Saad Mufti

CS 539 - HW 2

Conceptual and Theoretical Questions

Q1

To minimize the error function, take the derivative with respect to $\mathbf{w}$ and set it to 0:

$$0 = -\frac{1}{2}\sum_{n=1}^{N} 2 \cdot r_n \left(t_n - w^T \cdot \phi(x_n)\right)\phi(x_n)^T$$

$$0 = -\sum_{n=1}^{N} r_n t_n \phi(x_n)^T - r_n w \phi(x_n)\phi(x_n)^T$$

$$0 = \sum_{n=1}^{N} r_n t_n \phi(x_n)^T - w\sum_{n=1}^{N} r_n \phi(x_n)\phi(x_n)^T$$

$$w\sum_{n=1}^{N} r_n \phi(x_n)\phi(x_n)^T = \sum_{n=1}^{N} r_n t_n \phi(x_n)^T$$

Take the inverse to isolate w

$$w = \left(\sum_{n=1}^{N} r_n \phi(x_n)\phi(x_n)^T\right)^{-1} \sum_{n=1}^{N} r_n t_n \phi(x_n)^T$$

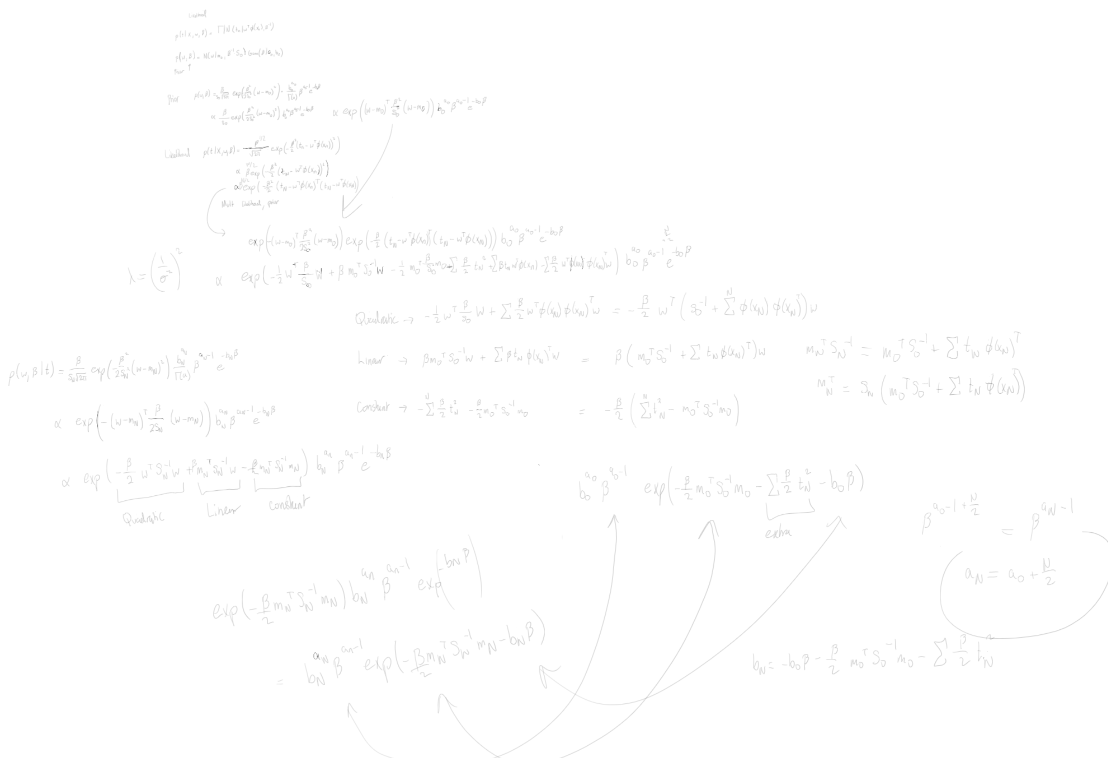
Because r_n is defined as the inverse of the variance of the nth data point, a large r_n indicates a small variance, indicating the high confidence in that data. A small weight would then of course indicate low certainty in that data with a high variance.

Alternatively, the weight r_n can simply be seen as how many times the nth data point has been observed, which is in line with the constraint $r_n > 0$, but is more fitting for integral values of r_n.

