ReLU Backpropagation Derivatives

explaining to me using derivatives relu backpropagation using example.

Understanding ReLU Backpropagation Using Derivatives

ReLU (Rectified Linear Unit) is a widely used activation function in neural networks, defined as:

$$\text{ReLU}(x) = \max(0, x)$$

The derivative of ReLU plays a crucial role in backpropagation, which is used to update weights in a neural network.

Step 1: Understanding ReLU and Its Derivative

The ReLU function is:

$$f(x) = \begin{cases} x, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases}$$

Derivative of ReLU

The derivative of ReLU is:

$$f'(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases}$$

This means:

- If x is positive, the gradient is **1** (ReLU behaves like a linear function).
- If x is negative or zero, the gradient is **0** (ReLU blocks the gradient, leading to the **dying ReLU problem** in some cases).

Step 2: Backpropagation with ReLU Example

Let's go through an example of a **simple neural network** with one hidden layer.

Network Structure

Consider a **single neuron** with:

- Input $x = 2$
- Weight $w = 3$
- Bias $b = 1$

- Activation function: ReLU

The forward pass computes:

$$z = wx + b = (3)(2) + 1 = 7$$

Applying ReLU:

$$a = \max(0, 7) = 7$$

Assume the loss function is:

$$L = (a - y)^2$$

where $y = 5$ (target output).

Step 3: Computing Gradients Using Backpropagation

To update w using **gradient descent**, we need:

$$\frac{dL}{dw}$$

Using the chain rule:

$$\frac{dL}{dw} = \frac{dL}{da} \cdot \frac{da}{dz} \cdot \frac{dz}{dw}$$

1. Compute $\frac{dL}{da}$

$$L = (a - y)^2$$

$$\frac{dL}{da} = 2(a - y) = 2(7 - 5) = 4$$

2. Compute $\frac{da}{dz}$ (Derivative of ReLU)

Since $z = 7$ (which is **positive**), we use:

$$\frac{da}{dz} = 1$$

3. Compute $\frac{dz}{dw}$

$$\frac{dz}{dw} = x = 2$$

Step 4: Update the Weight

$$\frac{dL}{dw} = 4 \times 1 \times 2 = 8$$

Using gradient descent with learning rate $\eta = 0.1$:

$$w_{\text{new}} = w - \eta \cdot \frac{dL}{dw}$$

$$w_{\text{new}} = 3 - 0.1 \times 8 = 2.2$$

Key Observations

1. **ReLU keeps gradients alive for positive values**, preventing vanishing gradients.
2. **For negative values, ReLU sets gradients to 0**, meaning neurons that get stuck in the negative region stop learning.
3. **Weight updates depend on the derivative of ReLU**, which is either **1 or 0**, leading to efficient computation.

Would you like a more complex example with a full backpropagation through multiple layers? 🚀

how would sigmoid do in this example instead of relu

Backpropagation with Sigmoid vs. ReLU: A Comparison Using the Same Example

Now, let's go through the same **forward and backward pass** using the **Sigmoid activation function** instead of ReLU and compare the results.

Step 1: Define the Sigmoid Activation Function

The **Sigmoid function** is:

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

Its derivative is:

$$\sigma'(z) = \sigma(z)(1 - \sigma(z))$$

Unlike ReLU, Sigmoid **never gives a zero gradient**, but it **saturates** when z is too large or too small, leading to **vanishing gradients**.

Step 2: Forward Pass with Sigmoid

We use the same inputs:

- $x = 2$
- $w = 3$

- $b = 1$

Compute:

$$z = wx + b = (3)(2) + 1 = 7$$

Apply Sigmoid:

$$a = \sigma(7) = \frac{1}{1 + e^{-7}} \approx 0.999$$

Assume the same loss function:

$$L = (a - y)^2, \quad y = 5$$

Step 3: Backpropagation

To update w, compute:

$$\frac{dL}{dw} = \frac{dL}{da} \cdot \frac{da}{dz} \cdot \frac{dz}{dw}$$

1. Compute $\frac{dL}{da}$

$$\frac{dL}{da} = 2(a - y) = 2(0.999 - 5) = -8.002$$

2. Compute $\frac{da}{dz}$ (Sigmoid Derivative)

$$\frac{da}{dz} = \sigma(z)(1 - \sigma(z)) = 0.999(1 - 0.999) = 0.000999$$

3. Compute $\frac{dz}{dw}$

$$\frac{dz}{dw} = x = 2$$

Step 4: Compute Gradient and Update Weights

$$\frac{dL}{dw} = (-8.002) \times (0.000999) \times (2) \approx -0.016$$

Using gradient descent with $\eta = 0.1$:

$$w_{\text{new}} = 3 - 0.1 \times (-0.016) = 3.0016$$

Comparison: ReLU vs. Sigmoid

Feature	ReLU	Sigmoid
Forward Output	$a = 7$	$a \approx 0.999$
Gradient	1	0.000999
Weight Update	$w_{\text{new}} = 2.2$	$w_{\text{new}} = 3.0016$
Gradient Strength	Strong (No Vanishing)	Very weak (Vanishing Gradient)

