Experiment 6: Harmonic Oscillator Part II. Physical Pendulum and Experiment 7: Waves on a Vibrating String

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Exploring harmonic oscillation in the physical pendulum and vibrating string systems

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Abstract

We explore the harmonic oscillation with two models: the first one being a physical pendulum swinging at small angle and the second one being a horizontal vibrating string. We study the oscillation of the pendulum with a rotation sensor under three conditions: undamped free motion, damped but undriven motion, and damped motion driven by a wave driver. The graph showing underdamped, critically damped and overdamped is plotted. The resonance frequency and the Q factor of the system is found. The oscillation of the string is studied by measuring the light intensity of a laser beam spot on the string. The vibration of the string is also driven by a wave driver. We found the wave speed on the stretched string and the corresponding resonance frequency for different order of normal mode of the vibration.

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Introduction

Damped oscillation happens when dissipative force causes the amplitude of oscillation to gradually reduce to 0. By applying a driving force to the damped system, we can maintain a constant-amplitude oscillation. Such motion is called forced oscillation or driven oscillation. When the frequency of the driving force is close to the system's natural frequency, the amplitude of oscillation reaches its peak. At this point, the driving frequency is called the resonance frequency.[1]

The first part of the experiment studies the damped and driven oscillation with the physical pendulum model. To record the underdamped, critically damped and overdamped motion, we gradually increase the damping force and record the rotation angle vs. time graphs. We expect the resonance frequency ω_R of the system to be close to the natural frequencyω_0. Therefore, we measured ω_0 of the pendulum, then use two ways to measure the actual ω_R. The first way is finding the driving frequencies ω_d corresponding to a symmetrical Lissajous graph. The second way is trying different ω_d and record the amplitudes of the oscillation. The ω_R is the ω_d corresponding to the peak amplitude. The two measured ω_R are then compared to each other and to the ω_0 to verify our prediction. Based on the two ω_R, the Q-factor of the pendulum system is also calculated.

The second part we study the driven oscillation on a vibrating string, on which every point is an oscillator. We first predict the wave speed v on strings with different mass density, then measure v by measuring the intensity of a light spot on a point of the string. Unlike the pendulum, the string has multiple ω_R corresponding to different orders of normal mode, in which all the points on the string move sinusoidally with the same frequency[1]. We will predict the ω_R for different normal modes with v measured, then measure the ω_R by plotting the Lissajous figure for a light spot on the string.

Method

Physical Pendulum

To set up the pendulum, we used a wave driver, which provides driving force when damped, driven oscillation is studied. The driving voltage and frequency can be adjusted through the DAQ system. A rotation sensor is used to record the rotation angle of the pendulum. The recorded angle and time stamps will be exported to the DAQ system for further analysis. A pendulum with long length is used to ensure the small angle approximation is valid. A set of damping magnets is used to provide the damping force. The gap between two magnets can be adjusted to provide different damping force. The figure below demonstrates how these apparatuses are set up for the experiment.

