ELEC3204 Wireless and Optical Communications

Wireless Channel Modelling, Wireless Systems, Space-Time Processing and Performance

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https://secure.ecs.soton.ac.uk/notes/elec3204/

References

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[3] L.-L. Yang, *Multicarrier Communications*, John Wiley & Sons. 2009.

[4] L. Hanzo, M. Münster, B.J. Choi and T. Keller, *OFDM and MC-CDMA for Broadband Multi-user Communications, WLANs and Broadcasting*, John Wiley and IEEE Press, 2003, p 980.

[5] L. Hanzo, L.-L. Yang, E.-L. Kuan, and K. Yen, *Single- and Multi-carrier DS-CDMA*. John Wiley and IEEE Press, 2003.

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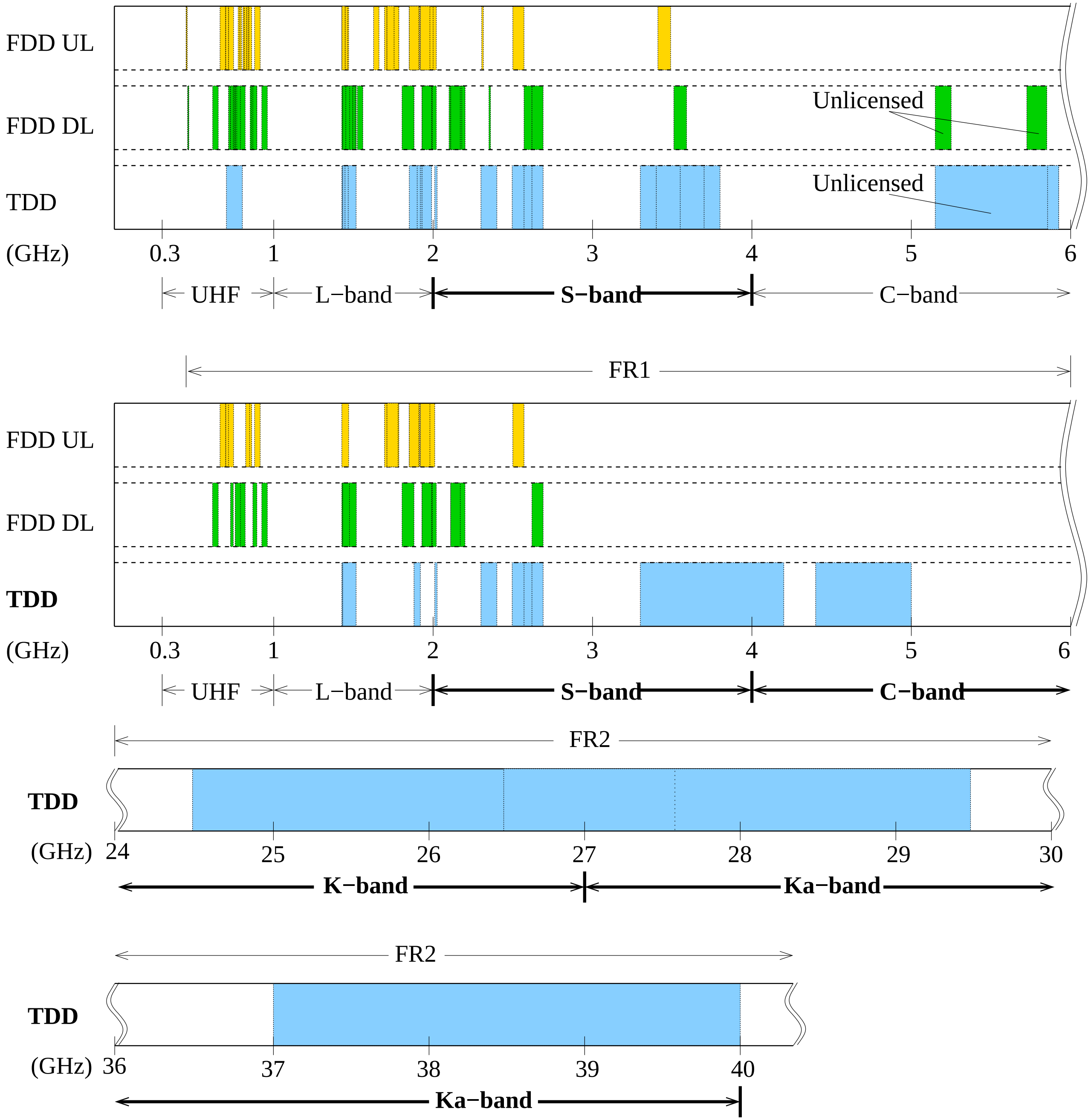
Main Targets

- Overall knowledge about wireless communications systems;
- Part 1 - Rob Maunder:
  - ✓ Wireless channel modeling;
  - ✓ Basic digital modulation/demodulation design;
  - ✓ Performance of wireless communications systems, error rate analysis;
  - ✓ Principles of diversity transmissions in wireless communications;
  - ✓ Principles of space-time processing.
- Part 2 - Lie-Liang Yang:
  - ✓ Principles of spread-spectrum multiple-access communications;
  - ✓ Direct-sequence (DS) code-division multiple-access (DS-CDMA);
  - ✓ Frequency-hopping (FH) SSMA;
  - ✓ Multicarrier CDMA;
  - ✓ Hybrid DS/FH SSMA.

Wireless Communications - Some Tips

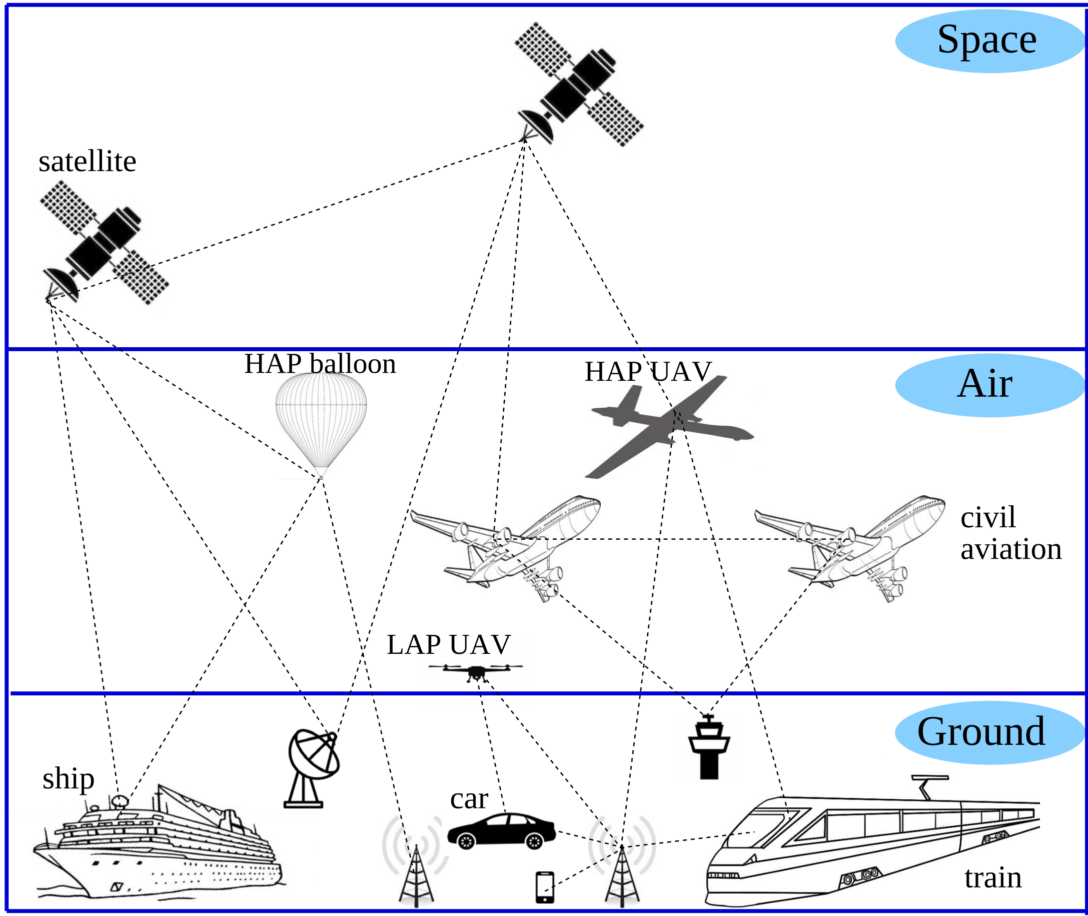
- Read slides before and after lectures.
- Relationship between figures, equations and definitions: make sure everything is logical to you.
- Sketch illustrative figures (e.g. Doppler effect); Derive equations (e.g. OFDM); Understand and accept definitions (e.g. channel modelling).
- Keep thinking; Keep asking questions; Keep reading.

From 4G LTE to 5G NR



- 1G in 1980s:
  - analog communication.
- 2G GSM in 1990s:
  - digital error-correction code.
  - success in standardization.
- 3G CDMA in 2000s:
  - proliferation of smart phones.
  - unprecedented internet access.
- 4G LTE in 2010s:
  - OFDM mitigates frequency selectivity.
  - MIMO improves performance.
  - Turbo coding.
- 5G NR:
  - new spectrum resources.
  - OFDM subcarrier spacing.
  - MIMO beamforming, LDPC, etc.

From 5G to 6G: Space-Air-Ground Integrated Network



Space	Altitude	GEO: 35786 km, MEO: 2000−35786 km, LEO: 180−2000 km
	Delay (S−G)	GEO: 0.12 s, MEO: 6.7−120 ms, LEO: 0.6−6.7 ms
	Mobility	GEO: stationary, MEO:2−24 hr orbital period, LEO: 2 hr orbital period (28080 km/h)
	Enabler	Backhaul: laser links, Access: dmWave, cmWave, mmWave.
	SWAP	Bulky steerable antennas, expensive links.
Air	Altitude	HAP: Above 18 km in stratosphere, Civil: 9−12km cruising, LAP: below 120m without licensing
	Mobility	Hypersoic UAV: >>1192km/h, Civil:up to 1080km/h cruising, High maneuver dynamics especially for UAV
	Enabler	radio links: ATC control, dmWave: ATC surveillance/data, mmWave: LAP UAV, laser links: HAP
	SWAP	Increasing demand but bandwidth−hungry. Size−limited and power−hungry especially for LAP UAV.
Ground	Mobility	Train: up to 603km/h (Japan 2015) Car: up to 447.19km/h (2017)
	Enabler	4G/5G: dmWave, cmWave, mmWave.
	SWAP	Bandwidth−Hungry due to regulations. Size−limited and power−hungry for handsets.

GEO: Geostationary Earth Obit   MEO: Medium Earth Obit   LEO: Low Earth Obit   S−G: Satellite to Ground
SWAP: Size, Weight And Power   HAP: High Altitude Platform   LAP: Low Altitude Platform   UAV: Unmanned Aerial Vehicle

- Compared to the technolgy-driven 1G-4G era, the current 5G era is more resource and business driven in terms of new spectrum and new URLLC and MTC applications.
- We anticipate that 6G may continue to follow the trend of better exploiting resources in both the spectral domain and in the space-air-ground domain.

Wireless Communications - Some Keywords

- Radio, satellite, transmitter, receiver, base-station, mobile phone, mobile terminal, antenna, transmission, signal, power, frequency, frequency band, bandwidth, narrow-band, wideband, broadband, ultra-wideband, capacity, GSM, UMTS, HSPA, LTE, WiMAX, WiFi, DVB-T, FDMA, TDMA, CDMA, SDMA, cell, etc.
- Channel, propagation, reflection, diffraction, scattering, dispersion, Doppler, noise, fading, path-loss, shadowing, modulation, demodulation, spread-spectrum, frequency-hopping, channel estimation, synchronisation, equalisation, diversity, space-time processing, etc.
- Service, quality, message, voice, image, video, data, bit error rate, reliability, error-correction coding, performance, signal-to-noise ratio (SNR), security, privacy, safety, etc.

Multiple-Access Techniques

In wireless communications, multiple users are supported by the so-called multiple-access techniques. Typical wireless multiple-access techniques include:

- ❏ Frequency-Division Multiple-Access (FDMA): Split the channels in the frequency domain;
- ❏ Time-Division Multiple-Access (TDMA): Split the channel in the time domain;
- ❏ Code-Division Multiple-Access (CDMA): Using signature waveforms for users to transmit information in the same frequency band at the same time;
- ❏ Space-Division Multiple-Access (SDMA): Split the channels in the space domain.

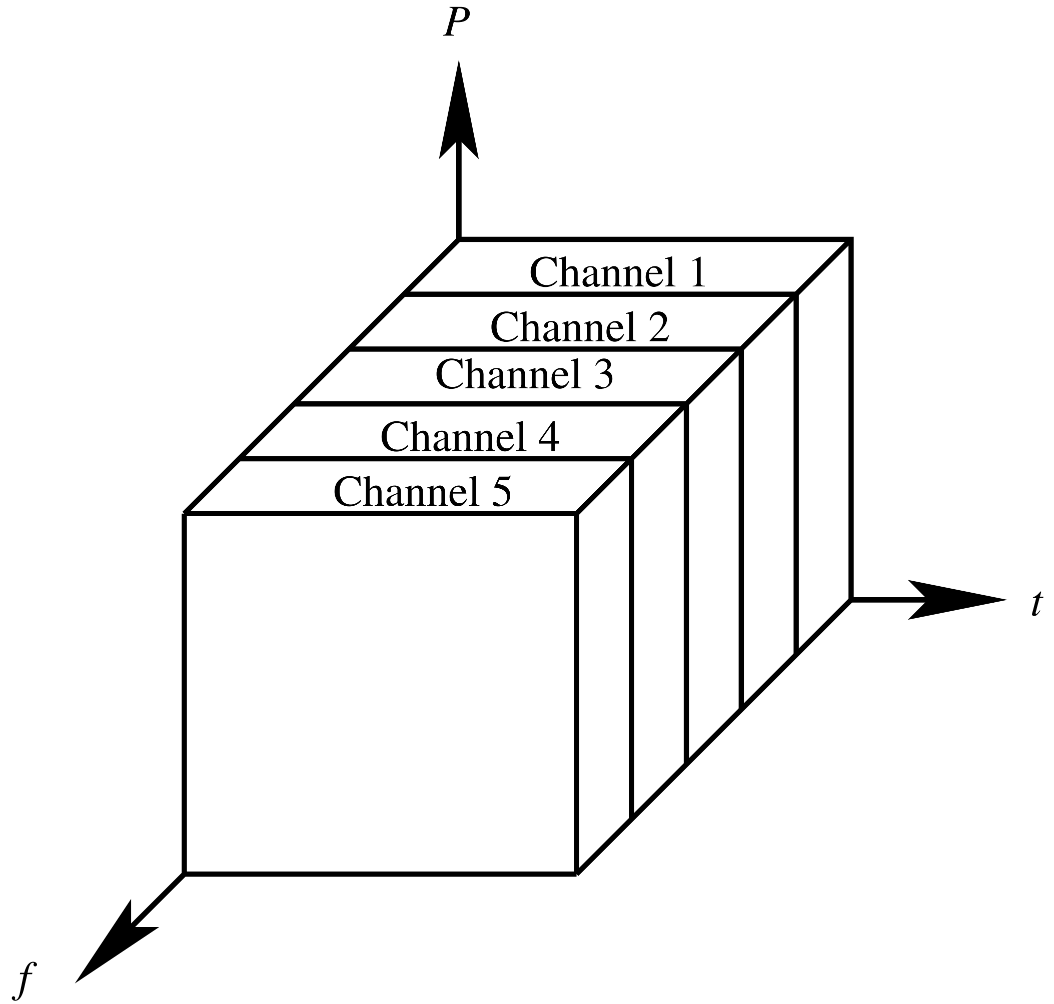


Figure 1: Illustration of channel configuration in FDMA systems. Different users transmit signals on different frequencies at the same time. ©John Wiley & Sons (L.-L. Yang, *Multi-carrier Communications*, John Wiley & Sons. 2009).

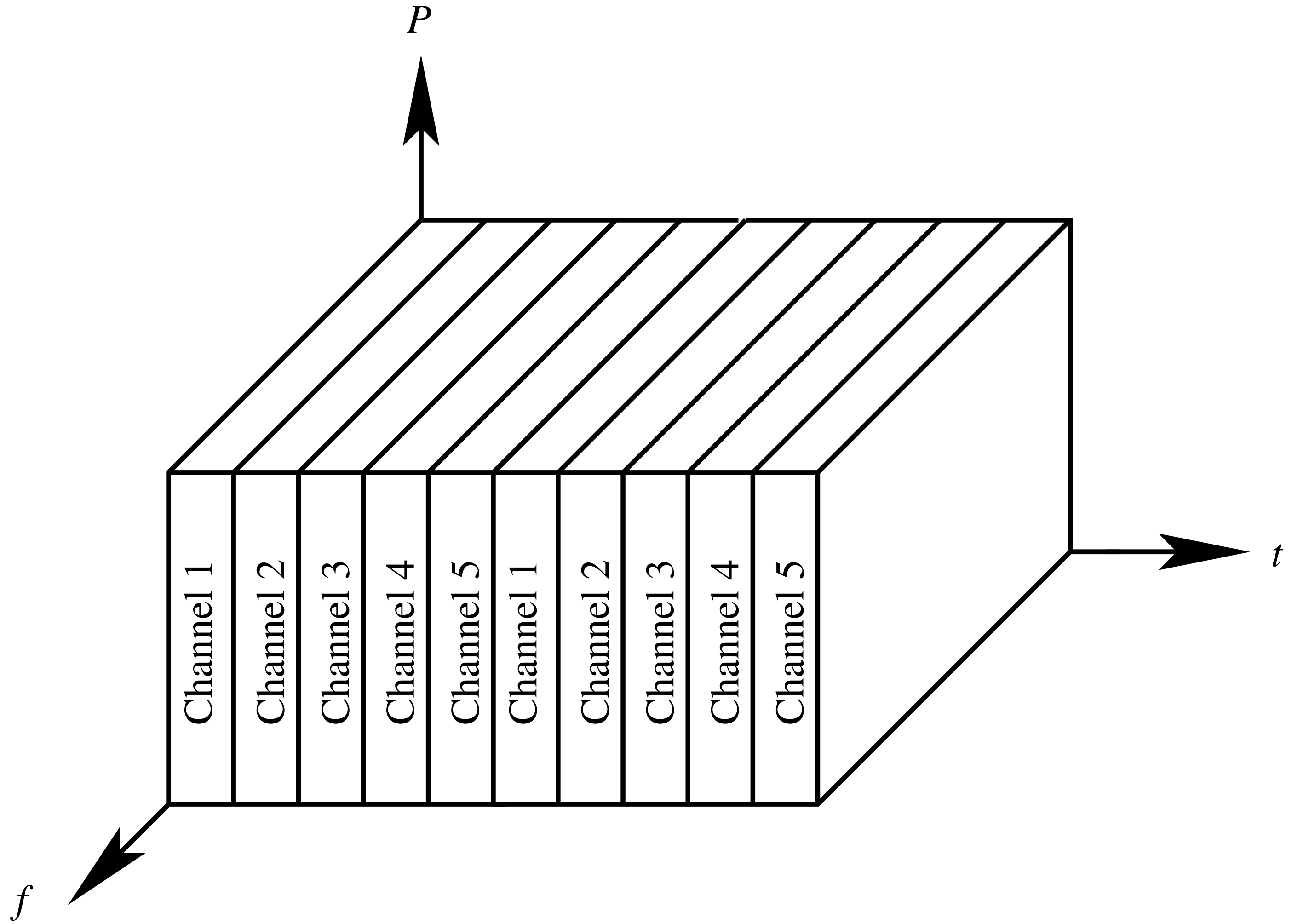


Figure 2: Illustration of channel configuration in TDMA systems. Different users transmit signals at different time-slots using the whole frequency-band available. ©John Wiley & Sons (L.-L. Yang, *Multicarrier Communications*, John Wiley & Sons. 2009).

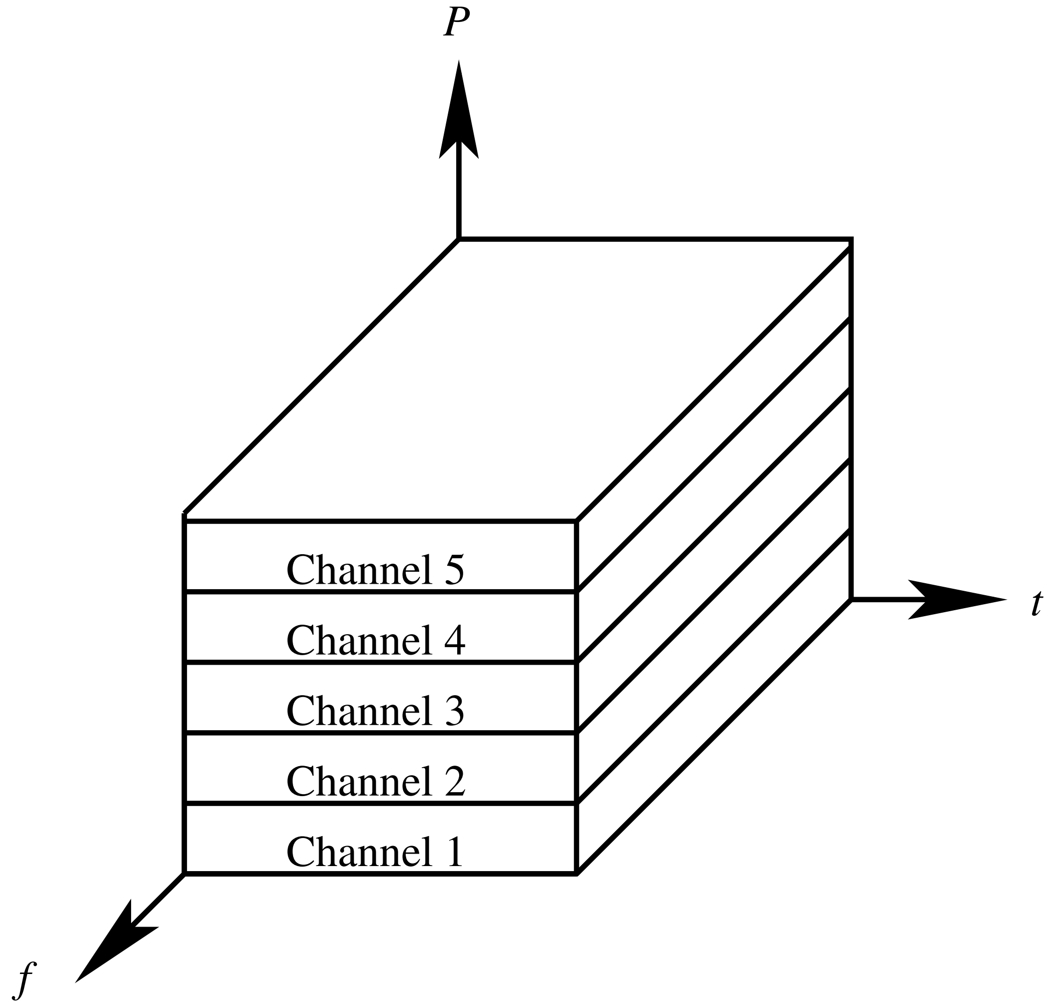


Figure 3: Illustration of channel configuration in CDMA systems. Different users are distinguished by their unique codes. All user signals are transmitted on the same frequency-band at the same time. ©John Wiley & Sons (L.-L. Yang, *Multicarrier Communications*, 2009).

