{1,2}$ provide the minimization of the variance of the estimates parameter when using the degree of the polynomial $S = 2$.

These coefficients are found as a solution of an appropriate system of the form (5) and can be represented as functions depending on the central moments of the random component ξ AR models:

$$k_{1,v}^{(p)}=\frac{\mu_4-\mu_2^2+2\mu_3\left(a_0+\sum_{p=1}^{P}a_p x_{v-p}\right)}{\mu_2\left(\mu_4-\mu_2^2\right)-\mu_3^2}x_{v-p},\quad k_{2,v}^{(p)}=-\frac{\mu_3^2}{\mu_2\left(\mu_4-\mu_2^2\right)-\mu_3^2}x_{v-p},\tag{10}$$

By substituting the coefficients (10) into (9), after certain transformations, the system of equations for finding estimates can be written in the form:

$$\sum_{v=1}^{N}\left\{x_{v-p}\left[A\left(a_0+\sum_{p=1}^{P}a_p x_{v-p}\right)^2+B_v\left(a_0+\sum_{p=1}^{P}a_p x_{v-p}\right)+C_v\right]\right\}=0,\quad p=\overrightarrow{0,Q-1},\tag{11}$$

where

$$A=\mu_3,\ B_v=\mu_4-\mu_2^2-x_v\mu_3,\ C_v=x_v^2\mu_3-x_v\left(\mu_4-\mu_2^2\right)-\mu_2\mu_3,\tag{12}$$

depends on sampled values x_v and focal points $\mu_2-\mu_4$ random component ξ.

Obviously, for the degree of the polynomial $S = 2$ PMM-estimates can be found only numerically, for example, using the Newton-Raphson iterative procedure [14].

It is proved that non-linear PMM parameters estimates are consistent and asymptotically unbiased [10]. In order to obtain analytical expressions describing the variances of PMM estimates, a matrix of the amount of extracted information about the parameter components is used when applying stochastic polynomials (1) of order S. Such a matrix consists of the elements [10]:

$$J_{SN}^{(p,q)} = \sum_{v=1}^{N} \sum_{i=1}^{S} k_{i,v}^{(p)} \frac{\partial}{\partial a_q} \Psi_{iv}, \quad p, q = \overrightarrow{0, Q-1}. \tag{13}$$

In a statistical sense, the amount of information obtained is conceptually a concept close to the amount of Fischer information. It is shown that the variances $\sigma^2_{(a_p)S}$ of PMM-estimates of vector parameter $\boldsymbol{\theta}$ in the asymptotic case (for $N \to \infty$) can be obtained as elements of the main diagonal of the variation matrix, which is inverse to the matrix composed of elements (13).

Using expressions for the optimal coefficients (10), it can be shown that the elements of the variation matrix of quadratic PMM-estimates for $S = 2$ differ from the elements of the matrix of linear estimates for $S = 1$ by a certain coefficient. It is the same for all components of the estimated vector parameter $\boldsymbol{\theta}$ and can be represented as [13]:

$$g_{2(Var)} = \frac{\sigma^2_{(a_p)S=2}}{\sigma^2_{(a_p)S=1}} = 1 - \frac{\gamma_3^2}{2 + \gamma_4}. \tag{14}$$

The transition from the moments to cumulant description in expression (14) is due to the fact that the deviations of the values of the cumulant coefficients of higher orders $\gamma_r = \kappa_r / \kappa_2^{r/2}$ from zero indicates the degree of difference from the Gaussian distribution of the random component of the AR model. Taking into account the well-known inequality $\gamma_4 + 2 \geq \gamma_3^2$ we can conclude that the coefficient of variance reduction $g_{2(Var)}$ is a dimensionless quantity that belongs to the range (0; 1]. Thus, with an increase in the asymmetry of the distribution of the random component of the AR model, the magnitude of the relative variance reduction can be quite significant.

4 Statistical Modeling

To verify the theoretical results, a set of functions has been developed in the R-Studio environment. Based on the method of multiple Monte Carlo tests, it implements a comparative analysis of the accuracy of different methods for estimating the parameters of AR models, the random component of which has an asymmetric distribution. It should be noted that the expression for the variance ratio (14), used as a theoretical criterion for the efficiency of PMM-estimates, is derived for two important practical constraints:

1) if there is a priori information about the values of the parameters (moments or cumulants up to the 4th order) determining the probabilistic properties of the random component of AR models;
2) with a statistical sample size asymptotically tending to infinity ($N \to \infty$).