Key Takeaways

1. **Sigmoid suffers from vanishing gradients** because its derivative is very small when z is large. This results in **slow learning**.
2. **ReLU provides strong gradients** when $z > 0$, making it more effective in deep networks.
3. **Sigmoid squashes values between (0,1)**, which can be useful for probability outputs but not for hidden layers.

Would you like me to extend this example with multiple layers? 🚀

does relu prevent exploding gradient ?

Great question. Let's unpack this clearly and deeply.

🔍 Does ReLU Prevent Exploding Gradient?

✅ Short Answer:

No, ReLU does not prevent exploding gradients by itself.
It helps **avoid vanishing gradients**, but **exploding gradients can still occur**, especially in deep networks or poorly initialized weights.

📘 Let's Understand Why

🔸 Exploding Gradient:

Occurs when gradients **accumulate and grow exponentially** during backpropagation, especially in **deep networks**.

Mathematically, during backprop, we multiply many Jacobians (derivatives of each layer):

$$\frac{dL}{dx} = \frac{dL}{da_n} \cdot \frac{da_n}{dz_n} \cdot \frac{dz_n}{da_{n-1}} \cdots \frac{dz_2}{da_1} \cdot \frac{da_1}{dz_1} \cdot \frac{dz_1}{dx}$$

If each term > 1, the overall gradient **blows up**.

🔍 What ReLU Does Well

ReLU:

$$f(x) = \max(0, x)$$

$$f'(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \le 0 \end{cases}$$

✅ Avoids Vanishing Gradients (unlike Sigmoid/Tanh):

- Derivative is **1**, not squashed to near-zero
- Keeps gradients flowing in positive regions

⚠️ But Why Doesn't It Prevent Exploding Gradients?

Because:

1. **ReLU doesn't cap gradients** — there's no upper bound.
2. If input weights are too large, **activations and gradients can still explode**.
3. In **deep ReLU networks**, if weights are poorly initialized or not regularized, the **magnitude of forward activations** can grow layer-by-layer → leading to exploding gradients during backprop.

🧪 Empirical Example

Let's say:

- You have 100 layers of ReLU.
- You initialize weights $w \sim \mathcal{N}(0, 1)$.
- Input $x = 1$

Forward pass:

$$a = w_1 x \to w_2 a \to w_3 a \to \cdots \to w_{100} a$$

Each multiplication by a weight could double or triple the magnitude → **output explodes** → **so do gradients**.

🛠️ How to Handle Exploding Gradients (with or without ReLU)

Technique	Description
Weight Initialization	Use **He Initialization** with ReLU:
$\mathcal{N}(0, \sqrt{2/n})$	
Gradient Clipping	Clip gradients to a max norm or value during backprop to stop explosions.
Normalization Layers	Use **BatchNorm**, **LayerNorm** to stabilize activations & gradients.
Regularization	L2 regularization (weight decay) can help control weight magnitudes.
Architecture Choices	Use residual connections (e.g., in ResNets) to help with gradient flow.

✅ Final Verdict

Question	Answer
Does ReLU prevent vanishing gradients?	✅ Yes (for $x > 0$)
Does ReLU prevent exploding gradients?	❌ No (not by itself)

Would you like a code example showing exploding gradients in a deep ReLU net and how He initialization can fix it?

using example and derivation explain why it does not prevent exploding gradient

Absolutely — let's walk through **why ReLU doesn't prevent exploding gradients**, with both:

- 🔬 A **mathematical derivation**
- 🧪 A **numerical example**

🧠 Concept Recap: Exploding Gradients

In deep neural networks, during backpropagation, gradients are propagated **back through multiple layers** by **chain rule**:

$$\frac{dL}{dw} = \frac{dL}{da} \cdot \frac{da}{dz} \cdot \frac{dz}{dw}$$

In a deep network:

$$\frac{dL}{dx} = \prod_{l=1}^{L} \frac{da^{(l)}}{dz^{(l)}} \cdot \frac{dz^{(l)}}{da^{(l-1)}}$$

If each partial derivative is > 1 in magnitude, then their **product explodes**.

