Summary on MIMO for ELEC3204

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Question-Driven General Steps

1) What is the type of MIMO?

- The 1^{st} Step is to identify system parameters and signal transmission pattern.

2) What's happened in wireless channel?

- The 2^{nd} Step is to model the signal from each transmit antenna to each receive antenna.

3) How to prepare for matrix operations?

- The 3^{rd} Step is to build MIMO matrix form.

4) What's the desired input-output relationship without repetitions in the input?

- The 4^{th} Step is to build equivalent STBC input-output model.
- This step may be skipped if it's not for single-user interference-free STBC.

5) How to cancel the effect of fading phases?

- The 5^{th} Step is correlation operation.

6) How to cancel the effect of inter-antenna or inter-user interference?

- The 6^{th} Step is decorrelating operation.
- This step may be skipped if there's no interference left from Step 5).

MIMO (Slides P113-114,130-132)

1) System Parameters and Signal Transmission Pattern

- Number of transmit antennas: M
- Number of receive antennas: N
- Number of modulated symbols: M
- Number of time slots: 1
- Signal transmission pattern: Table I

2) Received Signal Model

TABLE I: Basic MIMO transmission pattern

	Time Slot 1
Transmit Antenna 1	x_1
Transmit Antenna 2	x_2
$\vdots$	$\vdots$
Transmit Antenna M	x_M

- Signal received at the n-th receive antenna ($1 \leq n \leq N$):

$$y_n = h_{n1}x_1 + h_{n2}x_2 + \cdots + h_{nM}x_M + v_n = \sum_{m=1}^{M} h_{nm}x_m + v_n. \tag{1}$$

- h_{nm} denotes fading from the m-th transmit antenna to the n-th receive antenna.
- v_n denotes additive white Guassian noise (AWGN) at the n-th receive antenna.
- Example ($M = 2$,$N = 1$): $y_1 = h_{11}x_1 + h_{12}x_2 + v_1$.

3) MIMO Matrix Form

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}}_{\mathbf{Y}} = \underbrace{\begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M} \\ h_{21} & h_{22} & \cdots & h_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N1} & h_{N2} & \cdots & h_{NM} \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix}}_{\mathbf{X}} + \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}}_{\mathbf{V}} \tag{2}$$

- The sizes of $\mathbf{Y}$, $\mathbf{H}$, $\mathbf{X}$ and $\mathbf{V}$ are $(N \times 1)$, $(N \times M)$, $(M \times 1)$ and $(N \times 1)$, respectively.
- Example ($M = 2$,$N = 1$): $y_1 = \begin{bmatrix} h_{11} & h_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + v_1$.

4) Equivalent STBC Input-Output Model

- This step can be skipped because STBC is not used, and there is no repetitions in $\mathbf{X}$ of Eq. (2).

5) Correlation Operation

$$\widetilde{\mathbf{Y}} = \mathbf{H}^H\mathbf{Y} = \mathbf{H}^H\mathbf{H}\mathbf{X} + \mathbf{H}^H\mathbf{V} = \mathbf{R}\mathbf{X} + \mathbf{H}^H\mathbf{V}. \tag{3}$$

- $()^H$ refers to Hermitian transpose (conjugate transpose).
- The cross-correlation matrix $R = \mathbf{H}^H\mathbf{H}$ is of size $(M \times M)$.
- The size of $\widetilde{\mathbf{Y}}$ is $(M \times 1)$.

6) Decorrelating Operation

$$\mathbf{Z} = \mathbf{R}^{-1}\widetilde{\mathbf{Y}} = \mathbf{X} + \mathbf{R}^{-1}\mathbf{H}^H\mathbf{V}. \tag{4}$$

TABLE II: SIMO transmission pattern

	Time Slot 1
Transmit Antenna 1	x

- The sizes of $\mathbf{Z}$ is $(M \times 1)$. There is no interference any more. z_m can be directly used for detecting x_m, $1 \leq m \leq M$.
- Example ($M = 2$,$N = 1$): $\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \mathbf{R}^{-1}\mathbf{H}^H\mathbf{V}$. Here z_1 is a noise contaminated version of x_1m and z_2 is a noise contaminated version of x_2. They don't interfere with each other any more.
- The major disadvantages of having to do decorrelating operation: (i) matrix inversion imposes high computational complexity; (ii) matrix inversion may enlarge the noise power.