To overcome the first limitation, an adaptive approach is used, which is based on the above stated fact that the statistical properties of OLS residuals practically do not differ from the properties of the initial random component of the AR model. Therefore, when forming the system of PMM equations, instead of the a priori values of the central moments $\mu_2 - \mu_4$ you can use their posterior estimates.

To minimize the influence on the reliability of the factor associated with the finiteness of real samples, in addition to the experimental values of the variance ratios $\hat{g}_{2(Var)}$ additionally, two other statistical performance criteria are considered: the ratio of the mean square error $\hat{g}_{2(MSE)}$ and the ratio of absolute shifts $\hat{g}_{2(Bias)}$ estimates (relative to the true value) of informative parameters obtained by different methods.

When implementing statistical modeling, the simplest first-order AR(1) model (with a constant $a_0 = 1$ and information parameter $a_1 = 0.8$) was used as the object of research. Parameter values were estimated by different approaches: the least-squares method, the method based on the solution of Yule-Walker equations, the maximum likelihood method (optimized for a Gaussian model), and a quadratic modification of the polynomial maximization method. As an asymmetric random component of the AR model, we used a sequence of independent and equally distributed random variables having a gamma distribution with different values of the shape parameter, which were determined by the degree of asymmetry. In this case, the parameters (central moments up to the 4th order) of the random component, which are necessary to find the adaptive PMM-estimates, were considered a priori unknown, and in accordance with the adaptive algorithm, their posterior estimates based on the LSM residuals were used.

The set of experimental values of efficiency coefficients (presented in Tables 1, 2 and 3) were obtained for a series in $M = 10^4$ repeated experiments. An analysis of these results shows that the experimental values of the values of all the relative performance criteria under consideration are in the range from 0.35 to 0.8. Accordingly, this indicates a significant (20–65%) increase in the accuracy of the resulting estimates of the informative parameter of the studied AR model. Obviously, the value of the gain depends on the degree to which the distribution of the random component differs from the Gaussian distribution, which is numerically expressed primarily by the asymmetry coefficient. In this case, the analysis of the presented empirical values of the ratios of variances of estimates $\hat{g}_{2(Var)}$ indicates the adequacy of analytical calculations, since the maximum relative discrepancy between theoretical and experimental values is insignificant (at $N = 200$ does not exceed 3%).

Table 1. Relative efficiency of PMM-estimates at $S = 2$ compared to OLS-estimates (PMM estimates at $S = 1$)

Gamma distribution parameter (*shape*)	Theoretical values			Monte Carlo statistical simulation results								
	γ_3	γ_4	$g_{2(Var)}$	$\hat{g}_{2(Var)}$			$\hat{g}_{2(MSE)}$			$\hat{g}_{2(Bias)}$		
				N			N			N		
				50	100	200	50	100	200	50	100	200
1	2	6	0.5	0.48	0.46	0.47	0.37	0.41	0.45	0.43	0.41	0.44
2	1.4	3	0.6	0.65	0.59	0.59	0.55	0.55	0.57	0.6	0.55	0.56
4	1	1.5	0.71	0.78	0.73	0.7	0.69	0.7	0.68	0.74	0.71	0.69

Table 2. Relative efficiency of PMM-estimates at $S = 2$ comparison with estimates based on the Yule-Walker equations

Gamma distribution parameter (*shape*)	Theoretical values			Monte Carlo statistical simulation results								
	γ_3	γ_4	$g_2(Var)$	$\hat{g}_2(Var)$			$\hat{g}_2(MSE)$			$\hat{g}_2(Bias)$		
				N			N			N		
				50	100	200	50	100	200	50	100	200
1	2	6	0.5	0.49	0.45	0.47	0.35	0.38	0.42	0.34	0.34	0.36
2	1.4	3	0.6	0.67	0.62	0.58	0.55	0.54	0.54	0.48	0.48	0.45
4	1	1.5	0.71	0.8	0.74	0.7	0.59	0.65	0.66	0.6	0.57	0.57

Table 3. Relative efficiency of PMM-estimates at $S = 2$ compared to maximum likelihood estimates (Gaussian model).