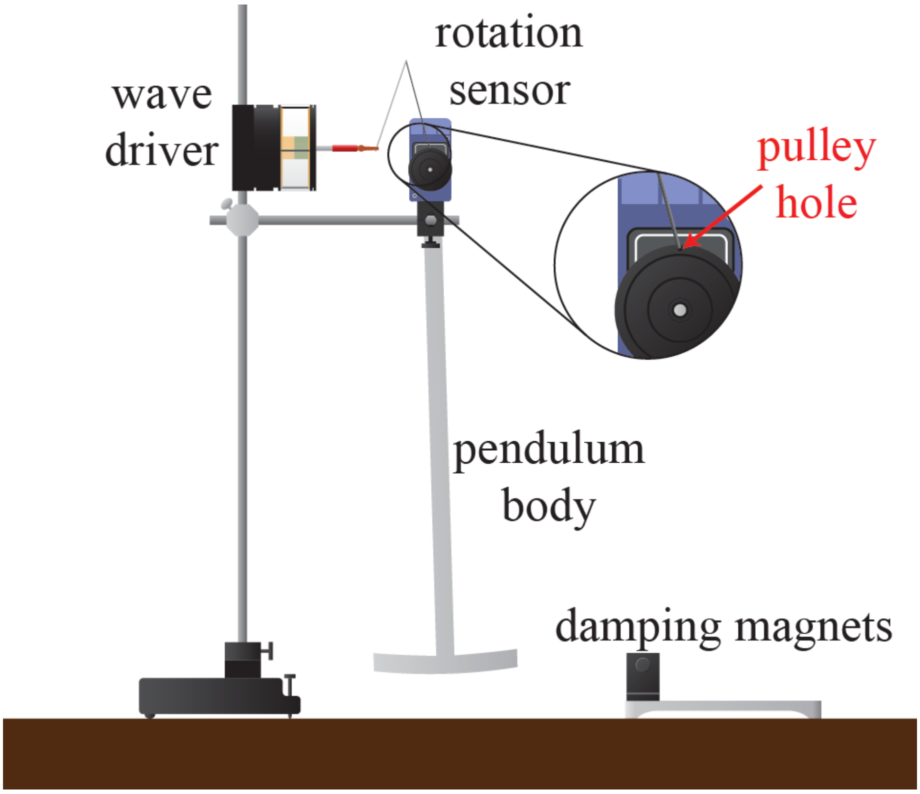

Figure 1. The set-up of the pendulum. The wave driver and the rotation sensor are connected by a V-shaped metal torsion spring so that the driving force can be passed on to the pendulum. The pendulum body is hanged below the rotation sensor. When damping force is needed, the damping magnets will be moved to the position that the blade of the pendulum is in between the two magnets. When the pendulum is set to oscillate, the maximum displacement is no longer than the length of the blade. Figure reproduced (with permission) from Fig. 6.1 by Campbell, W. C. *et al*.[2]

To study the nature of oscillation of this system, we start by measuring the frequency of the simplest motion, the undamped oscillation. The wave driver is turned off and damping magnets are removed. The pendulum is set to a small angle, then released. The rotation sensor records the time and rotation angle for 30 seconds.

Then, we will study the undriven, damped oscillation. To find overdamped, underdamped and critically damped oscillation, the pendulum is put between the damping magnets and set to oscillate when the gap between the magnets are 10mm, 20mm, 30mm, 40mm and 50mm. For each trial, the pendulum is released at the same position. The rotation sensor records the angle and time and the angle vs. time graph is plotted, with which we determine the nature of the oscillation. Normally, the 5 different gap widths give us the underdamped and overdamped oscillation. To find critically damped motion, the gap between magnets is then adjusted 1mm every time until the graph shows the critically damped property. The width of the gap is then recorded.

Then, to find the resonance frequency of the system, we adjust the gap between the damping magnets so that the oscillation damps out in 5-10 seconds. The damped, undriven oscillation for this gap width is recorded by the rotation sensor for analyzing the damping time constant. The DAQ system is set to plot the rotation angle vs. output voltage graph, also known as the Lissajous figures. The output voltage of the wave driver is set to 3V through DAQ system. The wave driver is then turned on with a certain frequency. To avoid systematic error, only until the oscillation achieves steady states after 10 seconds is the data recording started. Different frequencies are applied to get graphs of frequency lower than resonance frequency, higher than resonance frequency and at resonance frequency.

Finally, 10 different frequencies of the wave driver are applied, and the rotation angle is recorded for each trial. The amplitude of each frequency is calculated for later analysis and prediction of the resonance frequency.

Waves on a Vibrating String

To set up the vibrating string, we use a photodiode, a laser beam generator, a wave driver, an elastic string, a clamp, a pulley, and a mass. The clamp and the mass are used to fix the string at certain position. The wave driver will again provide driving force. The frequency, amplitude and the waveform can be adjusted through the DAQ system. The laser beam will project a light spot on the string, which will change its intensity as the string vibrate. The photodiode is used to detect the change in the light intensity.

