Experiment 5: Harmonic Oscillator Part I. Spring Oscillator

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Determining the damped and undamped oscillation frequency of a spring oscillator

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Abstract

We explore the frequency of oscillation of a spring oscillator by setting up a system of a vertically hanged spring with a magnetized mass attached at its end. The system is hanged on a force sensor which detects the motion of the system by measuring the tension of the spring. The oscillation frequency is measured with and without an aluminum tube around the mass, giving us the damped and undamped frequency. The two measured frequencies are then compared with the predicted frequencies calculated by the motion formula for oscillators. We found that our prediction for both damped and undamped oscillation are accurate.

Introduction

The simple harmonic motion happens when the restoring force is directly proportional to the displacement from the equilibrium position. Such motion is often demonstrated by a mass attached to a spring which obeys Hooke's law[1]. When the dissipative force is added into the freely oscillating system, the oscillation becomes damped oscillation, meaning the amplitude of oscillation will be decreasing. The dissipative force is called damping force and is proportional to the velocity of the oscillating body[1].

In this experiment, we will predict the damped and undamped frequency of the spring oscillator we set up. The spring constant k and the mass of the oscillating body m will be measured to predict the undamped frequency f_0. The damped frequency f_{damp} is related to f_0 and the Q-factor. The value of Q can be calculated using k, m and damping time τ, which is the time taken for the amplitude to decrease by a factor $\frac{1}{e}$. We will then measure the actual f_0 and f_{damp} by setting a spring with a magnetized mass to oscillate with and without an aluminum tube around it. The oscillation is recorded by a force sensor in the form of a force vs. time graph. From the time period between two maxima, we can obtain the period of oscillation and take reciprocal of it to get the actual frequency. Finally, the predicted frequencies and the measured frequencies will be compared to demonstrate the correctness of our predictions.

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Method

To set up the oscillator, we use the following equipment: a force sensor, a spring and a magnetized mass. The set-up is shown in the following figure:

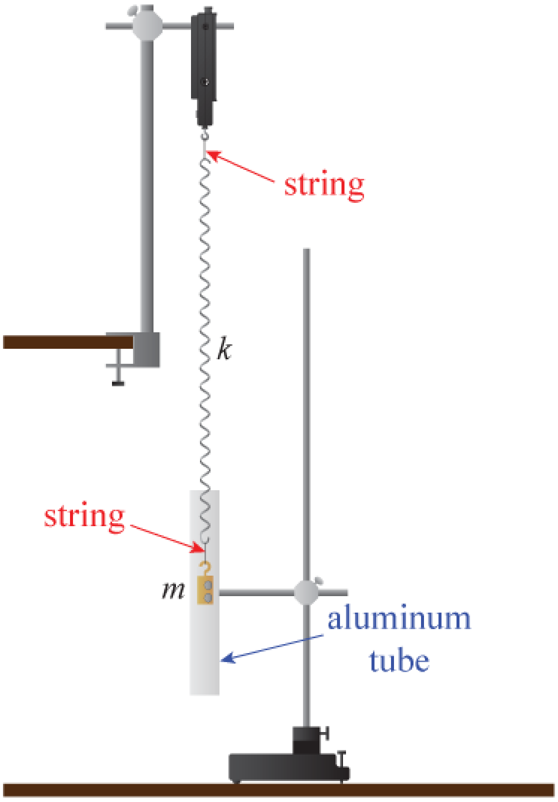

Figure 1. The set-up of the oscillator. The spring with magnetized mass attached is hanged vertically from the hook of the force sensor. The two ends of springs are connected to the force sensor and the mass with strings to prevent energy dissipation through the rotation of the spring. Without the aluminum tube, the system acts as a simple harmonic oscillator. With the tube, the magnetized mass will induce currents in the tube leading to a damping force proportion to the mass's velocity. Figure reproduced (with permission) from Fig. 4.1 by Campbell, W. C. *et al.*[2]

To predict the behavior of the oscillator, we measure the spring constant k and mass m of the magnetized mass. The spring constant is measured by attaching different masses at the end of the spring and measure the distance of the hanging mass from the ground. The distance from the ground is measured by a ruler with uncertainty 0.05cm. A linear best fit line of the distance vs. gravitational force provided by the mass is then plotted. The absolute value of the line is k. The mass m is measured with a scale of uncertainty 0.05g.

