Slide #6:

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Inverse of a Matrix:

if

$AB = BA = I$ (Identity Matrix)

then A is said to be invertible and B is called inverse of A.

Note: A Must be square matrix and $|A| \neq 0$.

Exp:

$$A = \begin{bmatrix} -1 & 2 \\ 3 & -6 \end{bmatrix}$$

$|A| = 6 - 6 = 0$

$|A| = 0$ so, its not invertible.

* $(AB)^{-1} = B^{-1}A^{-1}$
* $(A^T)^{-1} = (A^{-1})^T$ (Transpose)

* Inversion Algorithm

Example:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

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$$R \quad \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} R_2 - 2R_1 \\ R_3 - 1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & +1 & +5 & -2 & -1 \end{array}\right] \begin{array}{l} R_3 + 2R_2 \\ R_1 - 2R_2 \\ R_3 \times (-1) \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right] \begin{array}{l} R_1 - 9R_3 \\ R_2 + 3R_3 \end{array}$$

$$A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}$$

Inverse of a Matrix:

* Inversion Algorithm.

(iv) $\begin{bmatrix} -1 & 3 & -4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} -1 & 3 & -4 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

$$\underset{R}{\sim}\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

$$\underset{R}{\sim}\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & 1 & 0 & 0 \\ 0 & 10 & -7 & 2 & 1 & 0 \\ 0 & -10 & 7 & -4 & 0 & 1 \end{array}\right] \begin{array}{l} R_2+(-2)R_1 \\ R_3+4R_1 \end{array}$$

$$\underset{R}{\sim}\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 10 & -7 & 2 & 1 & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 \end{array}\right] R_3+R_2$$

$\Rightarrow$ Inverse do not exist.

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Practice Questions.

Question #2: Use Inverse Algorithm to find inverse of matrices:

$$(i)\quad \begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{bmatrix}$$

$$R, = \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 & 1 & 0 \\ 5 & -2 & -3 & 0 & 0 & 1 \end{array}\right]$$

$$R, = \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & -4/-4 & 4/-4 & -1/-4 & 1/-4 & 0 \\ 0 & -12 & 12 & -5 & 0 & 1 \end{array}\right] \begin{array}{l} R_2 - R_1 \\ R_3 - 5R_1 \\ R_2 \div -4 \end{array}$$

$$R, = \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1/4 & -1/4 & 0 \\ 0 & -12 & 12 & -5 & 0 & 1 \end{array}\right]$$

$$R, = \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1/2 & 1/2 & 0 \\ 0 & 1 & -1 & 1/4 & -1/4 & 0 \\ 0 & 0 & 0 & 7 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 - 2R_2 \\ R_3 + 12R_2 \end{array}$$

Inverse do not exist.

(v) $\begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$

Sol.

$$\left[\begin{array}{ccc|ccc} 2 & 5 & 5 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\underset{R}{\sim}\left[\begin{array}{ccc|ccc} 1 & 4 & 5 & 1 & 1 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 3 & 0 & 0 & 1 \end{array}\right] R_1+R_2$$

$$\underset{R}{\sim}\left[\begin{array}{ccc|ccc} 1 & 4 & 5 & 1 & 1 & 0 \\ 0 & 3 & 5 & 1 & 2 & 0 \\ 0 & -4 & -7 & -2 & -2 & 1 \end{array}\right] \begin{matrix} R_2+R_1 \\ R_3-2R_1 \end{matrix}$$

$$\underset{R}{\sim}\left[\begin{array}{ccc|ccc} 1 & 4 & 5 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & -1 \\ 0 & -4 & -7 & -2 & -2 & 1 \end{array}\right] -R_2-R_3$$

$$\underset{R}{\sim}\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & -1 & -1 & 1 \\ 0 & 1 & 2 & 1 & 0 & -1 \\ 0 & -4 & -7 & -2 & -2 & 1 \end{array}\right] R_1+R_3$$

$$\underset{R}{\sim}\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & -1 & -1 & 1 \\ 0 & 1 & 2 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 & -2 & -3 \end{array}\right] R_3+4R_2$$

$$R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -5 & -5 \\ 0 & 1 & 0 & -3 & 4 & 5 \\ 0 & 0 & 1 & 2 & -2 & -3 \end{array}\right] \begin{array}{l} R_1-2R_3 \\ R_2+2R_3 \end{array}$$

$$A^{-1} = \begin{bmatrix} 3 & -5 & -5 \\ -3 & 4 & 5 \\ 2 & -2 & -3 \end{bmatrix}$$ Ans.

$\Rightarrow$ Rest of Parts are same ::

Question # 3:

Find all values of c, if any, for which given matrix is invertible.