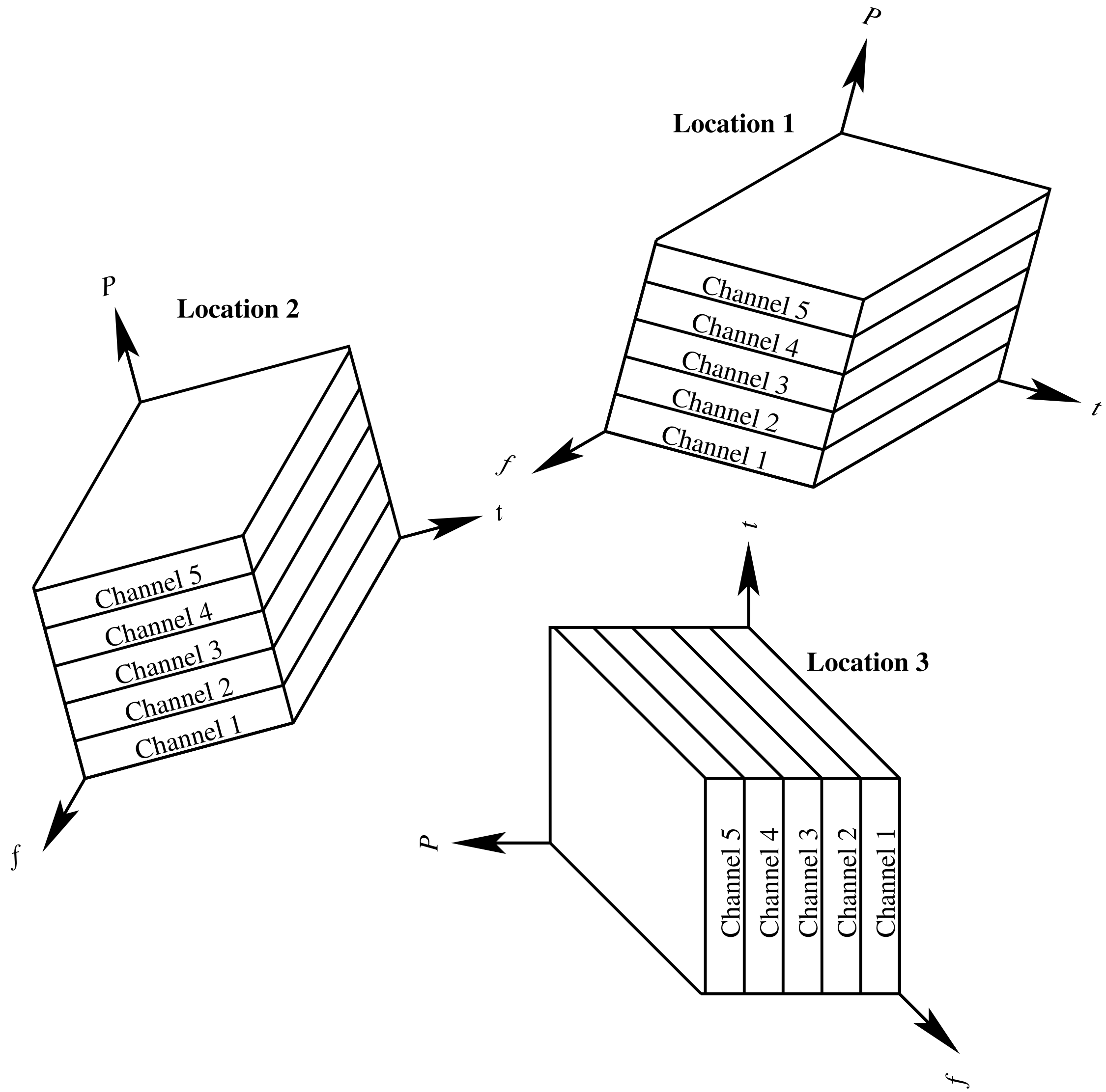
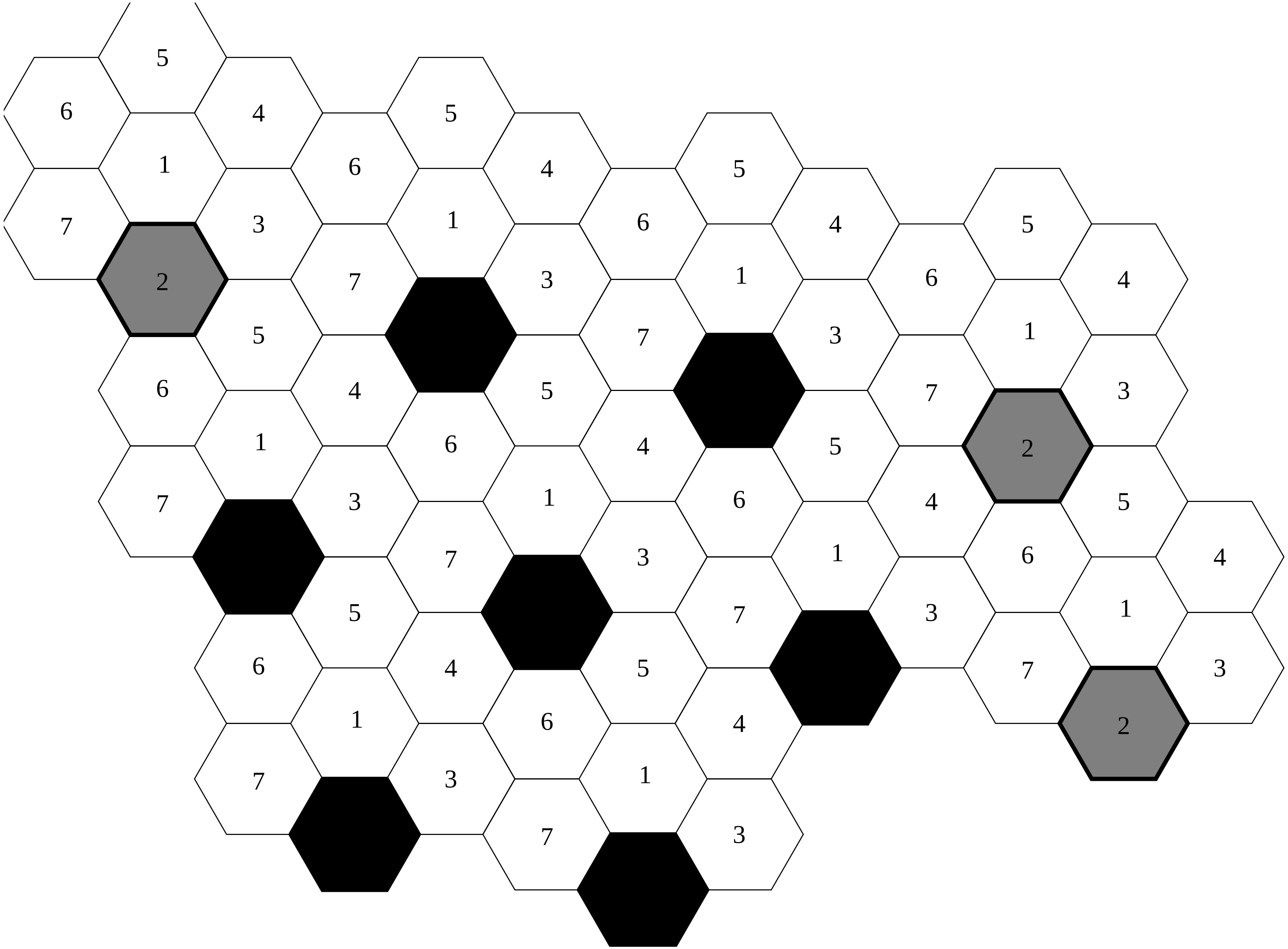


Figure 4: Illustration of channel configuration in SDMA systems. Different users or user sets can also be distinguished by their locations. ©John Wiley & Sons.



Carrier allocation: Hexagonal cells and seven-cell clusters.

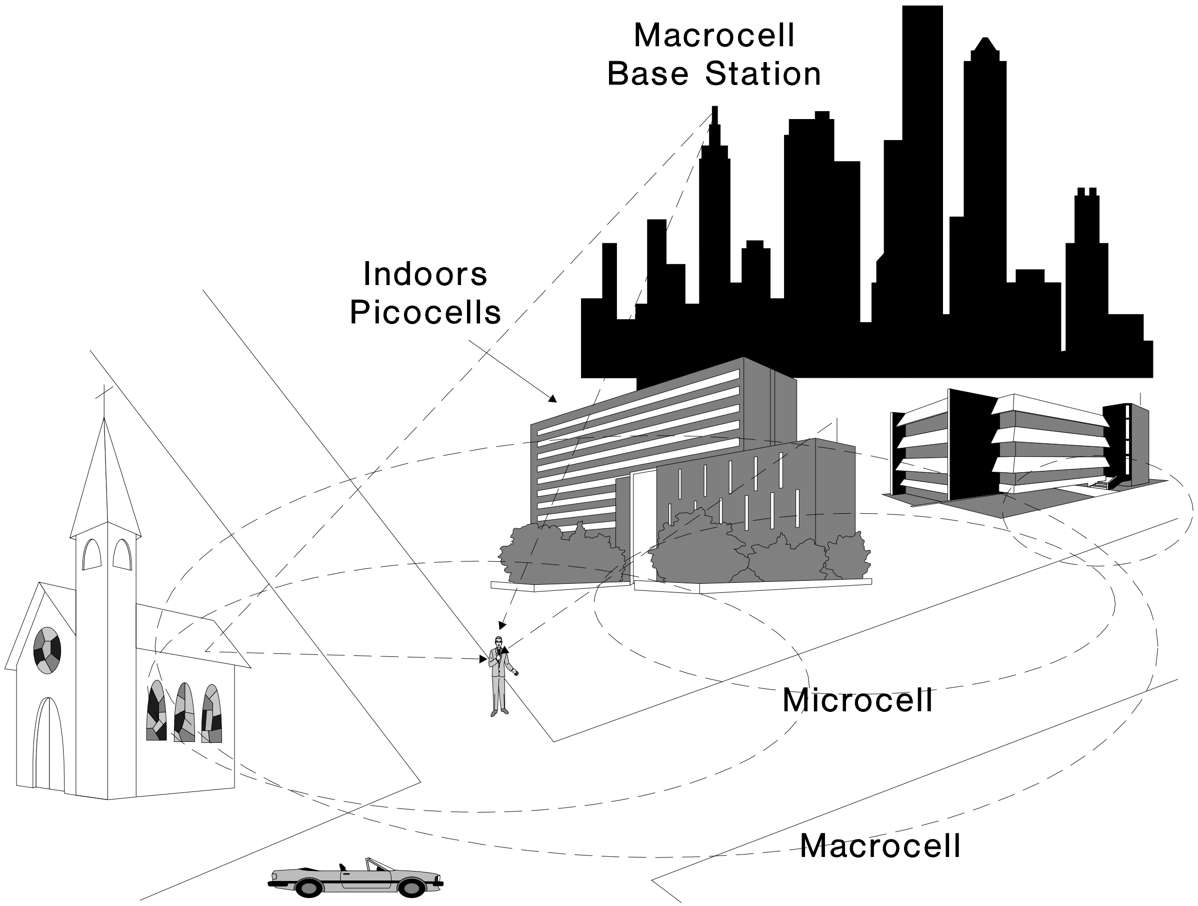


Figure 5: Various traffic cells.

Wireless Communications Systems - Classification

- Narrowband systems: bandwidth is less than the coherence bandwidth of the wireless channels, yielding frequency-nonselective (flat) fading;
- Wideband systems: bandwidth exceeds the coherence bandwidth of the wireless channels, yielding frequency-selective fading;
- Broadband systems: bandwidth is wider than that of the wideband systems, but lower than that of the UWB systems;
- Ultrawide bandwidth (UWB) systems: bandwidth exceeds at least 500 MHz or 20% of the center frequency.

Wireless Channel Modeling - Summary

- Propagation path-loss;
- Shadowing slow fading;
- Fast fading;
- Power-budget design in wireless communications systems.

Factors Affecting Wireless Signal Transmission

- ✔ **Propagation path-loss**: The strength of radio wave decreases as the distance between the transmitter and receiver increases;
- ✔ **Reflection**: When a radio wave propagating in one medium impinges upon another medium having different electrical properties, the wave is partially reflected and partially transmitted;
- ✔ **Diffraction** : Radio wave bends when it passes around an edge or through a slit. This bending is called diffraction;
- ✔ **Scattering**: When a radio wave impinges on a rough surface, the reflected energy is spread out (diffused) in all directions due to scattering;
- ✔ **Doppler effect**: When radio wave travels between two objects, the wavelength changes if one or both of them are moving. The Doppler effect is observed whenever the source of waves is moving with respect to an observer.
- ✔ **Dispersion**: Replicas of a radio signal that are reflected towards the receiver along different paths will arrive at different times, dispersing the signal in time.

Wireless Channel Modeling - Free Space Propagation

✔ The free space propagation model is usually used to predict received signal strength, when the transmitter and receiver have a clear, unobstructed line-of-sight (LoS) path between them.

✔ In free-space propagation environments the received signal power decays with the square of the propagation path length, and the received signal power can be expressed as

$$P_{\mathrm{r}}(d) = P_{\mathrm{t}} G_{\mathrm{T}} G_{\mathrm{R}} \left(\frac{\lambda}{4\pi d} \right)^2 = P_{\mathrm{t}} G_{\mathrm{T}} G_{\mathrm{R}} \left(\frac{c}{4\pi d f} \right)^2 \tag{1}$$

which shows that the received signal power decays with the 2nd power of the distance.

Table 1: Comparison between cmWave, mmWave and THz.

	cmWave	mmWave	THz
Boundaries (of interest)	f = 3GHz$\sim$ 30GHz λ = 10cm$\sim$ 1cm	f = 30GHz$\sim$ 100GHz λ = 10mm$\sim$ 3mm	f = 0.1THz$\sim$ 10THz λ = 3mm$\sim$30μm
Available Windows	C-band: 5.725 to 5.875 GHz	V-band: 57 to 64 GHz E-band: 76 to 81 GHz	Entire 10THz bandwidth subject to distance and absorption windows
Propagation Loss	Dominated by path loss.	Increased atmospheric absorption and penetration loss.	Similar to mmWave but water absorption becomes dominate.
Beamforming	Tx wide beams in 4G LTE	Tx&Rx narrow beams in 5G NR	Tx&Rx pencil beams
RF Chains	Fully-loaded	Fully-Connected	Array of Subarrays
Fading	Rich multipath, frequency-selective	Sparse multipath, frequency-selective	Stronger LoS, sparser multipath, near-flat
Example (I) on Distance	Fixed PL=76dB=$20\log_{10}(\frac{4\pi d}{\lambda})$: d = 50.2m$\sim$ 5.02m	Fixed PL=76dB=$20\log_{10}(\frac{4\pi d}{\lambda})$: d = 5.02m$\sim$ 1.5m	Fixed PL=76dB=$20\log_{10}(\frac{4\pi d}{\lambda})$: d = 1.5m$\sim$ 0.015m
Example (II) on Tx Gain	Fixed aperture A_e=80cm^2=$\frac{G_e\lambda^2}{4\pi}$: G_e = 10dB$\sim$ 30dB	Fixed aperture A_e=80cm^2=$\frac{G_e\lambda^2}{4\pi}$: G_e = 30dB$\sim$ 40.5dB	Fixed aperture A_e=80cm^2=$\frac{G_e\lambda^2}{4\pi}$: G_e = 40.5dB$\sim$ 80.5dB
Example (III) on Aperture	Fixed gain G_e = 30dB: A_e = 79.6dm^2 $\sim$ 80cm^2	Fixed gain G_e = 30dB: A_e = 80cm^2 $\sim$ 7.16cm^2	Fixed gain G_e = 30dB: A_e = 7.16cm^2 $\sim$ 0.07mm^2
Example (IV) on Angle	Fixed aperture A_e=80cm^2: $\Delta_e = \frac{\lambda}{\sqrt{A_e}} = 64^o \sim 6.4^o$	Fixed aperture A_e=80cm^2: $\Delta_e = \frac{\lambda}{\sqrt{A_e}} = 6.4^o \sim 1.9^o$	Fixed aperture A_e=80cm^2: $\Delta_e = \frac{\lambda}{\sqrt{A_e}} = 1.9^o \sim 0.02^o$

Free Space Propagation - Continued

The parameters in (1) are defined as follows:

- $P_{\mathrm{r}}(d)$ represents the received power in Watts at a distance of d from the transmitter, while P_{t} is the transmitted power;
- G_{T} is the transmitter antenna gain and G_{R} is the receiver antenna gain, which will be greater than 1 if the antennas are directional;
- λ is the wavelength in meters, while and d is the propagation path length.
- f is the frequency in Hertz and $c = 3 \cdot 10^8$ m/s is the speed of light,

Free Space Propagation - Continued

✔ The path-loss in free-space propagation environments can be expressed in dB as

$$L_{\mathrm{pl}} = -10\log_{10}\frac{P_{\mathrm{r}}(d)}{P_{\mathrm{t}}} \tag{2}$$

$$= -10\log_{10}\left[G_{\mathrm{T}}G_{\mathrm{R}}\left(\frac{c}{4\pi d f}\right)^2\right] \tag{3}$$

$$= 20\log_{10}d + 20\log_{10}f - 147.56$$
$$-10\log_{10}G_{\mathrm{T}} - 10\log_{10}G_{\mathrm{R}} \tag{4}$$

which shows that the received signal power decays 20dB/decade, when the receiver moves away from the transmitter.

Free Space Propagation - Continued

✔ Alternatively

$$L_{\mathrm{pl}} = -10\log_{10}\left[G_{\mathrm{T}}G_{\mathrm{R}}\left(\frac{c}{4\pi(d^{\mathrm{km}}\cdot 10^3)(f^{\mathrm{MHz}}\cdot 10^6)}\right)^2\right] \tag{5}$$

$$= 20\log_{10} d^{\mathrm{km}} + 20\log_{10} f^{\mathrm{MHz}} + 32.44 - 10\log_{10} G_{\mathrm{T}} - 10\log_{10} G_{\mathrm{R}} \tag{6}$$

Wireless Channel Modeling - Ground Reflection Model

- ✔ In practical wireless communications, however, the transmitted wireless signal does not experience free space propagation;
- ✔ Instead, the received signal constitutes the sum of many reflected multipath component signals possibly including a LoS component.
- ✔ In theory a more appropriate model assumes propagation over a flat reflecting surface, such as the earth, in addition to the LoS propagation, as shown in Fig.6.

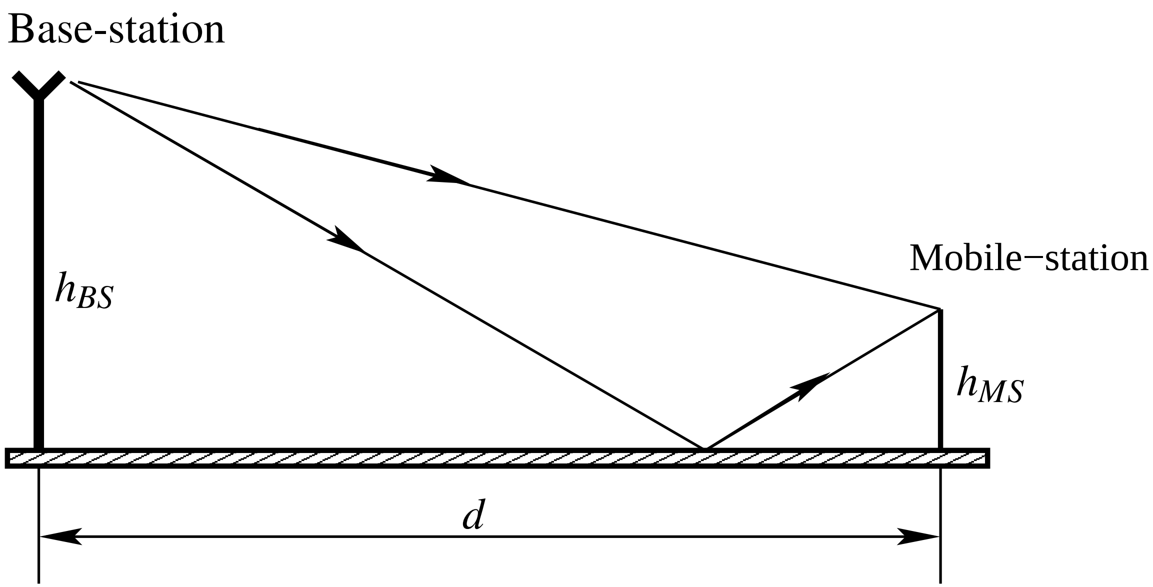


Figure 6: Radio signal propagation over flat reflecting surface.

For grazing incidence, the reflected wave is equal in magnitude to the incident wave and is π radians out of phase.

Ground Reflection Model - Continued

For the *Ground Reflection Model*, the received signal power is given by

$$P_{\mathrm{r}}(d) = P_{\mathrm{t}} G_{\mathrm{T}} G_{\mathrm{R}} \left(\frac{\lambda}{4\pi d} \right)^2 \cdot \left[2 \sin \left(\frac{\pi - \theta}{2} \right) \right]^2 \tag{7}$$

Here, $0 \leq \theta < 2\pi$ is the phase difference between the reflected and line-of-sight signals. Note that [7]

$$\frac{\pi - \theta}{2} \approx \frac{2\pi h_{\mathrm{BS}} h_{\mathrm{MS}}}{\lambda d} \tag{8}$$

when $d \gg h_{\mathrm{BS}} + h_{\mathrm{MS}}$, where h_{BS} and h_{MS} represent the heights of the BS and MS antennas in metres, respectively.

Ground Reflection Model - Continued

Combining 7 and 8 gives

$$P_r(d) \approx P_t G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2 \cdot \left[2\sin\left(\frac{2\pi h_{BS} h_{MS}}{\lambda d}\right)\right]^2 \qquad (9)$$

Note that $\sin(x) \approx x$, when x is small. Hence, when $\lambda d \gg h_{BS} h_{MS}$, (9) can be expressed as

$$P_r(d) \approx P_t G_T G_R \left(\frac{h_{BS} h_{MS}}{d^2}\right)^2 \qquad (10)$$

which shows that the received signal power decays with the 4th power of the distance.

Ground Reflection Model - Continued

✔ The propagation path-loss in ground reflection propagation environments can be expressed in dB as

$$
\begin{aligned}
L_{\text{pl}} &\approx -10\log_{10}\left[G_{\text{T}}G_{\text{R}}\left(\frac{h_{\text{BS}}h_{\text{MS}}}{d^2}\right)^2\right] && (11)\\
&\approx 40\log_{10}d - 20\log_{10}h_{\text{BS}} - 20\log_{10}h_{\text{MS}}\\
&\quad -10\log_{10}G_{\text{T}} - 10\log_{10}G_{\text{R}} && (12)\\
&\approx 120 + 40\log_{10}d^{\text{km}} - 20\log_{10}h_{\text{BS}}^{\text{m}} - 20\log_{10}h_{\text{MS}}^{\text{m}}\\
&\quad -10\log_{10}G_{\text{T}} - 10\log_{10}G_{\text{R}} && (13)
\end{aligned}
$$

which corresponds to a 40dB/decade decay when the receiver moves away from the transmitter.

Hata's Urban Path-Loss Model

✔ Hata's empirical model is a simple and widely used model for the path-loss in urban areas;

$$L_{\mathrm{Hu}} = 69.55 + 26.16 \log_{10} f^{\mathrm{MHz}} - 13.82 \log_{10} h_{\mathrm{BS}}^{\mathrm{m}} - a(h_{\mathrm{MS}}^{\mathrm{m}}) + (44.9 - 6.55 \log_{10} h_{\mathrm{BS}}^{\mathrm{m}}) \log_{10} d^{\mathrm{km}} \quad (14)$$

✔ Limitations for using Hata's model:

- Carrier frequency: $150 \leq f^{\mathrm{MHz}} \leq 1500$;
- BS antenna height: $30 \leq h_{\mathrm{BS}}^{\mathrm{m}} \leq 200$;
- MS antenna height: $1 \leq h_{\mathrm{MS}}^{\mathrm{m}} \leq 10$;
- Distance: $1 \leq d^{\mathrm{km}} \leq 20$.

Hata's Urban Path-Loss Model - Continued

In Hata's path-loss model of (14), $a(h_{\mathrm{MS}}^{\mathrm{m}})$ represents a terrain-dependent correction factor.

