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Polynomial Maximization Method for Estimation Parameters of Asymmetric Non-gaussian Moving Average Models

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Abstract. This paper considers the application of the Polynomial Maximization Method to find estimates of the parameters non-Gaussian Moving Average model. AQ1 This approach is adaptive and is based on the analysis of higher-order statistics. Case of asymmetry of the distribution of Moving Average processes is considered. It is shown that the asymptotic variance of estimates of the Polynomial Maximization Method (2nd order) analytical expressions that allow finding estimates and analyzing their uncertainty are obtained. This approach can be significantly less than the variance of the classic estimates based on minimize Conditional Sum of Squares or Maximum Likelihood (in Gaussian case). The increase in accuracy AQ2 depends on the values of the coefficient's asymmetry and kurtosis of residuals. The results of statistical modeling by the Monte Carlo Method confirm the effectiveness of the proposed approach.

Keywords: Estimation parameters · Moving average · Polynomial maximization · High-order statistics · Non-Gaussian processes

1 Introduction

It is known that MA (Moving Average) processes are a particular case of a broader class of ARMA (Auto-Regressive Moving Average) time series models. Initially they were developed to solve the problems of predicting the behavior of dynamic objects, they have subsequently found the widest spread for predicting geophysical, financial, biomedical, and other processes. For quite a long time, the theory of synthesis and analysis of time series models has been developing within the framework of linear filtration theory, theoretical foundations of which were laid in works of Wiener [1] and Kalman [2]. From the mathematical point of view, the linear predictor (independent variable) is optimal only when the processes have normal law of probability distribution. However, many researchers note the idealization of such an assumption, which, although simplifies the final solution, often does not correspond to realities of many practical problems [3–8].

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From the statistical point of view, a significant difference between the prediction errors (residuals) of the fitted model and the Gaussian idealization leads to an increase in the uncertainty of the resulting estimates of the parameters of such models. There are several ways to increase the estimation accuracy. One of them is the parametric approach based on the use of M-estimates obtained on the basis of the Maximum Likelihood Method (MML) or its various modifications. There are many distribution types used to describe the random component of ARMA models. For example, in [3] processes with gamma and lognormal innovations are studied, in [4] - Student's distribution, in [5] - mixtures of normal and Poisson distribution, in [6] - family of Exponential Power distributions, in [7] - poly-mixtures based on Laplace distributions, in [8] - Cauchy distribution, etc.

One of the key elements of the parametric approach is the need to solve the problem of identifying the type and finding estimates of the probability distribution parameters of the random component of time series models. A number of methods for joint [3] or iterative and adaptive estimation have been developed [9, 10].

An alternative approach to accounting for non-Gaussian processes is to use the apparatus of higher-order statistics (moments, cumulants or their functions). This approach is characterized by a substantial reduction in the level of necessary a priori information, as well as by algorithmic simplification in practical implementation. The price for this is sub-efficiency of obtained solutions, which is caused by partiality of such description (in comparison with probability density function). Examples of the use of this approach for solving the problems of identification of various predictive models and estimation of their informative parameters can be found in [11–15].

In this article we propose to use the Polynomial Maximization Method (PMM) [16] to find estimates of parameters of MA-models. This method, similarly, to MML, uses the principle of maximizing the functional from the sampled data in the vicinity of the true value of the estimated parameters. However, to form such a functional, not the density of distribution, but a description in the form of higher-order statistics is used. This study is a continuation of the works [17–19] in which the application of PMM to the problem of parameter estimation of linear and polynomial regression and autoregressive model at asymmetrically distributed non-Gaussian statistical data is considered.

2 Mathematical Formulation of the Problem

Consider a vector $\boldsymbol{X} = \{x_1, x_2, ..., x_N\}$ containing the values of the time series described by the moving average model.

$$x_v = \sum\nolimits_{q=1}^{Q} b_q \xi_{v-q} + \xi_v, \ v = \overrightarrow{1, N}, \tag{1}$$

where the samples ξ_v are a sequence of equally distributed independent random variables with zero mathematical expectation. Their distribution differs from the Gaussian (normal) law and has substantial asymmetry. An additional limitation is that the random variable ξ has finite moments μ_r up to order 4th. The general task is to find estimates of the vector parameter $\boldsymbol{\theta} = \left\{b_1, ...b_Q\right\}$ based on statistical analysis of the set of samples $\mathbf{X}$. In this case, the probability distribution law of a random variable ξ is a priori unknown.

