

⇒ Matrix Transformation

$$T: \mathbb{R}^n \rightarrow \mathbb{R}^m$$

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$T(\mathbb{R}^2) \rightarrow \mathbb{R}^2$$

$$T(\vec{x}) = A\vec{x}$$

$$\begin{array}{ccc} \mathbb{R}^n & & \mathbb{R}^m \\ \downarrow & & \downarrow \\ (\vec{x}) & & (T(\vec{x})) \end{array}$$

$$\begin{aligned} y &= T(\vec{x}) = A\vec{x} \\ y &= A\vec{x} \end{aligned}$$

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⇒ Translation

⇒ Formula

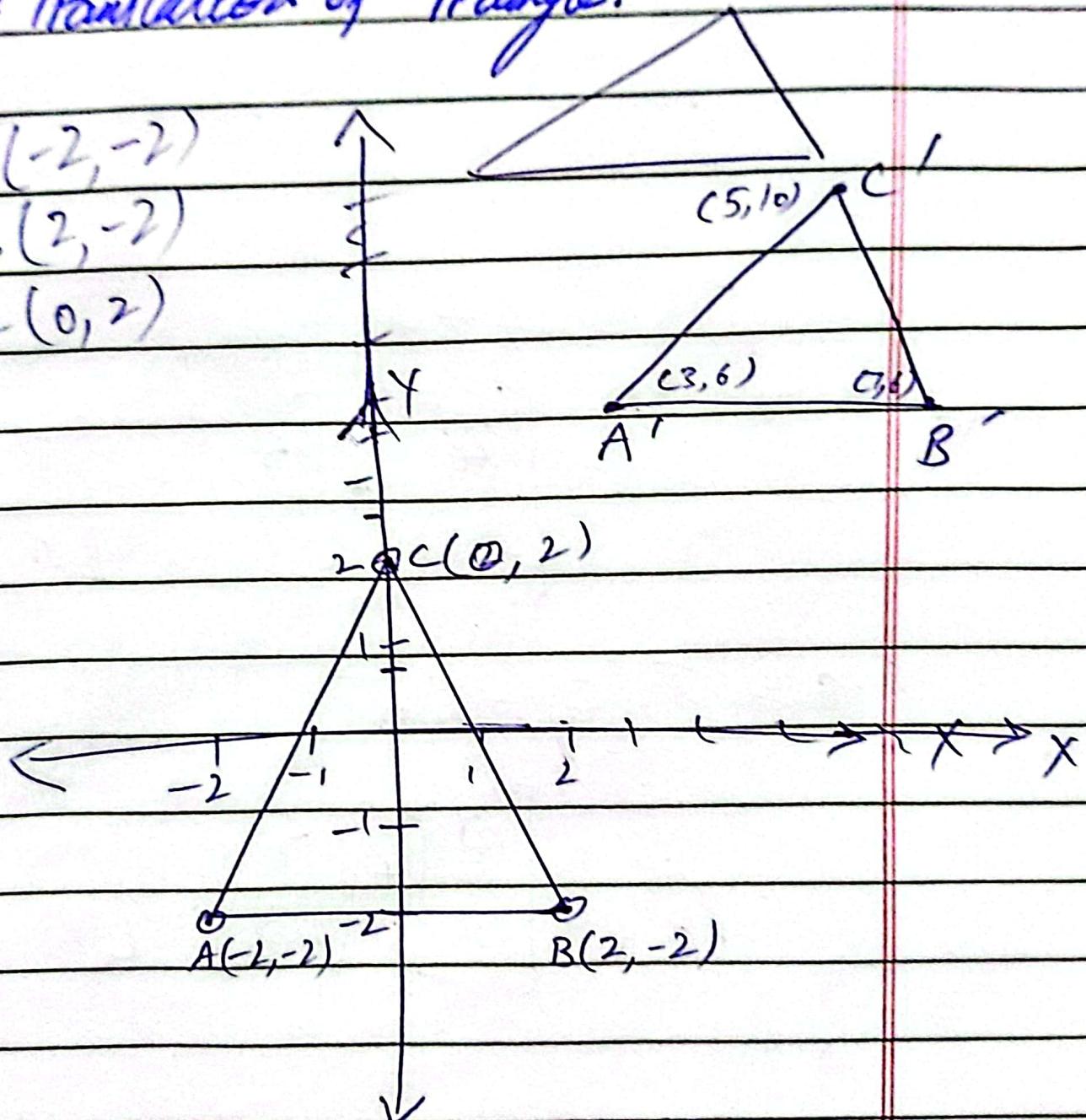
$$T(\vec{x}) = \vec{x} + \vec{a}$$

* Translation of Triangle:

$$A(-2, -2)$$

$$B(2, -2)$$

$$C(0, 2)$$



$$A' = T(A) = A + \vec{a}$$

Date _____

DATE: _____

$$= \begin{bmatrix} -2 \\ -2 \end{bmatrix} + \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$\vec{B}' = T(\vec{B}) = \vec{B} + \vec{a}$$

$$= \begin{bmatrix} 2 \\ -2 \end{bmatrix} + \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

$$\vec{C}' = T(\vec{C}) = \vec{C} + \vec{a}$$

$$= \begin{bmatrix} 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

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Transformation:

(Translation of Line & Circle)

$$T(\vec{x}) = \vec{x} + \vec{a}$$

$$3x - 4y = 2 \quad \text{--- (i)} \quad \vec{a} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$3x - 4y = 2$$