Figure 2. The set-up of the vibrating string. The string is horizontally stretched. The other end of the string goes through the pulley and has a mass hanged on it. The wave driver is 1-2mm away from the clamp. The laser beam spot is 1cm away from the pully apex. Figure reproduced (with permission) from Fig. 7.1 by Campbell, W. C. *et al.*[2]

Firstly, we will study the wave speeds on the string stretched by 3 different masses. The mass and unstretched total length of the string is first measured. Then the string is fixed with a clamp. When no mass is hanged, the unstretched useful length from the clamp to the other end is measured. For each trial, the mass is then hanged and set to still so that it won't be swinging and causing the string to vibrate. The stretched the length from the clamp to the mass is then measured. The vertical section of the string is also measured. The wave driver is then turned on with frequency 0.250Hz to produce square waves. The photodiode records the intensity of the laser light spot for 15 seconds and the data is exported to the DAQ system. After this, another mass is hanged, and the above steps are repeated. In the three trials we use masses 200g, 300g and 400g.

Then, the 400g mass is hanged on the other end of the string. The wave driver is changed to produce sinusoidal waves. The frequency of the wave driver is slowly changed until the string forms first to seventh normal mode. The Lissajous figure is again plotted to determine the precise frequency for each order of normal mode. The

frequencies for corresponding order of normal mode is then recorded.
Notice that we are assuming the wave start transmitting from the clamp to the pulley. However, the wave actually starts from the wave driver. To reduce this systematic error, the wave driver must be placed as close to the clamp as possible.

Analysis

Physical Pendulum

At the beginning of the experiment, we measured the natural angular frequency ω_0 of the system by taking the reciprocal of the average period of the oscillation. The period is measured by the time difference between the two maxima. We measure ω_0 to be:

$$\omega_0 = 4.65 \pm 0.07Hz$$

The uncertainty of ω_0 is calculated from the standard deviation of the period.
We then get three types of damped motion by changing the gap between the magnet damper. According to the lab manual[2], the damping satisfies the following relationship:

$\omega_0 > \frac{1}{\tau}$ correspond to underdamped oscillation,

$\omega_0 = \frac{1}{\tau}$ correspond to critically damped,

$\omega_0 < \frac{1}{\tau}$ correspond to overdamped oscillation,

where τ is the time taken for the amplitude to decrease by $\frac{1}{e}$.

Thus, we should expect that as the magnets get closer, the damping force increases, the τ decreases, the oscillation will change from underdamped to critically damped, then to overdamped. Next page shows the rotational angle vs. time graph we obtained for the three types of motion.

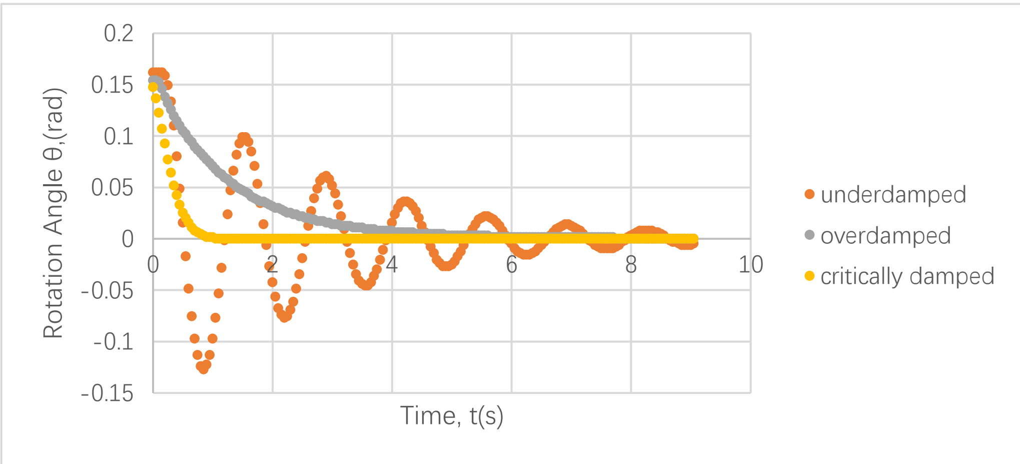

Figure 3. Underdamped, critically damped and overdamped oscillation. The three oscillations start at the same amplitude. The amplitude decreases the fastest at critically damped.