To make the actual measurement of the undamped frequency f_0, the aluminum tube is removed, and we pull down the mass then release it. The force sensor is the turned on with measuring frequency of 40Hz. The force sensor measures the tension of the spring and record the time of measurement. The force is then converted to output voltage signal and exported to the DAQ system with the time stamps. The sensor continues to

measure the oscillation for 50 seconds. After that, the graph of voltage vs. time graph is plotted. By finding the average period T between two maxima in the graph, we get f_0 by taking the reciprocal of T.
To measure f_{damp}, we repeat the steps for measuring undamped frequency, but with the aluminum tube around the mass. And we continue to measure until the oscillation stops. The model of damped oscillation describes f_{damp} to be related to f_0 and the Q factor. Here we use the measure to f_0 predict f_{damp}. The Q can be calculated from k, m and τ. By looking at the amplitudes in the voltage vs. time graph obtained when measuring f_{damp}, we can determine the time taken for the amplitude to decrease by $\frac{1}{e}$, which is τ. We can then get the predicted value of f_{damp}.
With the predicted values and measured values of f_{damp} and f_0, we can then verify if the models of damped and undamped oscillation fit the actual measurements.
To avoid systematic errors, the mass must be pulled vertically downwards each time. Since the force sensor is set in the vertical direction, it will only measure the vertical tension. If the mass is pulled to the side, then a portion of the force will not be recorded, and the amplitudes will therefore be incorrect. Throughout the measurement, the mass should stay in the aluminum tube without touching the tube. Otherwise the damping force will not be proportional to the velocity of the mass throughout the oscillation.

Analysis

The mass of the oscillating body m is measured to be:

$$m = 0.17430 \pm 0.00005 g$$

The spring constant k is found with the following graph:

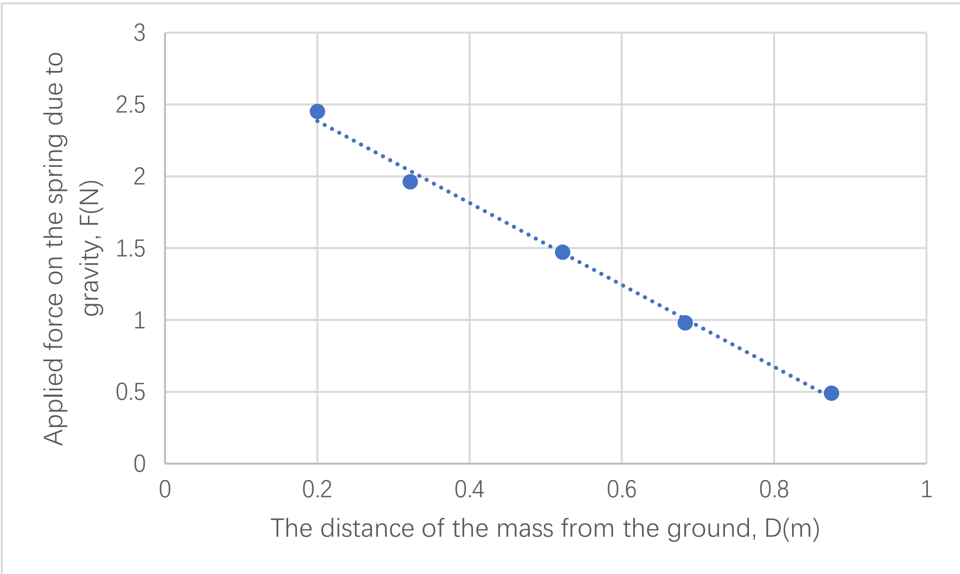

Figure 2. Determining the spring constant k. The data points represent the distances from the ground when different masses are hanged on the spring. The best fit line is $F = aD + b$, where $a = -2.852 \pm 0.001$. Since D is not the distance from the equilibrium position, which the tension is proportional to, the absolute value of a is taken to be k.

