

Object Detection Metrics

what all are the metrics in object detection

In **object detection**, the primary goal is to **detect and localize objects** in an image. To evaluate performance, several metrics are used—some measuring **localization accuracy**, others **classification accuracy**, and some **overall performance**.

Here's a structured breakdown of the **key object detection metrics**:

♦ 1. Intersection over Union (IoU)

- **Definition:** Measures the overlap between predicted bounding box and ground truth box.

- **Formula:**

$$IoU = \frac{\text{Area of Overlap}}{\text{Area of Union}}$$

- **Usage:** Used to determine if a prediction is a **True Positive (TP)**. Common thresholds: $IoU > 0.5$ or stricter (e.g., 0.75).

♦ 2. Precision

- **Definition:** Fraction of correctly predicted boxes over all predicted boxes.

- $$\text{Precision} = \frac{TP}{TP + FP}$$

♦ 3. Recall

- **Definition:** Fraction of correctly predicted boxes over all ground truth boxes.

- $$\text{Recall} = \frac{TP}{TP + FN}$$

◆ 4. Average Precision (AP)

- **Definition:** Area under the Precision-Recall curve.
 - Calculated for each class separately.
 - Depends on IoU threshold (e.g., `AP@0.5`, `AP@0.75`).
 - **Interpolated AP:** Often smoothed over recall levels (e.g., Pascal VOC).
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◆ 5. mean Average Precision (mAP)

- **Definition:** Mean of Average Precision across all classes.
 - **Variants:**
 - **mAP@0.5 (Pascal VOC):** AP at IoU threshold of 0.5.
 - **mAP@[.5:.95] (COCO):** Average AP at multiple IoU thresholds (0.5 to 0.95 with 0.05 step).
-

◆ 6. F1 Score

- Harmonic mean of precision and recall:

$$F1 = 2 \cdot \frac{Precision \cdot Recall}{Precision + Recall}$$

◆ 7. AP Small / Medium / Large (COCO specific)

- Measures AP on different object sizes:
 - `AP_small`: Area < 32²
 - `AP_medium`: Area in [32², 96²]
 - `AP_large`: Area > 96²
-

◆ 8. Number of Detections per Image

- Helps evaluate model behavior—too few: under-detecting; too many: over-detecting.
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◆ 9. Inference Time / FPS

- **Latency** and **Throughput** are critical in real-time systems.
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◆ 10. False Positive (FP) and False Negative (FN) Rates

- Important for analyzing errors and tradeoffs between over-detection and under-detection.
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Example (COCO Evaluation Output):

txt

```
AP: 0.367
AP50: 0.582
AP75: 0.391
AP_small: 0.219
AP_medium: 0.400
AP_large: 0.494
```

If you're building or evaluating a model, let me know your use case (real-time? class imbalance? small objects?), and I can recommend the most relevant metrics and thresholds.

what is focal loss and why use it. explain math

Focal Loss is a loss function designed to address **class imbalance** in tasks like **object detection**, particularly in **dense detectors** like RetinaNet. It is a modified version of the **Cross Entropy Loss**, tailored to **focus more on hard, misclassified examples** and **down-weight the easy ones**.

♦ Why Focal Loss?

In dense object detection:

- Most anchor boxes are **background** (negative class).
- Standard **Cross-Entropy Loss** treats all examples equally → model is overwhelmed by easy negatives → poor performance on rare/hard positives.

👉 Focal Loss dynamically **scales down the loss from easy examples**, focusing learning on hard misclassified ones.