✔ For small- and medium-sized cities, $a(h_{\mathrm{MS}}^{\mathrm{m}})$ was found to be

$$a(h_{\mathrm{MS}}^{\mathrm{m}}) = \left[1.1\log_{10}(f^{\mathrm{MHz}}) - 0.7\right] h_{\mathrm{MS}}^{\mathrm{m}} - \left[1.56\log_{10}(f^{\mathrm{MHz}}) - 0.8\right] \tag{15}$$

✔ For large cities, the correction factor $a(h_{\mathrm{MS}}^{\mathrm{m}})$ is frequency-parameterized and can be expressed as [7]

$$a(h_{\mathrm{MS}}^{\mathrm{m}}) = \begin{cases} 8.29\left[\log_{10}(1.54h_{\mathrm{MS}}^{\mathrm{m}})\right]^2 - 1.1, & f^{\mathrm{MHz}} \leq 200 \\ 3.2\left[\log_{10}(11.75h_{\mathrm{MS}}^{\mathrm{m}})\right]^2 - 4.97, & f^{\mathrm{MHz}} \geq 400 \end{cases} \tag{16}$$

Hata's Suburban and Rural Path-Loss Models

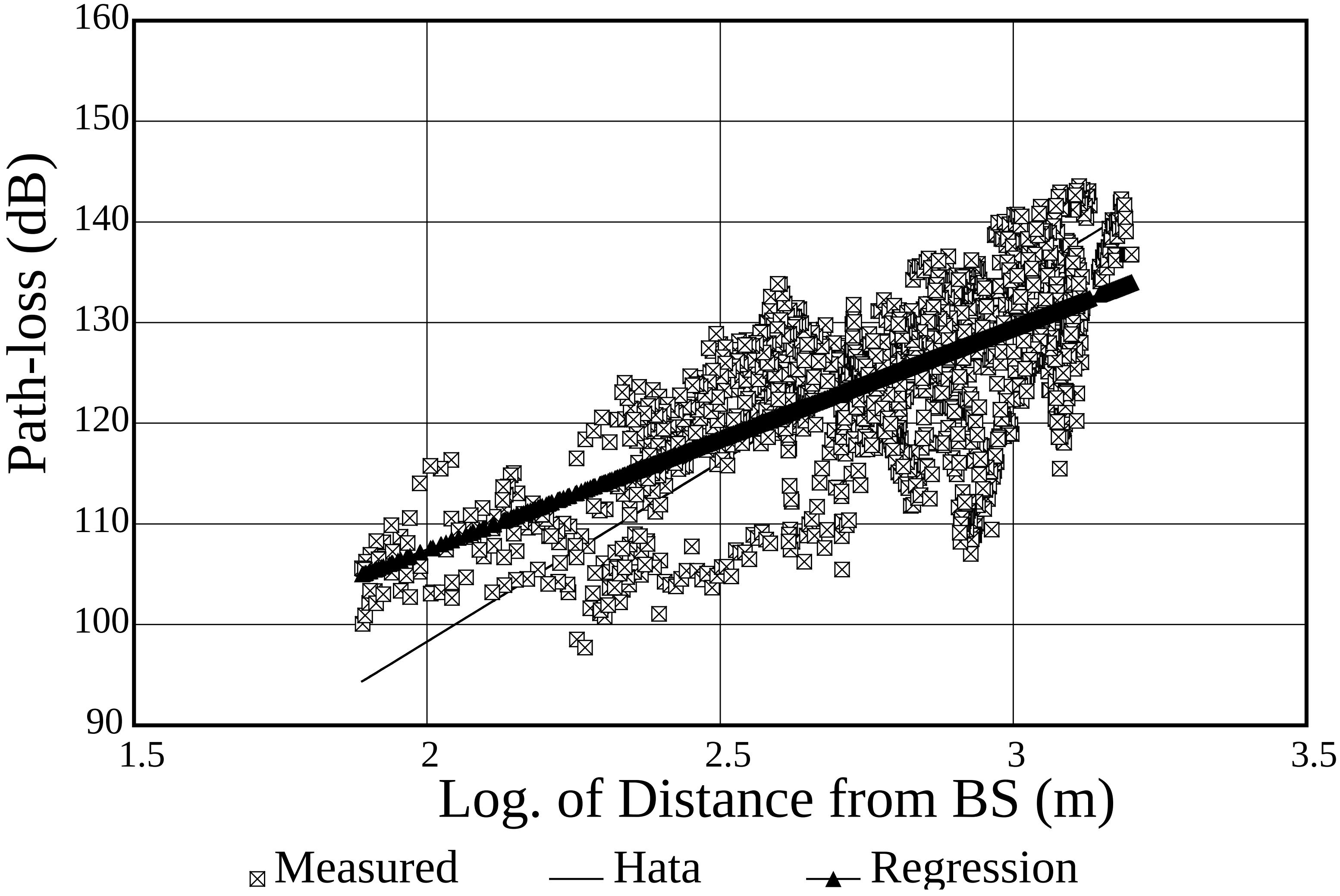
For suburban areas, a correction factor is applied to the urban path loss model,

$$L_{\mathrm{Hs}} = L_{\mathrm{Hu}} - 2\left[\log_{10}(f^{\mathrm{MHz}}/28)\right]^2 - 5.4 \tag{17}$$

Likewise, for rural areas

$$L_{\mathrm{Hr}} = L_{\mathrm{Hu}} - 4.78\left[\log_{10} f^{\mathrm{MHz}}\right]^2 + 18.33\log_{10} f^{\mathrm{MHz}} - 40.94 \tag{18}$$

PATH-LOSS REGRESSION



Typical microcellular path-loss regression line fitting and the corresponding Hata path-loss characteristic for an antenna elevation of 6.4 m and propagation frequency of 1.9 GHz ©Wiley 1999, Greenwood & Hanzo.

Wireless Channel Modeling - Shadowing Slow Fading

- ✔ Shadowing slow fading is mainly caused by terrain and topographical features in the vicinity of the mobile receiver, such as small hills and tall buildings;
- ✔ Wireless channels typically experience propagation path-loss, shadowing slow fading and fast fading. Hence, when our focus is on the shadowing slow-fading, the effects of both fast-fading and path-loss must be removed;
- ✔ **Local-mean**: The fast fading is removed by deriving the so-called local-mean, which is obtained by averaging the signal level over a distance of typically some 20 wavelength.

Shadowing Slow Fading - Continued

✔ Based on empirical measurements, it has been shown that the slow fading gain follows a *lognormal* distribution [7];

$$\mathrm{PDF}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \tag{19}$$

where

- x is the slow fading gain in dB;
- μ and σ are the mean and standard deviation of x, expressed in dB. The standard deviation σ typically ranges from 5 dB to 12 dB, depending on the specific communication scenario considered.

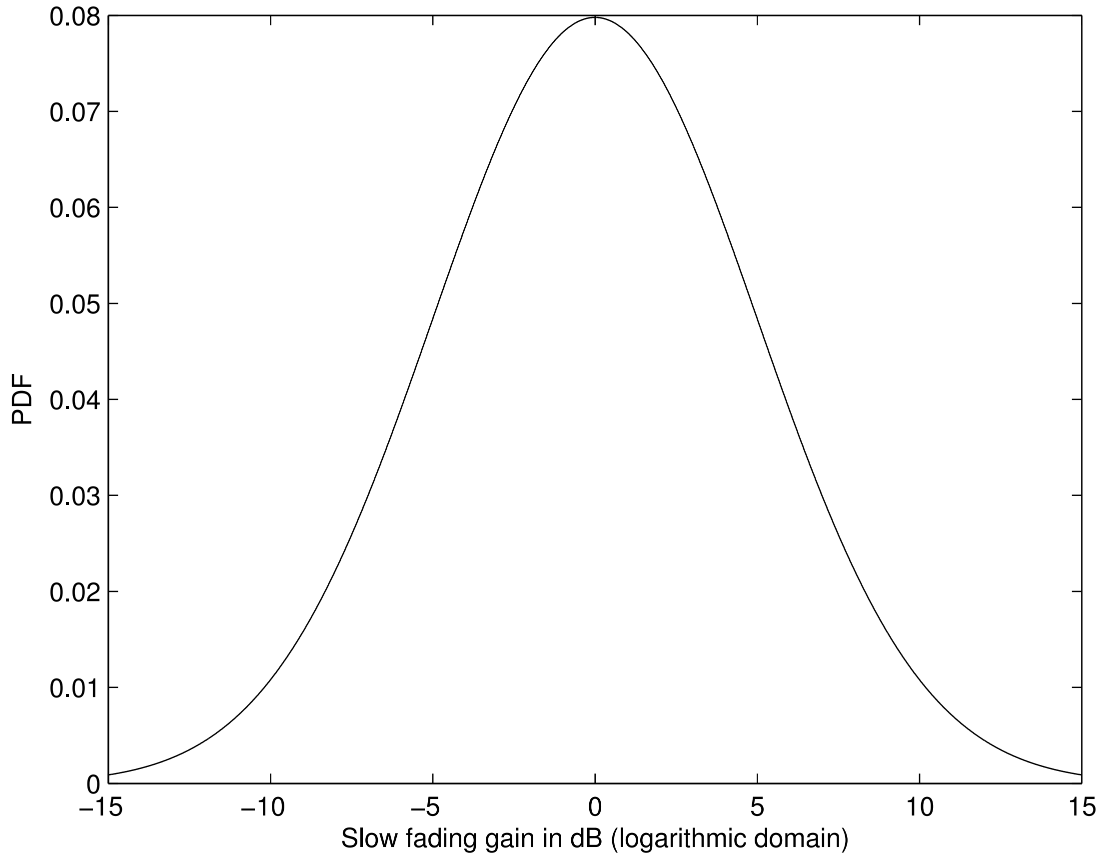


Figure 7: Slow-fading histogram, where the x-axis shows the slow fading gain in dB. Here, the mean is $\mu = 0$ dB and the standard deviation is $\sigma = 5$ dB.

Shadowing Slow Fading - Continued

✔ In the normal domain

$$\mathrm{PDF}(y) = \frac{10/\ln 10}{y\sigma\sqrt{2\pi}} \exp\left[-\frac{(10\log_{10} y - \mu)^2}{2\sigma^2}\right] \tag{20}$$

where

- y is the slow fading gain as a fraction, where $x = 10\log_{10} y$ dB;
- μ and σ are the mean and standard deviation of x in dB, as before.

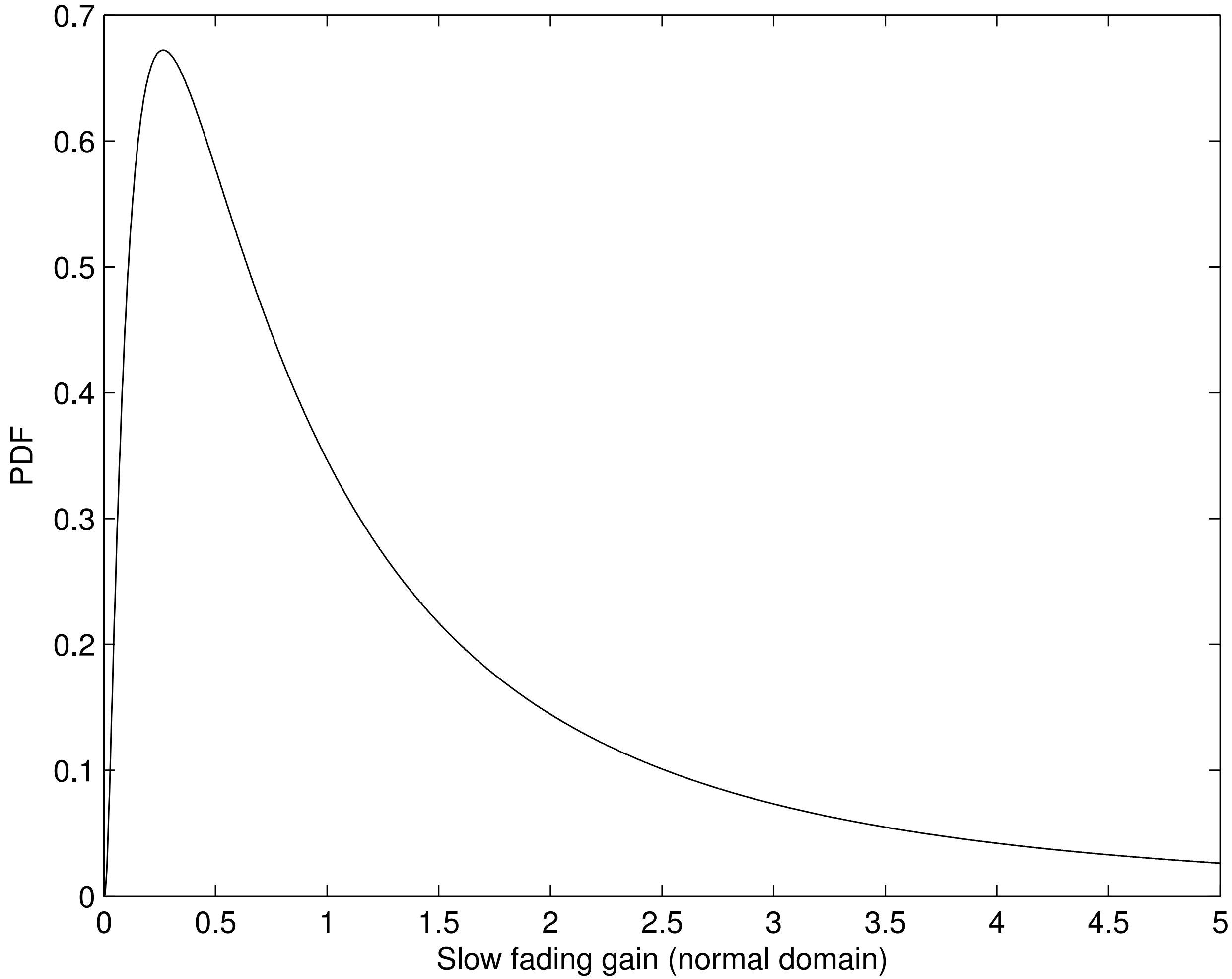


Figure 8: Lognormal distributed PDF, where the x-axis is shows the slow fading gain in the normal domain. Here, the mean is $\mu = 0$ dB and the standard deviation is $\sigma = 5$ dB.

Wireless Channel Modeling - Fast Fading

Fast fading is also referred to as *small-scale fading*, which accounts for the rapid variation of signal levels, when the user terminal moves within a small or local area. There are many physical factors in the radio propagation channel, which result in fast fading, which typically include

- ✔ Multipath propagation;
- ✔ Doppler effect;
- ✔ Carrier-frequency, bandwidth and symbol rate of the transmitted signal, etc.

Fast Fading - Continued

- ✔ Like slow fading, fast fading is associated with a multiplicative channel gain. However, the fast fading gain $a_i + ja_q$ is complex, having both in-phase a_i and quadrature a_q components.
- ✔ This complex gain has an amplitude a and a phase ϕ, where

$$a = \sqrt{a_i^2 + a_q^2}, \tag{21}$$

$$\phi = \begin{cases} \arctan\left(\frac{a_q}{a_i}\right) & \text{if } a_i \geq 0 \\ \arctan\left(\frac{a_q}{a_i}\right) - \pi & \text{otherwise} \end{cases}, \tag{22}$$

$$a_i = a\cos(\phi), \quad a_q = a\sin(\phi). \tag{23}$$

- ✔ The amplitudes of the complex gain is multiplied with that of the modulated and demodulated signals, while phase of the complex gain is added to that of the signals.

Fast Fading - Continued

- ✔ Regardless of the distributions of the various multi path gains, both components a_i and a_q of the complex channel gain will have normal (Gaussian) distributions due to the central limit theorem.
- ✔ Unlike the slow fading channel gain (which has a lognormal distribution), the distributions of a_i and a_q can go negative.
- ✔ The distribution means $\overline{a_i}$ and $\overline{a_q}$ will be non-zero if there is a line-of-sight path, where $s^2 = \overline{a_i}^2 + \overline{a_q}^2$ is the fraction of the transmitted power that is received by the line-of-sight path.
- ✔ We assume that the distributions of a_i and a_q have equal variances σ^2, where $2\sigma^2$ is the fraction of the transmitted power that is received due to scattering, giving $s^2 + 2\sigma^2 = 1$.
- ✔ Our aim is to determine the distribution of the amplitude $a = \sqrt{a_i^2 + a_q^2}$.

Fast Fading - Continued

- ✔ In general, for n normally distributed random constituent processes with means $\overline{a_i}$ and identical variances σ^2, the resultant process $y = \sum_{i=1}^{n} a_i^2$ has a χ^2 (chi-squared) distribution with a PDF given by:

$$p(y) = \frac{1}{2\sigma^2}\left(\frac{y}{s^2}\right)^{(n-2)/4} \cdot e^{-(s^2+y)/2\sigma^2} \cdot I_{(n/2)-1}\left(\sqrt{y}\frac{s}{\sigma^2}\right) \tag{24}$$

where $y \geq 0$ and $s^2 = \sum_{i=1}^{n} (\overline{a_i})^2$ is the noncentrality parameter computed from the means of the component processes $a_1 \cdots a_n$;

- ✔ n is called the degrees-of-freedom of the χ^2-distribution.

Fast Fading - Continued

- ✔ If the constituent processes have zero means, i.e., if $\overline{a_i} = 0$, the χ^2-distribution is referred to as the central χ^2-distribution; otherwise is referred to as the noncentral χ^2-distribution;
- ✔ In (24) $I_k(x)$ is the modified kth-order Bessel function of the first kind, given by

$$I_k(x) = \sum_{j=0}^{\infty} \frac{(x/2)^{k+2j}}{j!\Gamma(k+j+1)}, \; x \geq 0. \tag{25}$$

- ✔ The Γ function is defined as

$$\begin{aligned} \Gamma(p) &= \int_0^{\infty} t^{p-1} e^{-t} dt \quad \text{if } p > 0 \\ \Gamma(p) &= (p-1)! \qquad \text{if } p > 0 \text{ integer} \\ \Gamma(\frac{1}{2}) &= \sqrt{\pi} \; , \; \Gamma(\frac{3}{2}) = \frac{\sqrt{\pi}}{2}. \end{aligned} \tag{26}$$

Fast Fading - Continued

✔ In our case we have two quadrature components, that is,

- $n = 2$, $s^2 = (\overline{a_i})^2 + (\overline{a_q})^2$, $y = a_i^2 + a_q^2 = a^2$;

$$p_{\mathsf{Rice}}(y) = \frac{1}{2\sigma^2} e^{-(y+s^2)/2\sigma^2} I_o\left(\frac{\sqrt{y}s}{\sigma^2}\right) \tag{27}$$

- But, we want $p_{\mathsf{Rice}}(a)$, where $a = \sqrt{y}$.
- We can't just swap y for a^2, we need to make sure the intergral still comes to 1. Hence, we also need to multiply by $dy/da = 2a$

$$p_{\mathsf{Rice}}(a) = \frac{a}{\sigma^2} e^{-(a^2+s^2)/2\sigma^2} I_o\left(\frac{as}{\sigma^2}\right) \quad a \geq 0. \tag{28}$$

Fast Fading - Continued

- ✔ Formally introducing the Rician K-factor as

$$K = s^2/2\sigma^2 \tag{29}$$

renders the Rician distribution's PDF dependent on one parameter only, yielding:

$$p_{\text{Rice}}(a) = \frac{a}{\sigma^2} \cdot e^{-\frac{a^2}{2\sigma^2}} \cdot e^{-K} \cdot I_o\left(\frac{a}{\sigma} \cdot \sqrt{2K}\right), \tag{30}$$

- ✔ where K physically represents the ratio of the power received in the direct line-of-sight path to the total power received via indirect scattered paths.

Fast Fading - Continued

- ✔ When there is no dominant propagation path, $K = 0$, $e^{-K} = 1$ and $I_0(0) = 1$, yielding the worst-case Rayleigh PDF:

$$p_{\text{Rayleigh}}(a) = \frac{a}{\sigma^2} e^{-\frac{a^2}{2\sigma^2}}. \tag{31}$$

- ✔ In this case $2\sigma^2$ is the total fraction of the transmitted power that is received, since none is received by line-of-sight.

- ✔ Conversely, in the clear direct line-of-sight situation with no scattered power, $K = \infty$, yielding a Dirac delta-shaped PDF, representing a step-function-like cumulative distribution function (CDF).

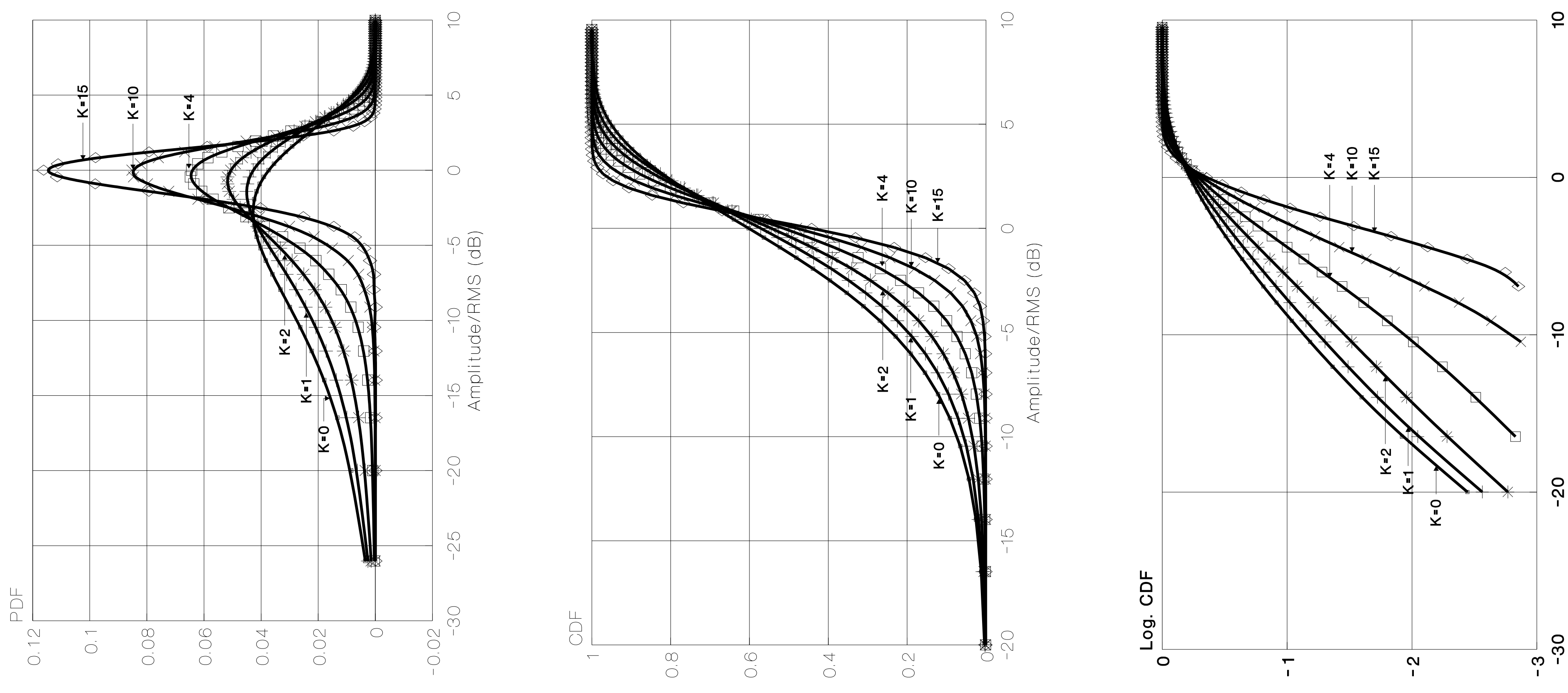


Figure 9: Rician PDFs. ⓒWebb & Hanzo.

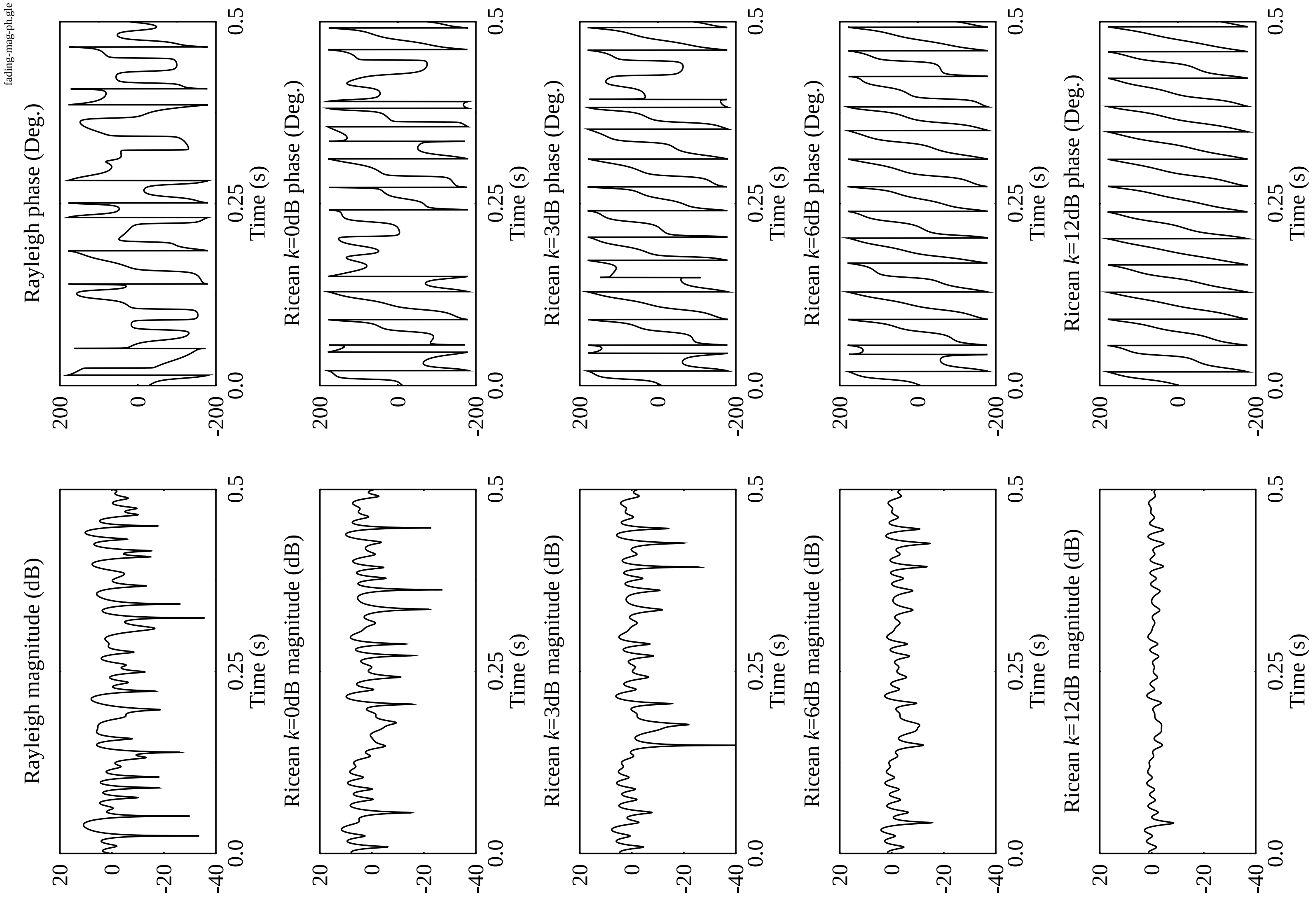


Figure 10: Typical Rayleigh and Rician fast-fading and phase profiles for a MS speed of 30 mph and carrier frequency of 900 MHz.