Put $x=1$

$$3(1) - 4y = 2$$

$$3 - 2 = 4y$$

$$y = \frac{1}{4}$$

$$(1, \frac{1}{4})$$

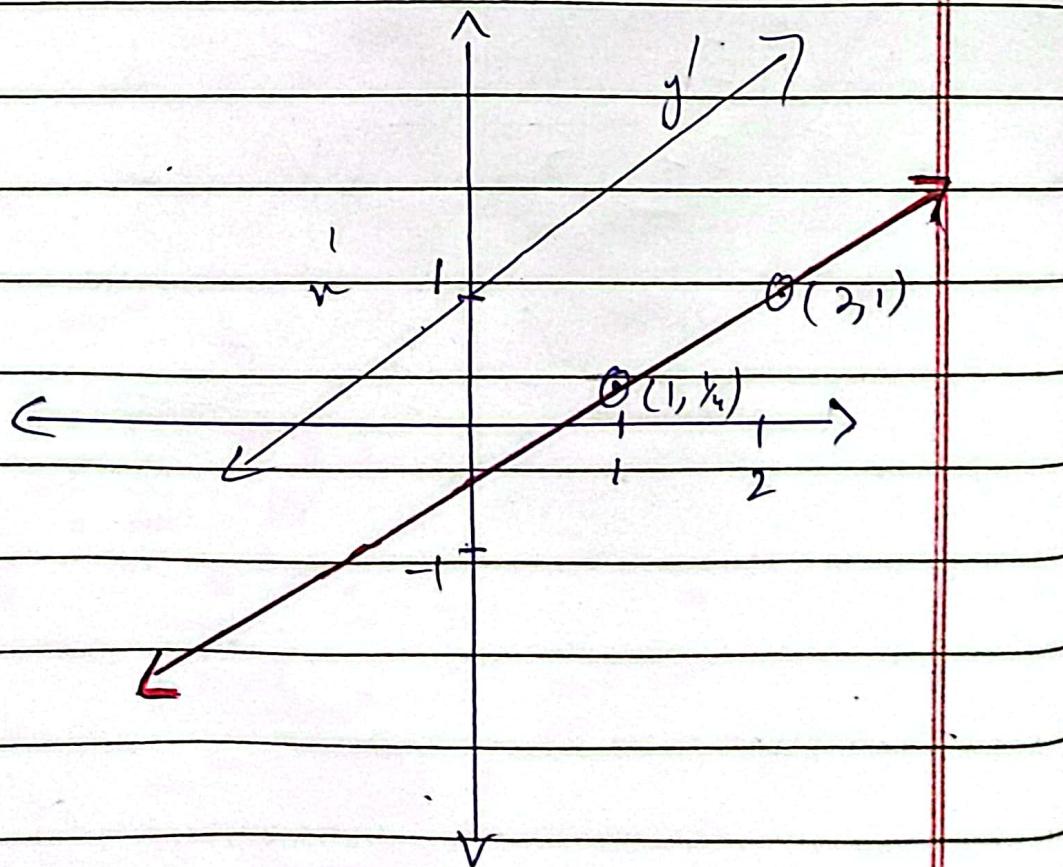
$$3x - 4y = 2$$

Put $x=2$

$$3(2) - 4y = 2$$

$$6 - 2 = 4y$$

$$(2, 1)$$



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$$\vec{a} = 2, 3$$

$$T(\vec{a}) = \vec{x} + \vec{a}$$

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x+2 \\ y+3 \end{bmatrix}$$

$$x = x' - 2$$

$$y = y' - 3$$

Put in eqn (i)

$$3(x' - 2) - 4(y' - 3) = 2$$

$$3x' - 6 - 4y' + 12 = 2$$

$$3x' - 4y' + 6 = 2 = 0$$

$$3x' - 4y' = -4$$

Put $x' = 1$

$$3(1) - 4y = -4$$

$$y = \frac{7}{4}$$

$$(1, \frac{7}{4})$$

Put $x' = 2$

$$3(2) - 4y = -4$$

$$x = \frac{5}{2}$$

$$(2, \frac{5}{2})$$

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Question:

$$y = 2n + 1 \quad \dots \text{(i)}$$

~~Set~~
 $\therefore n = 2, 3$

Put $n = 1$

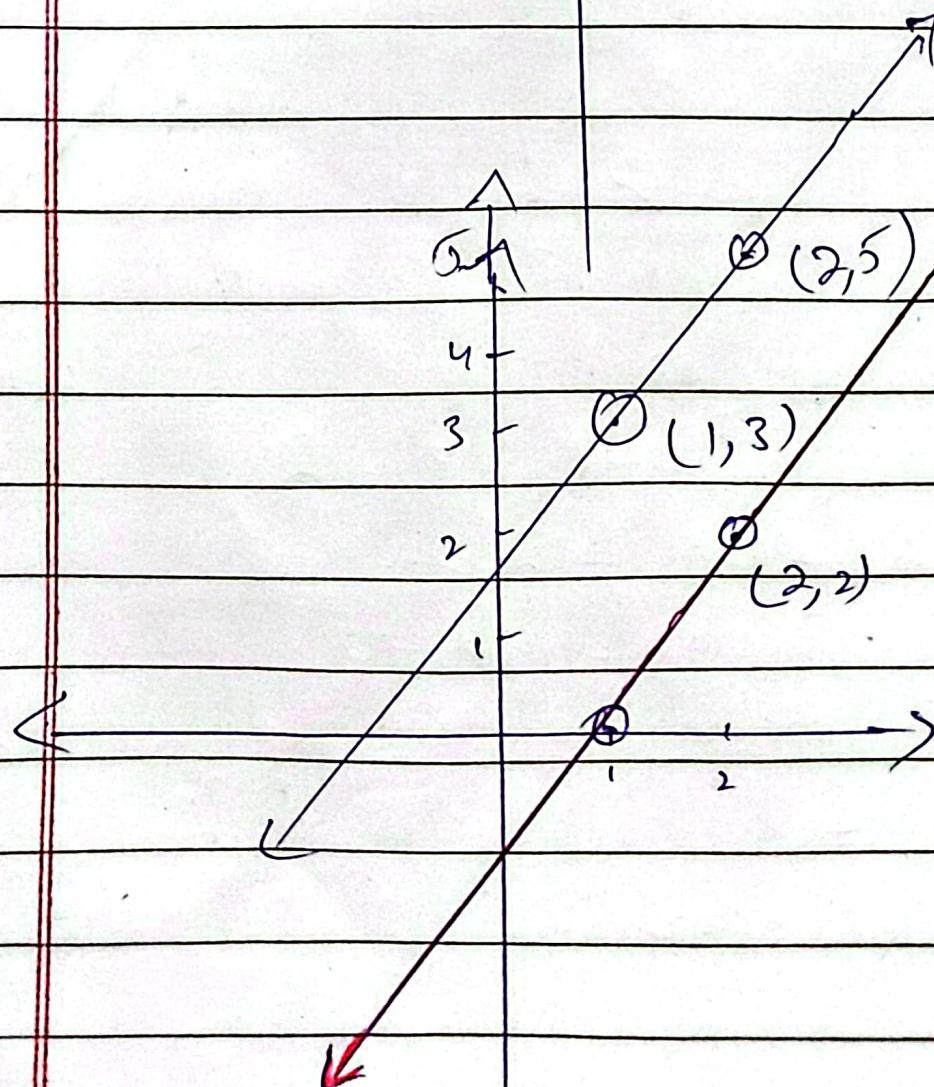
$$\begin{aligned} y &= 2(1) + 1 \\ &= 3 \end{aligned}$$

$$(1, 3)$$

Put $n = 2$

$$\begin{aligned} &= 2(2) + 1 \\ &= 5 \end{aligned}$$

$$(2, 5)$$



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$$T(\vec{a}) = \vec{n} + \vec{a}$$

$$T \begin{bmatrix} n \\ y \end{bmatrix} = \begin{bmatrix} n \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} n' \\ y' \end{bmatrix} = \begin{bmatrix} n+2 \\ y+3 \end{bmatrix}$$

$$n' = -2 + n'$$

$$y' = -3 + y$$

Put in (i)

$$(3+y) = 2(-2+n') + 1$$

~~$y = 2$~~

~~$-3+y' = -4 + 2n' + 1$~~

~~$-6 = 2n' + y'$~~

~~$-2 = -2n' + y'$~~

$y' = 2n' + p$

~~$2n' - y' = 2$~~

~~$y' = 2n' - 2$~~

$y' = 2(1) + p$

~~$y' = 3$~~

$y' = 2(2) + p$

~~$= 4 + p = 3$~~

(2, 5)

(1, 3)

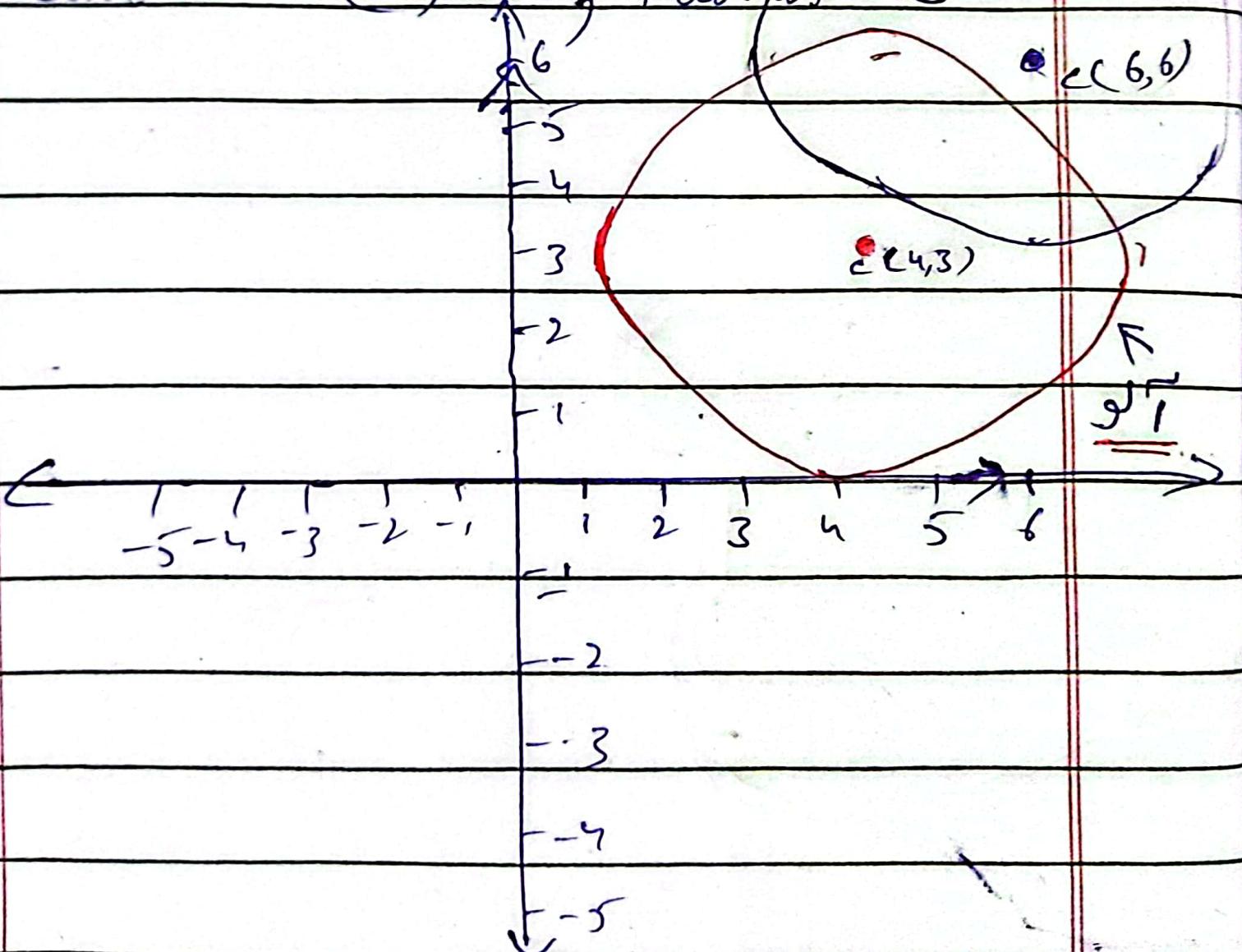
Translation of circle:

Ex 3: $\therefore (x-h)^2 + (y-k)^2 = r^2$

$$(x-4)^2 + (y-3)^2 = 9 \quad \text{--- (i)}$$

Centre ✓ vector (3,3) -

Centre = (4,3), radius = 3 -



$$T(\vec{v}) = \vec{v} + \vec{a}$$

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x+2 \\ y+3 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x+2 \\ y+3 \end{bmatrix}$$

$$x = x' - 2$$

$$y = y' - 3 \quad \text{Put in } \textcircled{i}$$

$$(x' - 2 - 4)^2 + (y' - 3 - 3)^2 = 9$$

$$(x' - 6)^2 + (y' - 6)^2 = 9$$

Centre (6, 6), r = 3

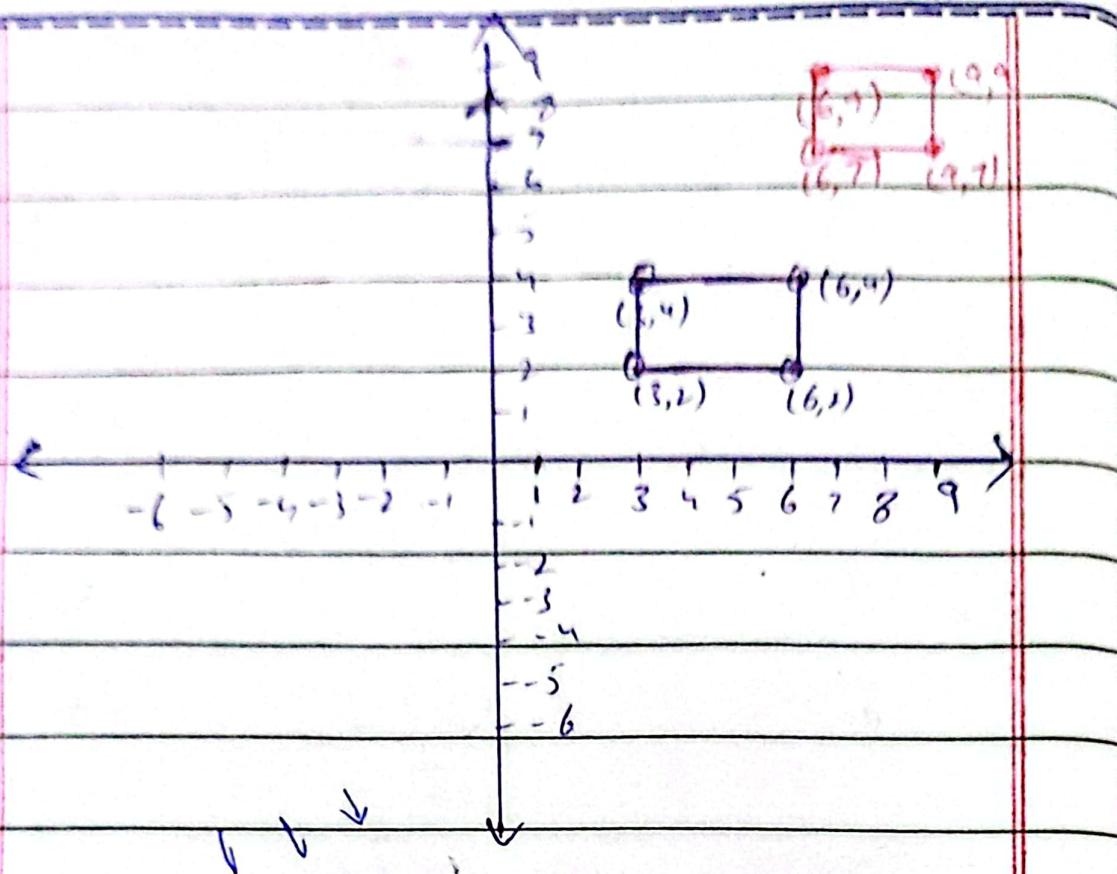


Q #2: Let A = (3, 4), B = (3, 2), C = (6, 2)

Let D(6, 4) form a rectangle.
Find translation through vector (3, 5).

