EE746 Assignment 1 Report

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Contents

1 Problem 1: Leaky Integrate and Fire Model

Assume that $C = 300pF$, $g_L = 30nS$, $V_T = 20mV$ and $E_L = 70mV$

1.1 Assume that a constant current I_0 is applied to the neuron. Write an expression for the steady state value of the membrane potential. Hence, determine the minimum value of the steady state current, I_c necessary to initiate a spike.

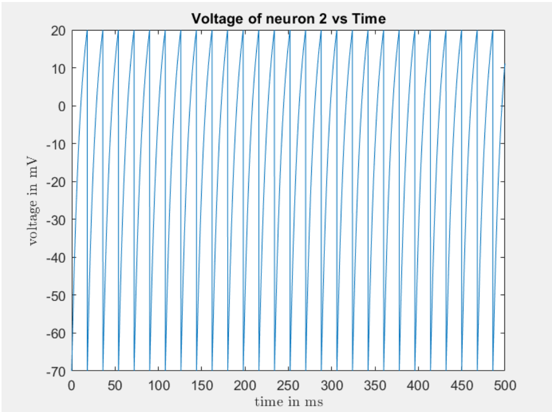
Assuming a constant current I_0 is applied to the neuron we obtain the following differential equation

$$\begin{aligned}
C\frac{dV(t)}{dt} &= -g_L(V(t) - E_L) + I_0 \\
\frac{dV}{dt} + \frac{g_L}{C}V &= \frac{I_0 + g_L E_L}{C} \\
V &= \frac{I_0}{g_L} + E_L + C' \exp\left(\frac{-g_L}{C}t\right) \text{ ,where C' is a constant of integration} \\
\text{as } t \to \infty, V &= \frac{I_0}{g_L} + E_L \geq V_T \text{ for neurons to fire} \\
I_0 &\geq g_L(V_T - E_L) = I_c
\end{aligned}$$

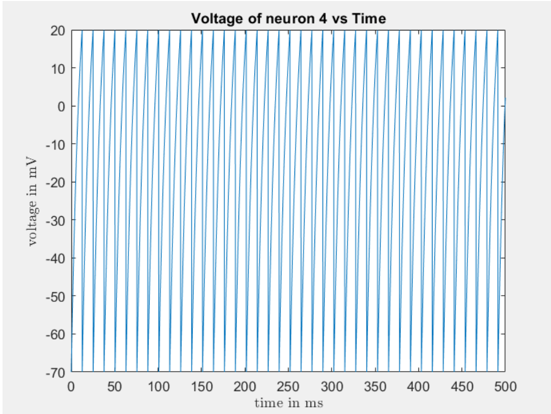
Thus, the minimum value of steady state current needed to initiate a spike is $I_c = g_L(V_T - E_L) = 2.7 * 10^{-9}A = 2.7nA$ using the given values of physical parameters.

1.2 In order to simulate the behavior of a set of N neurons, it is useful to define a N × 1 column vector to the store the membrane potentials of the N neurons. Write a program to solve the equivalent difference equation numerically using Runge-Kutta second order method for a set of N neurons driven by external current input. (You should not use a FOR loop to calculate the potential of the N neurons, rather, the potential of all the neurons should be calculated in one step.) Assume that the input to the program is a N × M column vector, representing the input current for the N neurons for M time-intervals where M = $T/\Delta t$. The output of your program should be a N × M matrix storing the values of the membrane potential for the N neurons, for the M time-intervals.

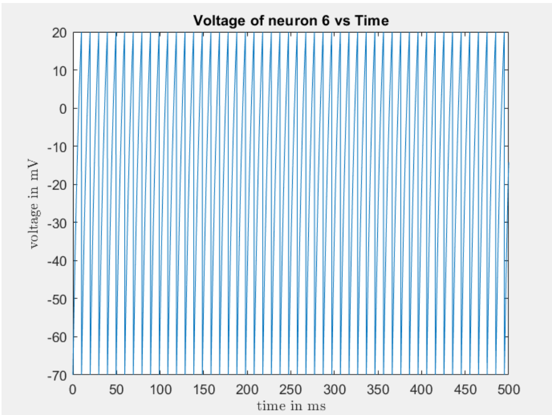
The second order Runge-Kutta solver performs the following updates
$V_{t+1} = V_t + \Delta t\, f(t + \frac{\Delta t}{2}, V_t + \frac{\Delta t}{2} f(t, V_t))$, where $f(t, V) = \frac{dV}{dt} = \frac{-g_L}{C}V + \frac{g_L E_L}{C} + \frac{I(t)}{C}$. The solver can simultaneously update the voltage values for all N neurons from values for the previous time-step. Since we do not have access to a continuous time function for current, the solver assumes that current values at midpoints of timesteps can be approximated by the average of current at the neighbouring timesteps. This, does not affect the results in the subsequent section however, since the current is taken to be constant.
The next stage is to check if any neuron's voltage has crossed the spiking threshold V_T and to reset it back to E_L. This is counted as one spike. The function LIF_RK2 runs this solver and returns a matrix of voltage values at each time step.