Summary: What has the receiver done? The receiver performs signal processing based on $\mathbf{Y}$ and $\mathbf{H}$ in order to obtain $\mathbf{Z}$, which is considered to be the noise contaminated version of $\mathbf{X}$ without fading and without interference.

SIMO (Slides P116)

1) **System Parameters and Signal Transmission Pattern**
    - Number of transmit antennas: 1
    - Number of receive antennas: N
    - Number of modulated symbols: 1
    - Number of time slots: 1
    - Signal transmission pattern: Table II
2) **Received Signal Model**
    - Signal received at the n-th receive antenna ($1 \leq n \leq N$):

$$y_n = h_n x + v_n. \tag{5}$$

3) **MIMO Matrix Form**

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}}_{\mathbf{Y}} = \underbrace{\begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_N \end{bmatrix}}_{\mathbf{H}} x + \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}}_{\mathbf{V}} \tag{6}$$

TABLE III: Alamouti's STBC transmission pattern

	Time Slot 1	Time Slot 2
Transmit Antenna 1	x_1	$-x_2^*$
Transmit Antenna 2	x_2	x_1^*

- The sizes of $\mathbf{Y}$, $\mathbf{H}$ and $\mathbf{V}$ are all $(N \times 1)$.

4) Equivalent STBC Input-Output Model

- This step can be skipped because STBC is not used.

5) Correlation Operation

$$z = \mathbf{H}^H\mathbf{Y} = \mathbf{H}^H\mathbf{H}x + \mathbf{H}^H\mathbf{V} = \begin{bmatrix} h_1^* & h_2^* & \cdots & h_N^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_N \end{bmatrix} x + \mathbf{H}^H\mathbf{V} = (\sum_{n=1}^{N} |h_n|^2)x + \mathbf{H}^H\mathbf{V}. \tag{7}$$

- $()^*$ refers to conjugate.
- The decision variable z is a noise contaminated version of x.
- z can be directly used for detecting x.
- The diversity order is N, because the power of N channels are added together in Eq. (7).

6) Decorrelating Operation

- This step is skipped because there's no interference.

Alamouti's STBC (Slides P123-P124)

1) System Parameters and Signal Transmission Pattern

- Number of transmit antennas: 2
- Number of receive antennas: 1
- Number of modulated symbols: 2
- Number of time slots: 2
- Signal transmission pattern: Table III

2) Received Signal Model

$$\begin{aligned} \text{Time Slot 1:} \quad & y_1 = h_1x_1 + h_2x_2 + v_1, \\ \text{Time Slot 2:} \quad & y_2 = -h_1x_2^* + h_2x_1^* + v_2. \end{aligned} \tag{8}$$

3) MIMO Matrix Form

$$\underbrace{\begin{bmatrix} y_1 & y_2 \end{bmatrix}}_{\mathbf{Y}} = \underbrace{\begin{bmatrix} h_1 & h_2 \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}}_{\mathbf{X}} + \underbrace{\begin{bmatrix} v_1 & v_2 \end{bmatrix}}_{\mathbf{V}} \tag{9}$$

- The sizes of $\mathbf{Y}$, $\mathbf{H}$, $\mathbf{X}$ and $\mathbf{V}$ are (1×2), (1×2), (2×2) and (1×2), respectively.

4) Equivalent STBC Input-Output Model

$$\underbrace{\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix}}_{\overline{\mathbf{Y}}} = \underbrace{\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}}_{\overline{\mathbf{H}}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\overline{\mathbf{X}}} + \underbrace{\begin{bmatrix} v_1 \\ v_2^* \end{bmatrix}}_{\overline{\mathbf{V}}} \tag{10}$$

- Unlike $\mathbf{X}$ in Eq. (9), $\overline{\mathbf{X}}$ in Eq. (10) only contains the two modulated symbols without repetitions.
- The purpose of this step is to obtain an input-output relationship, which has modulated symbols without repetitions as input, so that they can be easily detected in the next step.