Q2

Using Bayes' rule, the relationship between the prior, likelihood, and posterior distributions is known to be:

$$p(\mathbf{w},\beta|\mathbf{t}) \propto p(\mathbf{w},\beta)\cdot p(\mathbf{t}|\mathbf{X},w,\beta)$$



Work is shown above

Q3

If a dataset is lineary separable, then its classes can be exactly distinguished with a linear decision boundary. The probability of a class C_1 in a dataset using logistic regression is

$$p(C_1|\phi) = \sigma(w^T\phi)$$

Because the dataset is linearly separable, data of one class x_a may fall one side of the decision boundary with $w^T\phi(x_a) > 0$, and another class denoted by x_b may be found on the other side $w^T\phi(x_b) < 0$.

If $|w| \to \infty$, then the sigmoid function of the posterior for class a $\sigma(w^T\phi) \to 1$, which thus maximizes the likelihood. This would also be true for finding class b since $\sigma(x) = 1 - \sigma(-x)$

Q4

We have

$$H = \nabla\nabla E(w) = \sum_{n=1}^{N} y_n(1-y_n)\phi_n\phi_n^T = \Phi^T R\Phi \qquad (4.97, \text{Bishop})$$

For a symmetric matrix A to be positive definite, then any real valued vector z can be multiplied by A to get all entries being positive:

$$z^T Az = A_{ij} > 0$$

Begin by multiplying the double gradient by z as above to prove it is positive definite:

$$z^T \sum_{n=1}^{N} (y_n(1-y_n)\phi_n\phi_n^T)z$$

$$\sum_{n=1}^{N} y_n(1-y_n)z^T\phi_n\phi_n^T z$$

Notice that $z^T\phi_n = (\phi_n^T z)^T$, which is also a dot product (scalar quantity), which allows us to square that term, making it always greater than or equal to zero:

$$\sum_{n=1}^{N} y_n(1-y_n)(\phi_n^T z)^2$$

Now, because $1 > y_n > 0$, since that is the range of the sigmoid function in logistic regression, then $y_n(1-y_n) > 0$ as well, meaning the terms are all positive.

As for proving that the function is concave, it has just been proven that the double gradient is always greater than 0, which ensures $\nabla\nabla E(w) > 0$, which means it is concave by definition.

Q5

Part a

The PMF of the exponential distribution family can be written as

$$p(x|\eta) = h(x)g(\eta)\exp\left\{\eta^T y(x)\right\} \qquad \text{(2.194, Bishop)}$$

The functions correspond to the following:

$$h(x) = x^{\alpha-1}$$
$$y(x) = x$$
$$\eta^T = -\beta$$
$$g(\eta) = g(\beta) = \beta^\alpha$$

Part b

In order to find the maximum likelihood, take the log for convenience:

$$\log(L) = \sum^N -\log(T(v)) + v\log(\frac{xt_i}{y_i}) - \log y_i - \frac{vt_i}{y_i}$$

Expanding and removing constant terms:

$$-(v+1)\sum^N \log y_i - v\sum^N \frac{t_i}{y_i}$$

To find the gradient ascent steps, take the gradient with respect to w_0 and w_1:

$$\frac{\partial \log(L)}{\partial w_0} = \sum^N -(v+1) + vt_i y_i$$
$$\frac{\partial \log(L)}{\partial w_1} = \sum^N x_i(-(v+1) + vt_i y_i)$$

This leads to the following update rules:

$$w_0^{(r+1)} \leftarrow w_0^r + \propto \nabla w_0^r$$
$$w_1^{(r+1)} \leftarrow w_1^r + \propto \nabla w_1^r$$

To determine if the likelihood is concave or not, take the gradient with respect to w_0 and w_1:

Q6

Part a

The likelihood function would be the probability of observing each t_n, given the parameters w and ϕ, over a normal distribution, so that:

$$L(t_1, t_2, \dots t_n | w, \phi) = \prod_{i=1}^{N} N(t_i | w^T \phi(x), \sigma\epsilon)$$

The Laplacian prior is defined as

$$P(w|b) = \frac{1}{2b} \exp\left(-\frac{|w|}{b}\right)$$

Multiply the prior and likelihood:

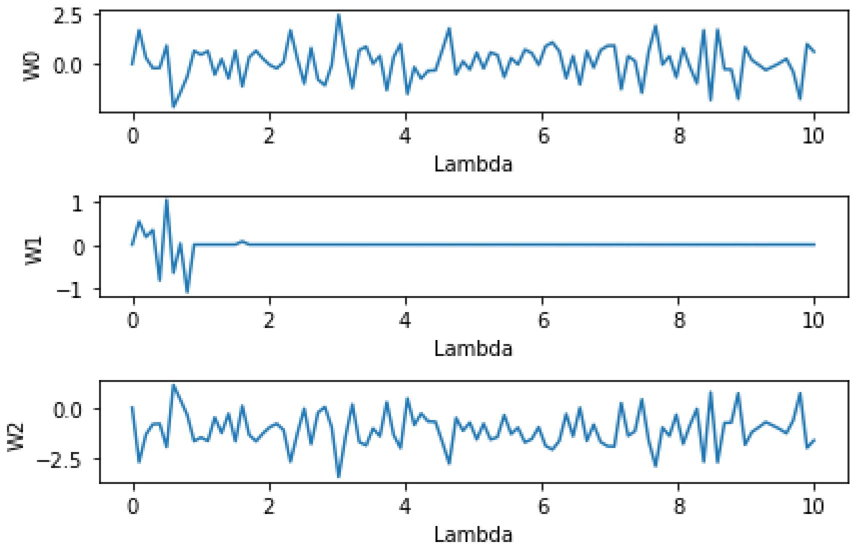
$$L(w|D, b) \propto L(t_1, t_2, \dots t_n | w, \phi) \cdot \frac{1}{2b} \exp\left(-\frac{|w|}{2b}\right)$$

Which follows the same general form as the lasso regularization formula:

$$-\frac{1}{2} \sum_i t_i - w^T \phi(x_i)^2 - \lambda \sum_j |w_i|$$

In [ ]:

```
# Part b
import numpy as np
import matplotlib.pyplot as plt

mu_ek = 0
sigma_ek = 0.1
ek = np.random.normal(loc=mu_ek, scale=np.sqrt(sigma_ek), size=(100,1))

mu_x = 0
sigma_x = 1
x_i = np.random.normal(loc=mu_x, scale=np.sqrt(sigma_x), size=(100,1))
t_i = 1 + (0.01 * x_i) - 2 *  np.multiply(x_i, x_i) + ek

lambda_1 = np.linspace(0, 10, num=100)
eps = 0.0001
phi_x = np.concatenate([np.ones((100, 1)), x_i, np.multiply(x_i,  x_i)], axis=
1)
alpha = 1e-4
ws = np.zeros((100, 3))
for l in np.arange(1, 100, step=1):
    t_lambda = lambda_1[l - 1]
    w_r = np.random.randn(3, 1)
    for iter in np.arange(1, 10000, step=1):
        err = t_i - np.matmul(phi_x, w_r) # 100, 1
        dw_0 = sum(err)

        d1 = 1 / max(eps, abs(w_r[1]))
        dw_1 = np.sum(phi_x[:, 1].T *err) - t_lambda * w_r[1] * d1

        d2 = 1 / max(eps, abs(w_r[2]))
        dw_2 = np.sum(phi_x[:, 2].T * err) - t_lambda * w_r[2] * d2

        w_r[0] = w_r[0] + alpha * dw_0
        w_r[1] = w_r[1] + alpha * dw_1
        w_r[2] = w_r[2] + alpha * dw_2
    ws[l,:] = w_r.reshape((3,))

fig_6, ax_6 = plt.subplots(3, 1)
ax_6[0].plot(lambda_1, ws[:, 0])
ax_6[0].set_ylabel("W0")
ax_6[0].set_xlabel("Lambda")

ax_6[1].plot(lambda_1, ws[:, 1])
ax_6[1].set_ylabel("W1")
ax_6[1].set_xlabel("Lambda")

ax_6[2].plot(lambda_1, ws[:, 2])
ax_6[2].set_ylabel("W2")
ax_6[2].set_xlabel("Lambda")
fig_6.tight_layout()
```