As we expected, as we reduce the distance between the magnets from 50mm to 10mm, the graph shift from underdamped to critically damped then to overdamped. The gap for critically damped oscillation is measured by a meter ruler:

$$D = 1.40 \pm 0.05 \text{ cm}$$

Where the uncertainty arises from the uncertainty of the ruler.

Then, we move on to find the resonance frequency of the driven oscillation. To predict the resonance frequency, we first measure the τ of the system to be:

$$\tau = 1.89 \pm 0.09\text{s}$$

Then, we apply the formula from the lab manual to predict the resonance frequency:

$$\omega_R = \sqrt{\omega_0{}^2 - 2\frac{1}{\tau^2}}$$

$$\omega_{R\,pred} = 4.59 \pm 0.07 Hz$$

The uncertainty is obtained by propagation of uncertainty from τ and ω_0.

To verify our prediction, we first used Lissajous figures to find the resonance frequency. When Lissajous figure becomes symmetrical, the system is at resonance, and the driving frequency is the resonance frequency.

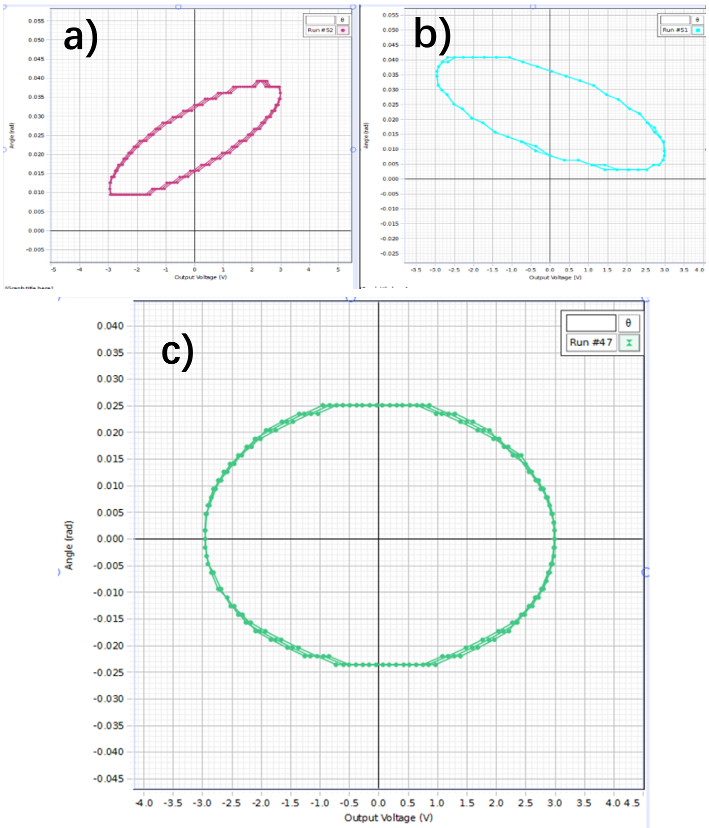

Figure 4. Finding resonance frequency with Lissajous figure. a) smaller than resonance frequency. b) greater than resonance frequency. c) at resonance frequency. When the frequency is not at resonance, the Lissajous figures are tilted. The symmetrical ellipse in c) shows that the driving frequency is the resonance frequency. The driving frequency is $f = 0.732 \pm 0.004\text{Hz}$.

When speaking of resonance frequency, we mean angular frequency. Therefore, to convert the resonance driving frequency to angular resonance frequency, we use the following equation:

$$\omega_{R1} = 2\pi f$$

We get:

$$\omega_{R1} = 4.60 \pm 0.02Hz$$

This value is very close to the predicted resonance frequency. If we take the error range into consideration, the measured value completely lies within the predicted range.