Gamma distribution parameter (*shape*)	Theoretical values			Monte Carlo statistical simulation results								
	γ_3	γ_4	$g_2(Var)$	$\hat{g}_2(Var)$			$\hat{g}_2(MSE)$			$\hat{g}_2(Bias)$		
				N			N			N		
				50	100	200	50	100	200	50	100	200
1	2	6	0.5	0.48	0.45	0.47	0.37	0.41	0.44	0.42	0.41	0.44
2	1.4	3	0.6	0.67	0.59	0.58	0.55	0.55	0.56	0.58	0.55	0.56
4	1	1.5	0.71	0.79	0.73	0.7	0.69	0.7	0.68	0.72	0.71	0.69

5 Experiment with Real Data

As an example of approbation of the developed approach to finding the PMM-estimates of parameters of AR models we used a set of time series data in the form of daily changes in the Dow Jones index (127 values from July 01, 2002 to December 31, 2002, Fig. 1a). Similar statistical data have been analyzed in [16], according to which for many rather long periods of time there is a slowly decaying positive selective autocorrelation function. And to ensure stationarity, it is advisable to consider the behavior of differential changes in this index (see Fig. 1b). As shown in Fig. 1c the resulting time series is adequately described by a first order AR model. However, the distribution of the random component differs from the Gaussian law and has a significant asymmetry of the distribution (Fig. 1d).

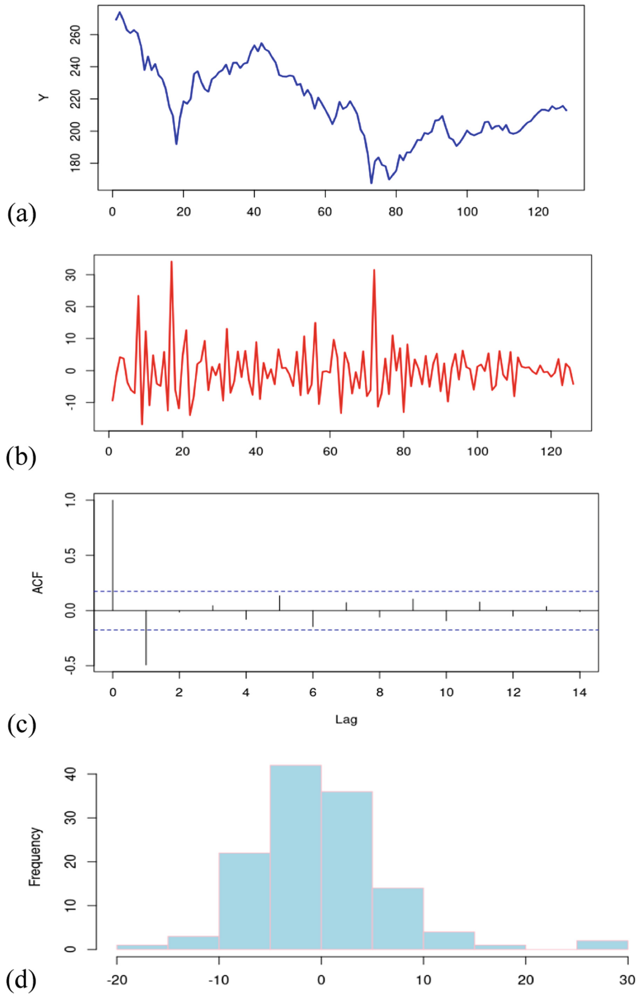

Fig. 1. Dow jones index: (a) raw data; (b) differential sequence; (c) autocorrelation function; (d) residuals histogram

The linear estimates of the informative parameter obtained by the classical least squares methods, the method based on the Yule-Walker equations, the maximum likelihood method (optimized for the Gaussian model) do not practically differ and are $\hat{a}_1^{(1)} = -0.49$. The estimate of this parameter by the quadratic modification by the polynomial maximization method is $\hat{a}_1^{(2)} = -0.43$. In this case, the values of the numerical characteristics of AR residues (skewness $\hat{\gamma}_3 = 1.1$ and kurtosis $\hat{\gamma}_4 = 3.7$) indicate a significant difference from the Gaussian distribution and permit to estimate potential asymptotic gain of the proposed approach at the level $\hat{g}_{2(Var)} = 0.77$.

Thereby, the practical importance of considering the non Gaussianity of real statistical data (in the form of asymmetry and kurtosis of residuals) consists in obtaining autoregressive models with less uncertainty of its parameters. This improves the predictive accuracy of the resulting models accordingly.