Application Questions

In [ ]:
```
# Imports

import pandas as pd
import numpy as np
from matplotlib import pyplot as plt
from itertools import combinations
from sklearn import preprocessing
from sklearn.linear_model import LogisticRegression, LinearRegression, Bayesia
nRidge
from sklearn.metrics import mean_squared_error, RocCurveDisplay
from sklearn.naive_bayes import BernoulliNB
from sklearn.model_selection import cross_val_score
from scipy import stats
import pymc3 as pm
```

Linear Regression Problem (25 pts)

In [ ]:

```python
# Dataset init

dfr = pd.read_excel("Real estate valuation data set.xlsx")
dfr.rename(columns={
    "X1 transaction date": "X1",
    "X2 house age": "X2",
    "X3 distance to the nearest MRT station": "X3",
    "X4 number of convenience stores": "X4",
    "X5 latitude": "X5",
    "X6 longitude": "X6",
    "Y house price of unit area": "Y"
}, inplace=True)
print(f"Zeroing X1, transaction date, at {min(dfr['X1'])}")
dfr["X1"] = dfr["X1"] - min(dfr["X1"])
dfr.head()
```

```
Zeroing X1, transaction date, at 2012.6666667
```

Out[ ]:

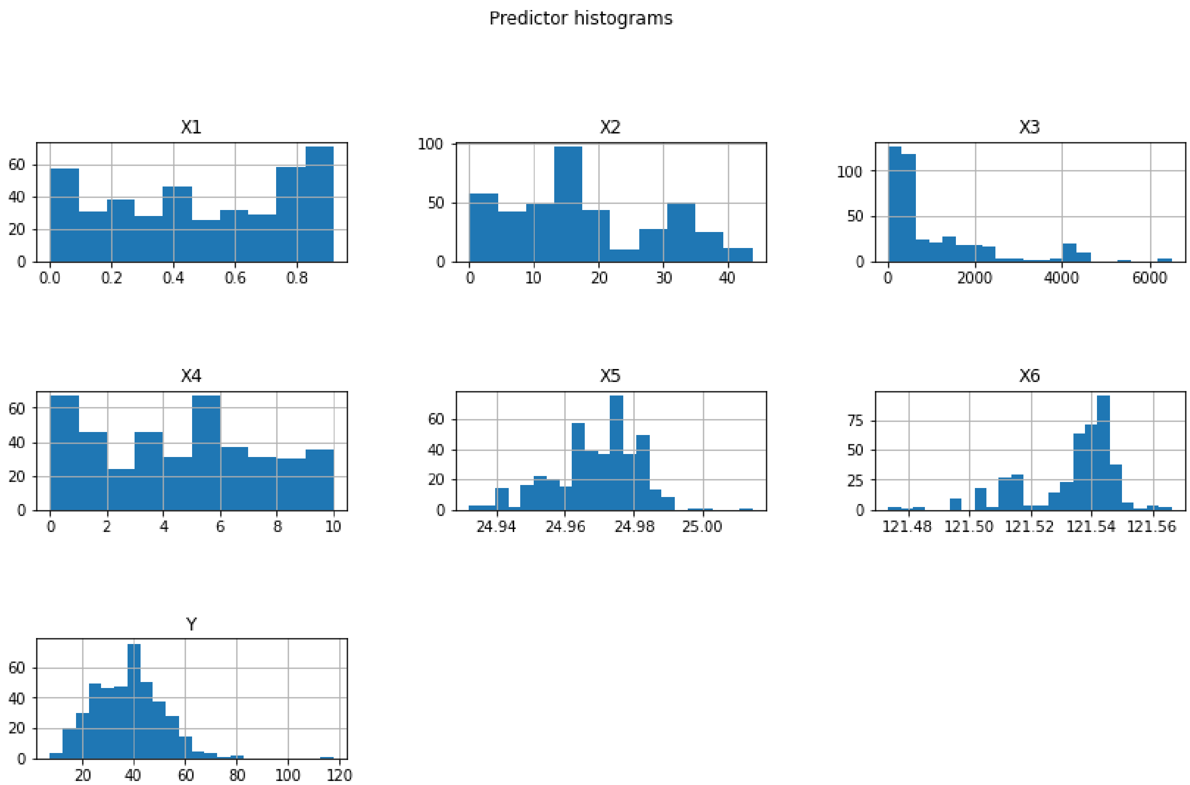
	No	X1	X2	X3	X4	X5	X6	Y
0	1	0.250000	32.0	84.87882	10	24.98298	121.54024	37.9
1	2	0.250000	19.5	306.59470	9	24.98034	121.53951	42.2
2	3	0.916667	13.3	561.98450	5	24.98746	121.54391	47.3
3	4	0.833333	13.3	561.98450	5	24.98746	121.54391	54.8
4	5	0.166667	5.0	390.56840	5	24.97937	121.54245	43.1