Wireless Channel Modeling - Power-Budget Design

Factors affecting the power-budget design:

- ✔ Propagation path-loss;
- ✔ Shadowing slow fading;
- ✔ Fast fading;
- ✔ Additive White Gaussian Noise.

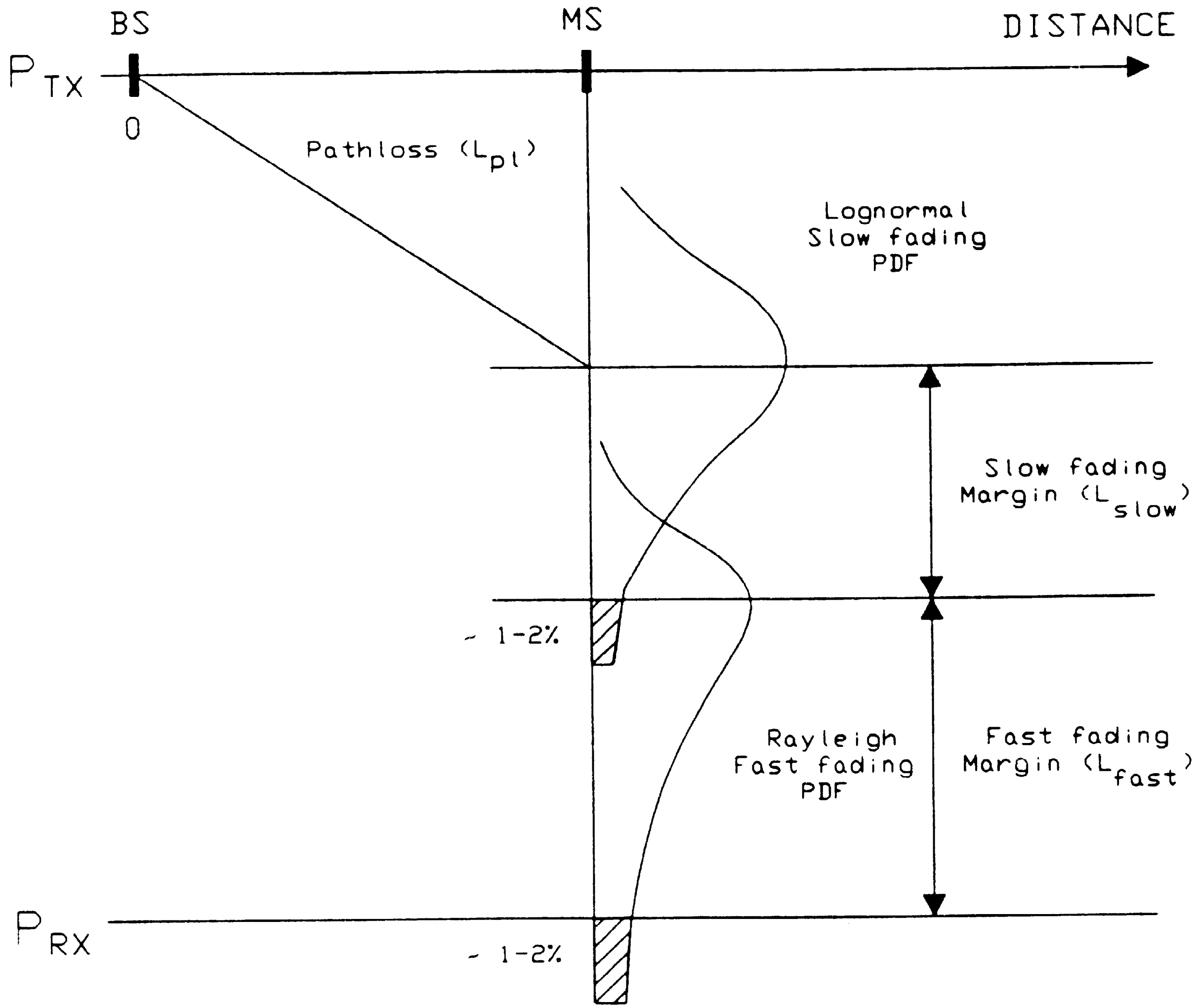


Figure 11: Power-budget design for mobile systems.

Power-Budget Design - Continued

- ✔ We deploy a correction factor $a(h_{MS})$ corresponding to the antenna elevation deduced from measurements, and then estimate the path-loss L_{pl} using the Hata-model;
- ✔ Using the characteristic slow fading variance of, say, σ=7 dB and a mean of $\mu = 0$ dB, assuming lognormal slow fading PDF and allowing for a 1.4% slow 'fading margin overload' probability we introduce a 'slow fading margin' of $L_{slow} = 2 \cdot \sigma$= 14 dB.
- ✔ Assuming a typical Rician fading with $K = 10$ and a fast 'fading margin overload' probability of 1% a 'fast fading margin' of L_{fast}=7 dB is inferred from Figure 9.
- ✔ Summing the pathloss and the two fading margin components from above yields a total pathloss of: $L_{total} = L_{pl} + L_{slow} + L_{fast}$. With the knowledge of the receiver sensitivity, i.e., the required minimum received power P_{rec} in dBm, this then allows us to compute the required minimum transmitted power as: $P_{tx} = P_{rec} + L_{total}$.

Power-Budget Design - An Example

- ✔ Let us compare the transmitted power requirements for an urban microcellular environment with cell radii of 300 m and 100 m, using the above mentioned L_{slow}=14 dB and L_{fast}=7 dB fading margins;
- ✔ Assume that our extensive measurement programme in this environment suggests a pathloss of L_{pl}=130 dB at a distance of 300 m from the BS, when the antenna elevation is AH=6.4 m, while at 100 m L_{pl}=110 dB.
- ✔ Then we have pathlosses of L_{300}=130 dB+14 dB+7 dB=151 dB and L_{100}=131 dB. Assuming a receiver sensitivity of -104 dBm, as in the Pan-European digital mobile radio system, the corresponding transmitted power requirements are: P_{300}=47 dBm $\approx$ 50 W and P_{100}=27 dBm $\approx$ 0.5 W.

Power-Budget Design - Continued

- ✔ Clearly, for this low antenna elevation microcellular scenario a cell radius of larger than 100 m is becoming unrealistic in terms of transmitted power;
- ✔ Increasing the antenna height substantially reduces the transmitted power requirement or extends the cell radius;
- ✔ When the BS antenna is elevated beyond the urban sky-line, substantially higher coverage area can be achieved at the same transmitted power;
- ✔ the fading gradually becomes Rayleigh, requiring a higher fading margin and thereby reducing the gains achieved.

Wireless Channel Modeling - Doppler Effect

Consider a MS traveling in a direction at an angle of α with respect to the signal received on the path from the BS.

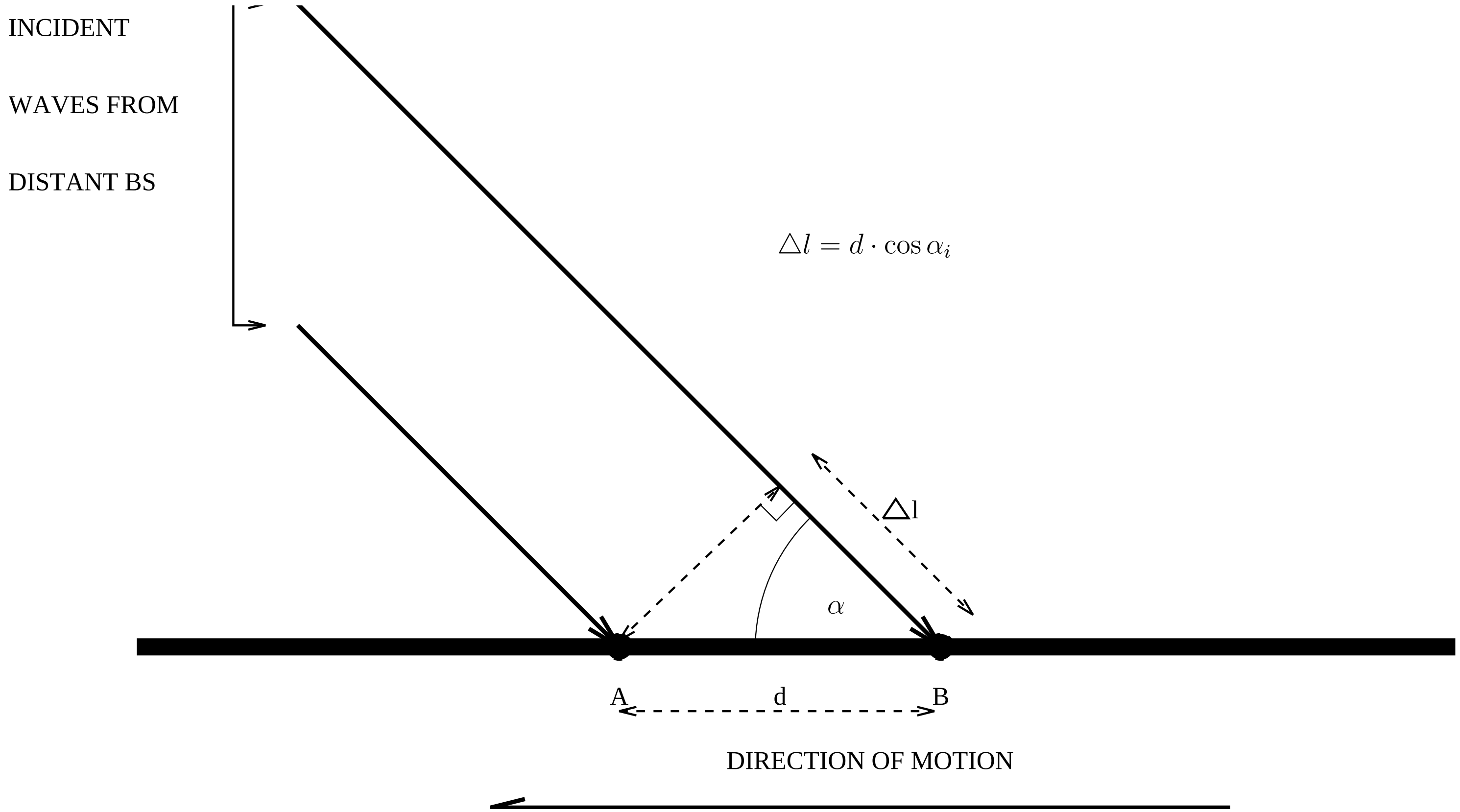


Figure 12: Relative path-length change due to the MS's movement. ©Webb & Hanzo.

Doppler Effect - Continued

- ✔ It takes $\triangle t = d/v$ seconds for the MS to travel the distance between B and A of d metres, when moving at a velocity of v m/s.
- ✔ If the flight of the modulated carrier wave is shortened by an amount of $\triangle l = \lambda = c/f_c$ metres, then its apparent phase is changed by $\triangle\Phi = -2\pi$ radians.
- ✔ The negative sign implies that the carrier wave's phase delay is reduced, since the flight is reduced when the MS is traveling toward the BS.
- ✔ For an arbitrary $\triangle l$ value we have $\triangle\Phi = -2\pi \cdot \triangle l/\lambda$.
- ✔ Substituting $\triangle l = d \cdot \cos\alpha = v\triangle t \cdot \cos\alpha$ gives

$$\triangle\Phi = -\frac{2\pi v \triangle t \cos\alpha}{\lambda}. \tag{32}$$

Doppler Frequency

✔ We define the Doppler frequency as the phase change due to the movement of the MS during the infinitesimal time interval $\triangle t$:

$$f_D = -\frac{1}{2\pi}\frac{\triangle\Phi}{\triangle t}. \tag{33}$$

✔ This quatifies how many wavelengths the flight is changed by per second due to the motion of the MS.

✔ When substituting Equation 32 into 33 we get:

$$f_D = \frac{v}{\lambda}\cos\alpha = f_m \cos\alpha, \tag{34}$$

where $f_m = v/\lambda = vf_c/c$ is the maximum Doppler frequency shift associated with the transmitted carrier frequency due to the MS's movement;

Doppler Effect - Continued

- ✔ Notice that a Doppler frequency can be positive or negative depending on α and that the maximum and minimum Doppler frequencies are $\pm f_m$;
- ✔ These extreme frequencies correspond to $\alpha = 0^\circ$ and 180°, when the ray is coming from directly in front of the MS or directly behind it, respectively.
- ✔ It is analogous to the change in the frequency of a whistle from a train perceived by a person standing on a railway station's platform, when the train is bearing down or receding from the person, respectively.

Doppler Effect - Continued

- ✔ Consider the case where there are multipaths coming in from all angles.
- ✔ The received power in an angle of $d\alpha$ around a particular direction α is given by $p(\alpha)d\alpha$, where $p(\alpha)$ is the power angular density.
- ✔ The received power is assumed to be uniformly distributed over the range of $0 \leq \alpha \leq 2\pi$.
- ✔ The total received power is assumed to be $\int_{\alpha=0}^{2\pi} p(\alpha)d\alpha = 1$.
- ✔ Therefore, $p(\alpha) = 1/2\pi$.

Doppler Effect - Continued

- ✔ The Doppler power spectral density $S(f_D)$ can be computed using Parseval's theorem.
- ✔ This equates the power $p(\alpha)d\alpha$ incident in the infinitesimal angle $d\alpha$ with the power received $S(f_D)df_D$ within an infinitesimal bandwidth df_D.
- ✔ Hence,

$$S(f_D) = \frac{1}{2\pi}\frac{d\alpha}{df_D}. \tag{35}$$

Doppler Effect - Continued

- ✔ Since $f_D = f_m \cos\alpha$ is an even function, $f_D = f_m \cos -\alpha$ and $\alpha = -\cos^{-1}\frac{f_D}{f_m}$.
- ✔ $\frac{d\cos^{-1}x}{dx} = -\frac{1}{\sqrt{1-x^2}}$ so $\frac{d\alpha}{df_D} = \frac{1}{f_m\sqrt{1-(f_D/f_m)^2}}$
- ✔ Therefore

$$S(f_D) = \frac{1}{2\pi f_m\sqrt{1-(f_D/f_m)^2}}. \qquad (36)$$

Doppler Effect - Continued

- ✔ The incident received power at the MS depends on the power gain of the antenna and the polarization used;
- ✔ Thus, the transmission of an unmodulated carrier is received as a "smeared" signal whose spectrum is not a single carrier frequency f_c but contains frequencies up to $f_c \pm f_m$.

Doppler Effect - Continued

- ✔ In general, we can express the received RF spectrum $S(f_D)$ for a particular MS speed, propagation frequency, antenna design, and polarization as [Gans72]:

$$S(f_D) = \frac{C}{\sqrt{1-(f_D/f_m)^2}}, \tag{37}$$

  where C is a constant that absorbs the $1/2\pi f_m$ multiplier in Equation 36.

- ✔ Notice that the Doppler spectrum of Equation 37 becomes $S(f_D = 0) = C$ at $f_D = 0$, while $S(f_D = f_m) = \infty$, when $f_D = f_m$.

- ✔ Between these extremes, $S(f_D)$ has a U shaped characteristic. Outside of them, there is no power.

Wireless Channel Modeling - Doppler Effect

The Doppler spectrum acts like a low pass filter on the fast fading channel gain, inducing the correlation seen in Figure 10.

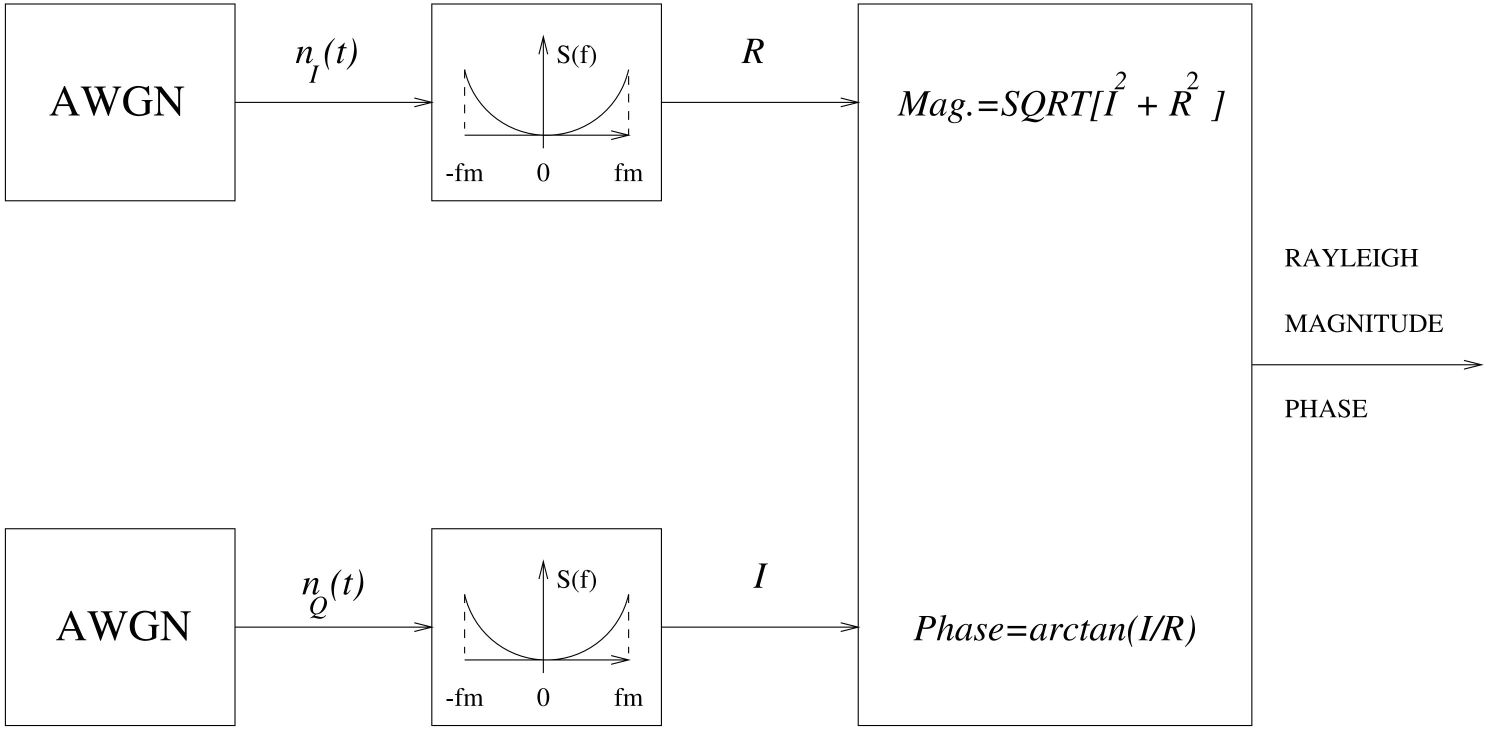
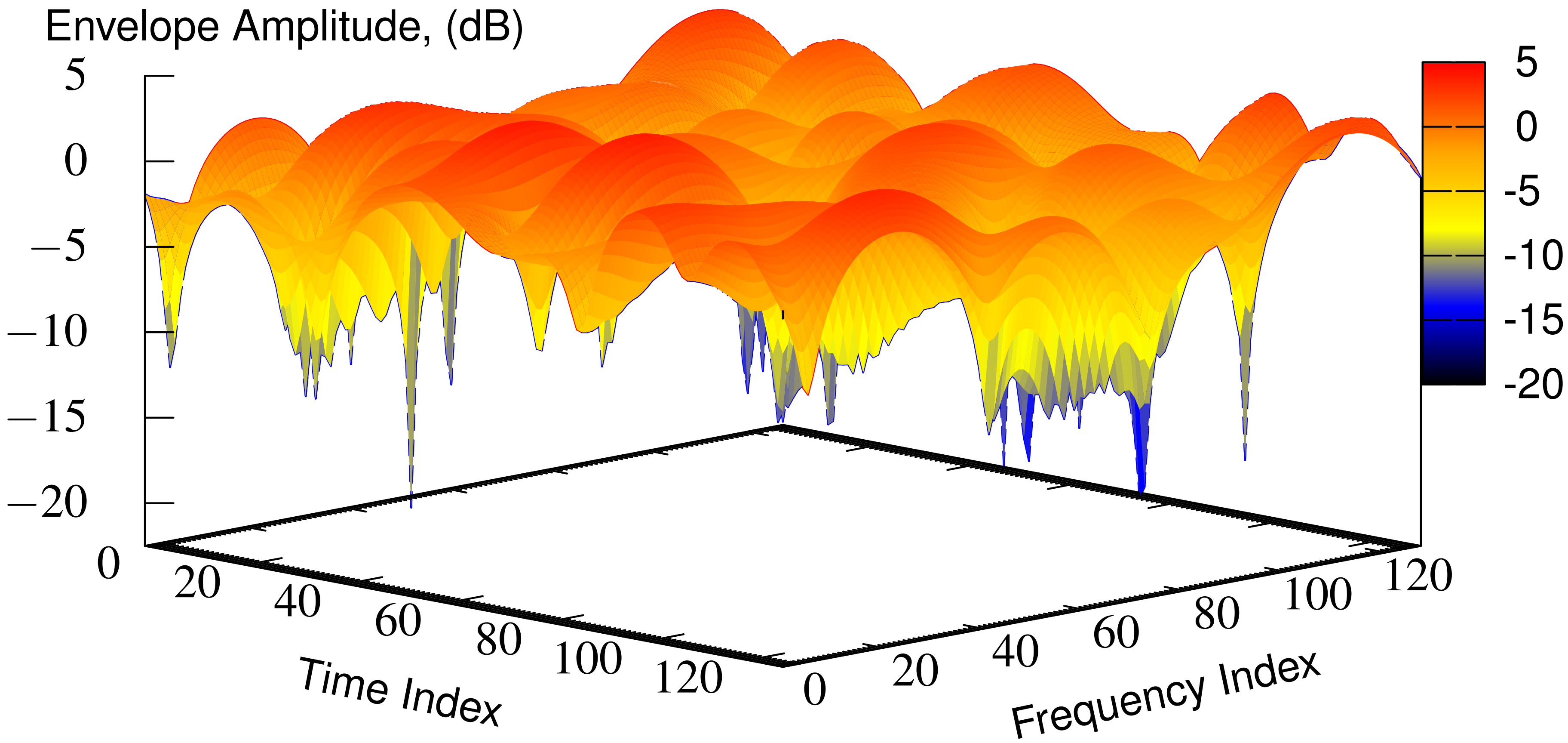


Figure 13: Baseband Rayleigh-fading simulation model. ©Webb & Hanzo.

Time -Freq. Properties of Wireless Channels



Wireless Channel Modeling - Some Important Concepts

- **Coherence bandwidth**: Coherence bandwidth is the range of frequencies over which any two frequency components experience strongly correlated fading. In wireless communications the maximum difference of the arrival time between the first arrival multipath replica and the last multipath replica determines the coherence bandwidth of the wireless channel;
- **Coherence time**: Coherence time is the time duration over which any two received signals having the similar frequencies experience strongly correlated fading. In wireless communications the maximum Doppler frequency shift determines the coherence time of the channel;

Wireless Channel Modeling - Some Important Concepts

- **Frequency non-selective fading (flat fading)**: When the bandwidth of a transmitted signal is significantly lower than the coherence bandwidth of the channel, such as in narrowband wireless systems, all the frequency components of the transmitted signal will be highly correlated and experience correlated fading. This phenomenon of fast fading is referred to as frequency non-selective fading or flat fading;
- **Frequency-selective fading**: When the bandwidth of a transmitted signal is significantly higher than the coherence bandwidth of the channel, for example, in wide-band, broadband or ultrawide bandwidth wireless systems, two frequency components having a frequency separation higher than the coherence bandwidth of the channel will conflict independent fading. This type of fast fading is referred to as frequency-selective fading.

Wireless Channel Modeling - Some Important Concepts

- **Time non-selective fading (slow fading)**: When the coherence time of the channel is significantly higher than the symbol duration, for example when transmitting a high data rate, the fading within one or several symbol durations will be highly correlated, resulting in all these symbols being faded similarly. The fast fading employing the above characteristics is referred to as time non-selective fading (slow fading);
- **Time-selective fading (fast fading)**: When the coherence time of the channel is significantly lower than the symbol duration in case of transmitting a low data rate, the front and rear parts of a given symbol will experience independent fading. The fast fading having this characteristics is referred to as time-selective fading (fast fading).