Date _____

Date _____



$$T(\vec{u}) = \vec{u} + \vec{a}$$

$$\begin{aligned} A' &= T(A) = A + \vec{a} \\ &= \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} \end{aligned}$$

$$T(A) = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$

$$\begin{aligned} B' &= T(B) = B + \vec{a} \\ &= \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} C' &= T(C) = C + \vec{a} \\ &= \begin{bmatrix} 6 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \end{bmatrix} \end{aligned}$$

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$$D' = T(D) = D + \vec{a}$$

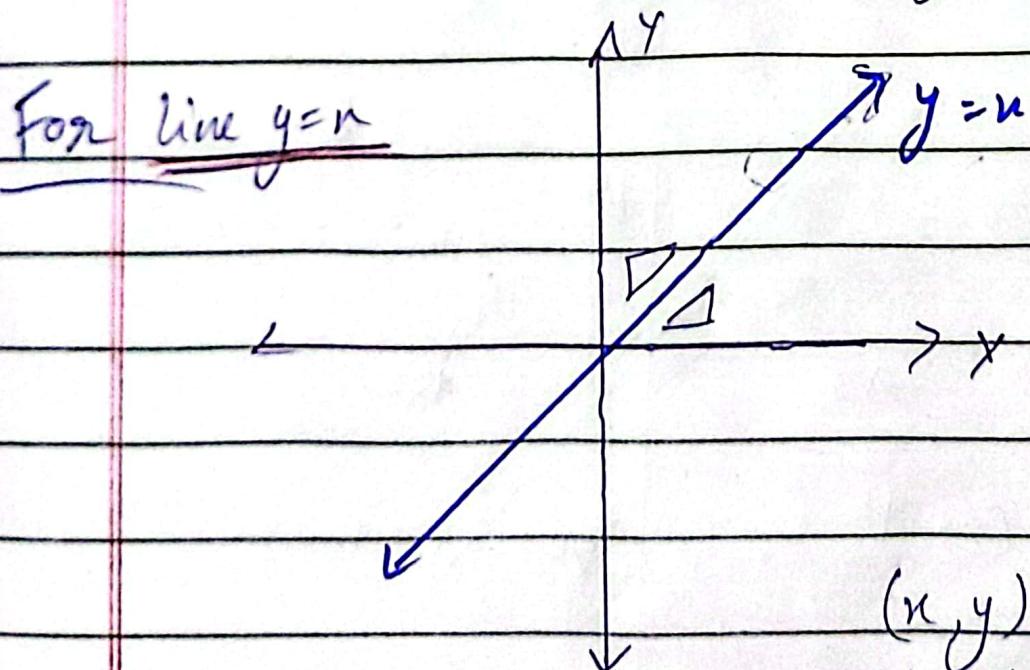
$$= \begin{bmatrix} 6 \\ 4 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} - \begin{bmatrix} 9 \\ 9 \end{bmatrix}$$

Euclidean Transformations.

1. Translation.
- \Rightarrow 2-Reflection.
- 3 - Rotation.

\Rightarrow Reflection.

- (i)- Reflection about y-axis ✓
- (ii) - Reflection about x-axis ↗
- (iii) Reflection about line $y = n$
- (iv) Reflection about line $y = -n$



$$(n, y) \Rightarrow (y, n)$$

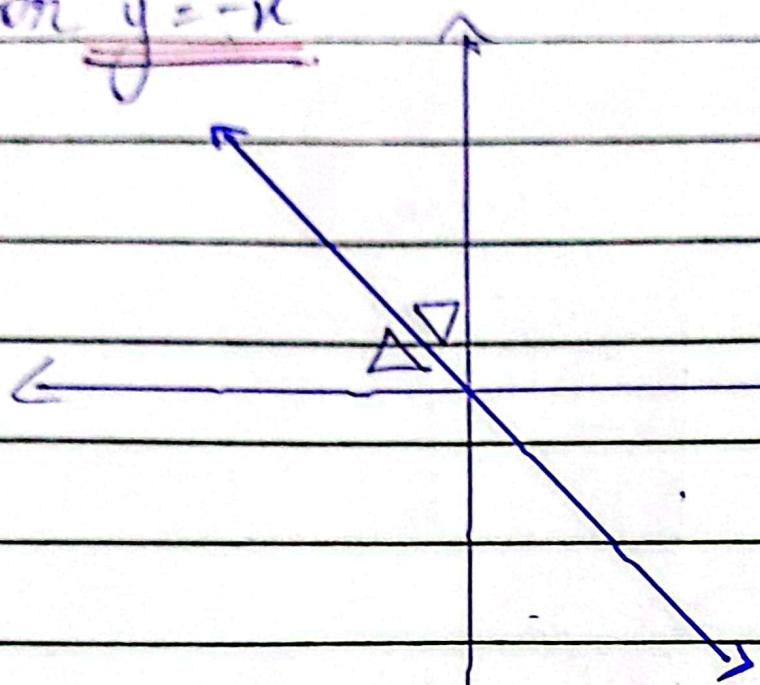
$$(1, 0) \Rightarrow (0, 1)$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (0, 1) \Rightarrow (1, 0)$$