In [ ]:

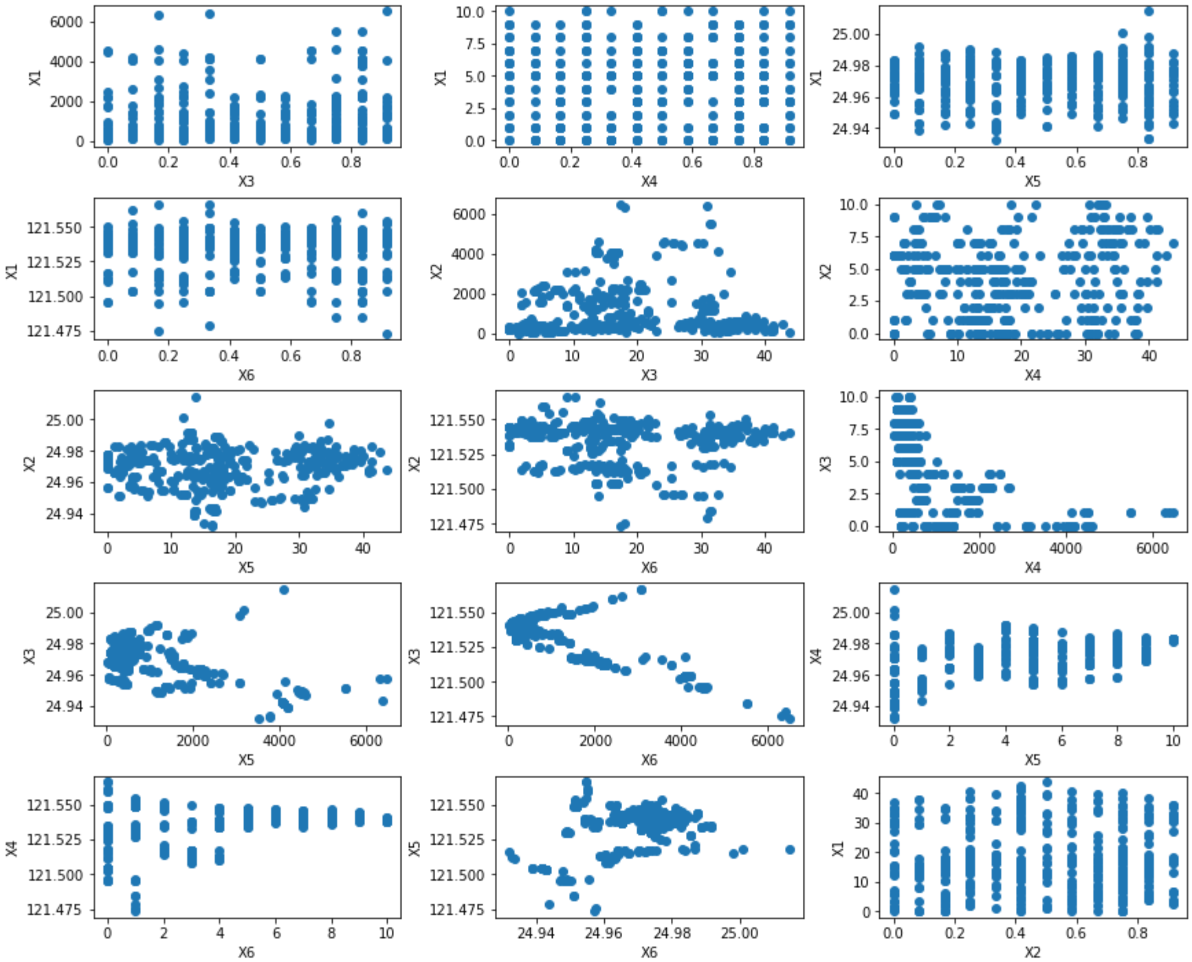
```python
# 1. Visualization

figure_hist, ax_hist = plt.subplots()
dfr.drop("No", axis=1).hist(ax=ax_hist, bins="auto")
figure_hist.suptitle("Predictor histograms")
figure_hist.tight_layout()
figure_hist.set_size_inches(12, 8)

fig_sc_preds, ax_preds = plt.subplots(nrows=5, ncols=3, figsize=(12, 10), cons
trained_layout=True)
fig_sc_resp, ax_resp = plt.subplots(nrows=2, ncols=3, figsize=(12, 10), constr
ained_layout=True)
combs = list(combinations(range(1,7), 2))

def lin_to_mat(cols, x):
    return (x // cols, x % cols)

for i in range(1, 7):
    ax_resp[lin_to_mat(3, i - 1)].scatter(dfr[f"X{i}"], dfr["Y"])
    ax_resp[lin_to_mat(3, i - 1)].set_xlabel(f"X{i}")
    ax_resp[lin_to_mat(3, i - 1)].set_ylabel("Y")
fig_sc_resp.suptitle("Scatter plots between predictors and response")

for i in range(len(combs)):
    ax_preds[lin_to_mat(3, i - 1)].scatter(dfr[f"X{combs[i][0]}"], dfr[f"X{com
bs[i][1]}"])
    ax_preds[lin_to_mat(3, i - 1)].set_ylabel(f"X{combs[i][0]}")
    ax_preds[lin_to_mat(3, i - 1)].set_xlabel(f"X{combs[i][1]}")
fig_sc_preds.suptitle("Scatter plots for predictors")

_="""
There appear to be some data points that differ from the rest of the predictor
data, such as in X3 and Y. X3 and X5 appear to have the strongest correlation
 with Y.
"""
```