Modulation and Transmission

- ✔ Overview;
- ✔ Constellation design;
- ✔ Detection and performance examples.

Modulation and Transmission

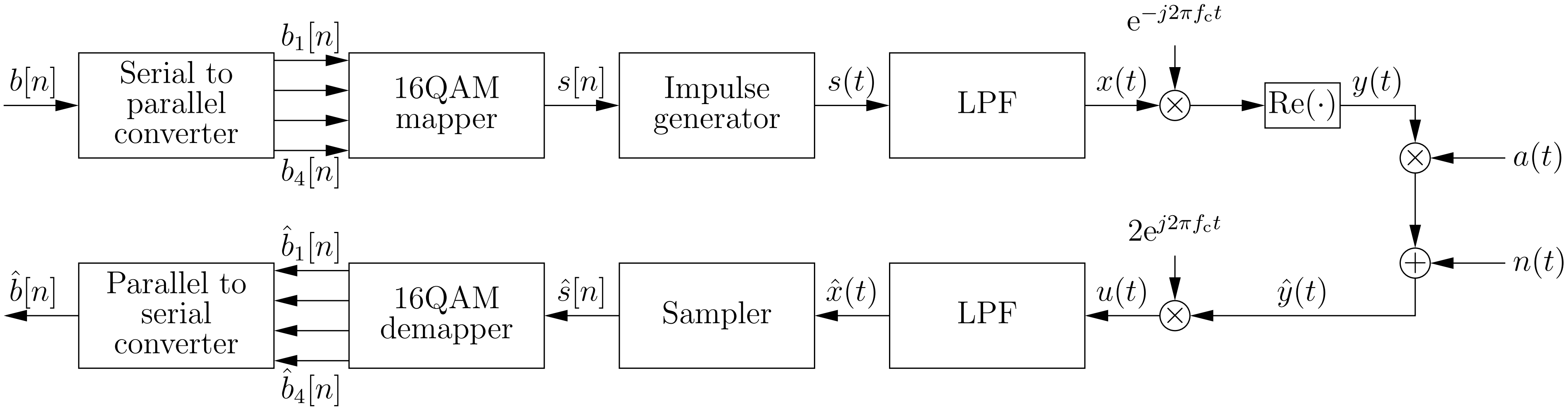


Figure 14: 16QAM modulator and demodulator.

16QAM modulator

- The serial to parallel converter decomposes the bit sequence $b[n]$ into $\log_2(M) = 4$ sequences.
- The 16QAM mapper combines the $\log_2(M) = 4$ binary values $b_1[n]$, $b_2[n]$, $b_3[n]$ and $b_4[n]$ into a single complex phasor $s[n]$. The 16QAM constellation diagram is used to map each of the $M = 16$ combinations of the $\log_2(M) = 4$ binary values to a different one of the $M = 16$ possible values of the complex phasor.
- The impulse generator converts the discrete time signal $s[n]$ into a continuous time signal $s(t)$. At each discrete time instant, an impulse is generated in the signal $s(t)$ having the same amplitude and phase as the corresponding phasor in $s[n]$.
- The LPF is a Nyquist filter, which limits the bandwidth of the impulse train $s(t)$, without causing its symbols to interfere with each other during their reconstruction. A root raised cosine filter that is matched with the receive filter may be employed for this purpose.
- The multiplier performs the quadrature amplitude modulation of the signal $x(t)$ onto the carrier wave. This is achieved by simply multiplying the two signals together.

16QAM demodulator

- The multiplier shifts the QAM signal $\hat{y}(t)$ down to the baseband and up to a frequency that is double the carrier frequency. This is achieved by simply multiplying $\hat{y}(t)$ with a locally generated carrier wave.

- The LPF is a Nyquist filter, which removes the component of $u(t)$ that is at double the carrier frequency and removes noise. A root raised cosine filter that is matched with the transmit filter may be employed for this purpose.

- The sampler converts the continuous time signal $\hat{x}(t)$ into a discrete time signal $\hat{s}[n]$. This is achieved by sampling the signal $\hat{x}(t)$ at each discrete time instant.

- The 16QAM demapper decomposes the complex phasor $\hat{s}[n]$ into the $\log_2(M) = 4$ binary values $\hat{b}_1[n]$, $\hat{b}_2[n]$, $\hat{b}_3[n]$ and $\hat{b}_4[n]$. The demapper must decide which binary values are most likely to be correct by comparing the received phasor $\hat{s}[n]$ with the $M = 16$ possible values of the transmitted phasor $s[n]$.

- The parallel to serial converter merges the $\log_2(M) = 4$ bits sequences into one.

Modulation and Transmission

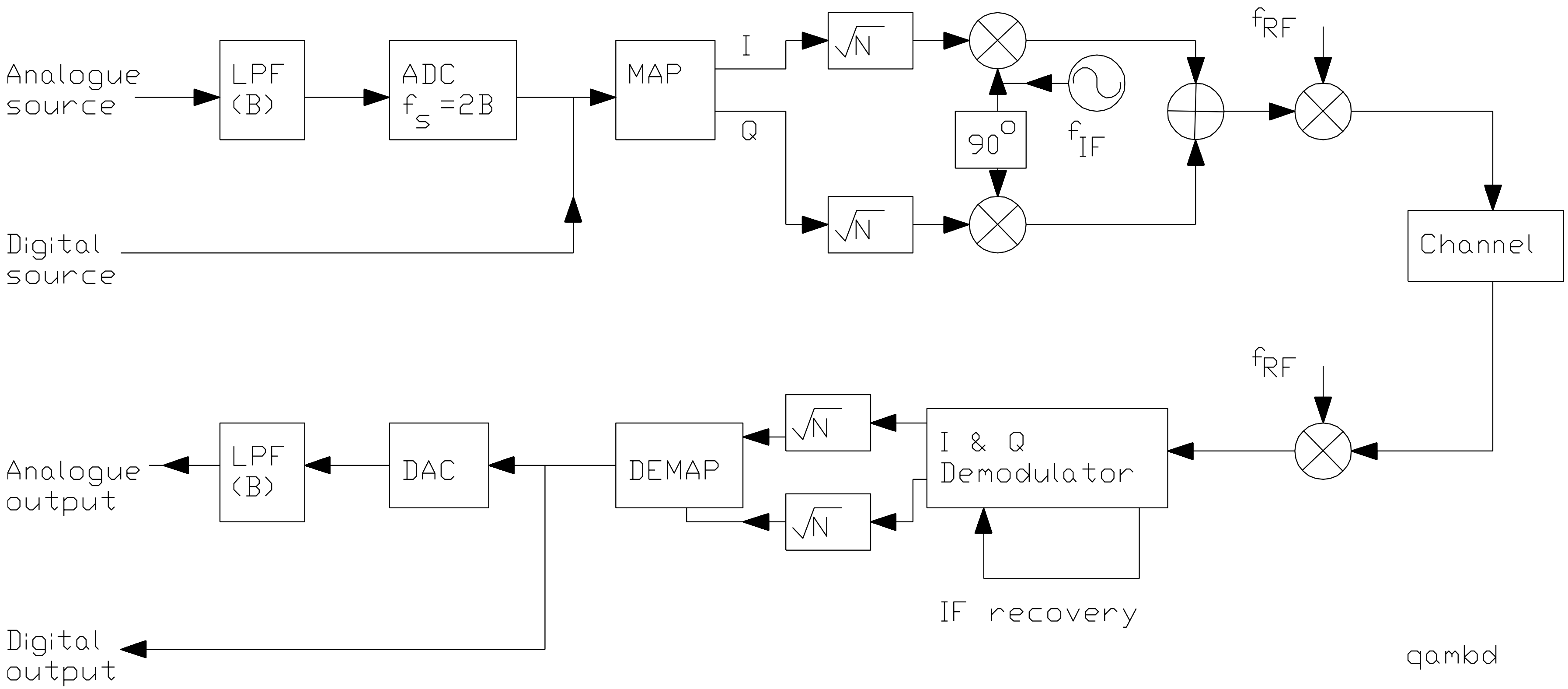


Figure 15: Another QAM modem schematic. ©Hanzo, Webb, Keller 2000 [2].

Modulation and Transmission

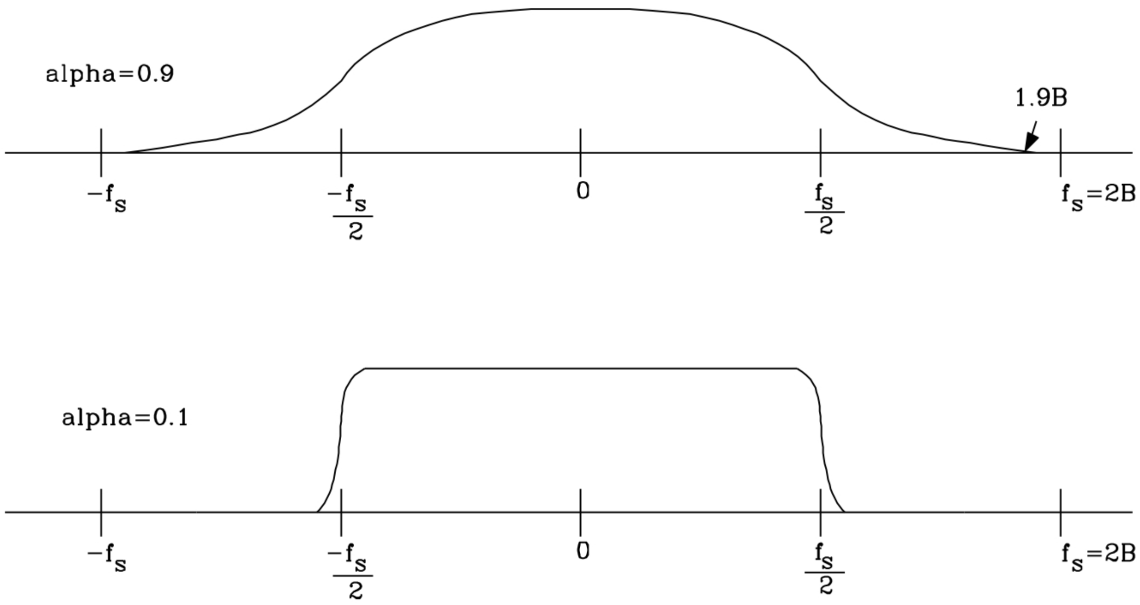


Figure 16: Stylized frequency response of two filters with $\alpha = 0.9$ and 0.1. ©Hanzo, Webb, Keller 2000 [2].

Modulation and Transmission

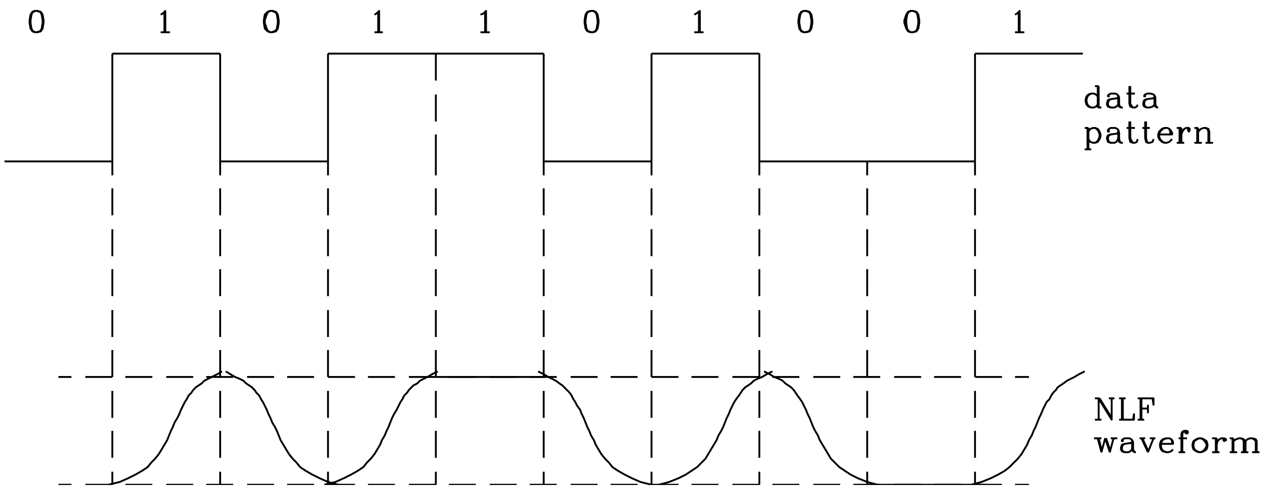


Figure 17: Stylized NLF waveforms. ©Hanzo, Webb, Keller 2000 [2].

Modulation and Transmission

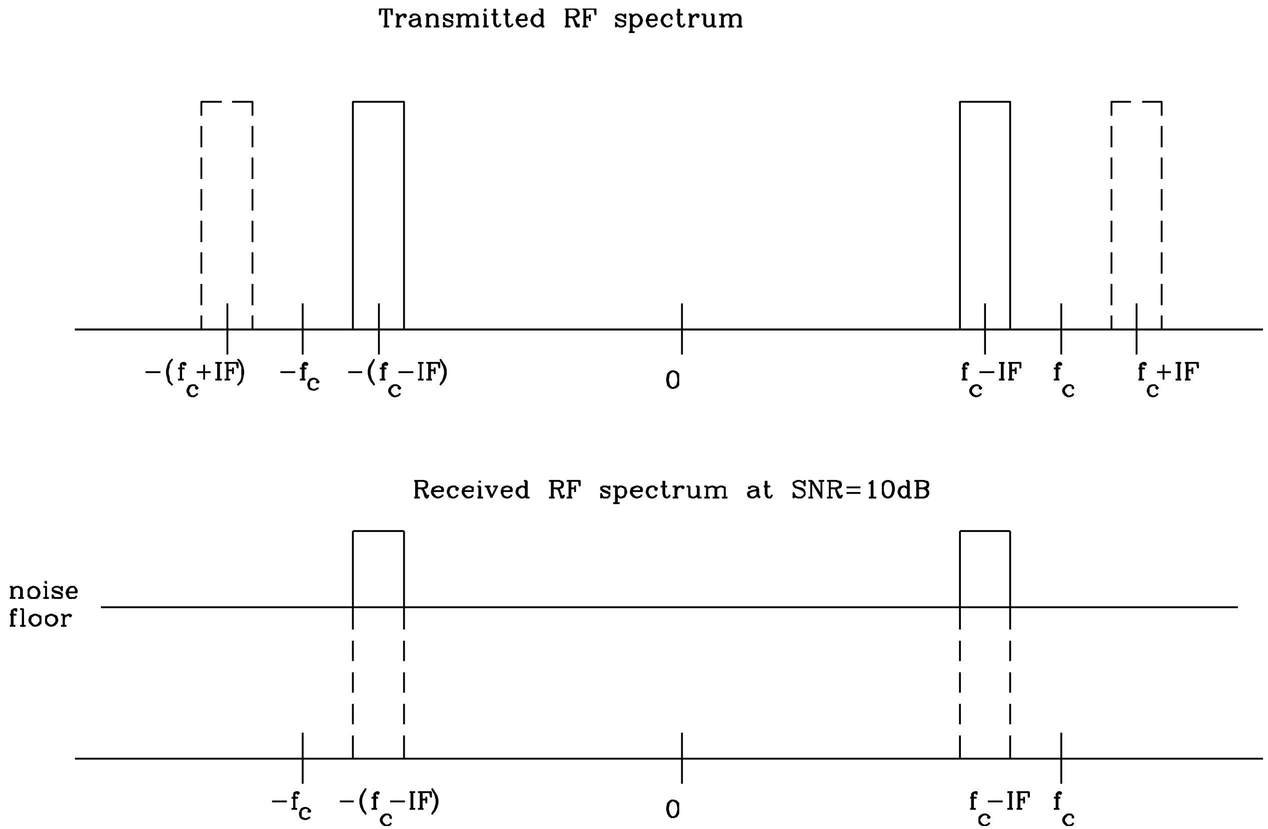


Figure 18: Stylized transmitted and received spectra. ©Hanzo, Webb, Keller 2000 [2].

Modulation and Transmission

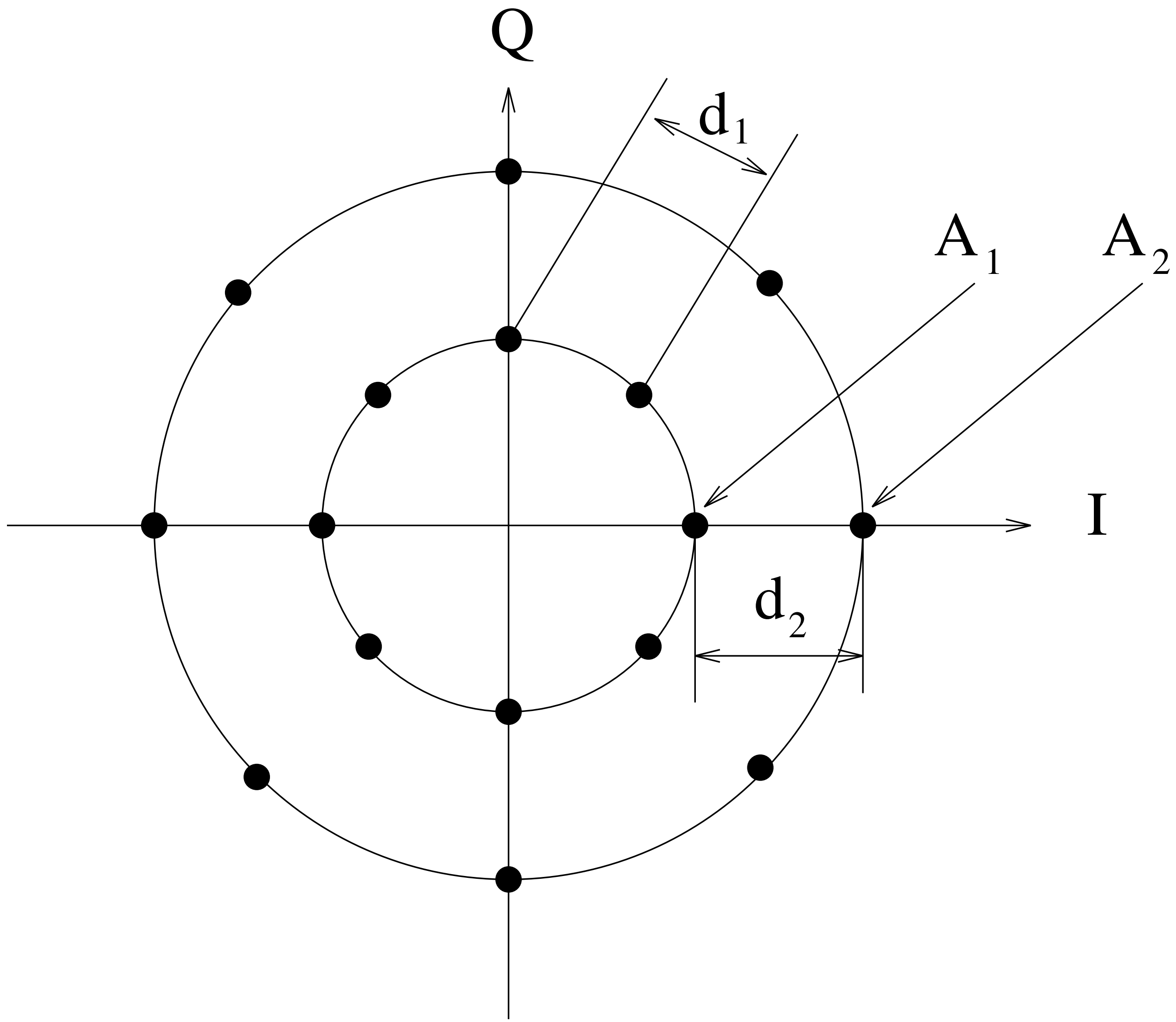


Figure 19: Star 16QAM constellation.

Modulation and Transmission

When designing a constellation, consideration must be given to:

☛ The *minimum Euclidean distance* among phasors, which is characteristic of the noise immunity of the scheme.

☛ The *minimum phase rotation* among constellation points, determining the phase jitter immunity and hence the scheme's resilience against clock recovery imperfections and channel phase rotations.

☛ The ratio of the *peak-to-average phasor energy*, which is a measure of robustness against nonlinear distortions introduced by the power amplifier.

Modulation and Transmission

- The minimum Euclidean distance among phasors is maximized and the BER is minimised if $d_1 = d_2 = A_2 - A_1$ in the star constellation of Figure 19. Using the geometry of Figure 19, we can write that:

$$\begin{aligned} \cos 67.5^\circ &= \frac{d_1}{2} \cdot \frac{1}{A_1} \\ d_1 &= 2 \cdot A_1 \cdot \cos 67.5^\circ \end{aligned}$$

and hence

$$A_2 - A_1 = d_1 = d_2 = 2 \cdot A_1 \cdot \cos 67.5^\circ.$$

- Upon dividing both sides by A_1 and introducing the ring ratio RR, we arrive at:

$$\begin{aligned} RR - 1 &= 2 \cdot \cos 67.5^\circ \\ RR &\approx 1.77. \end{aligned}$$

Modulation and Transmission

- Simulation results using a variety of ring ratios in the interval of $1.5 < RR < 3.5$ both over Rayleigh and AWGN channels showed [2] that the BER does not strongly depend on the ring ratio, exhibiting a flat BER minimum for RR values in the above range.

- Under the constraint of having identical distances among constellation points, when $d_1 = d_2 = d$, the average energy E_0 of the star constellation can be computed as follows:

$$E_0 = \frac{8 \cdot A_1^2 + 8 \cdot A_2^2}{16} = \frac{1}{2}(A_1^2 + A_2^2)$$

where

$$A_1 = \frac{d}{2 \cdot \cos 67.5^\circ} \approx \frac{d}{0.765} \approx 1.31d$$

$$A_2 \approx 1.77 \cdot A_1 \approx 2.3d$$

- The total energy becomes:

$$E_0 \approx 0.5 \cdot (5.3 + 1.72)d^2 \approx 3.5d^2.$$

Modulation and Transmission

☛ **The minimum distance** of the constellation for an average energy of E_0 becomes:

$$d_{min} \approx \sqrt{E_0/3.5} \approx 0.53 \cdot \sqrt{E_0},$$

while **the peak-to-average phasor energy ratio** is:

$$r \approx \frac{(2.3d)^2}{3.5d^2} \approx 1.5.$$

The minimum phase rotation θ_{min}, the minimum Euclidean distance d_{min}, and the peak-to-average energy ratio r are summarized in Table 1 for both of the above constellations.

Type	θ_{min}	d_{min}	r
Star	45°	$0.53\sqrt{E_0}$	1.5
Square	36.9°	$0.63 \cdot \sqrt{E_0}$	1.8

Table 1: Comparison of the Star and Square Constellations

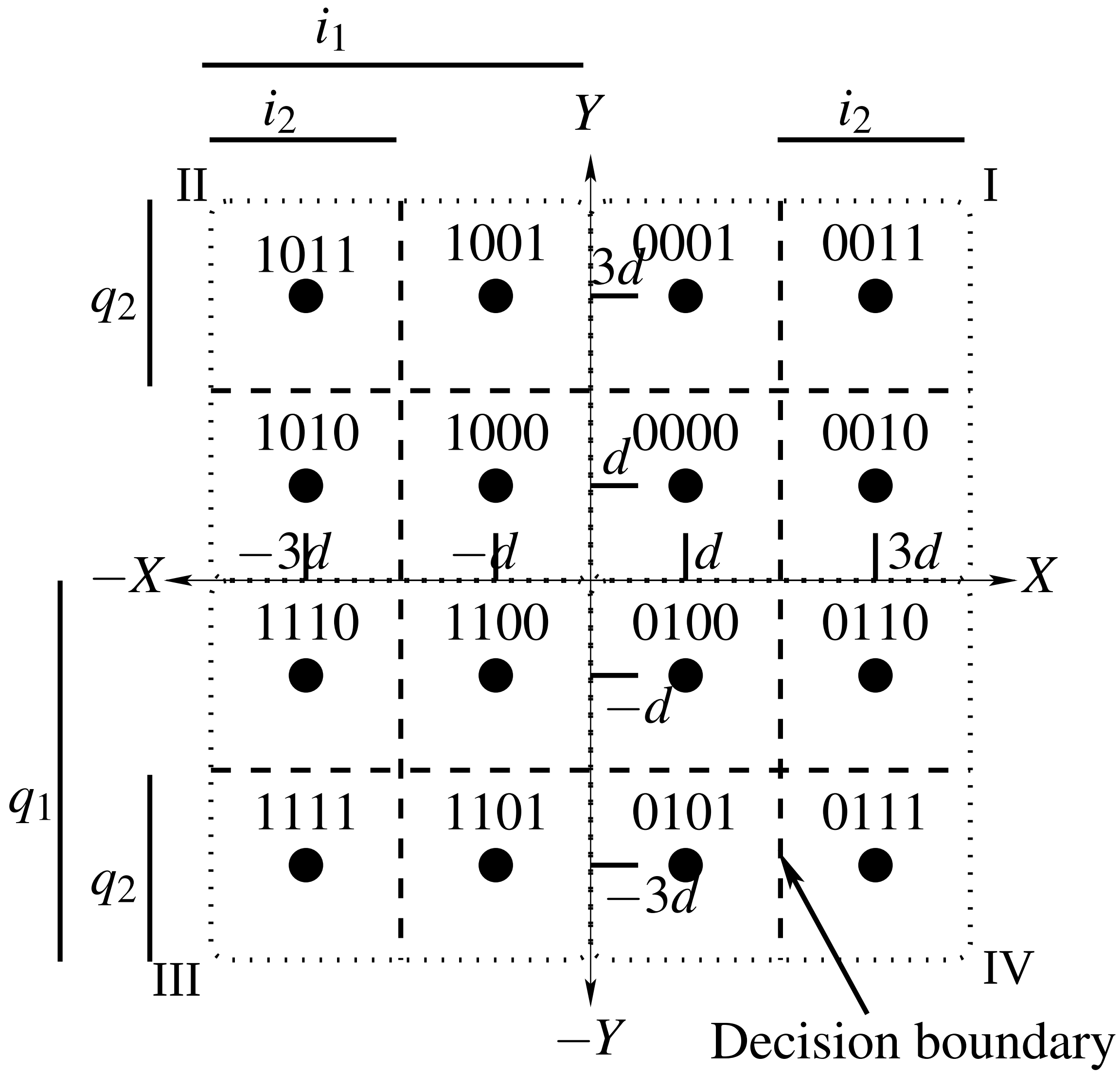


Figure 20: 16QAM square constellation. ©IEEE.