$$J = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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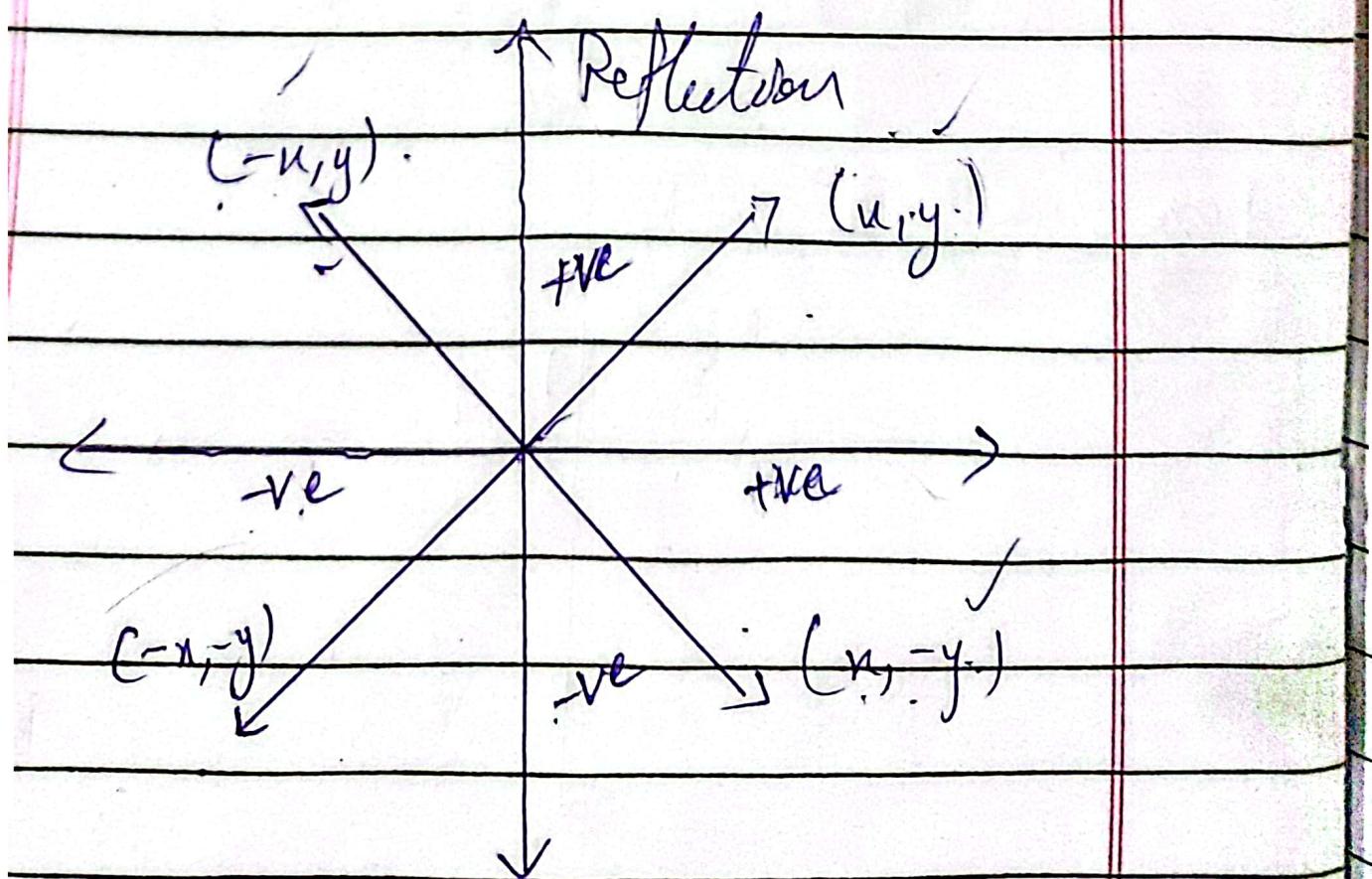
For $y = -x$ 

$$(x, y) \Rightarrow (-y, -x)$$

$$(1, 0) \Rightarrow (0, -1)$$

$$(0, 1) \Rightarrow (-1, 0)$$

$$A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$



DATE: _____

DATE: _____

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$e_1 = (1, 0)$$

$$e_2 = (0, 1)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$e_1 = (1, 0, 0)$$

$$e_2 = (0, 1, 0)$$

$$e_3 = (0, 0, 1)$$

About y-axis,

$$(x, y) \Rightarrow (-x, y)$$

$$(1, 0) \Rightarrow (-1, 0)$$

$$(0, 1) \Rightarrow (0, 1)$$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

