Experiment 2: Measurement of g

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2.Derivation

Let the velocity of the ball be v_0 when passing the first photogate, v_1 when passing the second photogate, v_2 when hitting the impact sensor.

With formula $\mathrm{v} = \mathrm{v}_0 + at$, we can get the following two equations:

$$\mathrm{v}_1 = \mathrm{v}_0 + gT_1 \tag{1}$$

$$\mathrm{v}_2 = \mathrm{v}_1 + gT_2 \tag{2}$$

Since the free falling of the ball is linear motion with constant acceleration, we can apply the formula: $\bar{v} = \frac{v_{final}+v_{initial}}{2} = \frac{\Delta x}{\Delta t}$

$$2 \times \frac{d}{T_1} = \mathrm{v}_1 + \mathrm{v}_0 \tag{3}$$

$$2 \times \frac{D}{T_2} = \mathrm{v}_1 + \mathrm{v}_2 \tag{4}$$

By substituting equation (1) and (2) into equation (4), we get:

$$2 \times \frac{D}{T_2} = \mathrm{v}_0 + \mathrm{v}_1 + gT_1 + gT_2 \tag{5}$$

By substituting equation (3) into equation (5), we get:

$$2 \times \frac{D}{T_2} = 2 \times \frac{d}{T_1} + gT_1 + gT_2 \tag{6}$$

Rearranging equation 6, we get:

$$\mathrm{g} = \frac{2}{(T_1 + T_2)}\left(\frac{D}{T_2} - \frac{d}{T_1}\right)$$

which is equation 2.1 in the lab manual.

3.Plots

Here is the plot of g vs. D:

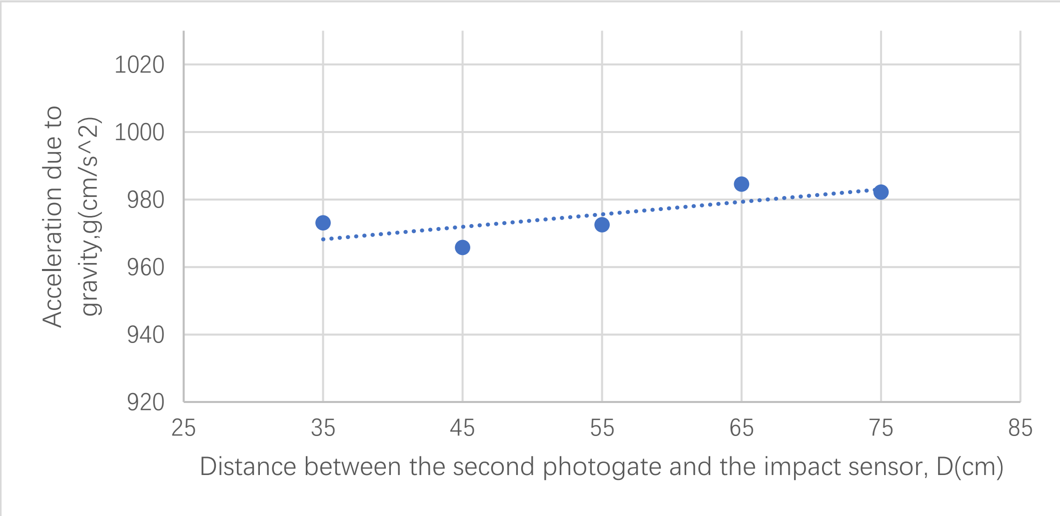

Figure 1: Dependence of acceleration due to gravity on the distance between the second photogate and the impact sensor. The dots from left to right describes g measured by dropping a ball through two photogates and an impact sensor at D equals to 35cm, 45cm, 55cm, 65cm and 75cm. The linear best fit line has the equation: g = aD+b, where a = 0.37±0.18 s^{-2}. The small a shows very weak dependence of g on D.