Modulation and Transmission

- Let us now derive the above characteristic parameters for the square constellation. Observe from Figure 19 that $\theta_{min} = 2\arctan(d/3d) = 36.9^{\circ}$, while the distance between phasors is $2 \cdot d$. Hence, the average phasor energy becomes:

$$\begin{aligned} E_0 &= \frac{1}{16}\left[4\cdot(d^2+d^2)+8(9d^2+d^2)+4\cdot(9d^2+9d^2)\right] \\ &= \frac{1}{16}(8d^2+80\cdot d^2+72d^2) \\ &= 10d^2. \end{aligned}$$

- Assuming the same average phasor energy E_0 as for the star constellation, we now have a minimum distance of

$$d_{min} = 2d = 2\cdot\sqrt{E_0/10} = \sqrt{E_0/2.5} \approx 0.63\cdot\sqrt{E_0}.$$

- Finally, the peak-to-average energy ratio r is given by:

$$r = \frac{18d^2}{10d^2} = 1.8.$$

Decision Theory

❶ Before analyzing the effects of errors, let us briefly review the roots of decision theory in the spirit of **Bayes' theorem**:

$$P(X|Y) \cdot P(Y) = P(Y|X) \cdot P(X) = P(X,Y), \tag{38}$$

where the random variables X and Y have probabilities of $P(X)$ and $P(Y)$, their joint probability is $P(X,Y)$, and their conditional probabilities are given by $P(X|Y)$ and $P(Y|X)$.

❷ In decision theory, the above theorem is invoked in order to infer from the noisy analog received sample y, what the most likely transmitted symbol was, assuming that the so-called *a-priori* probability $P(x)$ of the transmitted symbols $x_n, n = 1 \ldots M$ is known.

❸ Given the received sample y at the receiver, the conditional probability $P(x_n|y)$ quantifies the chance that x_n has been transmitted:

$$P(x_n|y) = \frac{P(y|x_n) \cdot P(x_n)}{P(y)}, \qquad n = 1 \ldots N \tag{39}$$

Decision Theory - Continued

❹ The probability of encountering a specific y value will be the sum of all possible combinations of receiving y, given that $x_n, n = 1 \ldots N$ was transmitted, which can be written as:

$$P(y) = \sum_{n=1}^{N} P(y|x_n) \cdot P(x_n) = \sum_{n=1}^{N} P(y, x_n). \tag{40}$$

❺ Let us now consider the case of binary phase shift keying (BPSK), where there are two legitimate transmitted values, x_1 and x_2 which are contaminated by noise. The conditional PDFs are shown in Fig.21.

Decision Theory - Continued

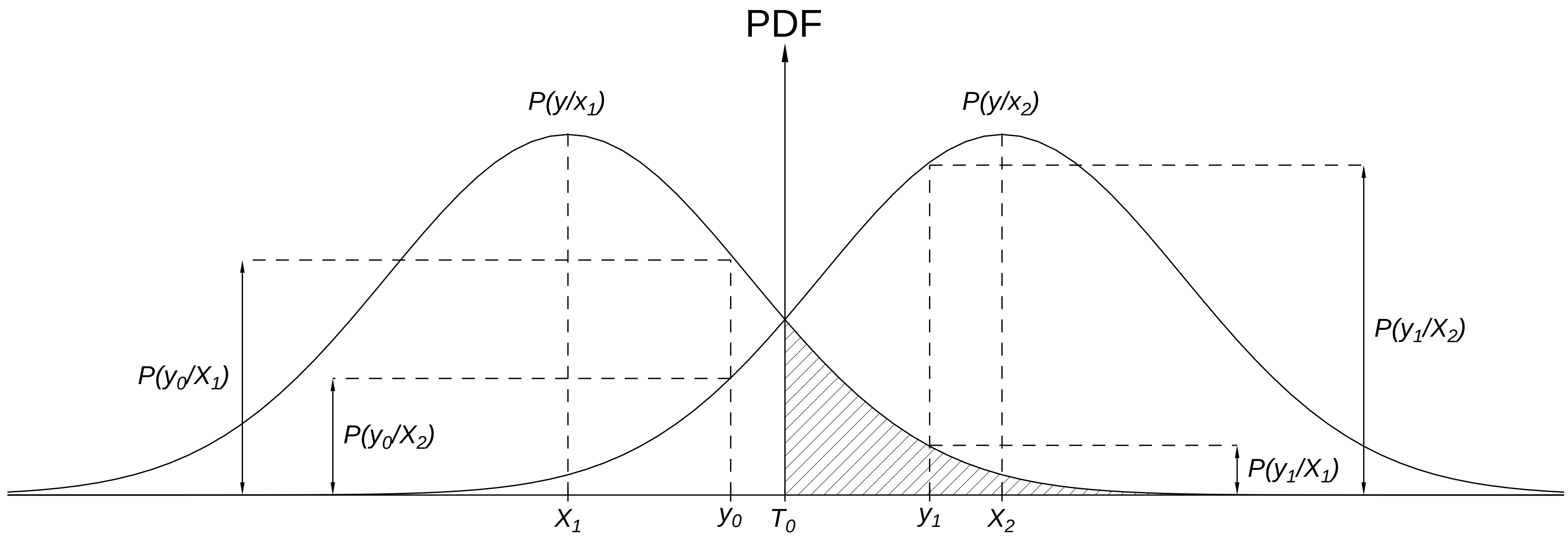


Figure 21: Probability density functions (PDFs) of the noisy received samples for BPSK, conditioned on the transmitted samples.

Decision Theory - Continued

❻ The conditional probability of receiving any particular noise-contaminated analog sample y, given that x_1 or x_2 was transmitted, is quantified by the Gaussian PDFs, described by:

$$P(y|x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(y-x)^2}{2\sigma^2}\right), \tag{41}$$

where $x = x_1$ or x_2 is the mean and σ^2 is the variance.

❼ According to the optimum decision theory, the optimum decision threshold above which x_2 is inferred is given by:

$$T_0 = \frac{x_1 + x_2}{2} \tag{42}$$

and below this threshold x_1 is assumed to have been transmitted. If $x_1 = -x_2$, then $T_0 = 0$ is the optimum decision threshold minimizing the bit error probability.

Error Performance - BPSK

- A decision error occurs, either when the amplitude y of the received sample exceeds the threshold T_0 given x_1 being transmitted, or when the amplitude y of the received sample is below the threshold T_0 given x_2 being transmitted:

$$P_e = P(x_1)P(y > T_0|x_1) + P(x_2)P(y < T_0|x_2). \tag{43}$$

- For the BPSK scheme, we have $x_1 = -p$, $x_2 = p$ and $T_0 = 0$, where p is the noise protection distance.
- Let's assume that $P(x_1) = P(x_2) = 0.5$.
- Owing to the symmetry of the situation, we have $P(y > T_0|x_1) = P(y < T_0|x_2)$.
- Therefore

$$P_e = P(y > 0|x_1). \tag{44}$$

Error Performance - BPSK

- We have

$$P_e = P(y > 0|x_1) = \int_0^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(y-x_1)^2}{2\sigma^2}\right) dy. \tag{45}$$

- Let $u = \frac{y+p}{\sigma}$, giving $\frac{du}{dy} = \frac{1}{\sigma}$ and $dy = \sigma du$.
- Substituting into 45 gives

$$P_e = \int_{\frac{p}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-u^2}{2}\right) du, \tag{46}$$

$$= Q\left(\frac{p}{\sigma}\right), \tag{47}$$

Decision Theory - Continued

❽ The Gaussian Q-function is given by:

$$Q(y) = \int_{y}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du, \tag{48}$$

❾ In physical terms, $Q(y)$ is the probability of a random number taken from a Gaussian distribution exceeding y number of standard deviations from the mean.

❿ It resembles the complementary cumulative density function (CDF) of the Gaussian distribution, as shown in 22.

Decision Theory - Continued

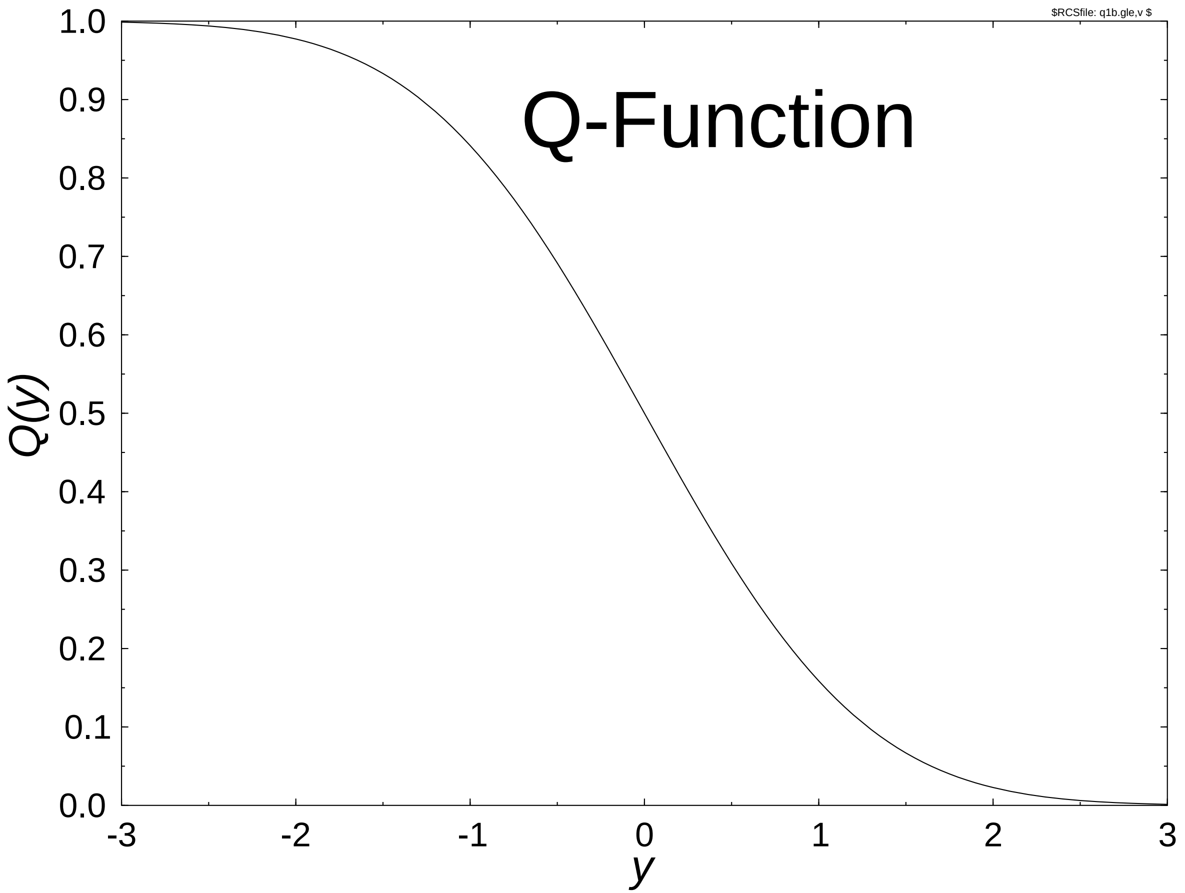


Figure 22: Gaussian Q-function.

Error Performance - BPSK

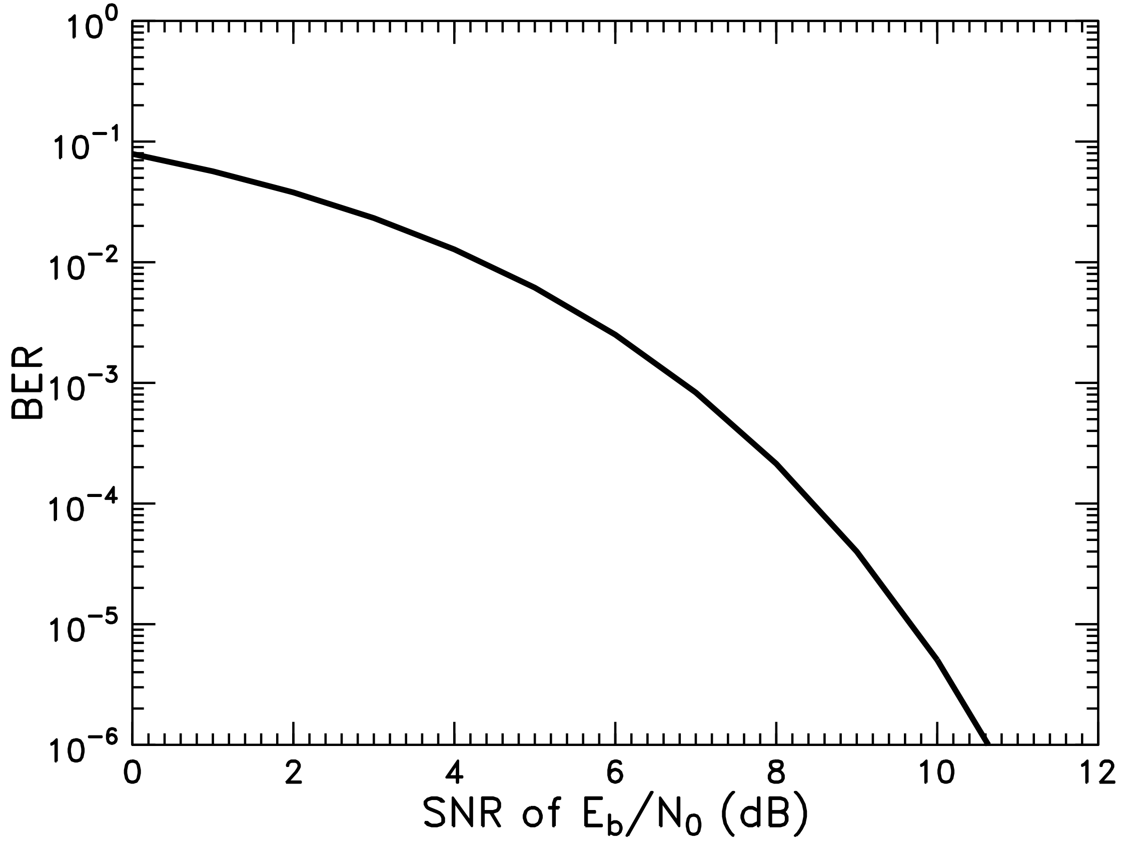


Figure 23: Bit error rate (BER) of BPSK, when communicating over AWGN channels.

16QAM - Transmission

- In general, the modulated signal can be represented by

$$s(t) = a(t)\cos[2\pi f_c t \; + \; \Theta(t)] = Re[a(t)e^{j[w_c t + \Theta(t)]}] \tag{49}$$

where the carrier $\cos(2\pi f_c t)$ is said to be amplitude modulated if its amplitude $a(t)$ is adjusted in accordance with the modulating signal, and it is said to be phase modulated if $\Theta(t)$ is varied in accordance with the modulating signal.

- In QAM the amplitude of the baseband modulating signal is determined by $a(t)$ and the phase by $\Theta(t)$. The in-phase component I is then given by

$$I(t) \; = \; a(t)\cos\Theta(t) \tag{50}$$

and the quadrature component Q by

$$Q(t) \; = \; a(t)\sin\Theta(t). \tag{51}$$

16QAM - Demodulation

❀ The received signal is given by

$$r(t) \;=\; a(t)\cos[2\pi f_c t \;+\; \Theta(t)] \;+\; n(t) \tag{52}$$

where $n(t)$ represents the AWGN, which has both an I and Q component, and can be expressed as

$$n(t) = \Re\{[n_I(t) + jn_Q(t)]\exp(j2\pi f_c t)\} \tag{53}$$

16QAM - Demodulation

- Detections of class 1 bits i_1 and q_1:

$$\begin{cases} \text{if} \quad I,Q \geq 0 \quad \text{then} \quad i_1,q_1 = 0 \\ \text{if} \quad I,Q < 0 \quad \text{then} \quad i_1,q_1 = 1 \end{cases}$$

- Detections of class 2 bits i_2 and q_2:

$$\begin{cases} \text{if} & & & I,Q & \geq & 2d & \text{then} & i_2,q_2 = 1 \\ \text{if} & -2d & \leq & I,Q & < & 2d & \text{then} & i_2,q_2 = 0 \\ \text{if} & -2d & > & I,Q & & & \text{then} & i_2,q_2 = 1. \end{cases}$$

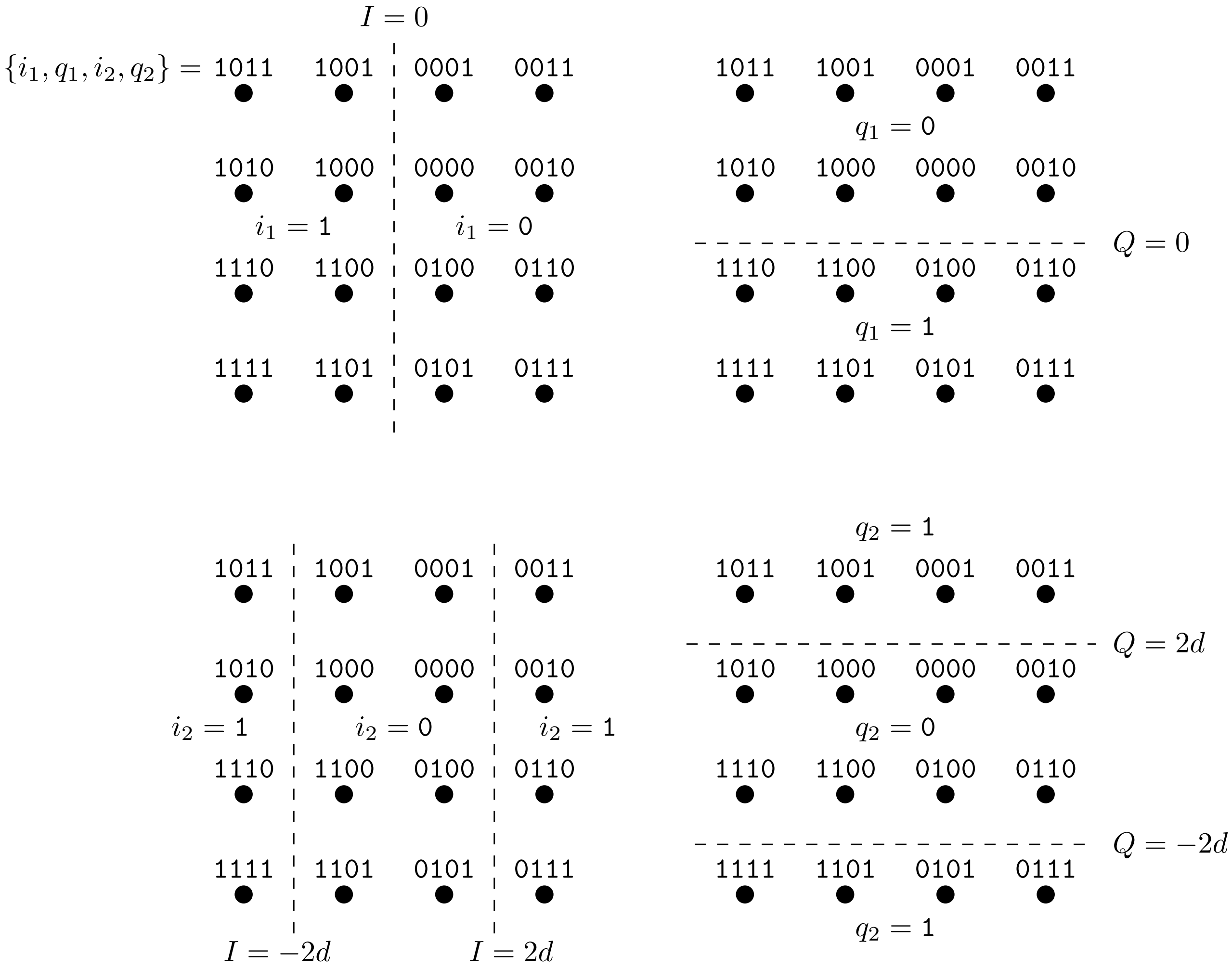


Figure 24: Decision boundaries for class 1 and class 2 bits.

Error Performance - 16QAM

- There are 64 points in Figure 24.
- 48 of these are at a Euclidean distance of d from a decision region that gives the wrong bit value.
- 32 are at a Euclidean distance of $3d$ from a decision region that gives the wrong bit value.
- 16 are at a Euclidean distance of $5d$ from a decision region that gives the *correct* bit value.

Error Performance - 16QAM

- The Q function $Q(x/\sigma)$ provides the probabilities of the noise moving the real and imaginary components of the received constellation point by a Euclidean distance of at least x.

- Hence the probability of a bit error for the square 16QAM is given by

$$P_e = \frac{48Q\left(\frac{d}{\sigma}\right)+32Q\left(\frac{3d}{\sigma}\right)-16Q\left(\frac{5d}{\sigma}\right)}{64} \tag{54}$$

$$= \frac{3}{4}Q\left(\sqrt{\frac{E_0}{5N_0}}\right)+\frac{1}{2}Q\left(3\sqrt{\frac{E_0}{5N_0}}\right)-\frac{1}{4}Q\left(5\sqrt{\frac{E_0}{5N_0}}\right). \tag{55}$$

where $N_0 = 2\sigma^2$ is the power spectral density of the AWGN and $E_0 = 10d^2$ for 16QAM.