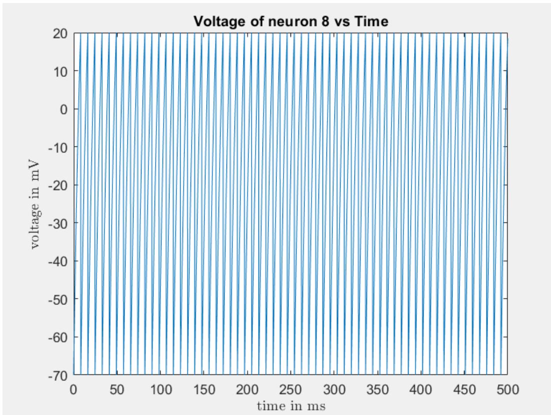
(a) $I_{app,2} = 1.2 * I_c$



(b) $I_{app,4} = 1.4 * I_c$



(c) $I_{app,6} = 1.6 * I_c$



(d) $I_{app,8} = 1.8 * I_c$

Figure 1: Membrane Potential vs time for neurons over 500ms under constant current

1.3 We would like to now use this framework to study the dynamics of LIF neurons. Assume that you have a population of 10 identical neurons, with each neuron receiving a constant current. Let the magnitude of the input current for the k^{th} neuron be given by the expression $I_{app,k} = (1 + k\alpha)I_c$ where $\alpha = 0.1$. (In this example, the current does not vary with time, so, all the values across any row of the input current matrix is a constant). Plot the membrane potential for neurons 2, 4, 6 and 8 from t = 0 to 500 ms. (Assume Δt = 0.1 ms and at t = 0, the neuron is in steady-state with $I_{app}(t) = 0$).

The plots show an exponentially rising voltage followed by spiking as soon as it reaches the threshold V_T. The exponential behaviour was expected from the solution to the differential equation that we obtained in the first part for constant current. The rate of spiking is seen to increase as the current is increased.

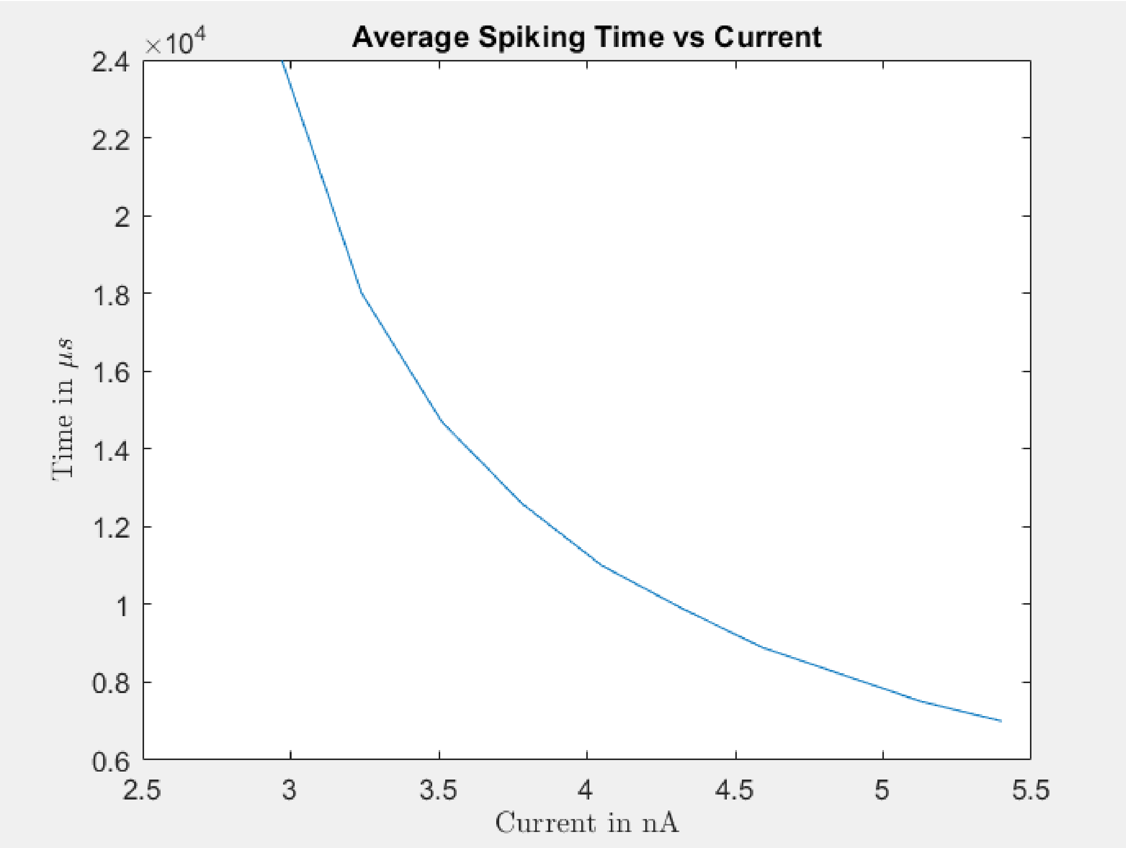


Figure 2: Average spiking time reduces as applied current increases

1.4 Plot the average time interval between spikes from (c) as a function of $I_{app,k}$.

We can see that the average time between spikes decreases as current increases. Equivalently, the rate of spiking increases as current increases.

2 Problem 2: Izhikevich Model

2.1 What are the steady state values of V and U for $I_{app} = 0$?

For finding the steady state values of U and V, the equation to solve would be as follows:

$$0 = k_z(V(t) - E_r)(V(t) - E_t) - U(t)$$
$$0 = a[b(V(t) - E_r) - U(t)]$$

On solving the above equations, we get the solutions as:

1. $U = 0,\ V = E_r$
2. $U = b(\frac{b}{k_z} + E_t - E_r),\ V = \frac{b}{k_z} + E_t$

2.2 Write the equivalent difference equations for (3) and (4).

The corresponding difference equations are:

$$C \cdot \frac{V(n+1) - V(n)}{\Delta t} = [k_z(V(n) - E_r)(V(n) - E_t) - U(n) + I_{app}(n)]$$
$$\frac{U(n+1) - U(n)}{\Delta t} = a[b(V(n) - E_r) - U(n)]$$

2.3 Write a program to solve the equivalent difference equation for a set of N Izhikevich model neurons using Runge-Kutta fourth order method. The neuron type should be a parameter in your function call, for each of the N neurons. You may use $\Delta t = 0.1ms$ and plot the response of the three neurons above from t = 0 to 500 ms, for I_{app} = 400, 500, 600 pA.