We expect the value of g to be independent of D. According to the formula for gravity force: $F = \frac{GMm}{R^2}$, the acceleration g becomes $\frac{GM}{R^2}$. This shows us that g should only depends on the mass of the earth (M) and the distance between the ball to the center of the earth (R). As we are dropping the ball, we are indeed reducing R. However, the original R is the radius of the earth, and the change in R is smaller than 1 meter, which is simply too limited to make any significant change in g. Therefore, we should expect that in our experiment, the value of g is constant at different height. That is, g should be independent of the dropping distance. A quantitative way to rule out the dependence is that when we are doing a linear fit between g and D, the gradient should be 0.

The experiment data shows us that 0 does not lies within the range of the slope of our actual best fit line, which we didn't expect. Such difference could be resulted by the imprecise measurement we made and the limited trials of data. If we take more precise measurements and measure g at different Ds, we should expect to see an almost 0 gradient.

Below is the plot of four sets of data collected using the photogate comb method:

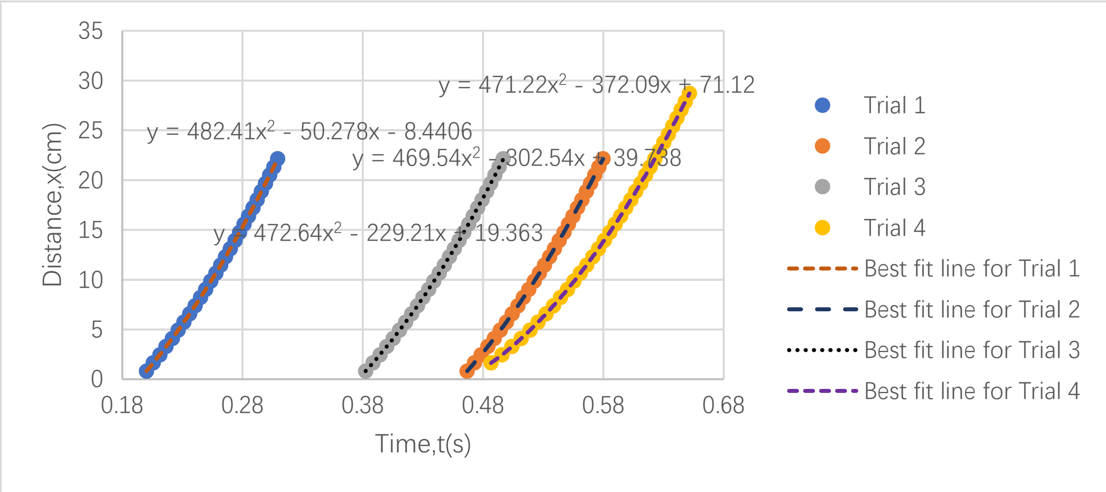

Figure 2. Quadratic fit for four sets of data collected using the photogate comb method. All sets share the same formula: $x = \frac{1}{2}gt^2 + v_0t + x_0$, where for trial 1, $\frac{1}{2}g = 482cm/s^2$, for trial 2, $\frac{1}{2}g = 470cm/s^2$, for trial 3, $\frac{1}{2}g = 473cm/s^2$, for trial 4, $\frac{1}{2}g = 471cm/s^2$.

4.Data Tables

Below is the table of ball drop results from 5 different heights:

Trial	Photogate spacing d(cm) $\delta d = 0.05$cm	Gap to impact sensor D(cm) $\delta D = 0.05$cm	Systematic uncertainty of acceleration δg_{system} (cm/s^2)	Statistical uncertainty of acceleration δg_{stat}(cm/s^2)	Measured acceleration g (cm/s^2)
1	8.00	35.00	9	9	973±18
2	8.00	45.00	8	9	966±17
3	8.00	55.00	6	7	972±13
4	8.00	65.00	5	4	985±9
5	8.00	75.00	5	2	982±7

The systematic error of each trial is calculated by the following method. First, select the best measurement among each trial. Then, instead of using D_{best}, d_{best}, to get the maximum value of g, we substitute $D_{max} = D_{best}+\delta D$, $d_{min} = d_{best}-\delta d$ into equation 2.1, to get minimum value of g, we substitute $D_{min} = D_{best}-\delta D$, $d_{max} = d_{best}+\delta d$ into equation 2.1. Finally, with formula:$\delta g_{system} = \frac{g_{max}-g_{min}}{2}$, we get the systematic error of g.