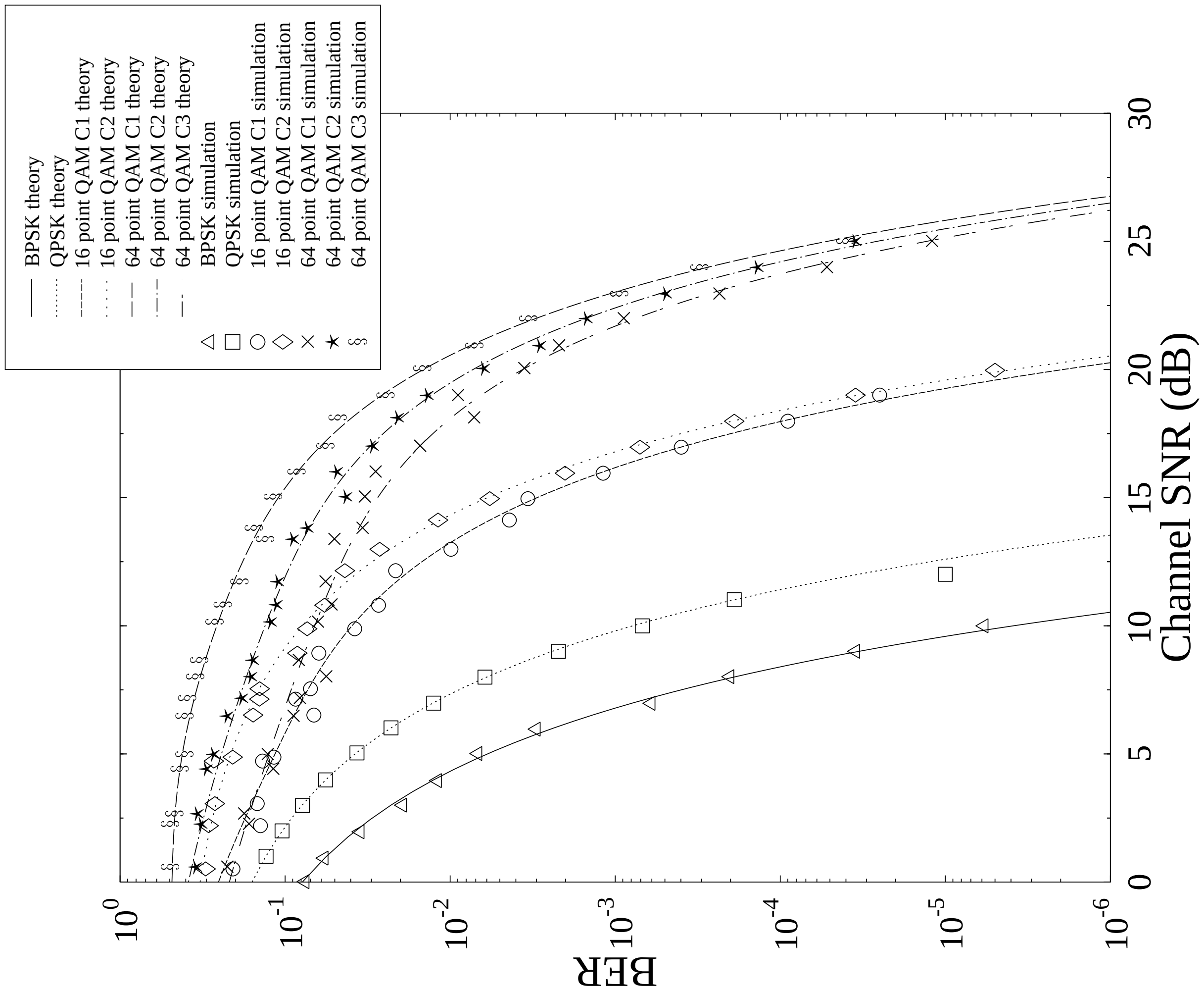


Figure 25: BPSK, QPSK, 16QAM, and 64QAM BER versus channel SNR performance over AWGN channels.

BPSK - Rayleigh Fading Channels

- When communicating over a Rayleigh fading channel, the amplitude and phase of the transmitted phasor is changed.
- The phase adjustment can be corrected for using channel estimation and synchronisation techniques.
- The amplitude is changed by a time-varying gain α having a Rayleigh distribution

$$f(\alpha) = \frac{2\alpha}{\Omega} \exp\left(-\frac{\alpha^2}{\Omega}\right), \ \alpha \geq 0 \tag{56}$$

- Here, Ω is the fraction of the transmitted power that is received.

BPSK - Rayleigh Fading Channels

➪ When using the BPSK constellation points $\pm p$, the BER for a given instantaneous fading envelope amplitude α is given by

$$P_e(\gamma) = Q\left(\frac{\alpha p}{\sigma}\right) = Q\left(\sqrt{2\gamma}\right) \tag{57}$$

Here $\gamma = \frac{\alpha^2 p^2}{2\sigma^2}$ represents the instantaneous SNR, and $\gamma_c = \frac{\Omega p^2}{2\sigma^2}$ is the average SNR, where $\gamma = \frac{\alpha^2}{\Omega}\gamma_c$.

BPSK - Rayleigh Fading Channels

➯ It can be shown that the PDF of $\gamma = \frac{\alpha^2}{\Omega}\gamma_c$ can be derived as

$$
\begin{aligned}
f(\gamma) &= f_\alpha(\alpha)\big|_{\alpha=\sqrt{\Omega\gamma/\gamma_c}} \times \frac{d\alpha}{d\gamma} \\
&= \frac{2\sqrt{\Omega\gamma/\gamma_c}}{\Omega}\exp\left(-\frac{\gamma}{\gamma_c}\right) \times \frac{1}{2\sqrt{\Omega\gamma/\gamma_c}}\frac{\Omega}{\gamma_c} \\
&= \frac{1}{\gamma_c}\exp\left(-\frac{\gamma}{\gamma_c}\right) \qquad (58)
\end{aligned}
$$

➯ The average BER is obtained by invoking the Rayleigh distribution, which can be expressed as

$$
\begin{aligned}
P_e &= \int_0^\infty Q\left(\sqrt{2\gamma}\right) f(\gamma) d\gamma \\
&= \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_c}{1+\gamma_c}}\right) \qquad (59)
\end{aligned}
$$

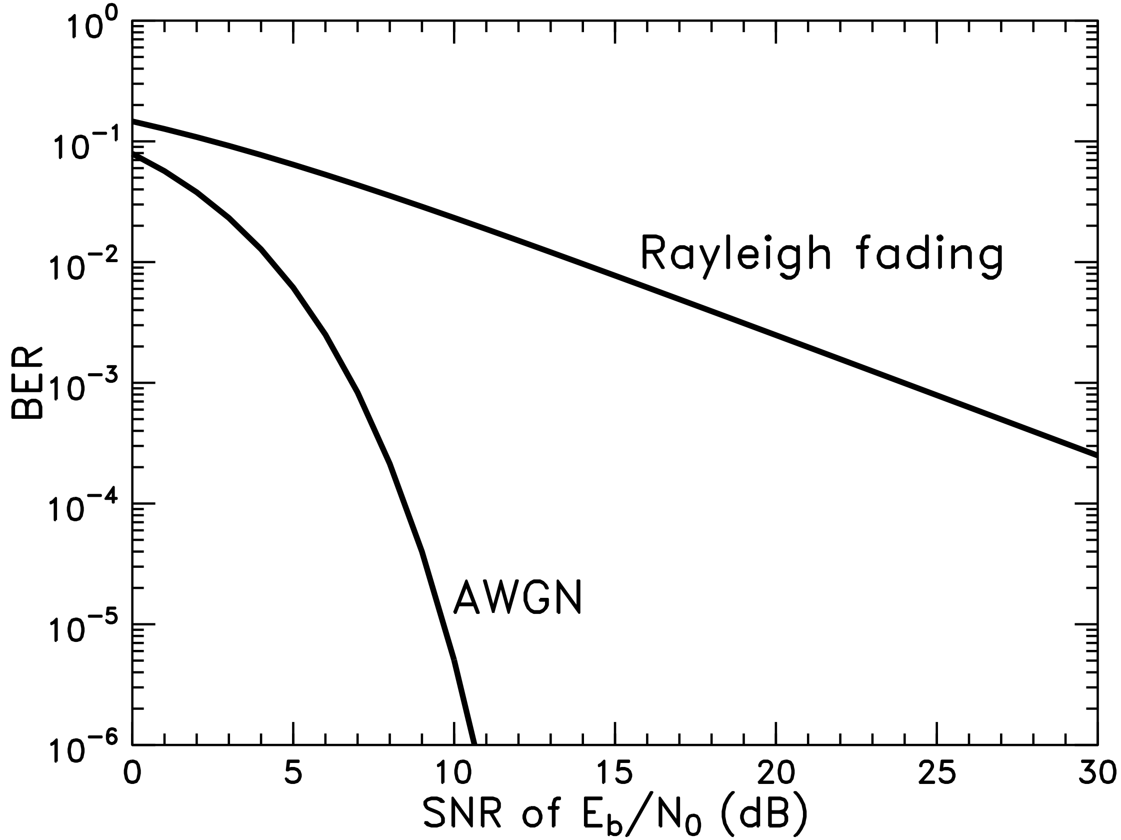


Figure 26: BER versus SNR per bit of E_b/N_0 performance of the BPSK system, when communicating over AWGN and flat Rayleigh fading channels.

Diversity Transmission

- ✔ Why using diversity transmission?
- ✔ Principles of
  - Frequency diversity;
  - Time diversity;
  - Spatial diversity.
- ✔ Antenna diversity:
  - Receive diversity;
  - Transmit diversity.

Why using diversity transmission?

Let us use an example to show the advantages of using diversity.

- ✔ **Case 1:** The receiver can only obtain one observation of the transmitted signal, which is expressed as $r_A(t) = \alpha s + n$, where n is the AWGN with a variance of σ^2 and α is the fading gain, where $E[\alpha^2] = \Omega$. Then, the instantaneous signal-to-noise ratio (SNR) is $\gamma = \alpha^2 E[s^2]/(2\sigma^2)$, while the average SNR is $\gamma_c = E[\alpha^2]E[s^2]/(2\sigma^2) = \Omega E[s^2]/(2\sigma^2)$;
- ✔ **Case 2:** The receiver can obtain two independent but attenuated observations of the same transmitted signal, which are expressed as $r_{B1}(t) = \alpha_1 s + n_1$ and $r_{B2}(t) = \alpha_2 s + n_2$, where n_1 and n_2 are AWGN with a common variance of σ^2, while α_1 and α_2 are the fading gains, where $E[\alpha_1^2] = E[\alpha_2^2] = 0.5\Omega$. Then, the total instantaneous SNR is $\gamma_B = (\alpha_1^2 + \alpha_2^2)E[s^2]/(2\sigma^2)$, while the average SNR of each observation is given by $\gamma_{c_1} = \gamma_{c_2} = E[\alpha_1^2]E[s^2]/(2\sigma^2) = 0.5\Omega E[s^2]/(2\sigma^2)$.

Why using diversity transmission? - Continued

From the above two cases, we have the observations:

- ✔ The total average SNR for both cases is the same, which is $\gamma_c = \Omega E[s^2]/(2\sigma^2)$;
- ✔ The instantaneous received SNR values for **Cases** 1 and 2 are:

$$\gamma = \frac{\alpha^2}{\Omega}\gamma_c, \tag{60}$$

$$\gamma_B = \frac{(\alpha_1^2 + \alpha_2^2)}{\Omega}\gamma_c \tag{61}$$

Why using diversity transmission? - Continued

✔ Assume that α, α_1 and α_2 obey the Rayleigh distribution having the PDFs given by

$$p_\alpha(y) = \frac{2y}{\Omega}\exp\left(-\frac{y^2}{\Omega}\right), \; y \geq 0 \tag{62}$$

$$p_{\alpha_1}(y) = p_{\alpha_2}(y) = \frac{2y}{0.5\Omega}\exp\left(-\frac{y^2}{0.5\Omega}\right), \; y \geq 0 \tag{63}$$

✔ Then, it can be shown that γ in (60) and γ_B in (61) obey the following PDFs:

$$\text{Central } \chi^2\text{-distribution } (n=2): \quad f(\gamma) = \frac{1}{\gamma_c}\exp\left(-\frac{\gamma}{\gamma_c}\right), \; \gamma \geq 0 \tag{64}$$

$$\text{Central } \chi^2\text{-distribution } (n=4): \quad f(\gamma_B) = \frac{\gamma_B}{(0.5\gamma_c)^2}\exp\left(-\frac{\gamma_B}{0.5\gamma_c}\right), \; \gamma \geq 0 \tag{65}$$

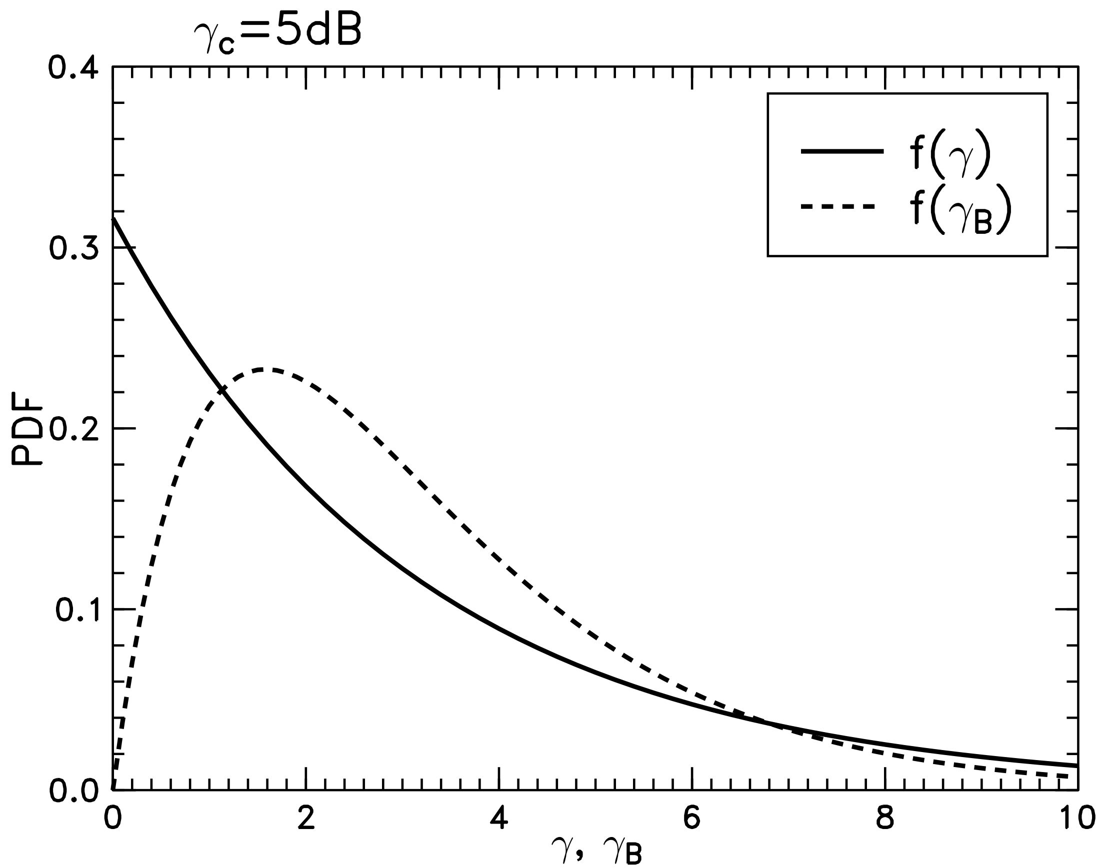


Figure 27: Illustration of the PDFs of γ and γ_B, shown in (64) and (65). ⓒJohn Wiley & Sons (L.-L. Yang, *Multicarrier Communications*, John Wiley & Sons. 2009).

Why using diversity transmission? - Continued

- ✔ According to Fig.27, we can explicitly observe that γ_B has a higher chance than γ to provide relatively high instantaneous SNRs, even though both of them have the same average received power;
- ✔ Correspondingly, when BPSK baseband modulation is assumed, the average BER of **Case** 1 should be higher than that of **Case** 2.

Diversity Transmission

- ✔ For the general case, where the receiver can obtain L number of observations having the same average power for the same transmitted signal, the PDF of the instantaneous received SNR is given by

$$f(\gamma) = \frac{\gamma^{L-1}}{(L-1)!\bar{\gamma}_l^L} \exp\left(-\frac{\gamma}{\bar{\gamma}_l}\right), \ \gamma \geq 0 \tag{66}$$

which is a central χ^2-distribution with $(n = 2L)$ degrees-of-freedom, where $\bar{\gamma}_l = \gamma_c / L$;

- ✔ We have known that, for the BPSK and for the received SNR of γ, the BER is given by $Q(\sqrt{2\gamma})$. Hence, the average BER of a L-diversity scheme is given by

$$\begin{aligned} P_e &= \int_0^\infty Q\left(\sqrt{2\gamma}\right) f(\gamma) d\gamma \\ &= \left[\frac{1-\mu}{2}\right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left[\frac{1+\mu}{2}\right]^k, \end{aligned} \tag{67}$$

where $\mu = \sqrt{\bar{\gamma}_l / (1+\bar{\gamma}_l)}$.

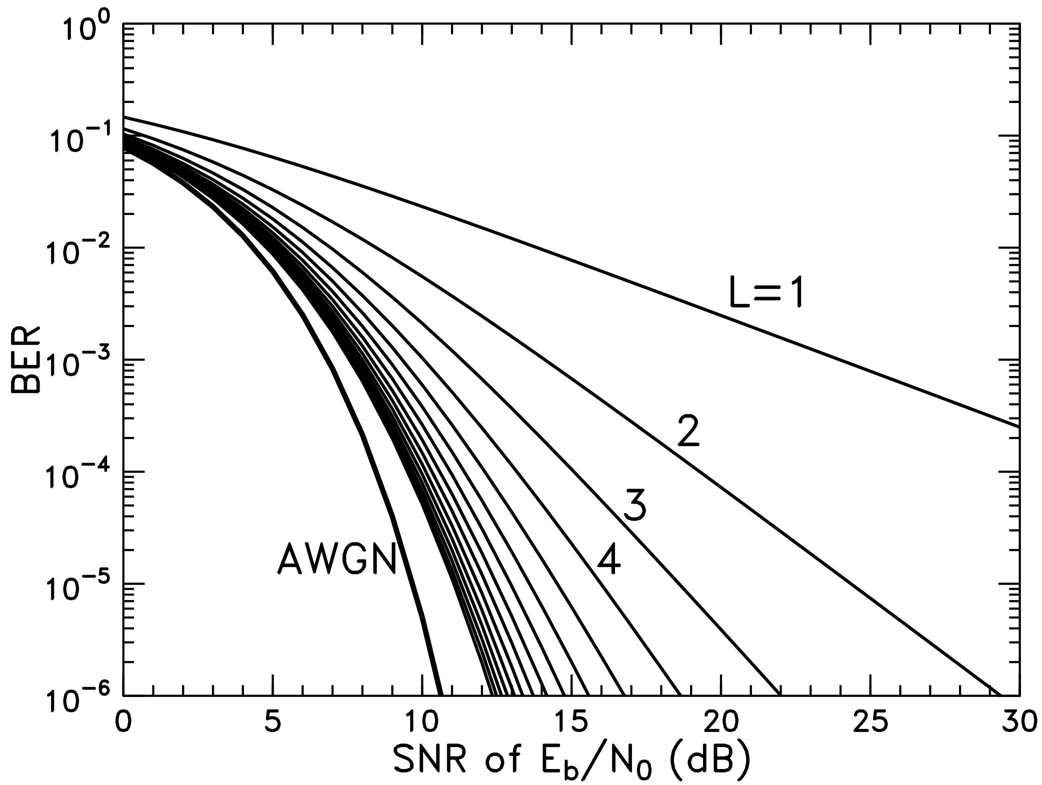


Figure 28: BER versus SNR per bit of E_b/N_0 performance of the BPSK system, when communicating over AWGN and multipath Rayleigh fading channels. ©John Wiley & Sons (L.-L. Yang, *Multicarrier Communications*, John Wiley & Sons. 2009).

Principles of Diversity Schemes

- **Frequency diversity**: When a signal is transmitted on two frequency bands separated at least by the coherence bandwidth of the channel, two independent observations of the transmitted signal can be obtained by the receiver from these two frequency bands, and a diversity of two can be achieved.
- **Time diversity**: When a signal is transmitted in two time slots separated at least by the coherence time of the channel, two independent observations of the transmitted signal can be obtained by the receiver from these two time slots, and a diversity of two can be achieved.
- **Spatial diversity**: When a transmitted signal is received by two receive antennas separated by a sufficient distance in space (usually $> 10\lambda$), the received signals of these two receive antennas are independent, and a diversity of two can be achieved.

Smart Antennas

- In recent years various smart antenna designs have emerged, which have found application in diverse scenarios;
- In **beamforming arrangements** typically $\lambda/2$-spaced antenna elements are used for the sake of creating a spatially selective beam, which allows the system to support multiple users within the same bandwidth and same time slot by separating them spatially;
- This spatial separation becomes however only feasible, if the corresponding users are separable in terms of the angle of arrival of their beams;

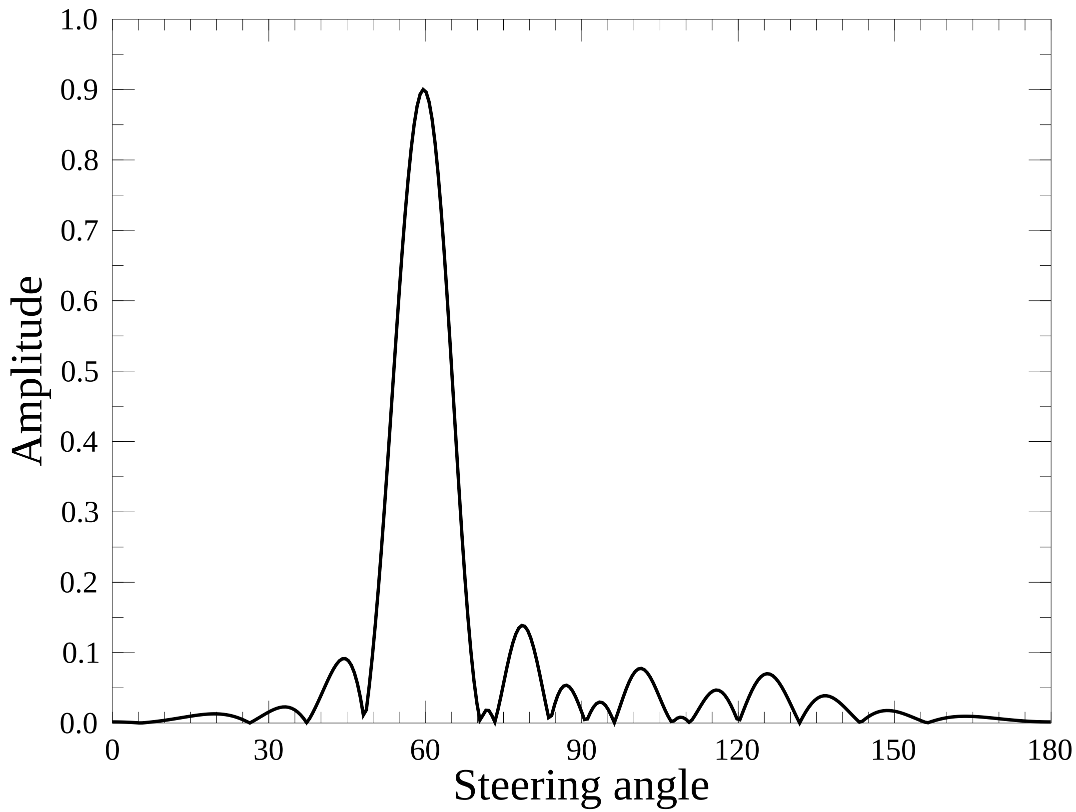


Figure 29: Beam-pattern of an antenna-array steering toward the user having a DOA of $\theta = 60$. In this figure, 16 users are distributed at the DOAs 0^o, 12^o, 24^o, 36^o, 48^o, 60^o, 72^o, 84^o, 96^o, 108^o, 120^o, 132^o, 144^o, 156^o, 168^o, 180^o, respectively.

Smart Antennas - Continued

- These beamforming schemes, which employ appropriately phased antenna array elements that are space at distances of $\lambda/2$ typically result in an improved SINR distribution and enhanced network capacity.
- In contrast to the $\lambda/2$-spaced phased array elements, in **diversity-motivated multiple-antenna schemes** the multiple antennas are positioned as far apart as possible, since they typically provide a higher diversity gain, when their fading channels are independent.
- Space diversity may be achieved by employment of multiple transmit antennas, multiple receive antennas or both.

Smart Antennas - Continued

- The aim of using multiple transmit/receive antennas is to provide both transmit as well as receive diversity and hence enhance the system's integrity/robustness.
- This results in a better physical-layer performance and hence a better network-layer performance, but they do not directly increase the achievable spectral efficiency.
- A third application of multiple antennas is often referred to as **Space Division Multiple Access** (SDMA), which exploits the unique, user-specific ”spatial signature” of the individual users for differentiating amongst them.

Smart Antennas - Continued

- In simple terms one could argue that both a conventional CDMA spreading code and the channel impulse response (CIR) affect the transmitted signal similarly.
- Hence, provided that the CIR is accurately estimated, it becomes known and certainly unique, although not orthogonal to the other CIRs of the other users.
- Nonetheless, it may be used for identifying users after channel estimation and hence for supporting several users within the same bandwidth.
- Provided that a powerful multiuser detector is available, the near-single user performance may be achieved, even many users are supported by a SDMA system.

Smart Antennas - Continued

- Hence the above method enhances the achievable spectral efficiency (capacity) directly.
- Finally, the single-user multiple-input multiple-output (MIMO) systems can also employ multiple transmit/receive antennas, aiming for increasing the capacity (number of bits per symbol) transmitted by the user.
- It has been shown that, if M of the number of transmit antennas and N of the number of receive antennas simultaneously become large, the capacity of the MIMO system is seen to grow linearly with $\min(M,N)$.