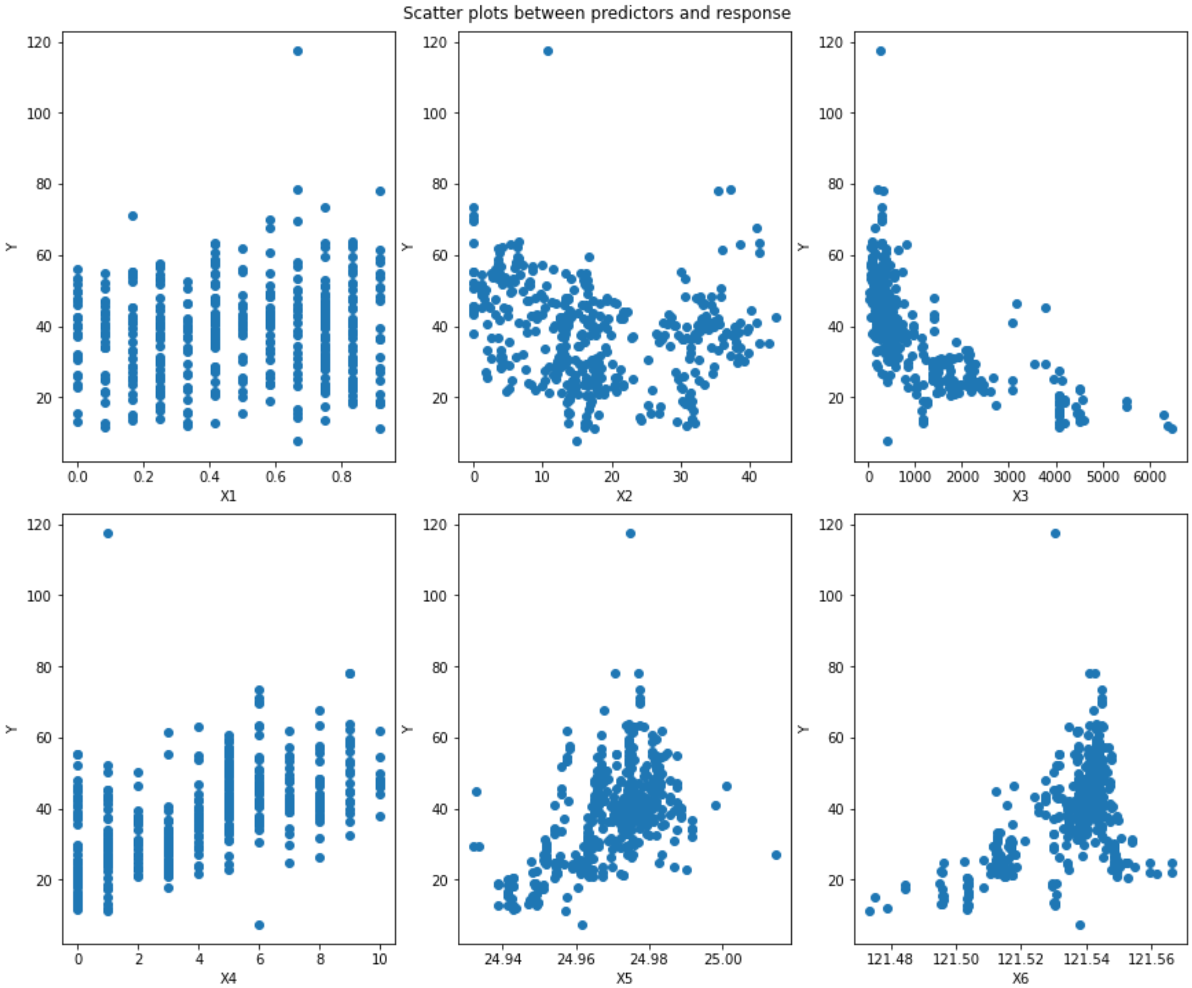
```
C:\Users\SAADMU~1\AppData\Local\Temp/ipykernel_28320/3992793283.py:4: UserWar
ning: To output multiple subplots, the figure containing the passed axes is b
eing cleared
  dfr.drop("No", axis=1).hist(ax=ax_hist, bins="auto")
```