3 Theoretical Results

The idea of using PMM to estimate the parameters of MA models is based on the mathematical analogy between model (1) and linear (by parameters) regression. Since the samples of a random variable ξ are statistically independent, the set of ξ_{v-q}, values can be formally treated as predictors of a multivariate linear regression of the current value of series x_v. In this way, to solve this problem we can use the results of [17], which deals with the application of PMM to find estimates of the vector parameter of a linear regression. Note that this estimation method is based on the property of maximization of the functional in the form of the stochastic polynomial

$$L_{SN} = \sum\nolimits_{v=1}^{N} \sum\nolimits_{i=1}^{S} \phi_i(x_v) \int^b k_{i,v}(a)dz - \sum\nolimits_{i=1}^{S} \sum\nolimits_{v=1}^{N} \int^b \Psi_{i,v} k_{i,v}(a)dz, \quad (2)$$

where

$$\Psi_{i,v} = E\{\phi_i(x_v)\},\ i = \overrightarrow{1,S},\ v = \overrightarrow{1,N} \quad (3)$$

in the proximity of the true value of the estimated parameter b.

If we use the power transformations $\phi_i(x_v) = (x_v)^i$ as basic functions, then the sequence of mathematical expectations (3) is a set of initial moments $\alpha_{i,v}$ of appropriate order. By analogy with the maximum likelihood method, the estimation of the parameter b can be found from the solution of the equation of the form

$$\left.\frac{d}{db} L_{SN}\right|_{b=\hat{b}} = \sum\nolimits_{i=1}^{S} \sum\nolimits_{v=0}^{N} k_{i,v} \left[(x_v)^i - \alpha_{i,v} \right] \Big|_{b=\hat{b}} = 0, \quad (4)$$

where the optimal coefficients $k_{i,v}$, which maximize the functional (2), are found from the solution of the system of linear algebraic equations

$$\sum\nolimits_{j=1}^{S} k_{j,v} F_{(i,j)v} = \frac{d}{db} \alpha_{i,v},\ i = \overrightarrow{1,S},\ v = \overrightarrow{1,N}, \quad (5)$$

where $F_{(i,j)v} = \alpha_{(i+j),v} - \alpha_{i,v}\alpha_{j,v},\ i,j = \overrightarrow{1,S}$.

This approach can be easily extended to the case of finding estimates of the vector parameter $\boldsymbol{\theta} = \{b_1, ...b_Q\}$ or this purpose, it is necessary to form Q polynomials of the general form (2) for each component of the vector parameter. Thus, the desired estimates can be found as a solution to a system of equations of the form

$$\sum\nolimits_{i=1}^{S} \sum\nolimits_{v=1}^{N} k_{i,v}^{(q)} \left[(x_v)^i - \alpha_{i,v} \right] \Big|_{b_q=\hat{b}_q} = 0,\ q = \overrightarrow{1,Q}. \quad (6)$$

Analysis of (6) shows that the required estimates of the parameter vector are found from the condition of equality to zero of the sums of weighted by optimal coefficients (minimizing the variance of estimates at the used degree of the stochastic polynomial) differences of theoretical and empirical values of moments of the observed statistical data. The main difficulty is the lack of a priori information about the theoretical values of the first $2S$ initial moments of $\alpha_{i,v}$, which depend both on the estimated parameters $\boldsymbol{\theta} = \{b_1, ...b_Q\}$ and on the moments μ_r of the random variable ξ.

As in [17], an adaptive approach can be applied to solve this problem. It consists in replacing the a priori μ_r values by a posteriori $\hat{\mu}_r$ estimates. They can be calculated based on the analysis of the residuals ε obtained after estimating the MA model parameters by the maximum likelihood method (under the assumption of distribution normality) or an iterative modification of the least squares method (minimization of conditional sums of squares). If the type of MA model and its order are identified correctly, then the sequence of residuals ε will be a random process of "white noise" type, the probabilistic properties of which are close to the distribution of random variable ξ. The values of uncorrelated ε_{v-q} samples are used as repressors in the original model (1).