6 Conclusions

The totality of the results obtained confirms the possibility of effective application of the polynomial maximization method for solving the problems of finding estimates of the informative parameters of autoregressive models for those situations when the nature of innovations (random component) has an asymmetric distribution.

Overall, the proposed approach to finding parameter estimates can be interpreted as adaptive and compromise in terms of practical implementation, as the proposed computational procedures potentially have less analytical complexity compared to the approach based on the maximum likelihood method and provide an improved accuracy (reduced variance of estimates) compared to linear estimates (least squares, Yula-Walker, etc.).

The next stage of the research should be to compare the effectiveness of the adaptive estimates of the polynomial maximization method and maximum likelihood, optimized for the corresponding non-Gaussian distributions.

References

1. Grunwald, G.K., Hyndman, R.J., Tedesco, L., Tweedie, R.L.: Theory and methods: non-Gaussian conditional linear AR (1) models. Aust. N. Z. J. Stat. **42**(4), 479–495 (2000)
2. Ozaki, T., Iino, M.: An innovation approach to non-Gaussian time series analysis. J. Appl. Probab. **38**(A), 78–92 (2001)
3. Bondon, P.: Estimation of autoregressive models with epsilon-skew-normal innovations. J. Multivar. Anal. **100**, 1761–1776 (2009)
4. Hürlimann, W.: On non-Gaussian AR(1) inflation modelling. J. Stat. Econom. Methods. **1**(1), 93–109 (2012)
5. Nguyen, H.D., McLachlan, G.J., Ullmann, J.F.P., Janke, A.L.: Laplace mixture autoregressive models. Stat. Probab. Lett. **110**, 18–24 (2016)
6. Akkaya, A.D., Tiku, M.L.: Time series AR(1) model for short-tailed distributions. Statistics **39**(2), 117–132 (2005)
7. Tikhonov, V.: Generalized autoregressive model of non-Gaussian processes. Radiotekhnika **132**, 78–82 (2003). (in Russian)
8. Swami, A., Mendel, J.M., Nikias, C.: Higher-order spectral analysis toolbox. In: MATLAB User Guide. The Math Works Inc. (2001)
9. Al-Smadi, A.: A new coefficient estimation method for autoregressive systems using cumulants. Int. J. Circuit Theory Appl. **29**(5), 511–516 (2001)
10. Kunchenko, Y.: Polynomial Parameter Estimations of Close to Gaussian Random variables. Shaker, Aachen (2002)
11. Warsza, Z.L., Zabolotnii, S.: Estimation of measurand parameters for data from asymmetric distributions by polynomial maximization method. In: Szewczyk, R., Zieliński, C., Kaliczyńska, M. (eds.) Automation 2018. AISC, vol. 743, pp. 746–757. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-77179-3_74

12. Zabolotnii, S.W., Warsza, Z.L.: Semi-parametric estimation of the change-point of parameters of non-Gaussian sequences by polynomial maximization method. In: Szewczyk, R., Zieliński, C., Kaliczyńska, M. (eds.) Challenges in Automation, Robotics and Measurement Techniques. AISC, vol. 440, pp. 903–919. Springer, Cham (2016). https://doi.org/10.1007/978-3-319-29357-8_80
13. Zabolotnii, S.W., Warsza, Z.L., Tkachenko, O.: Estimation of linear regression parameters of symmetric non-Gaussian errors by polynomial maximization method. In: Szewczyk, R., Zieliński, C., Kaliczyńska, M. (eds.) Automation 2019, vol. 920, pp. 636–649. Springer, Cham (2020). https://doi.org/10.1007/978-3-030-13273-6_59
14. Zabolotnii, S., Tkachenko, O., Warsza, Z.L.: Application of the polynomial maximization method for estimation parameters in the polynomial regression with non-Gaussian residuals. In: Szewczyk, R., Zieliński, C., Kaliczyńska, M. (eds.) Automation 2021: Recent Achievements in Automation, Robotics and Measurement Techniques. AISC, vol. 1390, pp. 402–415. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-74893-7_36
15. Bondon, P., Song, L.: AR processes with non-Gaussian asymmetric innovations. In: European Signal Processing Conference, pp. 1–5 (2013)
16. Brockwell, P.J., Davis, R.A.: Introduction to Time Series and Forecasting. Springer, New York (1996). https://doi.org/10.1007/b97391