Another method we applied to measure the resonance frequency is the plot the amplitude vs. driving frequency curve. The driving frequency at the peak amplitude is the resonance frequency. To obtain the amplitude of the oscillation, we take half of the difference between the maxima and minima of the rotation angle. We took in total 10 maxima and minima and take the average amplitude.

The figure in next page shows the graph we plotted:

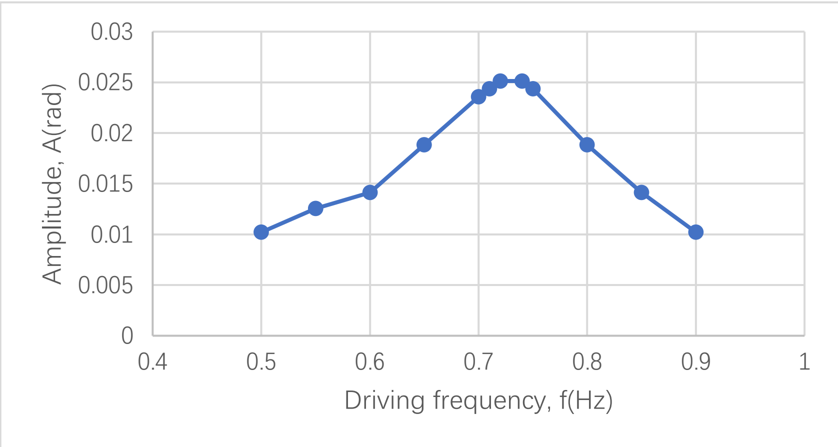

Figure 5. Finding resonance frequency by the amplitudes of oscillation. The 12 values ranging from 0.5-0.9Hz are used. The data points are connected by straight lines. The peak occurs between f=0.720Hz and f=0.740Hz. The two frequencies have the same amplitude. Thus, we deduce the resonance driving frequency to be $f = 0.73 \pm 0.01\text{Hz}$. The resonance angular frequency is $\omega_{R2} = 4.60 \pm 0.06Hz$

Again, the measured ω_R overlapped with the predicted value. This graph, however, is less precise as the Lissajous figure method. We cannot determine the exact peak of amplitude since the plot is simply connected by straight lines. Much information between the points are lost.

With the above measurement, we are also able to get the Q-factor of the system. Applying the formula from the lab manual[2]:$Q = \frac{1}{2}\tau\omega_R$, we get $Q_1 = 4.3 \pm 0.2$ by substituting the τ and ω_{R1} into it.

We may also get Q from Figure 5. When Q is substantially greater than 1, it can be approximate by the equation $Q \approx \frac{\omega_0}{\Delta\omega}$, where $\Delta\omega$ is the full width of the resonance, defined to be the frequency range over which the amplitude response is greater than or equal to $\frac{1}{\sqrt{2}}$ of its maximum value[2]. From Figure 5, $\Delta\omega = 1.09 \pm 0.04\text{Hz}$. The uncertainty arises from the difference between $\Delta\omega$ in Figure 5 and $\Delta\omega$ in a 9th order polynomial regression curve of the data points in Figure 5. Through this method, $Q_2 = 4.2 \pm 0.2$. The consistency between the two measured Qs indicates that our models of the pendulum system is correct. Meanwhile, Q_1 is probably more trustworthy than Q_2 as the measurement of $\Delta\omega$ is less precise due to the lack of data points.