In [17] it is shown that for the linear version of PMM (when using a stochastic polynomial of degree S = 1) the resulting system of Eq. (6) for estimating parameters of regression models is equivalent to the system of Least Squares Method (LSM) equations. However, it is known that the efficiency of such LSM estimations significantly decreases when the distribution of the random component of the regression model differs from the Gaussian law. Therefore, let us consider below a new approach to nonlinear estimation of MA model parameters based on the use of quadratic power stochastic polynomials.

When stochastic polynomials of order $S = 2$ are used, PMM estimates of MA model (1) can be found from solution of the system of equations

$$\sum_{v=1}^{N}\left\{k_{1,v}^{(q)}\left[x_v-\sum_{q=1}^{Q}b_q\varepsilon_{v-q}\right]+k_{2,v}^{(q)}\left[(x_v)^2-\left(\sum_{q=1}^{Q}b_q\varepsilon_{v-q}\right)^2-\mu_2\right]\right\}=0,\ \ q=\overrightarrow{1,Q}. \tag{7}$$

where: optimal coefficients $k_{i,v}^{(q)}$, $i=\overrightarrow{1,2}$ provide minimization of dispersion of estimates of components of required parameter using degree of polynomial $S = 2$. These coefficients are found as a solution of the corresponding system of the form (5) and can be represented as

$$k_{1,v}^{(q)}=\frac{\mu_4-\mu_2^2+2\mu_3\sum_{q=1}^{Q}b_q\varepsilon_{v-q}}{\mu_2\left(\mu_4-\mu_2^2\right)-\mu_3^2}\varepsilon_{v-q},\ k_{2,v}^{(q)}=-\frac{\mu_3^2}{\mu_2\left(\mu_4-\mu_2^2\right)-\mu_3^2}\varepsilon_{v-q}, \tag{8}$$

By substituting the coefficients (8) into (7), after certain transformations the system of equations for finding the estimates can be written

$$\sum_{v=1}^{N}\left\{\varepsilon_{v-p}\left[A\left(\sum_{q=1}^{Q}b_q\varepsilon_{v-q}\right)^2+B_v\sum_{q=1}^{Q}b_q\varepsilon_{v-q}+C_v\right]\right\}=0,\ \ q=\overrightarrow{1,Q}, \tag{9}$$

where

$$A=\mu_3,\ B_v=\mu_4-\mu_2^2-x_v\mu_3,\ C_v=x_v^2\mu_3-x_v\left(\mu_4-\mu_2^2\right)-\mu_2\mu_3 \tag{10}$$

depend on the sample values x_v and moments $\mu_2-\mu_4$ of the random component ξ of the model (1).

Obviously, at the degree of the polynomial $S = 2$ PMM-estimates can be found only numerically.

It has been proved [16] that nonlinear PMM parameter estimates are consistent and asymptotically unbiased. Expressions describing the variance of PMM-estimates for the asymptotic case (at $N \to \infty$) can be obtained as elements of the main diagonal of the variation matrix, which is the inverse of the matrix composed of elements

$$J_{SN}^{(q,p)} = \sum\nolimits_{v=1}^{N} \sum\nolimits_{i=1}^{S} k_{i,v}^{(q)} \frac{\partial}{\partial b_p} \alpha_{i,v}, \; q, p = \overrightarrow{1, Q}. \tag{11}$$

In a statistical sense, the amount of information extracted is conceptually close to the amount of information according to Fisher and tends to its limit value at the degree of polynomial $S \to \infty$ [16].

As shown in [17], the asymptotic values of variance of PMM-estimates of vector parameter, obtained by using the degree of polynomial $S = 1$, coincide with LSM-estimates, as well as MML-estimates (at normally distributed data). Therefore, a dimensionless coefficient

$$g_S^{(b_q)} = \frac{\sigma^2_{(b_q)S}}{\sigma^2_{(b_q)1}}. \tag{12}$$

can be used as a criterion of efficiency of PMM-estimates obtained by using polynomials of some degree S. This coefficient is the same for all components of the estimated vector parameter $\boldsymbol{\theta}$ and with $S = 2$ can be represented as

$$g_2^{(b_q)} = 1 - \frac{\mu_3^2}{\mu_2\left(\mu_4 - \mu_2^2\right)} = 1 - \frac{\gamma_3^2}{2 + \gamma_4}. \tag{13}$$

The transition in expression (13) from the moment description to the cumulant description is due to the well-known fact that the deviation of the values of the cumulant coefficients of higher orders $\gamma_r = \kappa_r \big/ \kappa_2^{r/2}$ from zero shows the degree of difference from the Gaussian distribution. Given the inequality $\gamma_4 + 2 \geq \gamma_3^2$, we can conclude that the variance reduction coefficient g_2 is a dimensionless value which belongs to the range (0; 1]. Thus, as the asymmetry of the ξ distribution increases, the relative reduction of the variance can be quite significant.