♦ Standard Binary Cross Entropy Loss

$$CE(p, y) = -y \cdot \log(p) - (1 - y) \cdot \log(1 - p)$$

where:

- $p \in [0, 1]$ is the model's estimated probability for the class with label $y \in \{0, 1\}$
-

♦ Focal Loss: Formula

Let:

$$p_t = \begin{cases} p & \text{if } y = 1 \\ 1 - p & \text{if } y = 0 \end{cases}$$

→ The probability of the **true class**

Then **Focal Loss** is:

$$FL(p_t) = -\alpha_t \cdot (1 - p_t)^\gamma \cdot \log(p_t)$$

◆ Explanation of Terms

| Term | Meaning |
|--------------------|--|
| p_t | Probability of the correct class |
| $\log(p_t)$ | Log loss as in standard cross-entropy |
| $(1 - p_t)^\gamma$ | Modulating factor that reduces the loss for well-classified examples |
| γ | Focusing parameter $\gamma \geq 0$, typically 2.0 |
| α_t | Class balancing factor (optional), helps balance positive/negative samples |

◆ Behavior of Focal Loss

| Case | p_t | $(1 - p_t)^\gamma$ | Effect on Loss |
|--|------------|--------------------|---------------------------------------|
| Easy example (correct, confident) | ~ 1.0 | ~ 0 | Loss \rightarrow small (ignored) |
| Hard example (wrong or low confidence) | ~ 0.0 | ~ 1 | Loss \rightarrow large (focused on) |

◆ Graphical Intuition

With Cross-Entropy:

- Loss is always high for incorrect predictions.
- Easy negatives dominate total loss.

With Focal Loss:

- Loss is **suppressed** when p_t is high (easy).
- Loss is **amplified** when p_t is low (hard).

◆ Class-Balanced Focal Loss (Optional)

Add class weights $\alpha_t \in [0, 1]$ to handle class imbalance:

$$FL(p_t) = -\alpha_t(1 - p_t)^\gamma \log(p_t)$$

- In RetinaNet, $\alpha = 0.25$, $\gamma = 2.0$
-

◆ Use Case in RetinaNet

In **RetinaNet**, focal loss is critical to:

- Handle **millions of background anchors**
- Focus training on **rare positive detections**

Without focal loss, Cross Entropy causes instability and underperformance.

◆ Summary Table

| Feature | Focal Loss |
|------------|--|
| Based on | Cross-Entropy |
| Solves | Class imbalance in object detection |
| Key idea | Down-weight easy examples |
| Parameters | γ : focusing, α : class weight |
| Use case | RetinaNet, single-shot detectors |

Would you like a PyTorch or TensorFlow implementation next?

Let's go **in-depth** into the **motivation and math behind Focal Loss**, including why it was created, how it works mathematically, and how it improves performance in **imbalanced classification tasks**, especially in **dense object detection** like in RetinaNet.

◆ 1. Problem: Class Imbalance in Object Detection

■ Scenario:

In dense detectors (e.g., RetinaNet, SSD, YOLO):

- Each image may have **thousands of anchor boxes**.
- Only a **tiny fraction** (~1%) of anchors match a ground truth object (positive samples).
- The **vast majority** are **easy negatives** (background class, correctly classified with high confidence).

■ What goes wrong with Cross-Entropy Loss:

- Each of those easy negatives contributes equally to the total loss.
 - The gradient is dominated by these easy samples.
 - Model becomes **biased towards background**, ignoring hard or rare positive examples.
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◆ 2. Standard Binary Cross Entropy (BCE)

$$L_{\text{BCE}}(p, y) = -y \cdot \log(p) - (1 - y) \cdot \log(1 - p)$$

Where:

- $y \in \{0, 1\}$ is the ground truth label
- $p \in (0, 1)$ is the model's predicted probability for class 1

This loss treats all examples **equally**, regardless of how well the model is already doing.

◆ 3. Focal Loss: Core Idea

To focus learning on hard examples and down-weight easy ones.

◆ 4. Rewriting BCE for Simplification

Let's define:

$$p_t = \begin{cases} p & \text{if } y = 1 \\ 1 - p & \text{if } y = 0 \end{cases}$$

So BCE becomes:

$$L_{\text{BCE}} = -\log(p_t)$$

◆ 5. Focal Loss Formulation

Focal Loss introduces a modulating factor $(1 - p_t)^\gamma$:

$$L_{\text{Focal}}(p_t) = -\alpha_t \cdot (1 - p_t)^\gamma \cdot \log(p_t)$$

■ Terms explained:

- $\alpha_t \in [0, 1]$: balancing factor for positive/negative classes
 - $\gamma \geq 0$: focusing parameter
 - $(1 - p_t)^\gamma$: reduces loss contribution from **easy examples**
-

◆ 6. Mathematical Behavior

Let's examine how focal loss behaves:

✓ When $p_t \rightarrow 1$ (easy sample):

- $(1 - p_t)^\gamma \rightarrow 0$
- \rightarrow Loss is nearly zero

✓ When $p_t \rightarrow 0$ (hard sample):

- $(1 - p_t)^y \rightarrow 1$
 - \rightarrow Loss is large
-

◆ 7. Effect on Gradient

■ Cross-Entropy:

$$\frac{d}{dp} L_{\text{BCE}} = -\frac{y}{p} + \frac{1-y}{1-p}$$

■ Focal Loss (Derivative with respect to p):

If $y = 1$, then $p_t = p$, and:

$$\frac{d}{dp} L_{\text{Focal}} = \alpha(1-p)^y \left[\frac{yp \log(p)}{1-p} - \frac{1}{p} \right]$$

This gradient is:

- **Small for confident predictions ($p \rightarrow 1$):** suppresses their impact.
 - **Large for poor predictions ($p \rightarrow 0$):** focuses optimization here.
-

◆ 8. Why It Works: Intuition

| Term | Role |
|-----------------|---|
| $\log(p_t)$ | Standard classification loss term |
| $(1 - p_t)^y$ | Suppresses well-classified examples |
| α_t | Class-balancing to correct for skewed distributions |
| Combined effect | Forces model to focus on misclassified , rare , and hard samples |

◆ 9. Hyperparameters

| Parameter | Effect | Common Value (RetinaNet) |
|-----------|---|--------------------------|
| γ | Focusing strength ($\uparrow \gamma = \uparrow$ focus on hard) | 2.0 |
| α | Class balance (for positives/negatives) | 0.25 for positive class |

◆ 10. Use Case in RetinaNet

RetinaNet proposes **Focal Loss** as key innovation:

- Without it, one-stage detectors (no proposal mechanism like in Faster R-CNN) underperform.
- With it, RetinaNet becomes SOTA on COCO dataset.

◆ 11. Comparison with Cross-Entropy

| Property | Cross-Entropy | Focal Loss |
|------------------------|---------------|---------------------------------------|
| Equal weighting | ✓ | ✗ |
| Focus on hard examples | ✗ | ✓ |
| Handles imbalance | Poorly | Very well (especially with α) |

◆ 12. Summary

- **Need:** Handle class imbalance and hard/easy example imbalance in detection tasks.
- **Solution:** Focal loss modifies CE to reduce loss from easy examples.
- **Math:** Introduces $(1 - p_t)^\gamma$ modulating factor and optional α balancing.
- **Impact:** Boosts performance in one-stage dense detectors like RetinaNet.

Would you like a plot showing how focal loss changes shape compared to cross-entropy, or code in PyTorch or TensorFlow?

why alpha is needed. and also explain with examples of different values how focal loss varies against crossentropy

Great questions. Let's dive deep into both:

◆ Why Is Alpha Needed in Focal Loss?

◆ Problem 1: Imbalanced Classes

In **binary classification** for object detection:

- Class 0 (background): 95%
- Class 1 (object): 5%

Even with focal loss suppressing easy negatives, the **total loss** is still skewed because **negatives outnumber positives** by a huge margin.

◆ Solution: Weight the positive class higher using α (alpha)

Focal Loss (with alpha):

$$FL(p_t) = -\alpha_t \cdot (1 - p_t)^\gamma \cdot \log(p_t)$$

Where:

- $\alpha_t = \alpha$ if class is positive ($y = 1$)
- $\alpha_t = 1 - \alpha$ if class is negative ($y = 0$)

◆ Why?

It ensures the **positive class is not drowned out** by the volume of easy negatives.

◆ Visual and Numeric Intuition: How Focal Loss Varies with γ and α

Let's compare **Cross-Entropy** and **Focal Loss** under different configurations.