The statistical error is obtained using the standard deviation formula (ii.13) in the lab manual:

$$\delta g = \frac{1}{\sqrt{N}}\sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(g_i - \bar{g})^2}$$

where N = 10.

We can tell from the table that the uncertainty is not dominated by neither the systematic uncertainty nor the statistical uncertainty, therefore, the final uncertainty is obtained by adding the two errors.

Below is the table of the results from 5 trials using photogate comb method:

Trial	Systematic uncertainty of acceleration δg_{system} (cm/s^2)	Statistical uncertainty of acceleration δg_{stat} (cm/s^2)	Measured acceleration g (cm/s^2)
1	3	2	965±5
2	3	1	939±4
3	3	1	945±4
4	3.3	0.3	942±3
5	3.3	0.4	949±4

In this experiment, we measured:

$$35\lambda \quad = 28.70 \pm 0.05\text{cm}$$

$$\lambda \quad = 0.8200 \pm 0.0014\text{cm}$$

To obtain the systematic error, for each trial I did two more fits. One for $\lambda = \lambda_{best} - \delta\lambda$ to get g_{min}, one for $\lambda = \lambda_{best} + \delta\lambda$ to get g_{max}. Then, with formula:$\delta g_{system} = \frac{g_{max} - g_{min}}{2}$, we get the systematic error of g.

The statistical error is calculated using the data analysis tool in Excel.

In the table, except for trial 4, the error is neither dominated by systematic error or statistical error. Thus, the error for all trials except trial 4 are calculated by adding systematic and statistical error. For trial 4, the systematic error is 10 times larger than the statistical error, so I ruled out the statistical error and only count systematic error into the final error. For this trial, the systematic error is in domination.

5.Conclusion

From the lab manual, we know that the expected value for g is $979.55\pm0.03 cm/s^2$. This value falls within the error range of the results from all trials of the ball drop method, while falling out of the range of the results from all trials of the photogate comb method. Therefore, we can conclude that the ball drop method is more accurate. However, the photogate comb method is more precise than the ball drop method, as the uncertainty of photogate comb is much smaller.

Except trial 4 in photogate comb method, I combined the systematic and statistical error, as they are both significant and no one is 10 times grater than the other. When the error is dominated by neither source of error, I decided to combine them. For trial 4 in photogate comb method, the systematic error is 10 times larger than the statistical error, making it the dominant error. Therefore, I only count in the systematic error and rule out the statistical error due to its insignificance.