4 Statistical Modeling

To verify the theoretical results, we modified a set of functions written in R, which implemented the procedure of multiple Monte Carlo tests of finding polynomial estimates of regression and autoregressive models [13–15]. Now it additionally allows to carry out the comparative analysis of accuracy of different methods of estimation of parameters of moving average models, the random component of which has non-Gaussian distribution.

When implementing this statistical modeling, two models were used as an object of research: MA(1) with parameter $b_1 = 0.4$ and MA(2) with parameters $b_1 = 0.4$; $b_2 = -0.2$. The value of informative parameters was estimated by means of built-in function R arima ( ), using two classical methods: “CSS” (minimization of Conditional Sum of Squares) and “ML” Maximum Likelihood (optimized for Gaussian model), as well as custom quadratic modification of Polynomial Maximization Method. Sequences of independent and equally distributed random variables with gamma distribution with different values of shape parameter were used as asymmetric random component of MA model, which were determined for the degree of asymmetry. The parameters (moments up to the 4th order) of the random component, which are necessary for finding adaptive PMM-estimates, were considered to be unknown a priori and instead of them a posteriori estimates were used

$$\hat{\mu}_r = \frac{1}{N}\sum\nolimits_{v=1}^{N} (\varepsilon_v)^i. \tag{14}$$

calculated from the residuals from the application of classical methods.

The sum of the experimental values of the efficiency coefficients obtained for the series $M = 10^4$ of multiple experiments are presented in Tables 1, 2, 3 and 4.

Table 1. Relative efficiency of the PMM estimates (S = 2) of parameters MA(1) model compared to CSS estimates

Gamma distribution parameter *shape*	Theoretical values			Monte Carlo statistical simulation results								
	γ_3	γ_4	g_2	$N = 50$			$N = 100$			$N = 200$		
				$\hat{\gamma}_3$	$\hat{\gamma}_4$	$\hat{g}_2^{(b_1)}$	$\hat{\gamma}_3$	$\hat{\gamma}_4$	$\hat{g}_2^{(b_1)}$	$\hat{\gamma}_3$	$\hat{\gamma}_4$	$\hat{g}_2^{(b_1)}$
1	2	6	**0.5**	1.6	3.1	**0.95**	1.8	4.1	**0.59**	1.9	4.9	**0.57**
2	1.4	3	**0.6**	1.2	1.6	**0.79**	1.3	2.2	**0.7**	1.3	2.5	**0.69**
4	1	1.5	**0.71**	0.8	0.8	**0.81**	0.9	1.1	**0.82**	1	1.3	**0.83**

Table 2. Relative efficiency of PMM-estimates (S = 2) of parameters MA(1) model compared to MML-estimates (Gaussian model).

Gamma distribution parameter *shape*	Theoretical Values			Monte Carlo statistical simulation results								
	γ_3	γ_4	g_2	$N = 50$			$N = 100$			$N = 200$		
				$\hat{\gamma}_3$	$\hat{\gamma}_4$	$\hat{g}_2^{(b_1)}$	$\hat{\gamma}_3$	$\hat{\gamma}_4$	$\hat{g}_2^{(b_1)}$	$\hat{\gamma}_3$	$\hat{\gamma}_4$	$\hat{g}_2^{(b_1)}$
1	2	6	**0.5**	1.6	3.1	**0.7**	1.8	4.1	**0.57**	1.9	4.8	**0.56**
2	1.4	3	**0.6**	1.1	1.6	**0.74**	1.3	2.1	**0.69**	1.3	2.5	**0.68**
4	1	1.5	**0.71**	0.8	0.8	**0.78**	0.9	1.1	**0.79**	0.9	1.3	**0.83**

The analysis of the empirical values of the efficiency coefficients presented in Tables 1, 2, 3 and 4 shows that polynomial estimates of the informative parameters studied by MA models are more accurate compared to classical estimates. The range of variance reduction is quite wide: from units of percent to twice the value. At the same time, the trends of accuracy changes depending on the degree of non-Gaussianity (numerically expressed by the value of skewness and kurtosis coefficients) in general