Out[ ]: `Text(0.5, 0.98, 'Scatter plots for predictors')`



Scatter plots for predictors

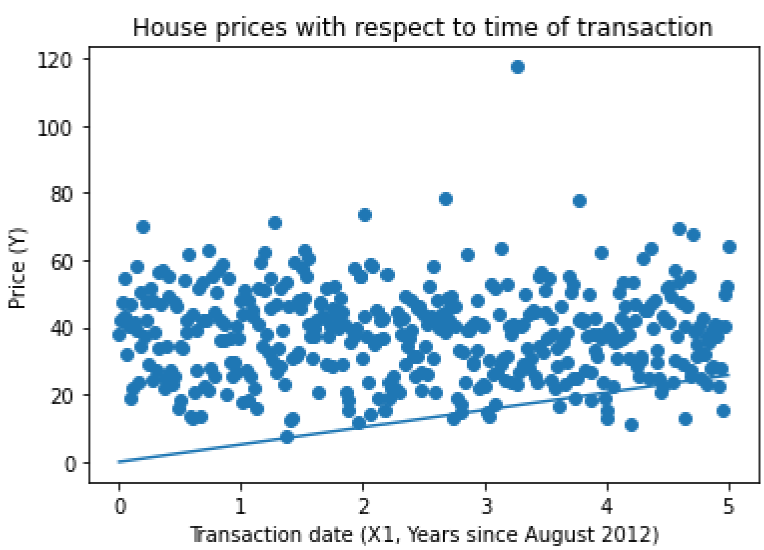



Scatter plots between predictors and response
Y
X1
X2
X3
X4
X5
X6

In [ ]:
```
# 2. Linear Regression Model
pred_names = ["X1", "X2", "X3", "X4", "X5", "X6"]
predictors = dfr[pred_names]

lin_model = LinearRegression().fit(X=predictors, y=dfr["Y"])
lin_y_preds = lin_model.predict(predictors)

def show_weights(names, weights):
    print("\n".join("{} weight: {}".format(*i) for i in list(zip(names, weight
s))))

show_weights(pred_names, lin_model.coef_)
print("\nRMSE: ", mean_squared_error(dfr["Y"], lin_y_preds, squared=False))

fig_time, ax_time = plt.subplots()
tt = np.linspace(0,5, num=414) ## Transaction times

ax_time.plot(tt, np.multiply(tt, lin_model.coef_[0]))
print(len(dfr["Y"]))
ax_time.scatter(tt, dfr["Y"])
ax_time.set_title("House prices with respect to time of transaction")
ax_time.set_ylabel("Price (Y)")
ax_time.set_xlabel("Transaction date (X1, Years since August 2012)")
```