🧪 Numerical Example with ReLU

Let's build a 3-layer network with ReLU activation and **no biases** for simplicity.

Forward Pass

Let's define:

- Input $x = 1$
- Weights: $w^{(1)} = w^{(2)} = w^{(3)} = 5$
- Each layer:
  $z^{(l)} = w^{(l)} \cdot a^{(l-1)}$
  $a^{(l)} = \mathrm{ReLU}(z^{(l)})$

Layer 1

$$z^{(1)} = 5 \cdot 1 = 5, \quad a^{(1)} = \mathrm{ReLU}(5) = 5$$

Layer 2

$$z^{(2)} = 5 \cdot 5 = 25, \quad a^{(2)} = \mathrm{ReLU}(25) = 25$$

Layer 3

$$z^{(3)} = 5 \cdot 25 = 125, \quad a^{(3)} = \mathrm{ReLU}(125) = 125$$

Loss (mean squared error to target $y = 0$):

$$L = \frac{1}{2}(a^{(3)} - y)^2 = \frac{1}{2}(125)^2 = 7812.5$$

🔁 Backpropagation

We want $\frac{dL}{dw^{(1)}}$, so use chain rule:

$$\frac{dL}{dw^{(1)}} = \frac{dL}{da^{(3)}} \cdot \frac{da^{(3)}}{dz^{(3)}} \cdot \frac{dz^{(3)}}{da^{(2)}} \cdot \frac{da^{(2)}}{dz^{(2)}} \cdot \frac{dz^{(2)}}{da^{(1)}} \cdot \frac{da^{(1)}}{dz^{(1)}} \cdot \frac{dz^{(1)}}{dw^{(1)}}$$

Break this down:

1. **Output Layer:**

$$\frac{dL}{da^{(3)}} = a^{(3)} - y = 125$$

$$\frac{da^{(3)}}{dz^{(3)}} = 1 \quad (\text{ReLU' since } z^{(3)} > 0)$$

$$\frac{dz^{(3)}}{da^{(2)}} = w^{(3)} = 5$$

2. **Hidden Layer 2:**

$$\frac{da^{(2)}}{dz^{(2)}} = 1 \quad (\text{ReLU}')$$

$$\frac{dz^{(2)}}{da^{(1)}} = w^{(2)} = 5$$

3. **Hidden Layer 1:**

$$\frac{da^{(1)}}{dz^{(1)}} = 1 \quad (\text{ReLU}')$$

$$\frac{dz^{(1)}}{dw^{(1)}} = x = 1$$

Putting all together:

$$\frac{dL}{dw^{(1)}} = 125 \cdot 1 \cdot 5 \cdot 1 \cdot 5 \cdot 1 \cdot 1 = \boxed{3125}$$

🎯 The gradient for $w^{(1)}$ is **3125**, which is **huge**.

🧮 Let’s Generalize This

Assume:

- Each weight $w = k$
- Each ReLU passes gradient = 1 (since $z > 0$)
- Network has L layers

Then the gradient becomes:

$$\frac{dL}{dw^{(1)}} = \text{Initial Loss Gradient} \cdot k^{L-1}$$

So for:

- $k = 5$, $L = 10$: gradient $\propto 5^9 = 1,953,125$ ✅ **Explodes**

- $k = 1$: gradient stays stable
- $k = 0.5$: gradient vanishes

⚠️ Why ReLU Doesn't Fix Exploding Gradient

✖️ What ReLU does not do:

- It does **not limit the magnitude** of forward activations or gradients.
- If weight magnitudes are large, forward pass activations **amplify** → so do gradients.

✅ What ReLU does do:

- Keeps gradient alive for $z > 0$ (helps with vanishing gradients).
- Efficient due to sparse activations (0s for negatives).

🛠️ Real Fix: Proper Initialization

To avoid exploding gradients in ReLU nets:

> **Use He Initialization:**

$$w \sim \mathcal{N}\left(0, \sqrt{\frac{2}{n_{\text{in}}}}\right)$$

Why? Because it keeps the **variance of activations and gradients** stable across layers.

✅ Summary Table

Concept	ReLU Helps?	Why?
Vanishing Gradient	✅ Yes	Derivative is 1 for $x > 0$
Exploding Gradient	❌ No	No control over magnitude of activations
Needs Proper Initialization	✅ Yes	Use **He Initialization**
Needs Gradient Clipping	✅ Sometimes	In very deep or recurrent nets

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Would you like to visualize exploding vs stable gradients with a deep ReLU net in code (e.g., PyTorch or Keras)?

i did not understand the role of he initialisation

Perfect — let's break down **He initialization** from first principles and show **why** it prevents exploding or vanishing gradients **mathematically and intuitively** using ReLU networks.