MIMO Communications Systems

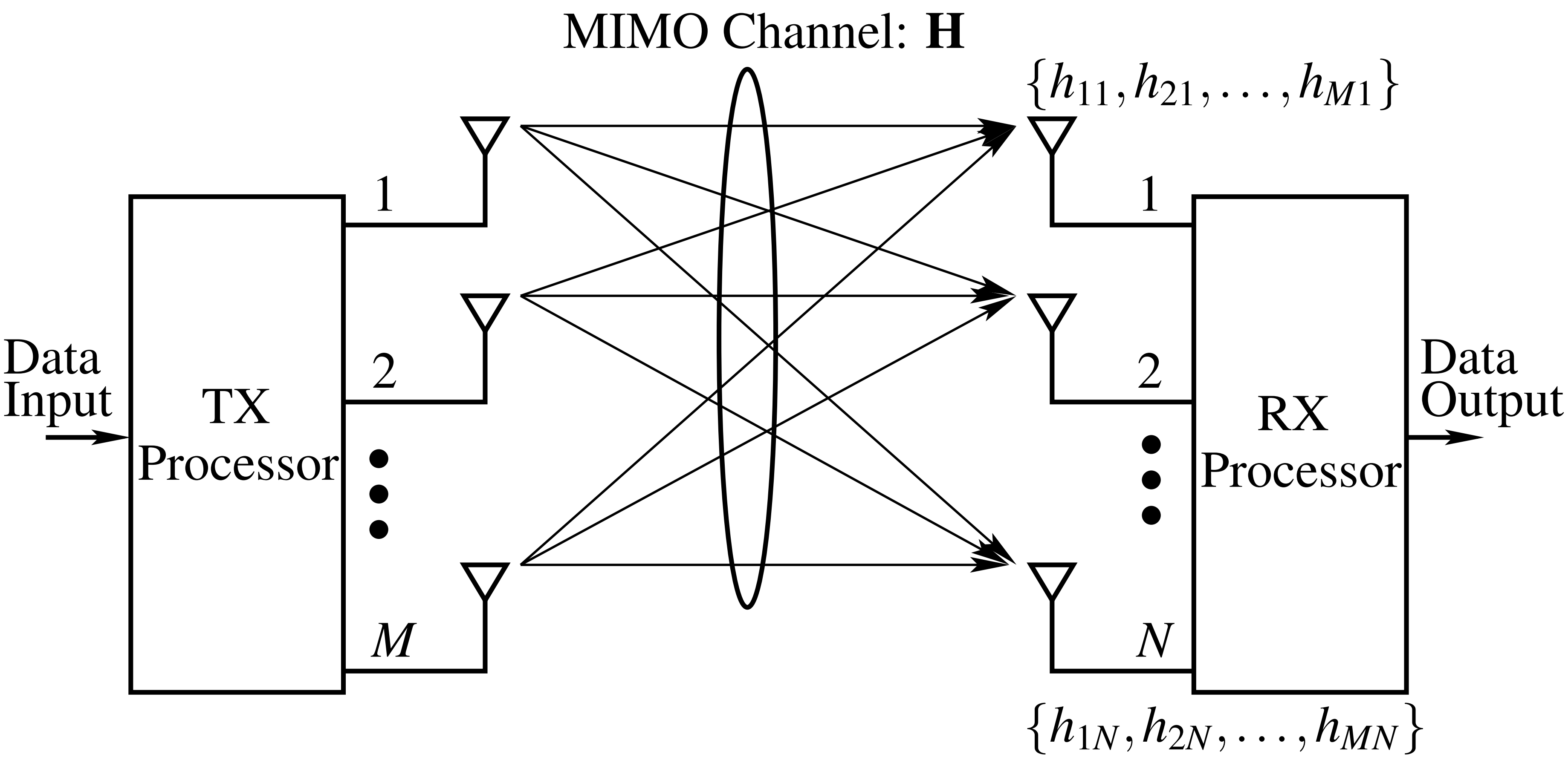


Figure 30: Schematic diagram of a MIMO wireless system.

MIMO: Received Signal Representation

Let the MIMO channel be represented by a $N \times M$ matrix $\mathbf{H}$. The $N \times 1$ received signal $\mathbf{y}$ can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{68}$$

where

- Input : $\boldsymbol{x} = [x_1, x_2, \ldots, x_M]^T$, which is the transmitted vector;
- Output : $\boldsymbol{y} = [y_1, y_2, \ldots, y_N]^T$, which is the received observations;
- $\boldsymbol{n} = [n_1, n_2, \ldots, n_N]^T$: the $N \times 1$ additive white complex Gaussian noise vector having zero-mean and a covariance matrix of $\sigma^2 \mathbf{I}_N$, where $\mathbf{I}_N$ is a $N \times N$ identity matrix;

MIMO: Received Signal Representation

MIMO channel matrix:

$$\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{h}_2, \ldots, \boldsymbol{h}_M] \tag{69}$$

$$= \begin{bmatrix} h_{11} & h_{12} & \ldots & h_{1M} \\ h_{21} & h_{22} & \ldots & h_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N1} & h_{N2} & \ldots & h_{NM} \end{bmatrix} \tag{70}$$

where $\boldsymbol{h}_m,\ m = 1, 2, \ldots, M$, represents the signature of symbol $x_m,\ m = 1, 2, \ldots, M$.

Diversity: Summary

- Single-Input Multiple-Output (SIMO) - Receive diversity, which is achieved by using one transmit antenna and N number of receive antennas;
- Multiple-Input Single-Output (MISO) - Transmit diversity, which is achieved by using M number of transmit antennas and one receive antenna;
- MIMO - MIMO diversity, which is achieved by using M number of transmit antennas and N number of receive antennas.

Diversity: SIMO System

- In SIMO systems the received signal can be expressed by

$$\mathbf{y} = \mathbf{H}x + \mathbf{n}, \tag{71}$$

where $\mathbf{y}$ is the received vector of size N, x is the transmitted symbol, and $\mathbf{H} = [h_1, h_2, \ldots, h_N]^T$;

- When multiplying $\mathbf{y}$ with $\mathbf{H}^H$, we obtain the decision variable for x, which is given by

$$Z = \mathbf{H}^H \mathbf{y} = \left(\sum_{n=1}^{N} |h_n|^2 \right) x + \mathbf{H}^H \mathbf{n}. \tag{72}$$

- Hence, the receive diversity order achieved is N.

Diversity: MISO System

❶ Closed-loop transmit diversity:

☛ Switched transmit diversity;

☛ Adaptive transmit diversity;

❷ Open-loop transmit diversity:

☛ Orthogonal transmit diversity;

☛ Space-time coding assisted transmit diversity;

Diversity: Switched Transmit Diversity

- The symbols are transmitted only over one antenna at any given time;
- The receiver uses the average received power from the common pilots from each antenna, and makes a decision as to from which antenna it would like the transmitter to transmit;
- The decision of the receiver is conveyed to the transmitter through a feedback channel.

Diversity: Adaptive Transmit Diversity

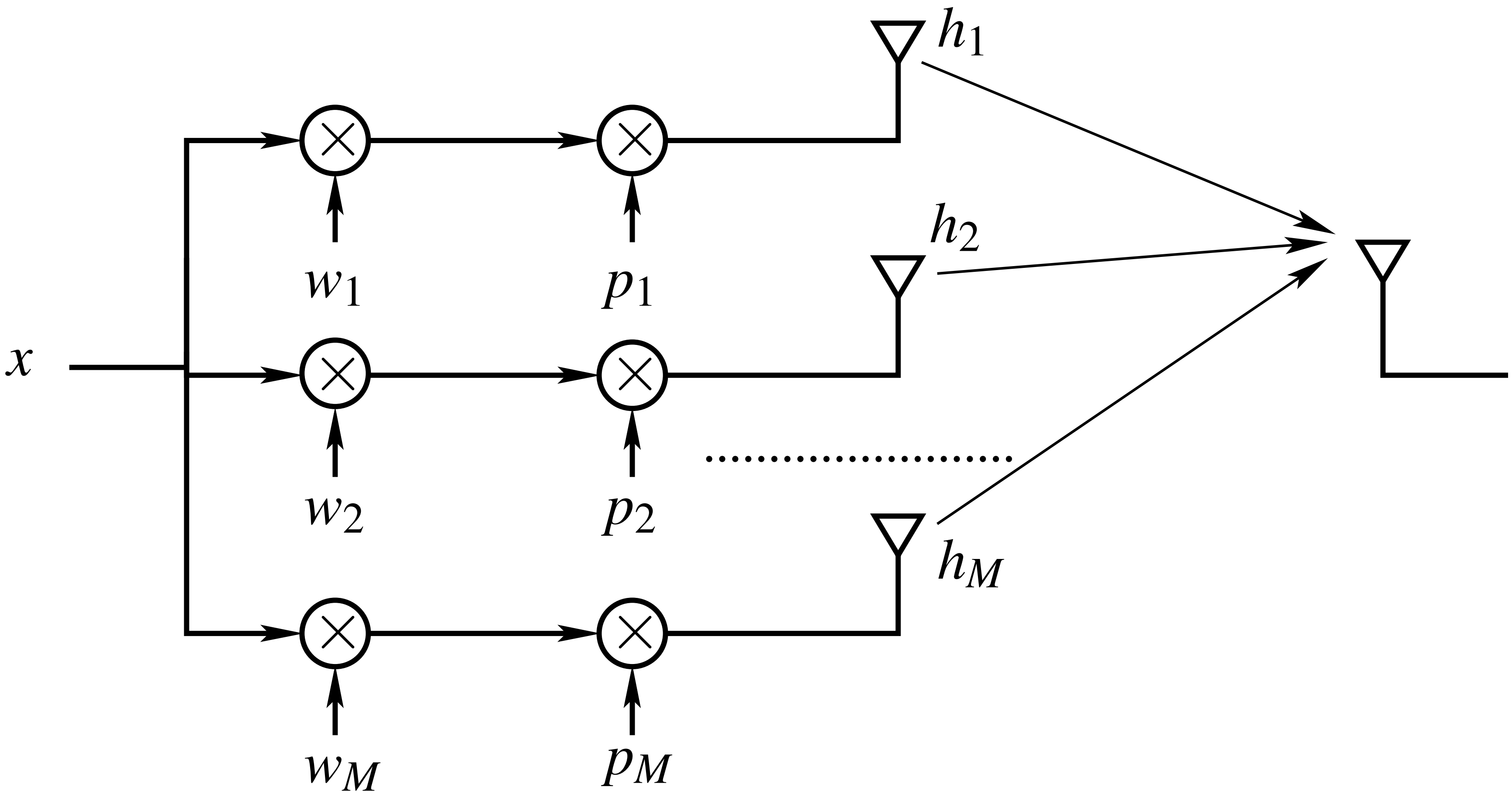


Figure 31: Block diagram of the adaptive transmit diversity scheme.

Diversity: Adaptive Transmit Diversity

- Based on Fig. 31, the received signal can be expressed as

$$y = \mathbf{HW}x + n, \tag{73}$$

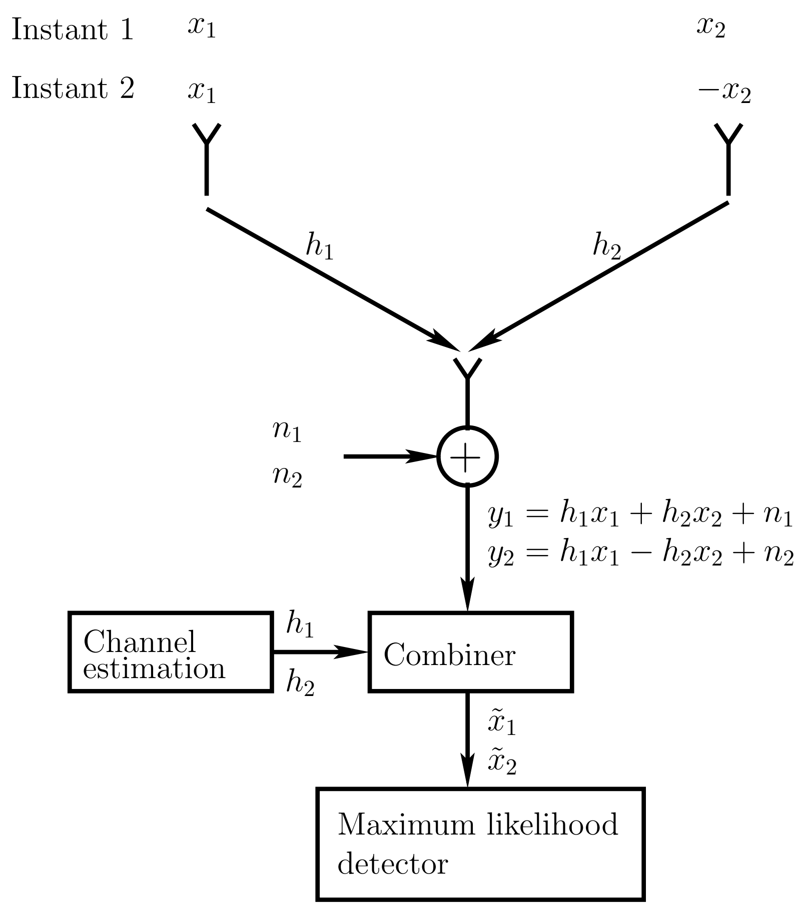
where $\mathbf{H} = [h_1, h_2, \ldots, h_M]$ is due to the channel, while $\mathbf{W} = [w_1, w_2, \ldots, w_M]^T$ represents the weights used by the transmitter;

- If the receiver is capable of estimating the channel vector of $\mathbf{H}$ with the aid of the pilots of $\mathbf{P} = [p_1, p_2, \ldots, p_M]^T$, and feeding back the channel vector to the transmitter, then, the optimum weights maximizing the SNR is given by $\mathbf{W} = \mathbf{H}^H$, and, consequently, the decision variable can be expressed as

$$Z = y|_{\mathbf{W}=\mathbf{H}^H} = \mathbf{HH}^H x + n = \left(\sum_{m=1}^{M} |h_m|^2\right) x + n. \tag{74}$$

- Hence, the transmit diversity order achieved is M.

A Simple Example of an orthogonal transmit diversity code:



Diversity: Orthogonal Transmit Diversity

- Let x_1 and x_2 represent two consecutive symbols to be transmitted. Over two symbol periods, these two symbols are transmitted using two antennas according to the scheme

$$\text{Antenna 1: } x_1, \quad x_1,$$
$$\text{Antenna 2: } x_2, \; -x_2.$$

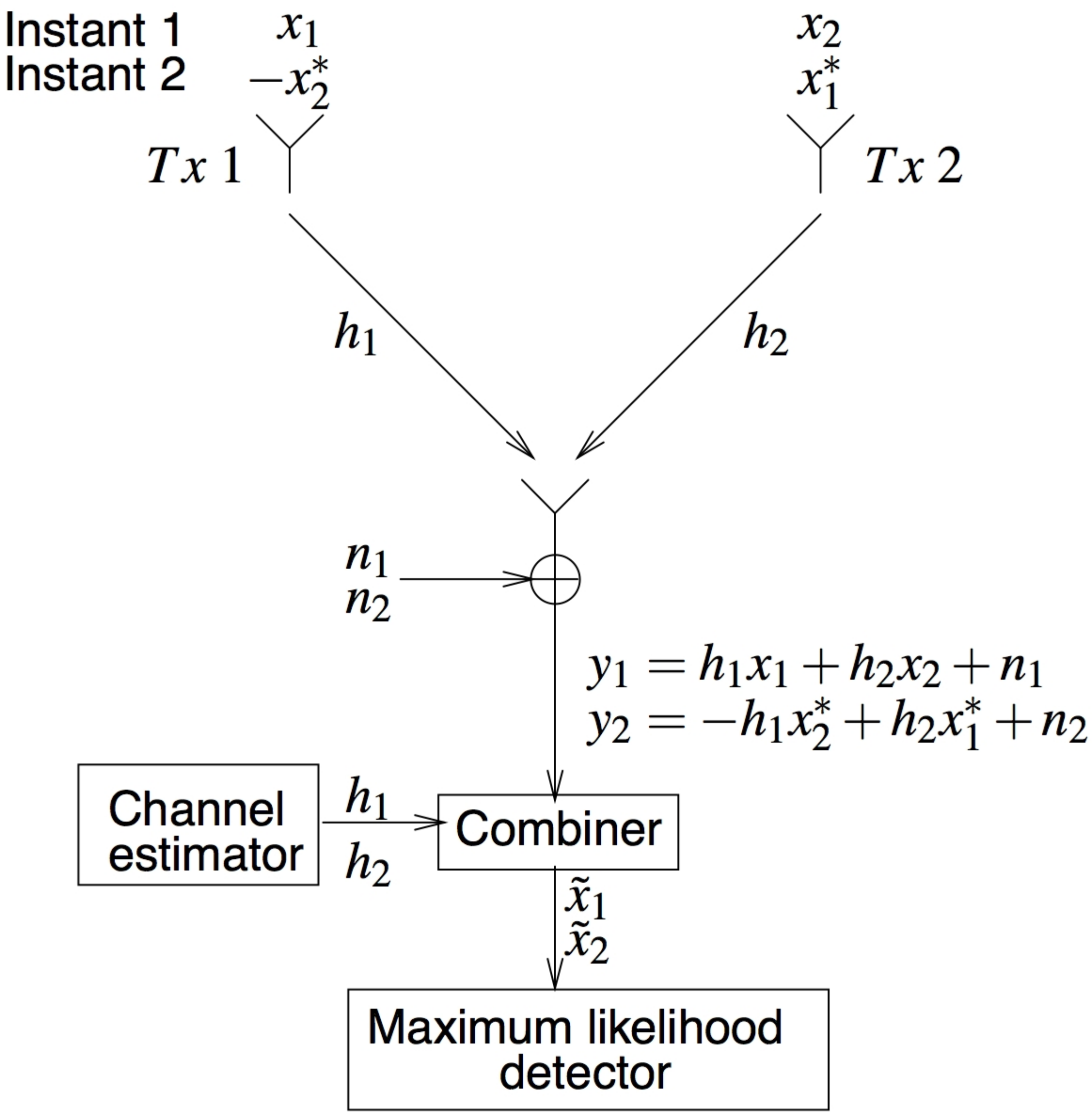
- Let y_1 and y_2 represent the received signal at two consecutive symbol intervals. Then, for the slow fading channels, we have

$$y_1 = h_1 x_1 + h_2 x_2 + n_1, \tag{75}$$
$$y_2 = h_1 x_1 - h_2 x_2 + n_2, \tag{76}$$

- The corresponding decision variables for x_1 and x_2 are given by

$$Z_1 = y_1 h_1^* + y_2 h_1^* = 2|h_1|^2 x_1 + h_1^*(n_1 + n_2), \tag{77}$$
$$Z_2 = y_1 h_2^* - y_2 h_2^* = 2|h_2|^2 x_2 + h_2^*(n_1 - n_2). \tag{78}$$

A Simple Example of the G_2 Alamouti Space-Time Block Code:



Diversity: Space-Time Coding

- Let x_1 and x_2 represent two consecutive symbols to be transmitted. Over two symbol periods, these two symbols are transmitted using two antennas according to the scheme

$$\begin{array}{ll} \text{Antenna 1:} & x_1, \quad -x_2^*, \\ \text{Antenna 2:} & x_2, \quad\ \ x_1^*. \end{array}$$

- Let y_1 and y_2 represent the received signal at two consecutive symbol intervals. Then, for the slow fading channels, we have

$$y_1 = h_1 x_1 + h_2 x_2 + n_1, \tag{79}$$

$$y_2 = -h_1 x_2^* + h_2 x_1^* + n_2, \tag{80}$$

- The corresponding decision variables for x_1 and x_2 are given by

$$Z_1 = y_1 h_1^* + y_2^* h_2 = (|h_1|^2 + |h_2|^2) x_1 + h_1^* n_1 + h_2 n_2^*, \tag{81}$$

$$Z_2 = y_1 h_2^* - y_2^* h_1 = (|h_1|^2 + |h_2|^2) x_2 + h_2^* n_1 - h_1 n_2^*. \tag{82}$$

MIMO Diversity - Conclusions

- In MIMO systems multiple transmit and receive antennas can be employed for achieving a high diversity order;
- In a rich scattering environment resulting in independent fading from each transmit antenna to each receive antenna, a MIMO system having M number of transmit antennas and N number of receive antennas is capable of achieving a diversity order of MN.

MIMO Diversity - An Example

- Let us assume that there are two transmit antennas and M number of receive antennas;
- Let x_1 and x_2 be two consecutive symbols to be transmitted using the Alamouti space-time coding scheme, i.e., the two symbols are transmitted using the following scheme:

$$\begin{aligned} &\text{Antenna 1:}\ \ x_1, \ \ -x_2^*, \\ &\text{Antenna 2:}\ \ x_2, \quad\ \ x_1^*. \end{aligned}$$

- Hence, for the mth receive antenna, the received signal vector can be written as

$$\mathbf{y}_m = \begin{bmatrix} y_{1m} \\ y_{2m}^* \end{bmatrix} = \underbrace{\begin{bmatrix} h_{1m} & h_{2m} \\ h_{2m}^* & -h_{1m}^* \end{bmatrix}}_{\mathbf{H}_m} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} n_{1m} \\ n_{2m}^* \end{bmatrix}}_{\mathbf{n}_m}. \tag{83}$$

MIMO Diversity - An Example (Continued)

- Let

$$\begin{aligned}
\mathbf{y} &= [\mathbf{y}_1^T, \mathbf{y}_2^T, \ldots, \mathbf{y}_M^T]^T, \quad (2M \times 1), && (84)\\
\mathbf{H} &= [\mathbf{H}_1^T, \mathbf{H}_2^T, \ldots, \mathbf{H}_M^T]^T, \quad (2M \times 2), && (85)\\
\mathbf{n} &= [\mathbf{n}_1^T, \mathbf{n}_2^T, \ldots, \mathbf{n}_M^T]^T, \quad (2M \times 1). && (86)
\end{aligned}$$

- Then, when considering the M number of receive antennas, the received signal associated with two consecutive symbol intervals can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}. \qquad (87)$$

MIMO Diversity - An Example (Continued)

- Since

$$\begin{aligned} \mathbf{H}_m^H \mathbf{H}_m &= \begin{bmatrix} |h_{1m}|^2 + |h_{2m}|^2 & 0 \\ 0 & |h_{1m}|^2 + |h_{2m}|^2 \end{bmatrix} \\ &= (|h_{1m}|^2 + |h_{2m}|^2)\mathbf{I}_2, \end{aligned} \tag{88}$$

- then, after multiplying $\mathbf{H}^H$ with $\mathbf{y}$, we obtain the decision variable for $\mathbf{x}$:

$$\begin{aligned} \mathbf{Z} &= \mathbf{H}^H \mathbf{y} \\ &= \sum_{m=1}^{M} (|h_{1m}|^2 + |h_{2m}|^2)\mathbf{x} + \mathbf{H}^H \mathbf{n}. \end{aligned} \tag{89}$$

- **Hence, the diversity order achieved is $2M$.**

SDMA: Signal Representation

- Let the MIMO channel related to the kth user through the relationship of

$$\mathbf{H}_k \mathbf{x}_k, \quad k = 1, 2, \ldots, K; \tag{90}$$

- We assume that each of the users employs the same number of transmit antennas and all the transmitted signals employ the same modulation scheme;
- Then, the received multiple-access signal can be expressed as

$$\mathbf{Y} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{x}_k + \mathbf{n}, \tag{91}$$

where $\mathbf{n}$ is assumed to be complex Gaussian distributed;

SDMA: Signal Representation (Continued)

- Equation (91) can be further represented as

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{n}, \tag{92}$$

where

$$\mathbf{H} = [\mathbf{H}_1, \mathbf{H}_2, \ldots, \mathbf{H}_K], \tag{93}$$

$$\mathbf{X} = \left[\mathbf{x}_1^T, \mathbf{x}_2^T, \ldots, \mathbf{x}_K^T\right]^T. \tag{94}$$

SDMA: Typical Characteristics

- Spatial signature - In SDMA systems users are identified by their channel state matrices of $\{\mathbf{H}_k\}$;
- The spatial signatures, i.e., the channel state matrices of $\{\mathbf{H}_k\}$ are usually time-varying;
- The maximum number of users supported by the SDMA system is determined by the ranks of the matrix of $\mathbf{H}$.

SDMA - Detection Examples

- **Correlation Single-User Detection (SUD)**: The correlation SUD is implemented by multiplying both sides of (92) by $\mathbf{H}^H$, which is expressed as

$$\tilde{\mathbf{y}} = \mathbf{H}^H \mathbf{Y} = \tilde{\mathbf{R}}\mathbf{X} + \tilde{\mathbf{n}}, \tag{95}$$

where $\tilde{\mathbf{R}}$ is the cross-correlation matrix of the channel state matrices of $\{\mathbf{H}_k\}$, and $\tilde{\mathbf{y}} = [\tilde{y}_1, \ldots, \tilde{y}_K]^T$.

- **Decorrelating Multiuser Detection (MUD)**: The decorrelating MUD is obtained by multiplying both sides of (95) with $\tilde{\mathbf{R}}^{-1}$, which is expressed as

$$\mathbf{Z} = \tilde{\mathbf{R}}^{-1}\tilde{\mathbf{y}} = \mathbf{X} + \tilde{\mathbf{R}}^{-1}\tilde{\mathbf{n}}. \tag{96}$$

Explicitly, all the **multiuser interference** (MUI) is removed.