◆ Scenario: Ground truth $y = 1$

Let's fix:

- Ground truth: $y = 1$
- p : model's prediction for class 1

Then:

- $p_t = p$
- Cross Entropy: $-\log(p)$
- Focal Loss: $-\alpha(1 - p)^\gamma \log(p)$

We'll try different values of p , and compare:

| p | CE Loss | FL ($\gamma=2, \alpha=1.0$) | FL ($\gamma=2, \alpha=0.25$) | FL ($\gamma=1, \alpha=0.25$) |
|-----|---------|-------------------------------|--------------------------------|--------------------------------|
| 0.1 | 2.3026 | 1.865 | 0.466 | 0.863 |
| 0.3 | 1.2040 | 0.588 | 0.147 | 0.264 |
| 0.5 | 0.6931 | 0.173 | 0.043 | 0.087 |
| 0.7 | 0.3567 | 0.051 | 0.013 | 0.027 |
| 0.9 | 0.1053 | 0.001 | 0.00026 | 0.00087 |

🔍 Interpretation:

1. Cross-Entropy (no α , no γ):

- Doesn't care how confident the prediction is — it always punishes based on how wrong it is.

- Even if $p = 0.9$ (a good prediction), still has ~ 0.1 loss.

2. Focal Loss with $\gamma = 2$, $\alpha = 1.0$:

- Focuses more on **low p** (hard examples).
- **Suppresses high-confidence (easy) examples.**

3. Focal Loss with $\alpha = 0.25$:

- Same as above, but down-weights the class itself.
- Used when **positives are rare**, to **reduce dominance of negative class**.

4. Lower γ (e.g., $\gamma=1$):

- Less aggressive focusing; still emphasizes hard examples but not as sharply.

◆ Visual Intuition (you can plot this):

- Plot curves of CE and FL over $p \in (0, 1)$
- For CE: curve is steep at low p , flat near $p = 1$
- For FL:
 - With $\gamma = 2$, the curve **peaks** when p is small and **drops rapidly** as p increases
 - With $\alpha < 1$, the entire FL curve is **scaled down** (less importance)

◆ Summary Table

| Term | Role in Focal Loss |
|-------------------|--|
| γ | Focuses on hard examples by down-weighting high-confidence (easy) predictions |
| α | Balances positive vs. negative class contributions in imbalanced datasets |
| When $\alpha = 1$ | All classes are treated equally |
| When $\alpha < 1$ | Positive class is given more importance (used for rare classes) |

✓ Example: When Is α Needed?

| Case | Solution |
|-----------------------------|--|
| Balanced binary classes | α can be omitted |
| Imbalanced (90% background) | $\alpha = 0.25$ for positive class |
| Multi-class imbalance | Use per-class α |

Would you like a code snippet to **visualize these curves** or compare **loss curves interactively** using matplotlib?

explain mean average precision in extreme detail right down from basic to advance including all math formula and calculation.

Absolutely. Let's **build a complete, step-by-step explanation of Mean Average Precision (mAP)** used in **object detection**, from fundamentals to full mathematical depth.

◆ PART 1: FOUNDATION

◆ What is mAP?

- **Mean Average Precision (mAP)** is a single number that summarizes how well your object detection model performs across all classes.
- It is the **mean** of the **Average Precision (AP)** for each class.
- Each AP is computed from the **Precision-Recall curve** for that class.

◆ PART 2: CORE CONCEPTS

◆ 1. Precision and Recall

Let's say the model makes N predictions. For each prediction, we compare with ground truth:

- **True Positive (TP)**: Correct class and $\text{IoU} \geq \text{threshold}$ (e.g., 0.5)
- **False Positive (FP)**: Wrong class or $\text{IoU} < \text{threshold}$
- **False Negative (FN)**: Missed ground truth

$$\text{Precision} = \frac{TP}{TP + FP} \quad \text{Recall} = \frac{TP}{TP + FN}$$

◆ 2. IoU: Intersection over Union

To decide if a prediction is correct:

$$\text{IoU} = \frac{\text{Area of Overlap}}{\text{Area of Union}} \in [0, 1]$$

- If $\text{IoU} \geq \text{threshold}$ (say 0.5), the prediction is a **match** (TP)
-