5) Correlation Operation

$$\mathbf{Z} = \overline{\mathbf{H}}^H \overline{\mathbf{Y}} = \overline{\mathbf{R}}\overline{\mathbf{X}} + \overline{\mathbf{H}}^H \overline{\mathbf{V}}. \tag{11}$$

- Thanks to the STBC orthogonality that leads to Eq. (10), the cross-correlation matrix becomes a scaled identity matrix:

$$\overline{\mathbf{R}} = \overline{\mathbf{H}}^H \overline{\mathbf{H}} = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} = \begin{bmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_1|^2 + |h_2|^2 \end{bmatrix} = (|h_1|^2 + |h_2|^2)\mathbf{I}_2 \tag{12}$$

- $\mathbf{I}_2$ refers to a (2×2) identity matrix.
- Replace $\overline{\mathbf{R}}$ in Eq. (11) by Eq. (12), we have:

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = (|h_1|^2 + |h_2|^2) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2^* \end{bmatrix}. \tag{13}$$

- The two decision variables become the noise contaminated versions of the two modulated symbols:

$$\begin{aligned} z_1 &= (|h_1|^2 + |h_2|^2)x_1 + h_1^* v_1 + h_2 v_2^*, \\ z_2 &= (|h_1|^2 + |h_2|^2)x_2 + h_2^* v_1 - h_1 v_2^*. \end{aligned} \tag{14}$$

- The diversity order is 2, because of $(|h_1|^2 + |h_2|^2)$.

6) Decorrelating Operation

- This step is skipped because there is no interference after Step 5).

Alamouti's STBC – MIMO Diversity (Slides P126-P128)

1) System Parameters and Signal Transmission Pattern

- Number of transmit antennas: 2
- Number of receive antennas: N
- Number of modulated symbols: 2
- Number of time slots: 2
- Signal transmission pattern: Table III

2) Received Signal Model

- Signal received at the n-th receive antenna ($1 \le n \le N$):

$$\begin{aligned} &\text{Time Slot 1:} \quad y_{n1} = h_{n1}x_1 + h_{n2}x_2 + v_{n1}, \\ &\text{Time Slot 2:} \quad y_{n2} = -h_{n1}x_2^* + h_{n2}x_1^* + v_{n2}. \end{aligned} \tag{15}$$

3) MIMO Matrix Form

- Signal received at the n-th receive antenna ($1 \le n \le N$):

$$\underbrace{\begin{bmatrix} y_{n1} & y_{n2} \end{bmatrix}}_{\mathbf{Y}_n} = \underbrace{\begin{bmatrix} h_{n1} & h_{n2} \end{bmatrix}}_{\mathbf{H}_n} \underbrace{\begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}}_{\mathbf{X}} + \underbrace{\begin{bmatrix} v_{n1} & v_{n2} \end{bmatrix}}_{\mathbf{V}_n} \tag{16}$$

- Let's stack all received signals in order to obtain the full system model:

$$\underbrace{\begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \\ \vdots \\ \mathbf{Y}_N \end{bmatrix}}_{\mathbf{Y}} = \underbrace{\begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_N \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}}_{\mathbf{X}} + \underbrace{\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \vdots \\ \mathbf{V}_N \end{bmatrix}}_{\mathbf{V}} \tag{17}$$

- The sizes of $\mathbf{Y}$, $\mathbf{H}$, $\mathbf{X}$ and $\mathbf{V}$ are $(N \times 2)$, $(N \times 2)$, (2×2) and $(N \times 2)$, respectively.