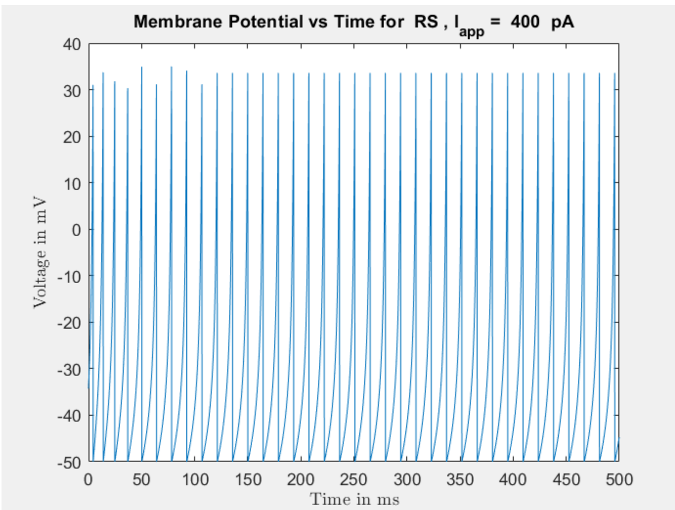


Figure 3: Type = "RS", I = 400pA

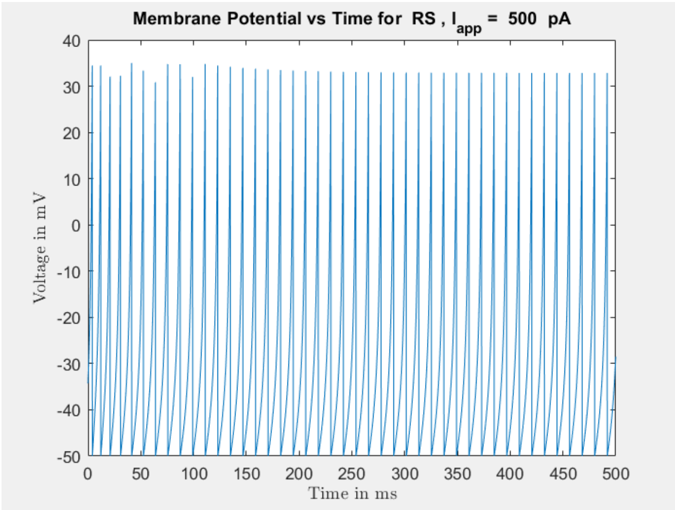


Figure 4: Type = "RS", I = 500pA

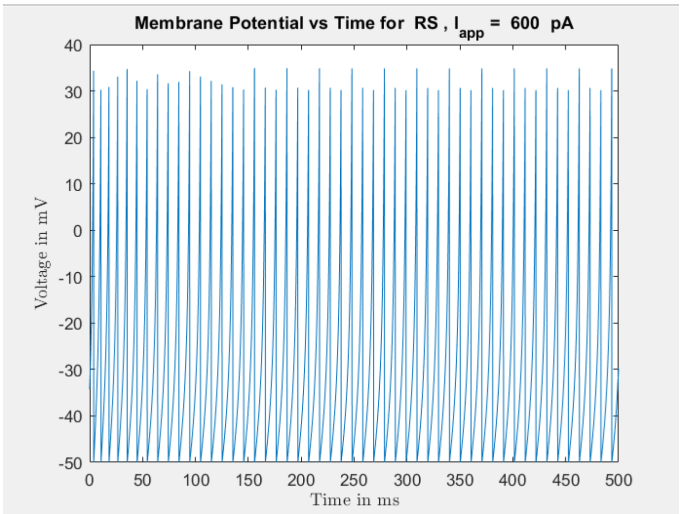


Figure 5: Type = "RS", I = 600pA

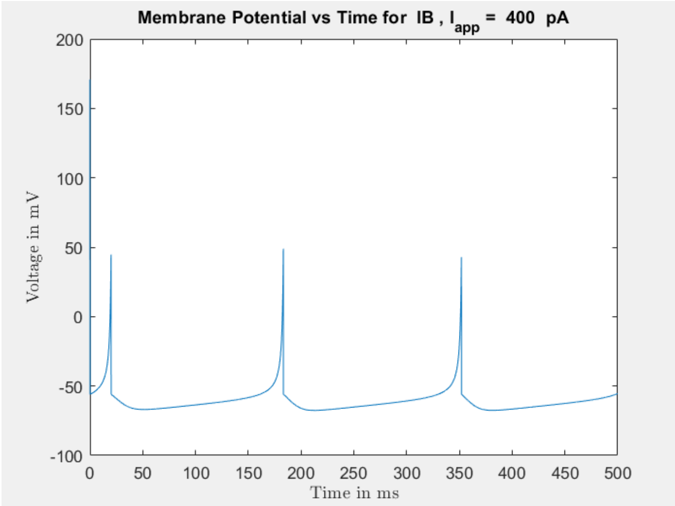


Figure 6: Type = "IB", I = 400pA

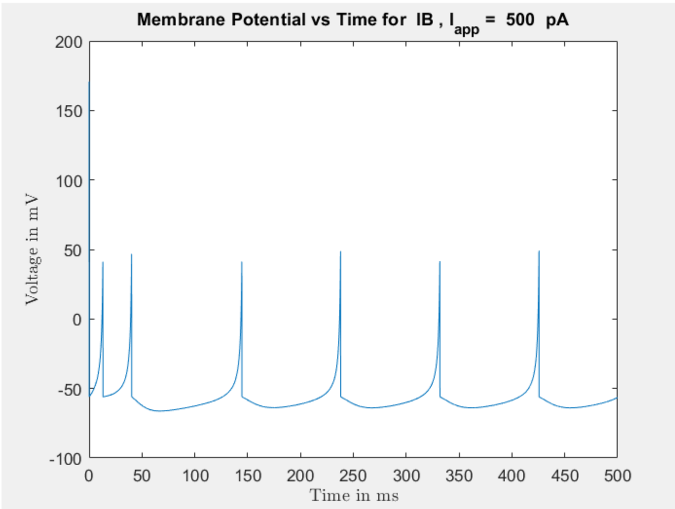


Figure 7: Type = "IB", I = 500pA

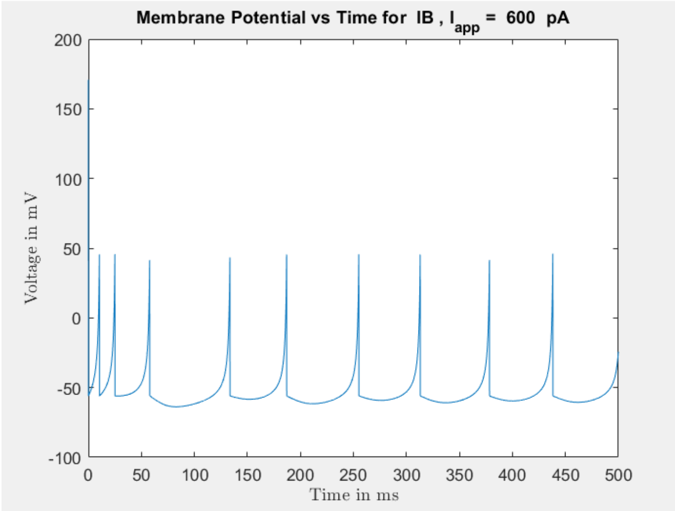


Figure 8: Type = "IB", I = 600pA

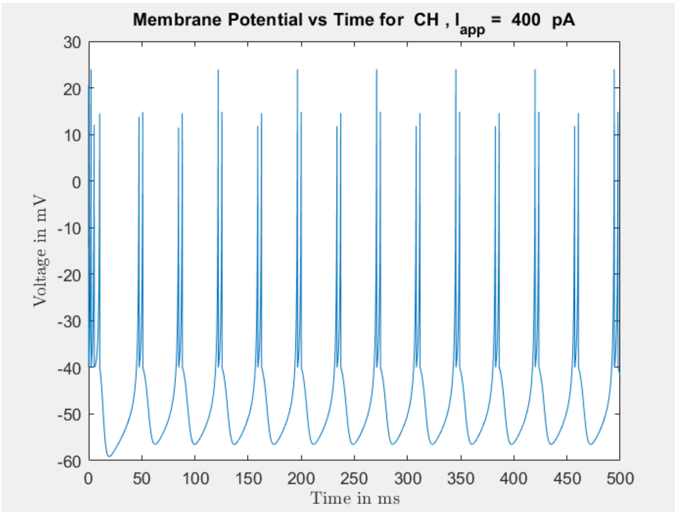


Figure 9: Type = "CH", I = 400pA

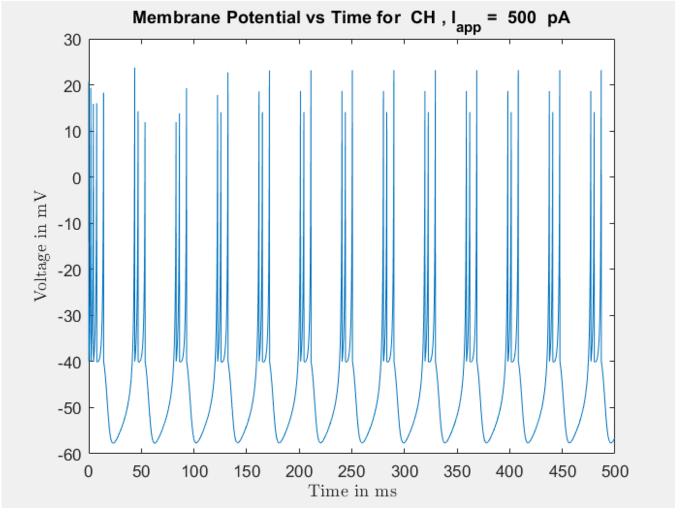


Figure 10: Type = "CH", I = 500pA