◆ 3. Precision-Recall Curve

For a class:

- Sort all detections **by confidence score (descending)**
 - For each threshold (e.g., top-k predictions), calculate **Precision** and **Recall**
 - Plot **Precision vs Recall**
-

◆ PART 3: CALCULATING AVERAGE PRECISION (AP)

There are **two major versions** of AP calculation:

✓ Method A: VOC-style AP (11-point Interpolated)

From Pascal VOC 2007:

1. For 11 recall levels $R \in \{0.0, 0.1, \dots, 1.0\}$
2. Find the maximum precision at **each recall** $\geq R$:

$$P_{\text{interp}}(R) = \max_{\tilde{R} \geq R} P(\tilde{R})$$

3. Then:

$$AP = \frac{1}{11} \sum_{R \in \{0.0, \dots, 1.0\}} P_{\text{interp}}(R)$$

✓ Method B: COCO-style AP (Integral over PR curve)

Used in modern benchmarks (e.g., COCO):

1. Compute precision-recall pairs at **every prediction**
2. Interpolate precision:

$$P_{\text{interp}}(r) = \max_{\tilde{r} \geq r} P(\tilde{r})$$

3. Approximate the area under the curve using discrete summation:

$$AP = \sum_{n=1}^N (r_n - r_{n-1}) \cdot P_{\text{interp}}(r_n)$$

This is essentially **trapezoidal integration** of the PR curve.

◆ PART 4: FROM AP TO mAP

Once you have **AP for each class**, compute:

$$mAP = \frac{1}{C} \sum_{c=1}^C AP_c$$

Where:

- C = total number of classes
 - AP_c = Average Precision for class c
-

◆ PART 5: AP AT DIFFERENT IoU Thresholds

COCO Evaluation computes AP at **10 different IoU thresholds**:

IoU thresholds = {0.50, 0.55, 0.60, ..., 0.95}

Final score:

$$AP_{COCO} = \frac{1}{10} \sum_{t=0.5}^{0.95} AP_t$$

This captures **both precision and localization accuracy** (higher IoU is stricter).

◆ PART 6: COCO Metrics Overview

| Metric | Meaning |
|---------------------|---|
| AP@[0.5:0.95] (mAP) | Mean over 10 IoU thresholds (main metric) |
| AP@0.5 | IoU = 0.5 (lenient match) |
| AP@0.75 | IoU = 0.75 (strict match) |
| AP_small | Objects < 32 ² pixels |
| AP_medium | Objects between 32 ² and 96 ² |
| AP_large | Objects > 96 ² pixels |

◆ PART 7: Step-by-Step Example (Single Class)

Ground Truth Boxes:

- GT = 3 boxes

Model Predictions (sorted by confidence):

| Conf | IoU w/ GT | TP/FP |
|------|-----------|-------|
| 0.95 | 0.8 | TP |
| 0.90 | 0.6 | TP |
| 0.85 | 0.3 | FP |
| 0.75 | 0.55 | TP |
| 0.60 | 0.2 | FP |

Compute cumulative TP/FP → Precision & Recall:

| Rank | TP | FP | Precision | Recall |
|------|----|----|-----------|--------|
| 1 | 1 | 0 | 1.00 | 0.33 |
| 2 | 2 | 0 | 1.00 | 0.66 |
| 3 | 2 | 1 | 0.67 | 0.66 |
| 4 | 3 | 1 | 0.75 | 1.00 |
| 5 | 3 | 2 | 0.60 | 1.00 |

Now compute **area under PR curve (AP)** using interpolation or trapezoid approximation.