About x-axis,

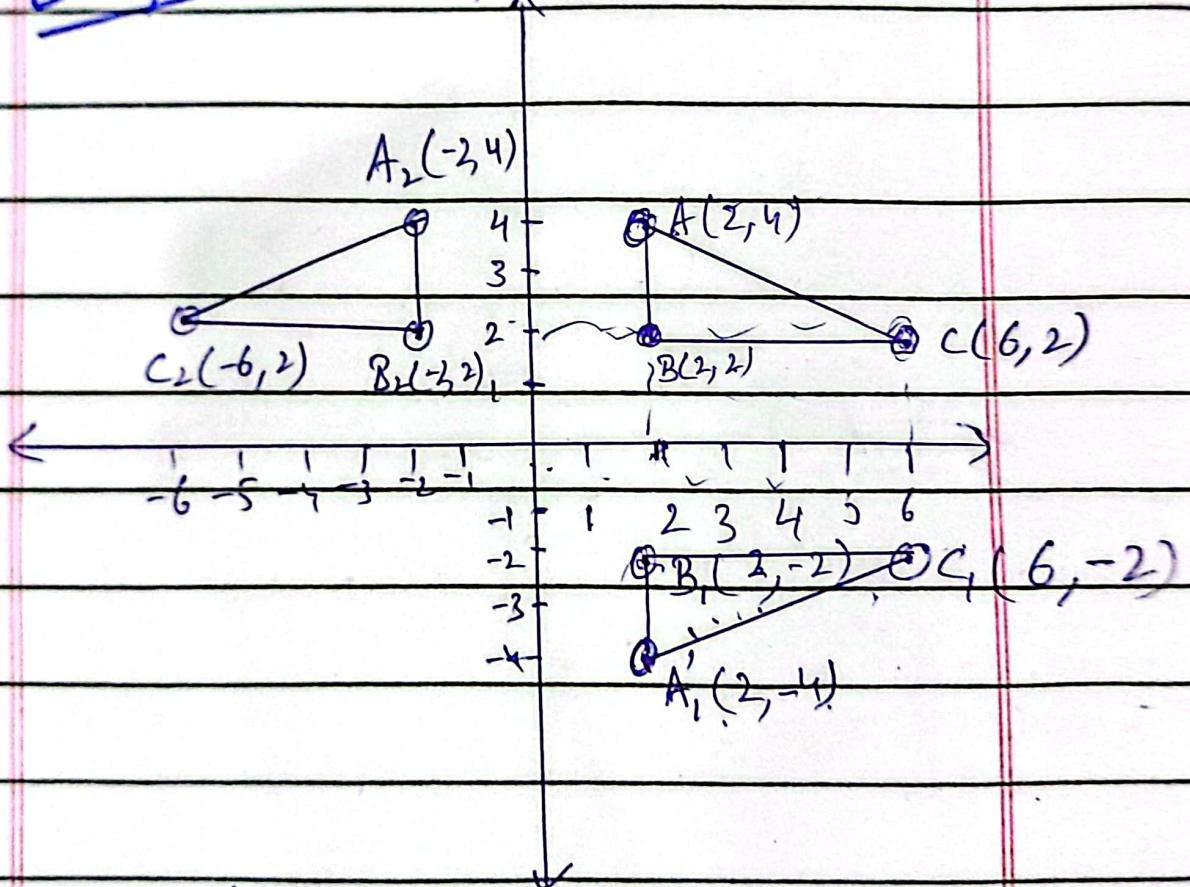
$$(x, y) \Rightarrow (x, -y)$$

$$e_1 = (1, 0) \Rightarrow (1, 0)$$

$$e_2 = (0, 1) \Rightarrow (0, -1)$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Example 1: A (2, 4), B (3, 2), C(6, 2)



→ Reflection About x-axis.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$T(\vec{n}) = A \vec{n}$$

$$A_1 = T \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -4 \end{bmatrix} \quad \text{—}$$

$$B_1 = T \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$C_1 = T \begin{bmatrix} 6 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$

→ Reflection About y-axis.

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{—}$$

$$A_2 = T \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

$$B_2 = T \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$C_2 = T \begin{bmatrix} 6 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$$

$$T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

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\Rightarrow Reflection About Line $y=n$

$$y = 2x + 1 \quad \text{--- (i)}$$

$$\text{Put } n=0$$

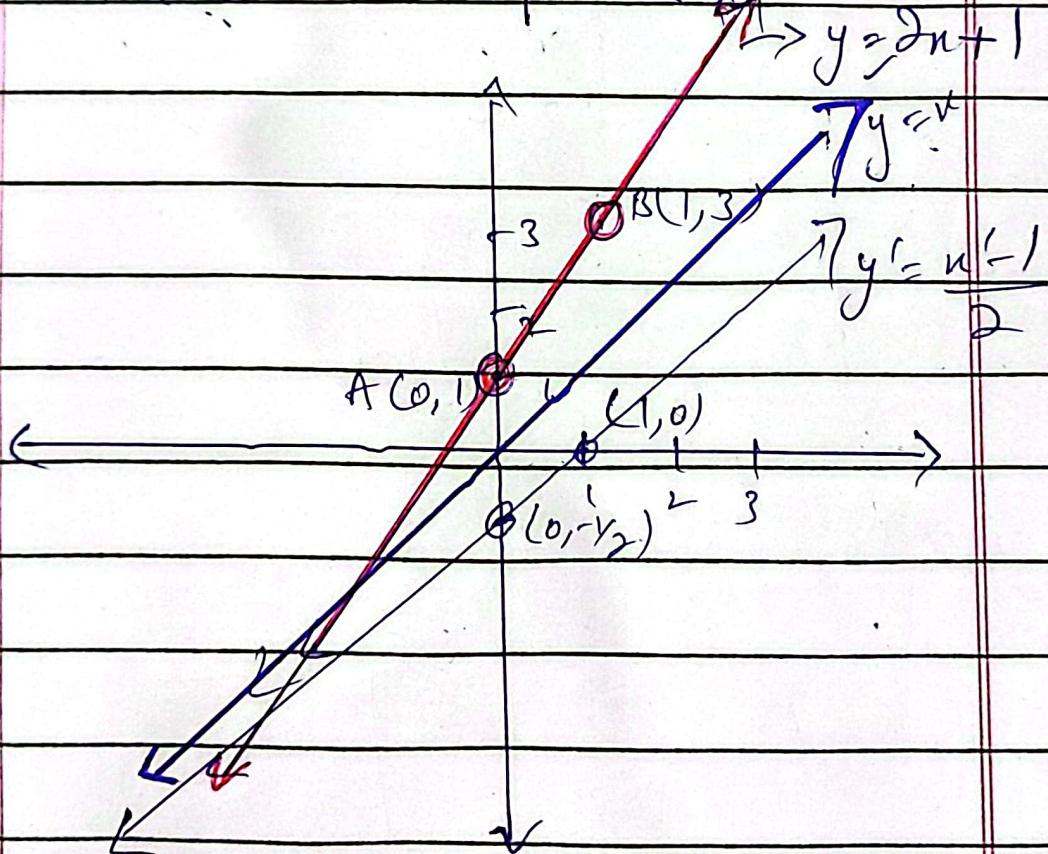
$$y = 2(0) + 1 \\ = 1$$

$$A(0, 1)$$

$$\text{Put } n=1$$

$$y = 2(1) + 1 \\ = 3$$

$$B(1, 3)$$



$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$T(\vec{n}) = A \vec{n}$$

DATE: _____

DATE: _____

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix}$$

$$\begin{aligned} x' &= y \\ y' &= x \end{aligned}$$

$$\begin{aligned} y &= x' \\ x &= y' \end{aligned}$$

Put in (i)