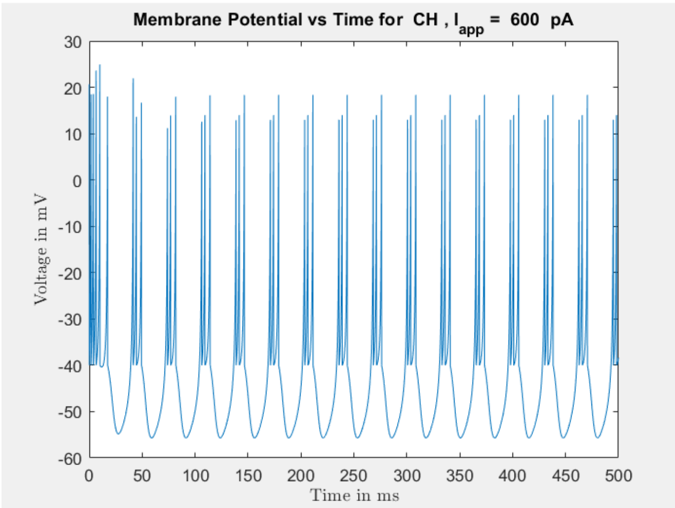


Figure 11: Type = "CH", I = 600pA

3 Problem 3: Adaptive Exponential Integrate-and-Fire Model

3.1 Write the equivalent difference equations for (5) and (6)

The given differential equations are
(5) $C\frac{dV(t)}{dt} = -g_L(V - E_L) + g_L\Delta_T \exp\left(\frac{V(t)-V_T}{\Delta_T}\right) - U(t) + I_{app}(t)$
(6) $\tau_\omega \frac{dU(t)}{dt} = a[V(t) - E_L] - U(t)$
the equivalent difference equations are
(5) $C\frac{V(n+1)-V(n)}{\Delta t} = -g_L(V(n) - E_L) + g_L\Delta_T \exp\left(\frac{V(n)-V_T}{\Delta_T}\right) - U(n) + I_{app}(n)$
(6) $\tau_\omega \frac{U(n+1)-U(n)}{\Delta t} = a[V(n) - E_L] - U(n)$

