

## Group Assignment

**Important:** Before you start, read the instructions

## 1 Problem statement

You need to design a control system for the pitch of an aircraft. The system is illustrated in Figure 1. The manipulated input is the elevator deflection angle,  $\delta$ ; changing the deflection rate affects the pitch of the aircraft. In Figure 1  $\alpha$  denotes the *angle of attack*, which is the angle between the longitudinal axis of the aircraft and its velocity vector. The pitch angle of the aircraft is denoted by  $\theta$ . The pitch rate is denoted by  $r$ .

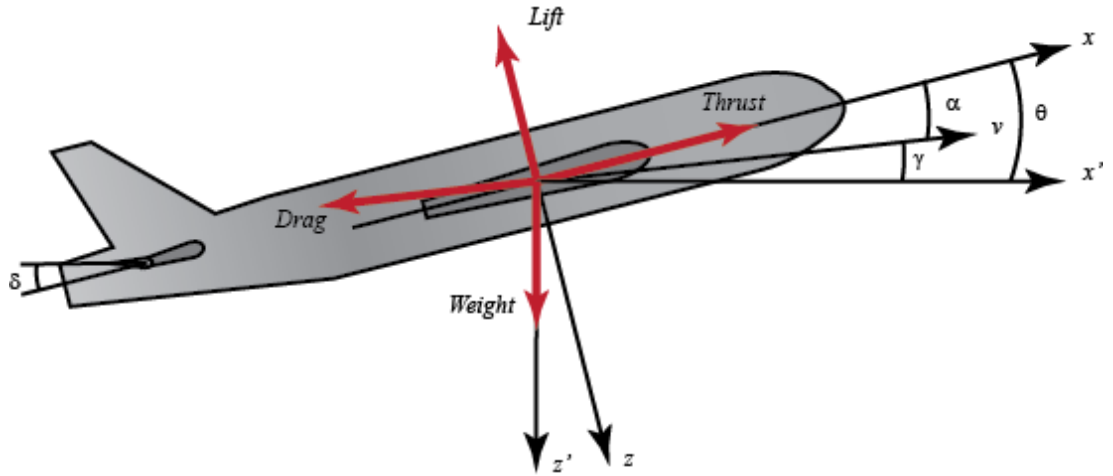


Figure 1: Illustration of an aircraft with its pitch angle,  $\theta$ , the angle of attack,  $\alpha$ , and the deflection angle of the elevators,  $\delta$ . The angle  $\gamma = \theta - \alpha$  is known as the *slope*.

The system dynamics is described by the following system of differential equations

$$\dot{\alpha} = -0.31\alpha + 56.7r + 0.231\delta, \quad (1a)$$

$$\dot{r} = -0.014\alpha - 0.426r + 0.0203\delta, \quad (1b)$$

$$\dot{\theta} = 56.7r, \quad (1c)$$

where all quantities are in SI units<sup>1</sup>. In addition, the deflection angle of the elevators is controlled by an actuator whose dynamics has been found to be of the first order with unit static gain and a time constant of 10 ms. The sensor has the transfer function  $G_m(s) = \exp(-0.005s)$ , i.e., the sensor introduces a delay of 5 ms.

<sup>1</sup>In Equation (1),  $\alpha$ ,  $\theta$  and,  $\delta$  are measured in radians — not degrees — and  $q$  is measured in radians per second.

The task of your team is to:

1. **Present a complete analysis of the given dynamics:** Your report should include a detailed study of the properties of the open-loop dynamics and your results should be accompanied by appropriate plots.
2. **Design an appropriate controller for the system:** Your controller needs to satisfy the standard requirements for a closed-loop control system. You need to make sure that the controlled system is BIBO-stable and that there is zero offset. Additionally, you may want to check how your closed-loop system will behave in presence of disturbances. You need to consider what sort of possible disturbances may be present. You should tune your controller to achieve reasonable stability margins and of course to exhibit a desirable (not highly oscillatory, not overly slow, not too aggressive) closed-loop behaviour.

You may use any part of the theory from the textbook. You can also use Python for your calculations and for plotting, but you will need to include a link to your code.

Here is a **hint** to get you started: Assuming zero initial conditions, apply the Laplace transform to each of the equations in (1). Then determine the transfer function of the system with the input being deflection angle of the elevators and the output being the pitch angle.

## 2 Marking

Your report will be marked for technical correctness and completeness of the proposed approach, justification, clarity, quality of presentation; this will account for 90% of the marks of your report. Your report must include an appendix or section entitled “project management and individual contributions” where you should provide evidence on (i) how you collaborated (what collaboration tools did you use? For example, did you use git/GitHub?) (ii) how you managed the work in this project? How were the tasks distributed? Did you use an issue tracker? (iii) any additional information project management you would like to include (e.g., planning), (iv) what were the contributions of each member of the team?

As mentioned in the previous paragraph, your report will be marked for its quality of presentation. To get a good mark, I would encourage you go through the slides on “how to write a good technical report.”

Marks will only be awarded based on the **evidence** you will provide. Examples of appropriate evidence can be meeting minutes, or a link to a GitHub repository, or other similar online collaborative development space. This will account for 10% of the marks of your report.