Waves on a Vibrating String

To start exploring the oscillation on a vibrating string, we start by predicting the speed of wave on it with the formula $v = \sqrt{\frac{T}{\mu}}$, where T is the tension on the string and μ is the linear mass density of the string. To obtain μ, we measured the unstretched full length $l_0 = 2.269 \pm 0.002\text{m}$ and useful length $l = 2.003 \pm 0.002\text{m}$ of the string, as well as the mass $m = 0.01430 \pm 0.00005\text{kg}$ of the string. With these data, we can calculate the stretched mass of the string: $m' = \frac{ml}{l_0}$. Then, we measure the stretched length l' of the string for different masses. With the relationship $\mu = \frac{m'}{l'}$, we get the linear mass density of the string when being stretched by different masses. To get T, we measure the mass of vertical section of the string by multiplying its length by μ, then multiply the sum of this mass and the hanging mass with gravitational constant $g = 9.81\text{m/s}^2$.

The table below shows the T, μ, and v for different applied mass:

Mass, m(kg) $\delta m = 0.00005$	Stretched length, l'(m) $\delta l' = 0.002$	Linear mass density, μ(kg/m) $\delta\mu = 0.00005$	Tension, T(N) $\delta T = 0.0002$	Wave speed, v(m/s)
0.20000	2.039	0.00619	1.9903	17.9 ± 0.1
0.30000	2.069	0.00610	2.9717	22.0 ± 0.2
0.40000	2.099	0.00603	3.9530	25.6 ± 0.2

To measure the actual wave speed, we plot the following light intensity vs. time graphs for the three different masses. The graphs are shown on the next page.