3.2 What are the steady state values of V and U for $I_{app} = 0$? (Determine the answer numerically, such that the value of V is accurate within ± 1).

The numerical solution proceeds as follows. Set $\frac{dU}{dt} = 0, \frac{dV}{dt} = 0, I_{app} = 0$, substitute the expression for $U(t)$ from equation (6) into (5) and get an equation purely in terms of $V(t)$, use the symbolic math toolbox in MATLAB to numerically solve for V(t) [which is a constant at steady state].

Setting derivatives, I_{app} to 0

(5) $0 = -g_L(V - E_L) + g_L\Delta_T \exp\left(\frac{V(t)-V_T}{\Delta_T}\right) - U(t)$
(6) $0 = a[V(t) - E_L] - U(t)$
Substituting U(t) from (6) into (5)
$0 = -g_L(V - E_L) + g_L\Delta T \exp\left(\frac{V(t)-V_T}{\Delta T}\right) - a[V(t) - E_L]$
Solving using symbolic math toolbox, the exact solutions come out to be

1. RS: V = - 2*lambertw(0, -(5*exp(-10))/6) - 70 = -69.9999 mV
2. IB: V = - 2*lambertw(0, -(9*exp(-4))/11) - 58 = -57.9696 mV
3. CH: V = - 2*lambertw(0, -(5*exp(-4))/6) - 58 = -57.9690 mV

We can also find the steady state value of U using equation (6), $U(t) = a[V(t) - E_L]$

3.3 Write a program to solve the equivalent difference equation for a set of N AEF model neurons using Euler method. The neuron type should be a parameter in your function call, for each of the N neurons. You may use $\Delta t = 0.1$ ms and plot the response of the three neurons above from t = 0 to 500 ms, for I_{app} = 250, 350, 450 pA

The Euler Method is a first order method for solving ODEs. The results of simulations for various types of neurons using this model are as follows.