🧠 What is the Core Problem?

In deep networks, especially with many layers, two things can go wrong during **forward** and **backward** passes:

- 🔻 **Vanishing gradients**: gradients shrink → weights stop updating
- 🔺 **Exploding gradients**: gradients grow exponentially → training becomes unstable

The root cause: improper scaling of weight initialization → causes activations and gradients to either blow up or vanish across layers.

🧮 Let's Derive the Problem

Assume:

- We have a network with **ReLU activations**
- Each layer:

$$z^{(l)} = w^{(l)} \cdot a^{(l-1)} \quad , \quad a^{(l)} = \mathrm{ReLU}(z^{(l)})$$

- The inputs $x \sim \mathcal{N}(0,1)$
- Weights $w \sim \mathcal{N}(0,\sigma^2)$

❓ What Happens to the Variance of Activations?

Let's track how the **variance** of activations changes as we move from layer to layer.

Suppose:

- $a^{(l-1)}$ has variance $\mathrm{Var}[a^{(l-1)}] = v$
- We want to compute $\mathrm{Var}[a^{(l)}]$

Since:

$$z^{(l)} = \sum_{i=1}^{n} w_i a_i^{(l-1)}$$

Then:

$$\mathrm{Var}[z^{(l)}] = n \cdot \mathrm{Var}[w] \cdot \mathrm{Var}[a^{(l-1)}]$$

Now, ReLU activation is applied:

$$a^{(l)} = \mathrm{ReLU}(z^{(l)})$$

And for a Gaussian input $z \sim \mathcal{N}(0, \sigma^2)$, ReLU zeroes out half the values → so:

$$\mathrm{Var}[\mathrm{ReLU}(z)] = \frac{1}{2}\mathrm{Var}[z] = \frac{1}{2}n \cdot \mathrm{Var}[w] \cdot \mathrm{Var}[a^{(l-1)}]$$

🔁 **This recursion happens layer-by-layer!**

🚨 If Initialization is Wrong

If $\mathrm{Var}[w]$ is too small:

- Variance of activations **shrinks exponentially** → 🔻 vanishing gradients

If $\mathrm{Var}[w]$ is too large:

- Variance of activations **explodes** → 🔺 exploding gradients

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✅ He Initialization to the Rescue

🎯 Goal:

Keep the **variance of activations constant** across layers
→ i.e. prevent both vanishing and exploding

To keep:

$$\mathrm{Var}[a^{(l)}] \approx \mathrm{Var}[a^{(l-1)}]$$

Set:

$$\frac{1}{2}n \cdot \mathrm{Var}[w] = 1 \Rightarrow \mathrm{Var}[w] = \frac{2}{n}$$

✅ Therefore:

He initialization:

$$w \sim \mathcal{N}\left(0, \frac{2}{n_{\text{in}}}\right)$$

Where:

- n_{in} = number of input units to the layer

This keeps:

- **Forward activations** stable
- **Backward gradients** stable (through chain rule)
- Works **specifically well for ReLU**

📊 Contrast With Other Initializations

Initialization	Formula	Works Well With
Xavier/Glorot	$\mathcal{N}(0, \frac{1}{n_{\text{in}}+n_{\text{out}}})$	Tanh, Sigmoid
He Initialization	$\mathcal{N}(0, \frac{2}{n_{\text{in}}})$	ReLU, LeakyReLU

🧪 Simple Code Demo (PyTorch)

```python
import torch.nn as nn

# He Initialization for ReLU network
def init_weights_he(m):
    if isinstance(m, nn.Linear):
        nn.init.kaiming_normal_(m.weight, nonlinearity='relu')

model = nn.Sequential(
    nn.Linear(256, 128),
    nn.ReLU(),
    nn.Linear(128, 64),
    nn.ReLU(),
    nn.Linear(64, 10)
)

model.apply(init_weights_he)
```

✅ Summary

Concept	Explanation
Why exploding/vanishing happens	Multiplicative gradient chain across layers
Role of He Initialization	Sets weight variance to keep activations' variance constant
Why it's ideal for ReLU	ReLU zeros out half activations → needs compensation via larger init variance (×2)
Variance rule	$\mathrm{Var}[w] = \frac{2}{n_{\text{in}}}$

Would you like a Colab-ready notebook to **visualize how variance explodes or stabilizes across layers** with and without He init?