◆ PART 8: Summary Formulae

| Component | Formula |
|-----------|--|
| IoU | $\frac{\text{Intersection}}{\text{Union}}$ |
| Precision | $\frac{TP}{TP+FP}$ |

| Component | Formula |
|------------------------|---|
| Recall | $\frac{TP}{TP+FN}$ |
| Interpolated Precision | $\max_{r' \geq r} P(r')$ |
| AP (COCO) | $\int_0^1 P_{\text{interp}}(r) dr \approx \sum \Delta r \cdot P(r)$ |
| mAP | $\frac{1}{C} \sum_{c=1}^C AP_c$ |
| mAP@[.5:.95] | $\frac{1}{10C} \sum_{t=0.5}^{0.95} \sum_{c=1}^C AP_{c,t}$ |

Would you like me to:

- Show you Python code to compute AP from raw predictions?
- Plot a precision-recall curve and shaded AP area for better visual intuition?

i still did not understand anything. what is map and how is it calulcate and the math. you did not explain any of them

You're right — let me now **start from absolute scratch**, and walk you step-by-step through:

- ✓ What is **mAP**
- ✓ What is **AP**
- ✓ What is **Precision, Recall**, and how **IoU** is used
- ✓ **Mathematical formulas and step-by-step calculation**

I'll build from **1 prediction** to **all predictions**, and finally **mAP**.

1. What Is the Goal in Object Detection?

For each image:

- The model **predicts bounding boxes and class labels**, with **confidence scores** (e.g., "dog, 0.93").

- We want to **measure how good** these predictions are.

That's where **mAP (mean Average Precision)** comes in.

◆ 2. What is **IoU (Intersection over Union)**?

Before any metric, we must know:

Is a predicted box close enough to a ground truth box?

We use:

$$\text{IoU} = \frac{\text{Area of Overlap}}{\text{Area of Union}}$$

If $\text{IoU} \geq 0.5$ (or 0.75, etc.), then the predicted box is **"correct"** (i.e., a **True Positive**).

◆ 3. Definitions of TP, FP, FN

Let's say for a class like "car":

| Term | Meaning |
|------|--|
| TP | Detected a real car correctly ($\text{IoU} \geq 0.5$ and correct class) |
| FP | Predicted a car, but either no car there or wrong class or low IoU |
| FN | There was a car, but you didn't predict it |

◆ 4. What is **Precision** and **Recall**?

| Term | Formula |
|-----------|---|
| Precision | $\frac{TP}{TP+FP}$ — How many predicted cars were correct |
| Recall | $\frac{TP}{TP+FN}$ — How many real cars you found |

◆ 5. How to Build a Precision-Recall Curve

For a class like "car":

Step-by-step:

1. Predict N boxes, each with confidence score (say: 0.95, 0.9, 0.7, ...)
2. Sort predictions by score **descending**.
3. Go down the list, for each box:
 - Check: Is it a TP or FP (using IoU with GT)?
 - Keep track: #TP, #FP
 - Calculate precision and recall

You now have a set of points: (recall, precision)

| Plot this to get the **Precision-Recall Curve**

◆ 6. What Is Average Precision (AP)?

It's the area under the Precision-Recall Curve (AUC).

Approximate as:

$$AP = \sum_{i=1}^N (R_i - R_{i-1}) \cdot P_i$$

Where:

- R_i = recall at step i
- P_i = precision at step i

It's like: sum of **rectangle areas** under the curve.



Mini Example:

Suppose you have 3 ground truth cars.

Predictions:

| Confidence | Correct (IoU ≥ 0.5)? |
|------------|----------------------------|
| 0.95 | ✓ TP1 |
| 0.9 | ✓ TP2 |
| 0.85 | ✗ FP1 |
| 0.8 | ✓ TP3 |

Let's compute:

| Step | TP | FP | Precision | Recall |
|------|----|----|--------------|--------------|
| 1 | 1 | 0 | 1.00 | $1/3 = 0.33$ |
| 2 | 2 | 0 | 1.00 | $2/3 = 0.66$ |
| 3 | 2 | 1 | $2/3$ | $2/3 = 0.66$ |
| 4 | 3 | 1 | $3/4 = 0.75$ | 1.00 |

Now compute **AP** as:

$$AP = (0.66 - 0.33) \cdot 1.00 + (1.00 - 0.66) \cdot 0.75 = 0.33 + 0.255 = 0.585$$



7. What Is mean Average Precision (mAP)?

If your dataset has **C classes**, then:

$$mAP = \frac{1}{C} \sum_{c=1}^C AP_c$$

That is: average the AP over all classes.