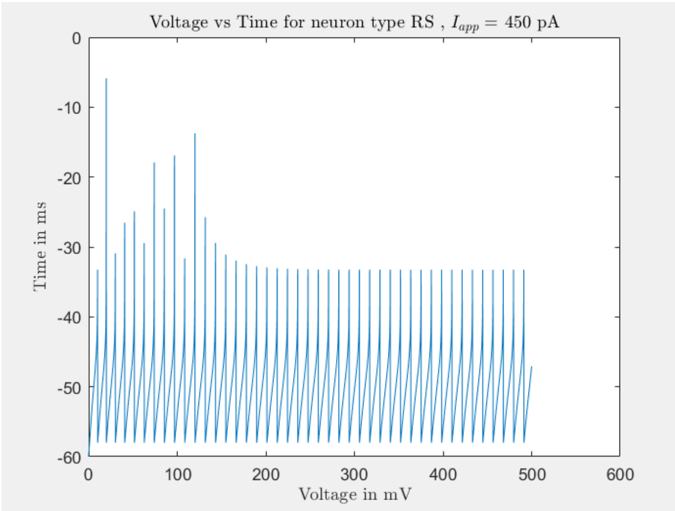


Figure 12: Type = "RS", I = 450pA

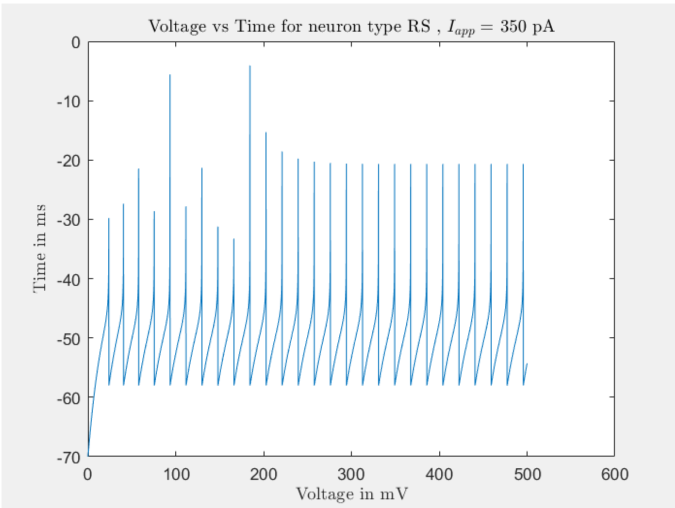


Figure 13: Type = "RS", I = 350pA

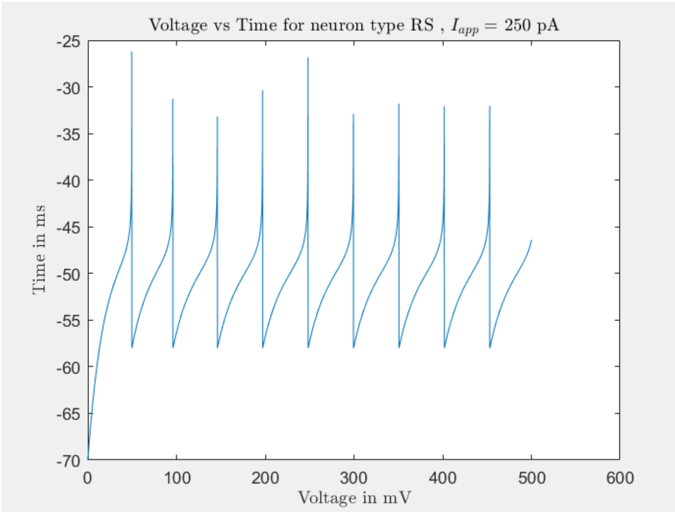


Figure 14: Type = "RS", I = 250pA

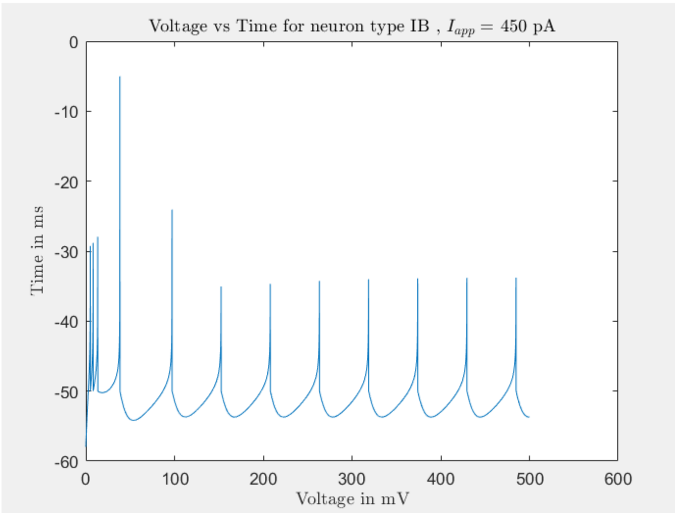


Figure 15: Type = "IB", I = 450pA

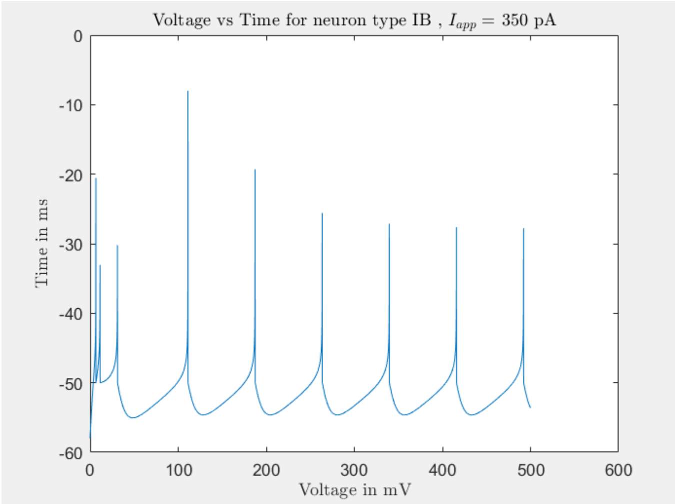


Figure 16: Type = ”IB”, I = 350pA

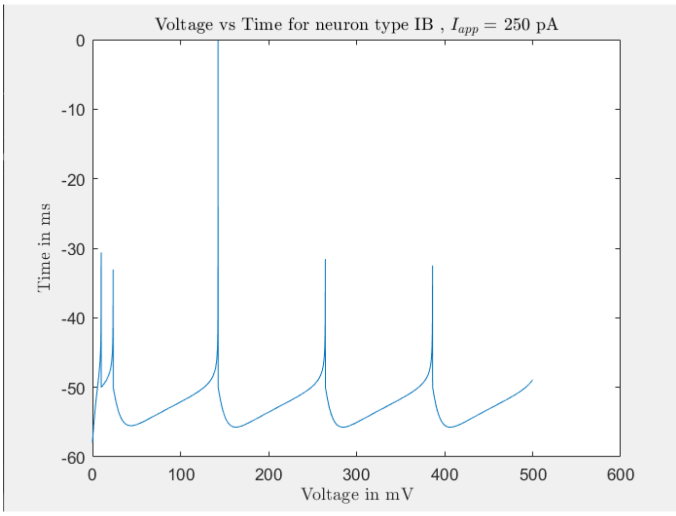


Figure 17: Type = ”IB”, I = 250pA

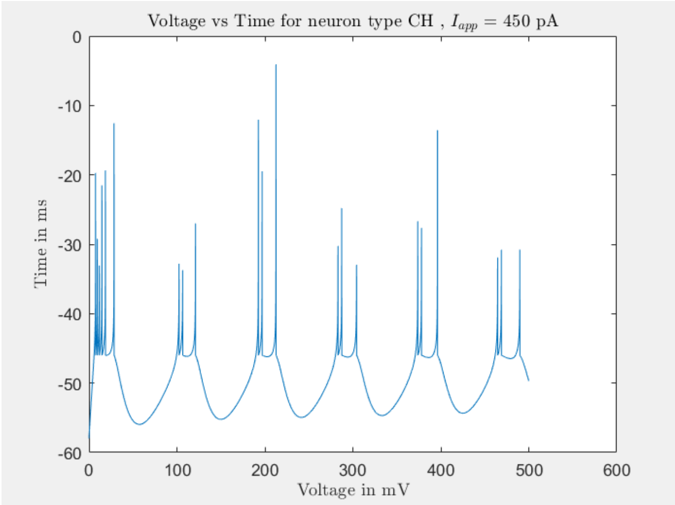


Figure 18: Type = "CH", I = 450pA

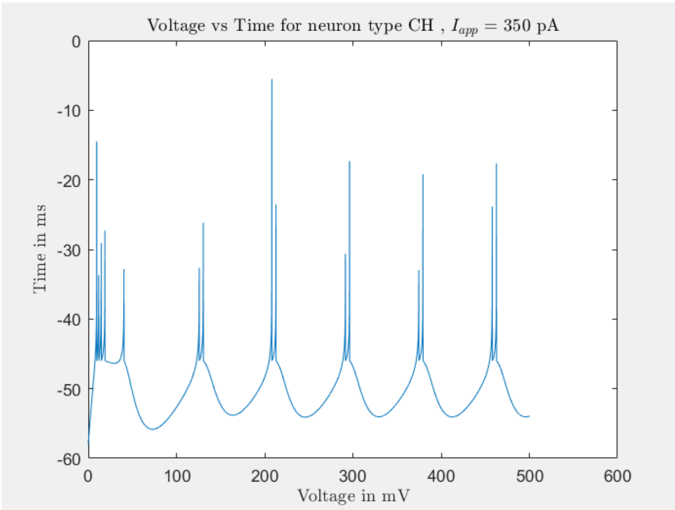


Figure 19: Type = "CH", I = 350pA

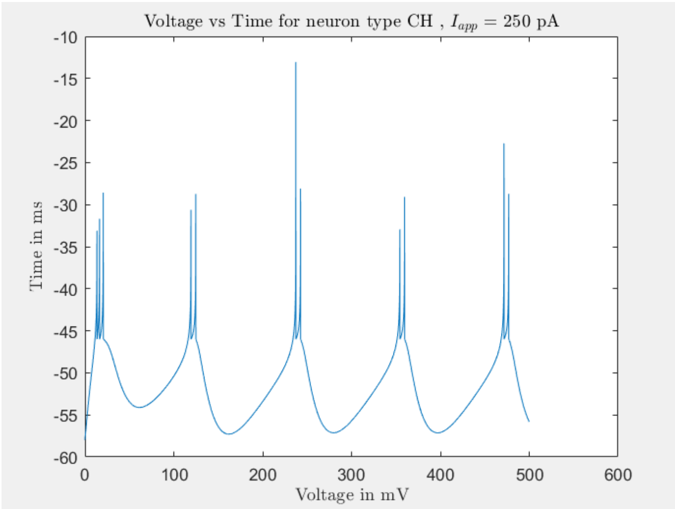


Figure 20: Type = "CH", I = 250pA

4 Problem 4: Spike energy based on Hodgkin-Huxley neuron model

The dynamics of the membrane potential V (t), in the Hodgkin-Huxley neuron model is given by the equations,

$$C\frac{dV(t)}{dt} = -i_{Na}(t) - i_K(t) - i_l + I_{ext}(t) \tag{1}$$

where,

$$\begin{aligned} i_{Na}(t) &= g_{Na}m^3h(V(t) - E_{Na}) \\ i_K(t) &= g_K n^4(V(t) - E_K) \\ i_l(t) &= g_l(V(t) - E_l) \end{aligned}$$

The variables n, m and h lie in the interval [0, 1] and obey the equation