(i) $\begin{bmatrix} c & c & 1 \\ 1 & 1 & c \\ 0 & 1 & c \end{bmatrix}$

Sol:

$$\begin{vmatrix} c & c & 1 \\ 1 & 1 & c \\ 0 & 1 & c \end{vmatrix} \neq 0$$ Expand by row 1

$$c\begin{vmatrix} 1 & c \\ 1 & c \end{vmatrix} - c\begin{vmatrix} 1 & c \\ 0 & c \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \neq 0$$

$$c(0) - c(c) + 1(1) \neq 0$$

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$1 \neq c^2$

$c \neq \pm 1$

(vi) $\begin{bmatrix} c & c & -1 \\ 1 & 1 & 2c \\ 0 & 1 & c \end{bmatrix}$

$\begin{vmatrix} c & c & -1 \\ 1 & 1 & 2c \\ 0 & 1 & c \end{vmatrix} \neq 0$

Expand by column 1:

$c \begin{vmatrix} 1 & 2c \\ 1 & c \end{vmatrix} - 1 \begin{vmatrix} c & -1 \\ 1 & c \end{vmatrix} + 0 \begin{vmatrix} c & -1 \\ 1 & 2c \end{vmatrix} \neq 0$

$c(c-2c) - 1(c^2+1) + 0 \neq 0$

$\cancel{c^2} - 2c^2 - \cancel{c^2} - 1 \neq 0$

$-2c^2 \neq 1$

$c^2 \neq \frac{1}{-2}$

Sine square root of -ve value do no exist

so,

No value of c exists here.

(iii) $\begin{bmatrix} c & c & c \\ 1 & c & c \\ 1 & 1 & c \end{bmatrix}$

$[\quad \begin{vmatrix} c & c & c \\ 1 & c & c \\ 1 & 1 & c \end{vmatrix} \neq 0$

Expand by row 1

$$c\begin{vmatrix} c & c \\ 1 & c \end{vmatrix} - c\begin{vmatrix} 1 & c \\ 1 & c \end{vmatrix} + c\begin{vmatrix} 1 & c \\ 1 & 1 \end{vmatrix} \neq 0$$

$$c(c^2 - c) - c(c - c) + c(1 - c) \neq 0$$

$$c^3 - c^2 - 0 + c - c^2 \neq 0$$

$$c^3 - 2c^2 + c \neq 0$$

$$c(c^2 - 2c + 1) \neq 0$$

$\boxed{c \neq 0}$, $c^2 - 2c + 1 \neq 0$

, $(c - 1)^2 \neq 0$

, $\sqrt{(c-1)^2} \neq \sqrt{0}$

, $c - 1 \neq 0$

$\boxed{c \neq 0}$, $\boxed{c \neq 1}$ ✓

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Question # 4. Find det A & det B using reduced matrix.

$$A = \begin{bmatrix} -3 & 0 & 7 \\ 2 & 5 & 1 \\ -1 & 0 & 5 \end{bmatrix}$$

$$\det A = \begin{vmatrix} -3 & 0 & 7 \\ 2 & 5 & 1 \\ -1 & 0 & 5 \end{vmatrix}$$

Expand by row 1

$$-3\begin{vmatrix} 5 & 1 \\ 0 & 5 \end{vmatrix} - 0\begin{vmatrix} 2 & 1 \\ -1 & 5 \end{vmatrix} + 7\begin{vmatrix} 2 & 5 \\ -1 & 0 \end{vmatrix}$$

$$= -3(25) - 0 + 7(5)$$

$\det A = -40$ Ans.

$$B = \begin{vmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{vmatrix}$$

Expand by column 3.

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$$= 0\begin{vmatrix} 2 & 2 & -2 \\ 4 & 1 & 0 \\ 2 & 10 & 2 \end{vmatrix} + 0\begin{vmatrix} 3 & 3 & 5 \\ 4 & 1 & 0 \\ 2 & 10 & 2 \end{vmatrix} + (-3)\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix}$$

$$+ 3\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix}$$

$$= 0 + 0 - 3\left[3\begin{vmatrix} 2 & -2 \\ 10 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & -2 \\ 2 & 2 \end{vmatrix} + 5\begin{vmatrix} 2 & 2 \\ 2 & 10 \end{vmatrix}\right]$$

$$+ 3\left[3\begin{vmatrix} 2 & -2 \\ 1 & 0 \end{vmatrix} - 3\begin{vmatrix} 2 & -2 \\ 4 & 0 \end{vmatrix} + 5\begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix}\right]$$

$$= -3(24 - 24 + 80) + 3(6 - 24 - 30)$$

$$= -240 - 144$$

$$\det B = -384 \quad \text{Ans.}$$

Question #5:

if $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -6$ then $\begin{bmatrix} 3a & 3b & 3c \\ -d & -e & -f \\ 4g & 4h & 4i \end{bmatrix} = ?$

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Sol

$$= \begin{vmatrix} 3a & 3b & 3c \\ -d & -e & -f \\ 4g & 4h & 4i \end{vmatrix}$$

Taking 3 as common from row 1.

$$= 3\begin{vmatrix} a & b & c \\ -d & -e & -f \\ 4g & 4h & 4i \end{vmatrix}$$

Taking -1 as common from row 2.

$$= 3.(-1)\begin{vmatrix} a & b & c \\ d & e & f \\ 4g & 4h & 4i \end{vmatrix}$$

Taking 4 as common from row 3.

$$= -3.(4)\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= -12\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$ Putting the value

$$= -12(-6) = 72$$ **Ans.**