Mini-Report

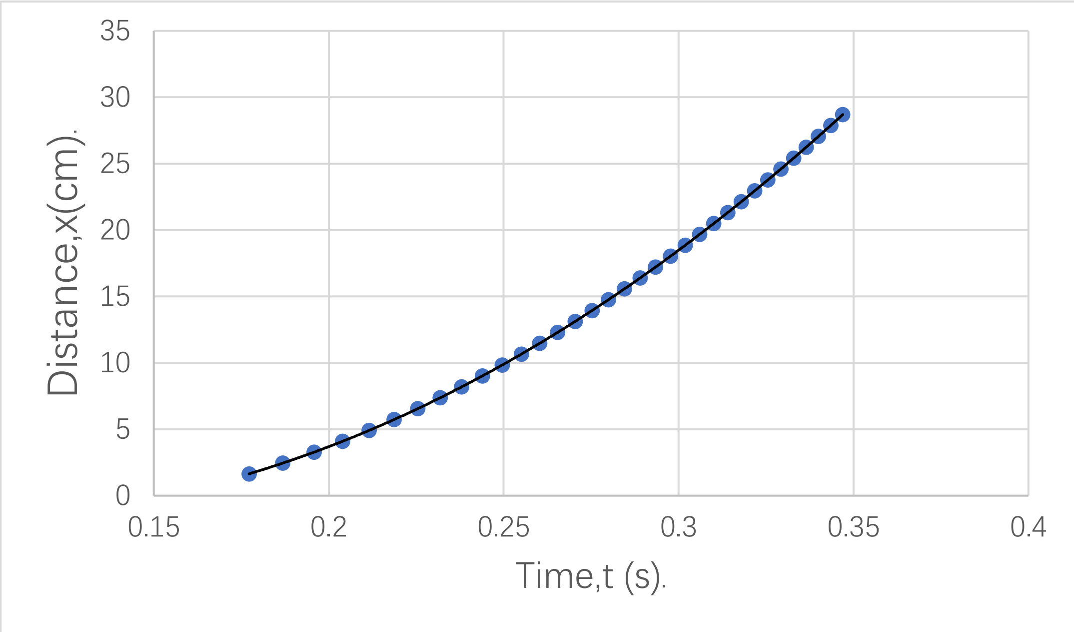

Figure 3. Measurement of acceleration due to gravity using photogate comb method. The data points represent the time when the comb blocks the photogate and the distance traveled at the time. The equation for the fit curve is $x = \frac{1}{2}gt^2 + v_0t + x_0$, where $\frac{1}{2}g = 474.4 \pm 0.2 \text{cm/s}^2$. There is a strong quadratic relationship between time and distance as every data point fall onto the quadratic fit line, whose constant second derivative represents a half of the acceleration due to gravity.

The data points in Figure 3 are obtained by dropping a "comb" through a photogate. Each time when the comb blocks the photogate, the time will be recorded. The distance traveled between two successive time records is exactly the width of a slot on the comb, denoted by λ_{best}. Therefore, the difference of y-coordinate of two successive data points in the figure is always equal to $\lambda_{best} = 0.8200\text{cm}$. The difference in x-coordinate of two successive data points continues to decrease as time increases. This is because the velocity of the comb increases with time, meaning that it takes shorter time for the comb to drop through the same distance. There are in total 34 data points in the graph. The comb has 35 slots. As we ruled out the first block time because the first slot has different width than the others, there are 34 block events recorded, which matches the data. As the formula for free fall is: $x = \frac{1}{2}gt^2 + v_0 t + x_0$, we use quadratic fit for our distance vs. time graph. The fit curve is generated by Excel. The constant of the t^2 in the quadratic fit curve is $\frac{1}{2}$g. Therefore, g shown by this figure is 948.8±0.4cm/s^2. This g is smaller than the expected value 979.55±0.03cm/s^2. There several potential causes for this mismatch. First, as the comb falls, it could be not falling straightly. As the comb tilts, the width of slots might increase. The following figure shows how this happens:

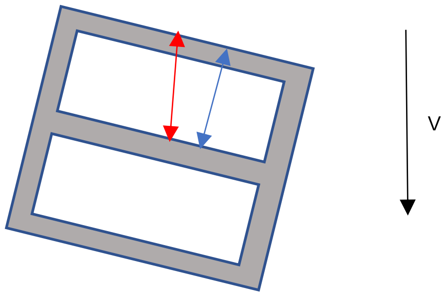

Figure 4. Two slots of a tilted comb. The red arrow is the actual width measured by the photogate, while the blue arrow is the width we used to plot the graph. The dark arrow indicates the direction of the comb's velocity.

The λ_{best} we used is smaller than the actual λ_{best}, making the quadratic fit curve "flatter" than it should be. Thus, we might get smaller g than expected. Another source of error is the air resistance. The drag force from the air might make the net force experience by the comb smaller than the gravity force, resulting the acceleration of comb smaller than g.

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