Table 3. Relative efficiency of the PMM estimates (S = 2) of parameters MA(2) model compared to CSS estimates

Gamma distribution parameter *shape*	Theoretical values			Monte Carlo statistical simulation results								
	γ_3	γ_4	g_2	$N = 50$			$N = 100$			$N = 200$		
				$\hat{\gamma}_3$	$\hat{\gamma}_4$	$\hat{g}_2^{(b_1)}$	$\hat{\gamma}_3$	$\hat{\gamma}_4$	$\hat{g}_2^{(b_1)}$	$\hat{\gamma}_3$	$\hat{\gamma}_4$	$\hat{g}_2^{(b_1)}$
						$\hat{g}_2^{(b_2)}$			$\hat{g}_2^{(b_2)}$			$\hat{g}_2^{(b_2)}$
1	2	6	**0.5**	1.5	2.8	**0.98**	1.7	4.1	**0.55**	1.9	4.9	**0.49**
						0.66			**0.68**			**0.67**
2	1.4	3	**0.6**	1.1	1.5	**0.57**	1.2	2.1	**0.59**	1.3	2.5	**0.61**
						0.75			**0.75**			**0.79**
4	1	1.5	**0.71**	0.8	0.8	**0.69**	0.9	1	**0.74**	0.9	1.2	**0.73**
						0.81			**0.87**			**0.91**

Table 4. Relative efficiency of the PMM estimates ($S = 2$) of parameters MA(2) model compared to MML-estimates (Gaussian model)

Gamma distribution parameter *Shape*	Theoretical values			Monte Carlo statistical simulation results								
	γ_3	γ_4	g_2	$N = 50$			$N = 100$			$N = 200$		
				$\hat{\gamma}_3$	$\hat{\gamma}_4$	$\hat{g}_2^{(b_1)}$	$\hat{\gamma}_3$	$\hat{\gamma}_4$	$\hat{g}_2^{(b_1)}$	$\hat{\gamma}_3$	$\hat{\gamma}_4$	$\hat{g}_2^{(b_1)}$
						$\hat{g}_2^{(b_2)}$			$\hat{g}_2^{(b_2)}$			$\hat{g}_2^{(b_2)}$
1	2	6	**0.5**	1.5	2.9	**0.9**	1.7	4	**0.48**	1.9	4.9	**0.51**
						0.69			**0.64**			**0.67**
2	1.4	3	**0.6**	1.1	1.5	**0.62**	1.2	2.1	**0.63**	1.3	2.5	**0.61**
						0.76			**0.78**			**0.8**
4	1	1.5	**0.71**	0.7	0.8	**0.7**	0.9	1.1	**0.71**	0.9	1.3	**0.73**
						0.82			**0.88**			**0.92**

correlate with the theoretical dependence (13). Significant differences are observed only for small values of the sample size of statistical data N. This can be explained by the factor that with a small amount of statistical data, the posterior estimates of the parameters $\hat{\gamma}_3$ and $\hat{\gamma}_4$ have a fairly high variance, as well as a significant bias (as evidenced by the tabular data). However, with N growth, this problem is leveled, and the experimental values asymptotically approach the theoretical ones.

5 Conclusions

The set of obtained results confirms the possibility of effective application of the Polynomial Maximization Method for solving problems of finding estimates of informative parameters of Moving Average models for those situations when the nature of the random component has a non-Gaussian asymmetric distribution.

In general, the proposed approach to finding parameter estimates can be interpreted as adaptive and compromise in terms of practical implementation. The algorithm for obtaining polynomial estimates does not require a priori knowledge of the probability distribution law. To implement it, it is sufficient to obtain information about the values of a limited set of higher-order statistics. Consequently, it has significantly less implementation complexity compared to the approach based on the Maximum Likelihood Method. At the same time, polynomial estimates are characterized by higher accuracy (according to the variance ratio criterion) compared to the estimates of classical methods optimized for Gaussian probability model.

The next research tasks in this direction may be:

- consideration of the option of estimating parameters of MA models with symmetry of non-Gaussian statistical data distribution
- comparison of the efficiency of adaptive estimates of the Polynomial Maximization Method and Maximum Likelihood Method optimized for the corresponding non-Gaussian distributions
- polynomial estimation of parameters of more complex types of non-Gaussian time series models (ARMA, GARH, etc.).

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