$$\frac{dx}{dt} = \alpha_x(t)(1-x) - \beta_x(t)x$$

$\alpha_n, \alpha_m, \alpha_h$ as well as $\beta_m, \beta_n, \beta_h$ are all functions of voltage, V(t).

4.1 Solve the equivalent difference equations for above and determine the ion currents and the membrane potential for a step current waveform described as follows: $I_{ext}(t) = I_0[H(t-2T) - H(t-3T)]$ where H(x) is the Heaviside step function defined as H(x) = 1 if $x \geq 0$ and H(x) = 0 otherwise. You should set your initial conditions such that there are no spikes when there is no external input current to the neuron. Assume $I_0 = 15\mu A/cm^2$, Δt = 0.01ms, T = 30ms

We will first evaluate the equations in the steady state(resting state) and calculate $V_{resting}$.

Making the following substitutions:

$\frac{dV}{dt} = 0$ and $I_{app} = 0$ as well as $\frac{dm}{dt} = 0, \frac{dn}{dt} = 0, \frac{dh}{dt} = 0$
and solving the resultant system of equations using MATLAB yields the following solution:

$$V_{dc} = -65.1560308mV$$

To evaluate the spikes in the model, a for loop with time steps of δt = 0.01ms is used in the equivalent difference form of the equations and the values are recorded and plotted.