4) Equivalent STBC Input-Output Model

- For the n-th receive antenna ($1 \le n \le N$):

$$\underbrace{\begin{bmatrix} y_{n1} \\ y_{n2}^* \end{bmatrix}}_{\overline{\mathbf{Y}}_n} = \underbrace{\begin{bmatrix} h_{n1} & h_{n2} \\ h_{n2}^* & -h_{n1}^* \end{bmatrix}}_{\overline{\mathbf{H}}_n} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\overline{\mathbf{X}}} + \underbrace{\begin{bmatrix} v_{n1} \\ v_{n2}^* \end{bmatrix}}_{\overline{\mathbf{V}}_n} \tag{18}$$

- Once again, let's stack all received signals in order to obtain the full system model:

$$\underbrace{\begin{bmatrix} \overline{\mathbf{Y}}_1 \\ \overline{\mathbf{Y}}_2 \\ \vdots \\ \overline{\mathbf{Y}}_N \end{bmatrix}}_{\overline{\mathbf{Y}}} = \underbrace{\begin{bmatrix} \overline{\mathbf{H}}_1 \\ \overline{\mathbf{H}}_2 \\ \vdots \\ \overline{\mathbf{H}}_N \end{bmatrix}}_{\overline{\mathbf{H}}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\overline{\mathbf{X}}} + \underbrace{\begin{bmatrix} \overline{\mathbf{V}}_1 \\ \overline{\mathbf{V}}_2 \\ \vdots \\ \overline{\mathbf{V}}_N \end{bmatrix}}_{\overline{\mathbf{V}}} \tag{19}$$

- The sizes of $\overline{\mathbf{Y}}$, $\overline{\mathbf{H}}$, $\overline{\mathbf{X}}$ and $\overline{\mathbf{V}}$ are $(2N \times 1)$, $(2N \times 2)$, (2×1) and $(2N \times 1)$, respectively.

5) Correlation Operation

$$\mathbf{Z} = \overline{\mathbf{H}}^H \overline{\mathbf{Y}} = \overline{\mathbf{R}}\overline{\mathbf{X}} + \overline{\mathbf{H}}^H \overline{\mathbf{V}}. \tag{20}$$

- Thanks to the STBC orthogonality that leads to Eq. (19), the cross-correlation matrix becomes a scaled identity matrix:

$$\overline{\mathbf{R}} = \overline{\mathbf{H}}^H \overline{\mathbf{H}} = \begin{bmatrix} \overline{\mathbf{H}}_1^H & \overline{\mathbf{H}}_2^H & \cdots & \overline{\mathbf{H}}_N^H \end{bmatrix} \begin{bmatrix} \overline{\mathbf{H}}_1 \\ \overline{\mathbf{H}}_2 \\ \vdots \\ \overline{\mathbf{H}}_N \end{bmatrix} = \sum_{n=1}^{N} \overline{\mathbf{H}}_n^H \overline{\mathbf{H}}_n = \sum_{n=1}^{N} (|h_{n1}|^2 + |h_{n2}|^2) \mathbf{I}_2 \tag{21}$$

- Replace $\overline{\mathbf{R}}$ in Eq. (20) by Eq. (21), we have:

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \sum_{n=1}^{N} (|h_{n1}|^2 + |h_{n2}|^2) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overline{\mathbf{H}}^H \overline{\mathbf{V}}. \tag{22}$$

- The diversity order is 2N, because of $\sum_{n=1}^{N} (|h_{n1}|^2 + |h_{n2}|^2)$.

6) Decorrelating Operation

- This step is skipped because there is no interference after Step 5).

How to apply these steps in coursework?

1) System Parameters and Signal Transmission Pattern

- Please note that all matrices in coursework are the transpose versions of the matrices in this document. For example, in this document, we use $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{V}$ for MIMO model, while in coursework, we have the form of $\mathbf{Y} = \mathbf{X}\mathbf{H} + \mathbf{N}$.
- There are different ways to construct STBCs for $M = 4$. The first option may be based on Alamouti's STBC but re-arrange zeros to reduce interference, such as using the following

patterns for different users:

$$\begin{bmatrix} x_1 & x_2 & 0 & 0 \\ -x_2^* & x_1^* & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & x_1 & x_2 \\ 0 & 0 & -x_2^* & x_1^* \end{bmatrix}. \tag{23}$$