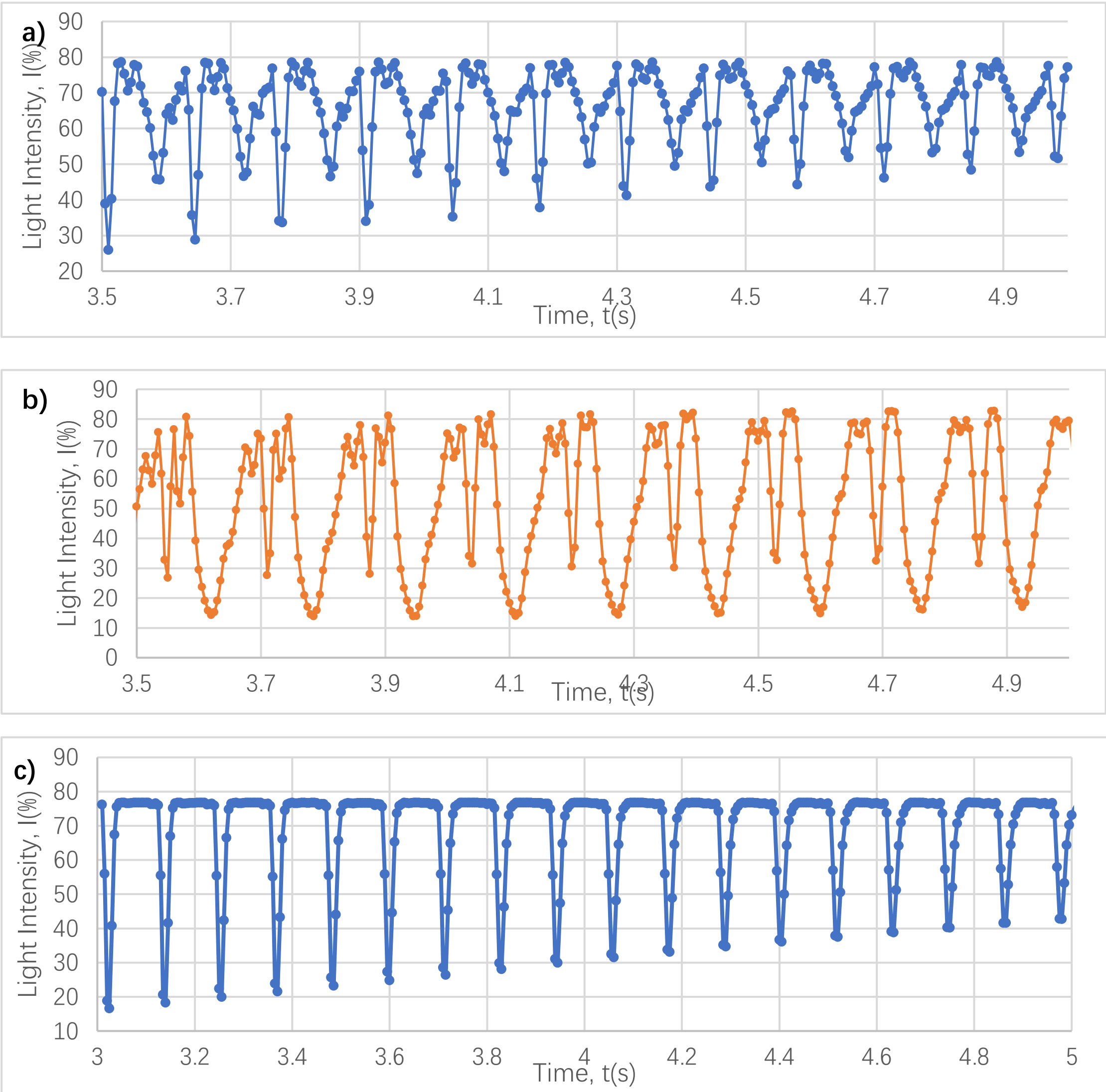

Figure 6. Measuring wave speed. a)m=200g b)m=300g c)m=400g. By finding the period Δt of the repeated patterns, we get the wave speed with equation $v = \frac{2L}{\Delta t}$, where L is the distance from the pulley apex to the clamp, measured to be $L = 1.500 \pm 0.002$m. The measured v is: a)19.1 ± 0.3m/s. b) 22.4 ± 0.4m/s c)25 ± 2m/s.

By comparing the measured and predicted v, we found that the prediction for m=300g and m=400g is very accurate. However, the measured and predicted v for m=200g differs. This could be caused by the error when determining the period from Figure 7. Part a of Figure 7 is much messier when compared to the other two graphs, making it harder to get accurate speed from the pattern in the graph.

With the measurement of wave speed, we then move on to predict and measure the resonance frequency of the vibrating string. We kept the 400g mass hanging on the other end of the string and used the wave speed for this mass to predict the resonance frequency for the nth normal node with formula $f = n\frac{v}{2L}$.

The table below shows the predict resonance frequencies:

n	Predicted frequency, $f_{predicted}(Hz)$
1	8.3 ± 0.5
3	25 ± 1
5	41 ± 2
7	58 ± 3

Then, to measure the actual resonance frequency, we will again apply the Lissajous figure method that we used for the pendulum. Different frequencies are tried until the Lissajous figure becomes symmetrical.

The figure below demonstrates the symmetrical Lissajous figure for the fundamental mode:

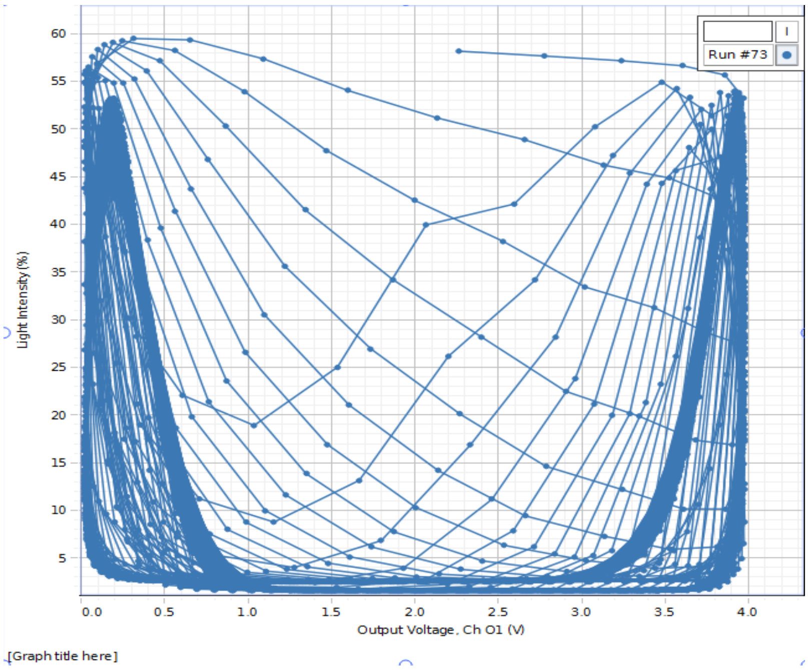

Figure 7. Finding resonance frequency with Lissajous figures. The driving frequency when this figure is plotted is $f = 8.603 \pm 0.004$Hz. Ignoring the noise before the vibrating string reaches the steady state, we can tell that the Lissajous figure is symmetrical by looking at the equal height of the two peaks.

With Lissajous figures, the resonance frequency of 1st, 3rd,5th,7th normal mode is measured, presented in the table below:

n	Measured frequency, $f_{measured}(Hz)$ $\delta f = 0.004$
1	8.603
3	26.020
5	43.195
7	61.770

By comparing the measured values and the predicted values, we can tell that all the measured values are slightly higher than the prediction. This is possibly caused by the inaccurate measurement of L. We assume L to be the distance from the clamp to the pully apex. However, the laser spot and the wave driver are several millimeters away from the two ends. The measured L is therefore longer than the actual value, resulting the predicted values to be smaller.

Conclusion

The experiment aims to explore the harmonic oscillation in the physical pendulum and the vibrating string. By predicting various properties of the two systems and compare the measured values with predictions, we are able to verify our models for the oscillations.

For the physical pendulum, we first explore the three regimes of the damped oscillations: underdamped, critically damped and overdamped. The graph of motion of the three regimes are plotted, and the gap of damping magnets for critically damped motion is found to be 1.40 ± 0.05 cm. We measured the natural frequency and the damping time of the system. With these two values we predicted the resonance frequency of the system. To verify our prediction, we measured the resonance frequency with two methods: the Lissajous figures and the maximum amplitude. The first method measures $\omega_{R1} = 4.60 \pm 0.02Hz$ and the second $\omega_{R2} = 4.60 \pm 0.06Hz$. We believe the Lissajous figures method yields more precise measurement. The precision of the maximum amplitude method can be improved by taking more data points and fit the data points with higher order polynomial regression curves. The Q-factor of the system is also measured with two method: the first uses the damping time and resonance frequency, and the second uses the full width of resonance. The corresponding Q-factors measured by the first and second methods are: $Q_1 = 4.3 \pm 0.2$, $Q_2 = 4.2 \pm 0.2$. We believe the first methods yields better measurement with the same reason for the resonance frequency measurement.

For the vibrating string, we first predict and measured the wave speed for three hanging masses. The prediction and the measurement fit very well for all masses except the lightest one. The vibration pattern becomes messier when the hanging mass is light. The possible reason for the problem is that when the string is less stretched, its vibration can be easily disturbed by other factors in the environment, such as wind and the swinging of the hanged mass. For future experiment, the heavier masses should be used to yield better results. The wave speed for 400g hanging mass is measured to be $25 \pm 2\mathrm{m/s}$. With this mass, we move on to calculate the resonance frequencies for 1st,3rd, 5th, and 7th normal mode. The predicted resonance frequency for fundamental mode is 8.3 ± 0.5 Hz. With the Lissajous figures method, the resonance frequencies are measured and compared to the prediction. While the fundamental mode overlapped with the prediction (f = 8.603 ± 0.004Hz), all other resonance frequencies are slightly higher, due to the inaccurate stretched length measurement. For future experiment, we may use longer strings or put wave drivers and laser spot closer to the two ends to reduce the systematic error.

Overall, our predictions with the physical models are successful as they fit the majority of the measurements. This shows that our models and equations for the harmonic oscillator are accurate in both pendulum and strings.

Bibliography

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2. Campbell, W. C. *et al.* Physics 4AL: Mechanics Lab Manual (ver. June 27, 2018). (Univ. California Los Angeles, Los Angeles, California).