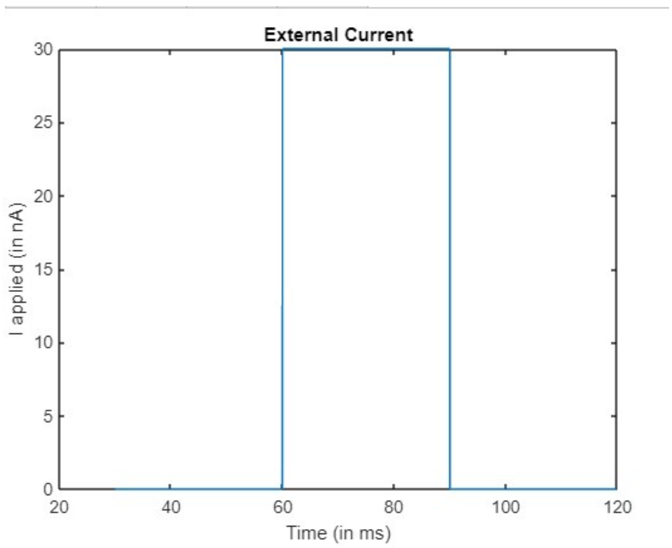


Figure 21: External input current

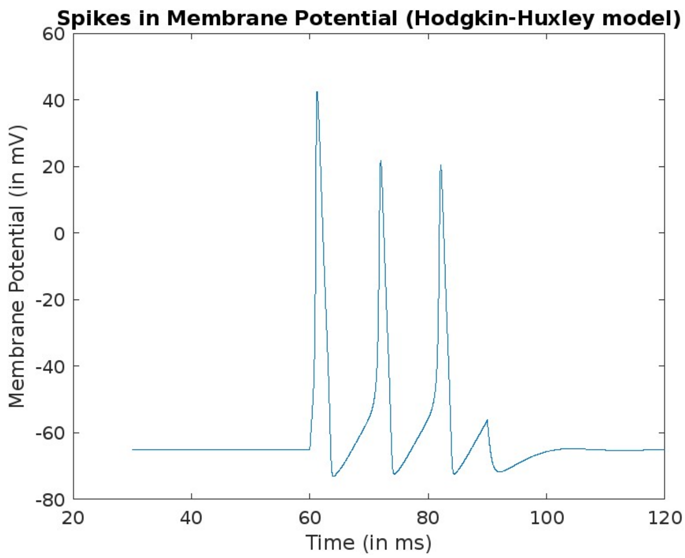


Figure 22: Membrane Voltage vs Time

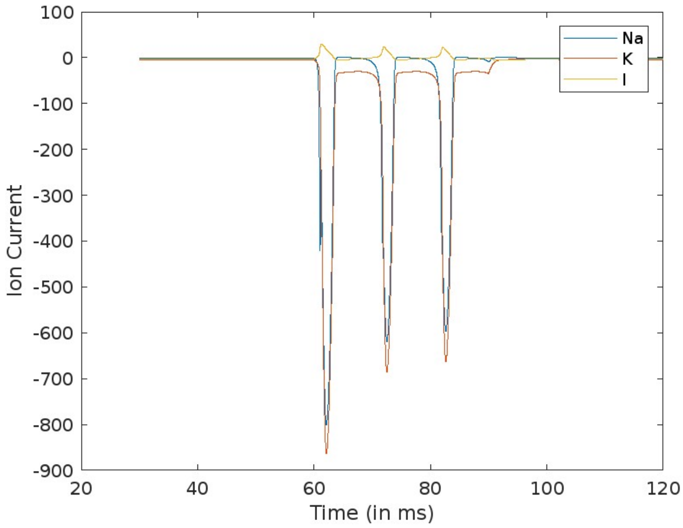


Figure 23: Ion Currents vs Time

4.2 Plot the relative magnitudes of the instantaneous power dissipated in the three ion channels and in the membrane capacitor as a function of time for one cycle of the action potential

The instantaneous power dissipated (per unit area) in the various ion channels can be approximated by the expression

$$P_x(t) = i_x(t)(V(t) - E_x)$$

Similarly, the power spent in charging/discharging the membrane capacitance (per unit area) can be approximated as

$$CV(t)\frac{dV(t)}{dt} = (-i_{Na}(t) - i_K(t) - i_l + I_{ext}(t))V(t)$$

We use the above expressions in the loop to calculate the instantaneous power dissipated and record the values.

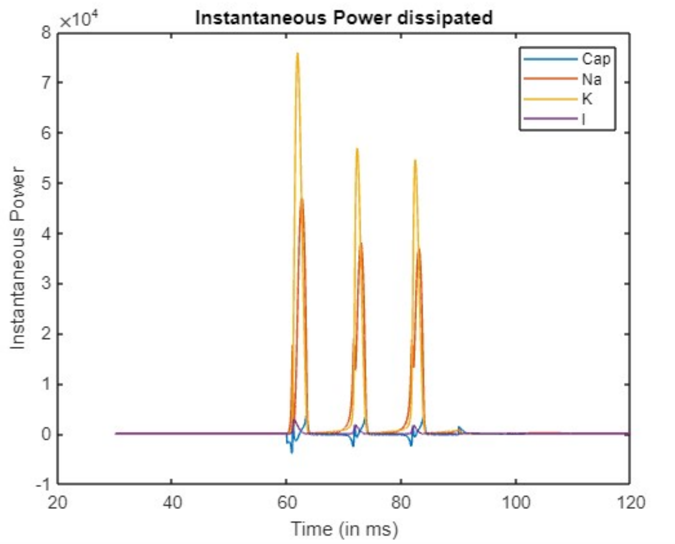


Figure 24: Instantaneous Power consumed

4.3 Numerically integrate the power in (b) to determine the total energy dissipated in one cycle of the action potential for a patch of the cell membrane with area of 1 µm2

We have plots resembling three complete spikes and one half/incomplete spike.
To calculate the total energy dissipated, we sum up all the instantaneous power values for the whole time period.

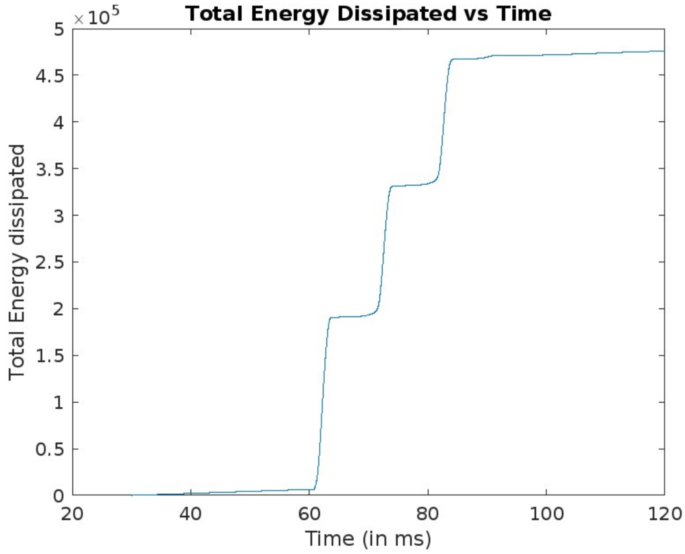


Figure 25: Total Energy dissipated vs Time