- The second option is called amicable-orthogonal STBC (AO-STBC) constructed as shown by Eq. (85) of [1]. Explicitly, the pattern for $M = 4$ is:

$$\begin{bmatrix} x_1 & x_2 & x_3 & 0 \\ -x_2^* & x_1^* & 0 & x_3 \\ -x_3^* & 0 & x_1^* & -x_2 \\ 0 & -x_3^* & x_2^* & x_1 \end{bmatrix}, \tag{24}$$

  where you need to modulate one QPSK symbol for x_1 and two BPSK symbols for x_2, x_3, in order to retain the required throughput. Please note that the conjudate of a BPSK symbol does not change anything, i.e. $x_2^* = x_2$ and $x_3^* = x_3$. Therefore, the conjugate format of x_2 and x_3 are the same as that of x_1 in each row of Eq. (24), yielding:

$$\begin{bmatrix} x_1 & x_2 & x_3 & 0 \\ -x_2^* & x_1^* & 0 & x_3^* \\ -x_3^* & 0 & x_1^* & -x_2^* \\ 0 & -x_3 & x_2 & x_1 \end{bmatrix}, \tag{25}$$

- The third option is called quasi-orthogonal STBC (QO-STBC) constructed as shown by Eq. (86) of [1]. Explicitly, the pattern for $M = 4$ is:

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix}, \tag{26}$$

  where you need to modulate BPSK symbols for all. This is called ”quasi-orthogonal”, because there is interference between the (2×2) blocks in Eq. (26). This implies that correlation operation cannot lead to the ideal identity matrix any more.
- There might be other options on signal transmission codeword!

2) Received Signal Model

- Fig. 1 of the coursework document.

3) **MIMO Matrix Form**

- Eqs.(1)-(4) of the coursework document.
- We need to obtain $\mathbf{Y} = \mathbf{X}\mathbf{H} + \mathbf{N}$.

4) **Equivalent STBC Input-Output Model**

- This model is different for different patterns.
- We need to obtain $\overline{\mathbf{Y}} = \overline{\mathbf{X}\mathbf{H}} + \overline{\mathbf{N}}$, where there is no repetitions in $\overline{\mathbf{X}}$.
- Alternatively, this step can be skipped, and the model of Step 3) can be used for correlation and decorrelating operations.

5) **Correlation Operation**

- Based on the model of Step 3): $\mathbf{Y}\mathbf{H}^H = \mathbf{X}\mathbf{R} + \mathbf{V}\mathbf{H}^H$, where $\mathbf{R} = \mathbf{H}\mathbf{H}^H$.
- Based on the model of Step 4): $\overline{\mathbf{Y}\mathbf{H}}^H = \overline{\mathbf{X}\mathbf{R}} + \overline{\mathbf{V}\mathbf{H}}^H$, where $\overline{\mathbf{R}} = \overline{\mathbf{H}\mathbf{H}}^H$.
- Both $\mathbf{R}$ and $\overline{\mathbf{R}}$ are not scaled identity matrix because of inter-user interference.

6) **Decorrelating Operation**

- Based on the model of Step 3): $\mathbf{Z} = \mathbf{Y}\mathbf{H}^H(\mathbf{R})^{-1} = \mathbf{X} + \mathbf{V}\mathbf{H}^H(\mathbf{R})^{-1}$. Following this, $\mathbf{Z}$ can be used for detecting $\mathbf{X}$, but there are repetitions in $\mathbf{X}$. One method is to only use the first row of $\mathbf{Z}$ to detect the first row of $\mathbf{X}$. Another method is to take average on repetitions.
- Based on the model of Step 4): $\overline{\mathbf{Z}} = \overline{\mathbf{Y}\mathbf{H}}^H(\overline{\mathbf{R}})^{-1} = \overline{\mathbf{X}} + \overline{\mathbf{V}\mathbf{H}}^H(\overline{\mathbf{R}})^{-1}$. Following this, $\overline{\mathbf{Z}}$ can be used for detecting $\overline{\mathbf{X}}$, where there's no repetitions.

References

[1] C. Xu, S. Sugiura, S. X. Ng, P. Zhang, L. Wang, and L. Hanzo, "Two decades of MIMO design tradeoffs and reduced-complexity MIMO detection in near-capacity systems," *IEEE Access*, vol. 5, pp. 18564–18632, 2017.