$$\cancel{y} \quad x' = 2y' + 1$$

$$\frac{x'-1}{2} = y' - \textcircled{u}$$

$$\text{Put } x'=0$$

$$\frac{0-1}{2} = y'$$

$$y' = -\frac{1}{2}$$

$$A'(0, -\frac{1}{2})$$

$$\text{Put } x'=1$$

$$\frac{1-1}{2} = y'$$

$$y' = 0$$

$$B'(1, 0)$$

Ques. $T(\vec{r}) = A\vec{x}$ Date: _____

⇒ Reflection of Circle:

General Formula:

$$(x-h)^2 + (y-k)^2 = r^2$$

where $(h, k) \Rightarrow$ centre of Circle

$r \Rightarrow$ radius of circle.

Ex:

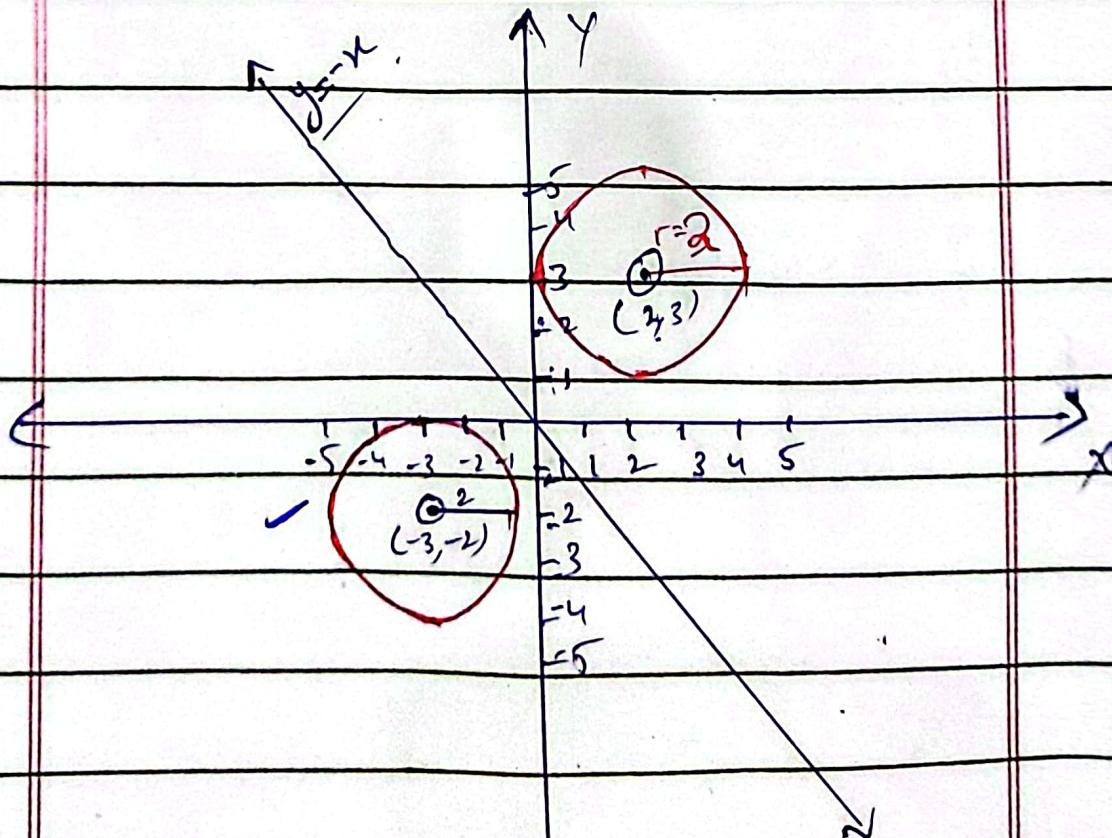
let $(x-2)^2 + (y-3)^2 = 4 \quad \text{--- (i)}$

Find its reflection along line $y = -x$

Sol.

Centre. $(2, 3)$

radius = 2



Date: _____

DATE: _____

$$T(\vec{v}) = A(\vec{v})$$

$$A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$T \begin{bmatrix} u \\ y \end{bmatrix} = A \begin{bmatrix} u \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} u \\ y \end{bmatrix}$$

$$\begin{bmatrix} u' \\ y' \end{bmatrix} = \begin{bmatrix} -y \\ -u \end{bmatrix}$$

$$y = -u', \quad \text{put in ① eqn.}$$

$$x = -y$$

$$(-y+2)^2 + (-u'+3)^2 = 4$$

~~$(-x+3)^2 + (y+2)^2 = 4$~~

$$(-(y'+2))^2 + (-(u'+3))^2 = 4$$

$$(y'+2)^2 + (u'+3)^2 = 4$$

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Centre $(-3, -2)$, Radius = 2.