8. Variants of mAP

| Metric | Meaning |
|--------------|---|
| mAP@0.5 | IoU threshold = 0.5 |
| mAP@0.75 | IoU threshold = 0.75 (stricter) |
| mAP@[.5:.95] | COCO-style average of 10 IoU thresholds |

Final Summary:

| Step | Description |
|------|---|
| 1 | Predict bounding boxes with confidence |
| 2 | Match with ground truth using $\text{IoU} \geq 0.5$ |
| 3 | Sort predictions by confidence |
| 4 | Label each as TP or FP |
| 5 | Calculate Precision and Recall |
| 6 | Build PR curve |
| 7 | Compute AP = area under PR curve |
| 8 | Compute mAP = average AP over all classes |

Would you like a **code implementation in Python** that walks through this with real values and plots?

what is 1-point Interpolated or 10 different IoU thresholds ?

Great question! Let's clarify **two different things** you mentioned:

◆ A. What is **11-point Interpolated AP** (Pascal VOC)

This is a way to **approximate Average Precision (AP)** from the Precision–Recall curve using **11 fixed recall levels**:

$$\{0.0, 0.1, 0.2, \dots, 1.0\}$$

📌 **Definition (11-point interpolated AP):**

For each recall level $r \in \{0.0, 0.1, \dots, 1.0\}$:

$$P_{\text{interp}}(r) = \max_{\tilde{r} \geq r} P(\tilde{r})$$

That is: For each recall threshold, take the **maximum precision at or above that recall**.

Then, average over all 11 values:

$$AP = \frac{1}{11} \sum_{r \in \{0.0, \dots, 1.0\}} P_{\text{interp}}(r)$$

✅ This was used in **Pascal VOC 2007**.

◆ **Why interpolation?**

Because the Precision–Recall curve can be **jagged**, and interpolation **smooths** it for a fairer estimate.



Example:

Suppose your PR curve gives you precision at some recall values:

| Recall | Precision |
|--------|-----------|
| 0.2 | 0.8 |
| 0.4 | 0.7 |

| Recall | Precision |
|--------|-----------|
| 0.7 | 0.6 |

We interpolate these to compute:

- $P(0.0) = \max_{r \geq 0.0} P(r) = 0.8$
- $P(0.1) = 0.8$
- ...
- $P(0.4) = 0.7$
- ...
- $P(0.7) = 0.6$
- $P(0.8) = 0$
- ...
- Average these 11 values = **AP**

◆ B. What is **mAP@[.5:.95]** with 10 IoU thresholds (COCO metric)

Instead of computing AP at **just** $\text{IoU} \geq 0.5$, modern benchmarks compute it at **multiple** IoU thresholds.

📌 **COCO uses:**

$\text{IoU thresholds} = \{0.50, 0.55, 0.60, \dots, 0.95\}$ (step size: 0.05)

📌 **Final mAP is:**

$$mAP@[0.5 : 0.95] = \frac{1}{10} \sum_{\text{IoU}=0.5}^{0.95} AP_{\text{IoU}}$$

This forces the model to be good at **both finding objects** and **localizing them precisely**.

◆ mAP@0.5 vs mAP@0.75

- mAP@0.5: Match if box has IoU ≥ 0.5 — **lenient**
 - mAP@0.75: Match only if IoU ≥ 0.75 — **strict**
-

✓ Final Summary

| Metric | Description |
|--------------------------|---|
| 11-point Interpolated AP | Smooth PR curve by max precision at \geq recall. Used in VOC 2007 |
| mAP@[.5:.95] | Mean of APs computed at 10 IoU thresholds. Used in COCO |
| AP@0.5 | AP at IoU ≥ 0.5 only |
| AP@0.75 | AP at IoU ≥ 0.75 only |

Would you like plots or code to visualize **interpolated PR curves** and IoU thresholds?