$$k = 2.852 \pm 0.001 \text{ N/m}$$

With k and m, we may calculate the predicted f_0 with the formula provided by the lab manual[2]:

$$f_{0\,predicted} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\delta f_{0\,predicted} = \sqrt{\left(\frac{1}{4\pi}k^{-\frac{1}{2}}m^{-\frac{1}{2}}\delta k\right)^2 + \left(-\frac{1}{4\pi}k^{\frac{1}{2}}m^{-\frac{3}{2}}\delta m\right)^2}$$

$$f_{0\,predicted} = 0.64 \pm 0.01Hz$$

Then, to measure the actual f_0, we plot the voltage vs. time graph:

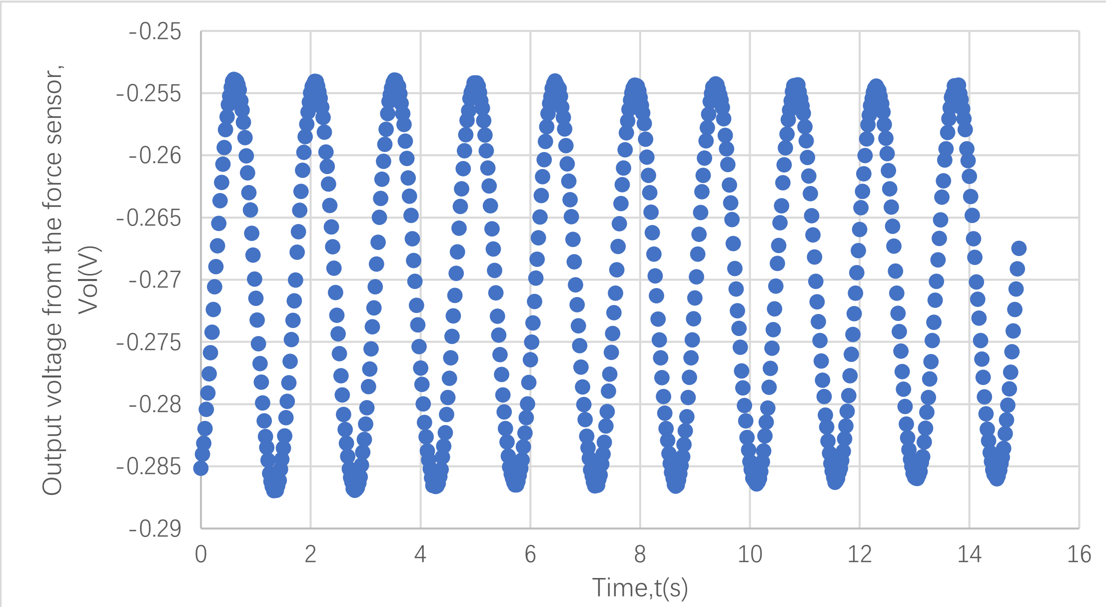

Figure 3. Measurement of f_0. The figure shows the first ten period of the free oscillation. The data points are not centered around 0 due to the offset of the force sensor. To determine the period T, I got T_1 as the time difference between the 1st and 6th maxima divided by 5, T_2 between 2nd and 7th, and so on until T_{12}. T is the average of the 12 periods and δT is the standard deviation, which is $T = 1.461 \pm 0.004$s. The periods after T_{12} is not calculated as the oscillation is slightly damped due to air friction.

The frequency is the reciprocal of T, which is:

$$f_{0\,measured} = 0.68 \pm 0.04Hz$$

The error is calculated by the propagation of uncertainties formula given by the lab manual.

Then, with the aluminum tube, we get the voltage vs. time graph for the damped oscillation:

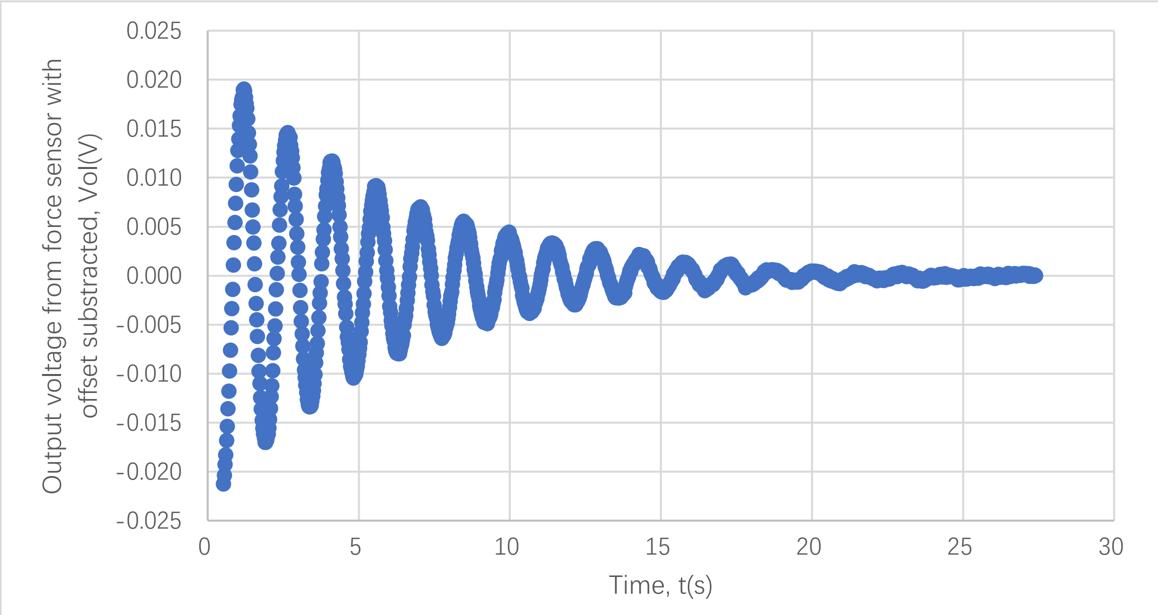

Figure 4. Measurement of f_{damp}. The figure shows the oscillation until the amplitude becomes 0. To calculate period T, the same method in Figure 3 is applied. However, only the first 13 maxima are used as the ones after are harder to determine due to the decreasing amplitude. The calculated average period is $T = 1.462 \pm 0.008s$. The offset of the force sensor is calculated using the 100 data points after the oscillation stops. It is subtracted from all the data so that the oscillation is centered around 0.

Again, by taking the reciprocal of T, we get the damped frequency with uncertainty calculated by the propagation of uncertainties formula:

$$f_{damp\ measured} = 0.68 \pm 0.08Hz$$

To get the predicted value of f_{damp}, we will apply the formula from the lab manual[2]:

$$f_{damp} = f_0\sqrt{1 - \frac{1}{4Q^2}}$$

To obtain Q, we may derive the relationship between Q and τ from the formula given in the lab manual[2].Below is the derivation process:

The lab manual gives the following 2 equations:

$$\tau = \frac{2m}{b}$$

$$Q = \frac{\sqrt{km}}{b}$$

From the first one, we get:

$$b = \frac{2m}{\tau}$$

Substitute it into the second equation in the lab manual:

$$Q = \frac{\tau\sqrt{km}}{2m} = \frac{\tau\sqrt{k}}{2\sqrt{m}}$$

The following formula gives us the way to calculate τ :

$$\tau = -\frac{T}{\ln[\frac{V(t+T)}{V(t)}]}$$

where T is the period of the damped oscillation, which is already measured, and $\frac{V(t+T)}{V(t)}$ is the ratio between two successive maxima. To obtain the ratio, we will analyze Figure 4 and plot the ratio between the nth and (n-1)th maxima, for n=2,3,…10. The maxima after 10 are not used for analysis as the error becomes very large due to the decreasing amplitude.

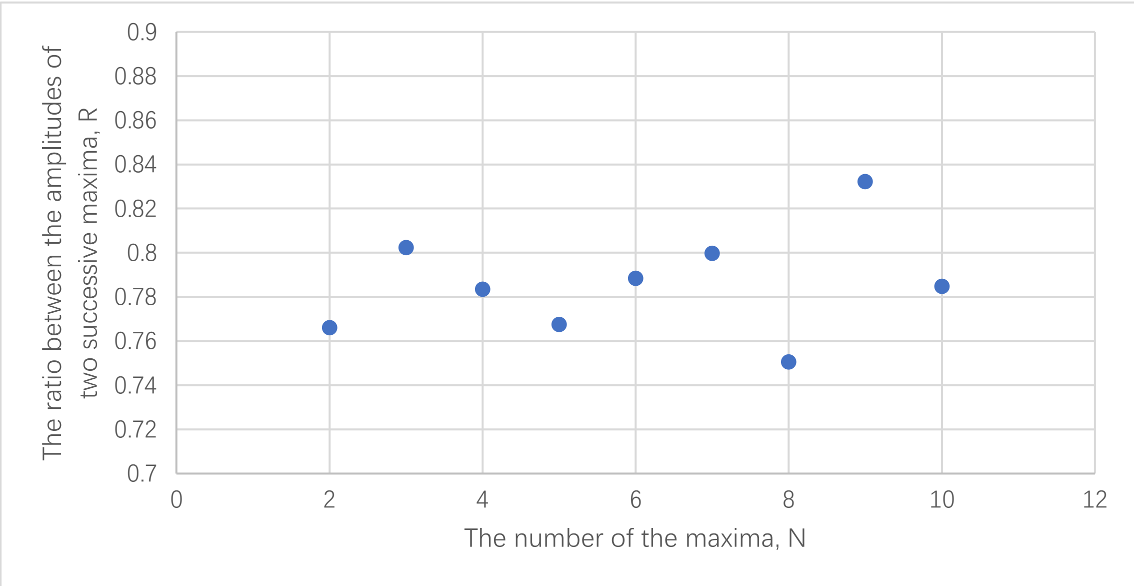

Figure 5. Determining τ. The data points represent the ratio of amplitudes between the N^{th} and $(N-1)^{th}$ maxima. The ratios center around 0.79 and there is no significant linear trend, meaning that the damping force is proportional to the velocity. Notice that the ratios deviate larger from the 8^{th} maxima as the decreasing amplitude caused larger error.

For each of the 9 ratios I calculated, I used the formula to calculate their corresponding τ. With 9 different τ, I take an average of them and the standard deviation becomes its error.

$$\tau = 6.2 \pm 0.8 \text{ s}$$

The Q factor therefore is:

$$Q = 12 \pm 2$$

The predicted frequency is therefore:

$$f_{damp\ predicted} = 0.68 \pm 0.04\ Hz$$

The error is calculated by the propagation of uncertainties formula given by the lab manual.

Conclusions

This experiment explores the damped and undamped oscillations by predicting the frequencies of the two types of oscillation using specific physics models and verifying the prediction with measured values.

The table below demonstrate the predicted and measured frequencies of damped and undamped oscillation:

	Undamped frequency, f_0(Hz)	Damped frequency, f_{damp}(Hz)
Predicted	0.64 ± 0.01	0.68 ± 0.04
Measured	0.68 ± 0.04	0.68 ± 0.08

If we compare the predicted and measured values for each set of frequency, we would find that both sets match. The best values for predicted and measured damped frequency matched exactly, showing that the model for damped oscillation is accurate. The best values between the predicted and measure frequencies of undamped oscillation are different. But after taking uncertainty into account, they match on the edge of the error range. The difference might be caused by the error in measuring the spring constant k. Since we measure the extension of the spring using a meter ruler, the ruler will unavoidably be tilted, giving imprecise measurements. The measured extension is larger than the actual value, meaning k is smaller than the actual value. As predicted f_0 is proportional to $\sqrt{k}$, it becomes smaller, which matches with the values presented above. In future experiment, the measurement of the spring constant can be improved by hanging the spring horizontally and extend it with a mass through a pulley. The extension of the spring can thus be measured horizontally with a ruler, which allows us to align the ruler with the spring more precisely to give better readings.

When we compare the measured damped and undamped oscillation frequencies, we can see that the difference is not significant. The best values of the frequencies are the same. This may be resulted by the small damping force. The Q factor in this experiment is 12, which means the ratio between damped and undamped frequencies $\sqrt{1 - \frac{1}{4Q^2}}$ is around 0.999. This value is very close to 1, therefore the reduce in frequency due to damping force becomes insignificant. For future experiment, a stronger magnet can be used to induce larger damping force to make the difference more significant.

Bibliography

1. Young, H. D. & Freedman, R. A. *University physics with modern physics*. (Addison-Wesley, 2015).
2. Campbell, W. C. *et al.* Physics 4AL: Mechanics Lab Manual (ver. June 27, 2018). (Univ. California Los Angeles, Los Angeles, California).