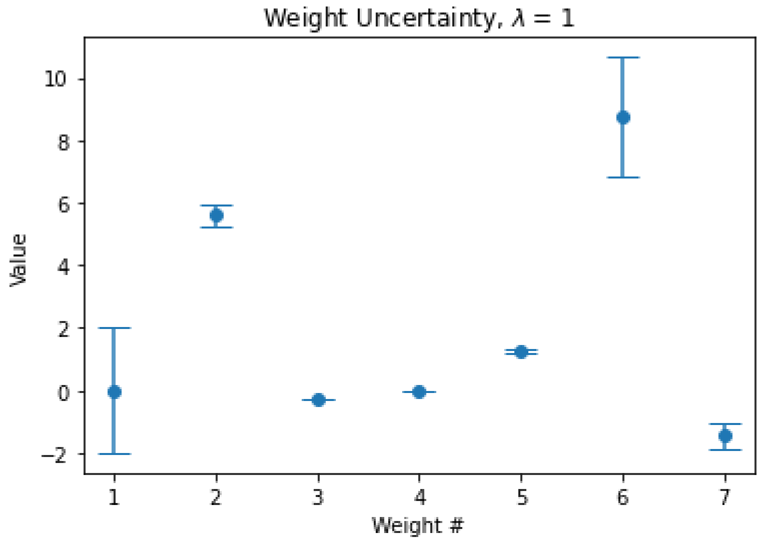
```
X1 weight: 5.149017210936201
X2 weight: -0.2696967345199864
X3 weight: -0.004487508250365868
X4 weight: 1.1333249810148465
X5 weight: 225.47014318107793
X6 weight: -12.42906116553292

RMSE:  8.782312975361108
414
```

Out[ ]:
```
Text(0.5, 0, 'Transaction date (X1, Years since August 2012)')
```

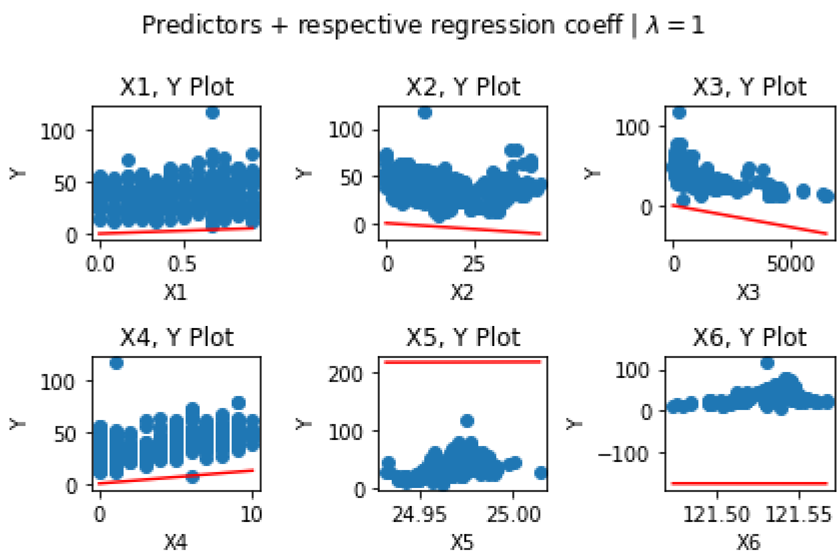


In [ ]:
```
# 3. Bayesian Linear Regression Model

# x must be a dataframe
def bayesian_reg(x, t, lambda_reg, basis=lambda v: v, basis_params=None):
    alpha = lambda_reg ** 2
    beta = lambda_reg
    def posterior(phi, t):
        # Prior here is defined as N(w|mu=0, cov=a^-1 * I)
        cov_n = np.linalg.pinv(alpha * np.eye(phi.shape[1]) + beta * phi.T.dot
(phi))
        m_n = beta * cov_n.dot(phi.T).dot(t)
        return m_n, cov_n

    def predict_dist(phi, m_n, cov_n, beta):
        cov = beta ** -1 + np.sum(phi.dot(cov_n) * phi, axis=1)
        mu = phi.dot(m_n)
        return mu, cov

    def design_mat(x, basis):
        if(basis_params == None):
            return np.concatenate([np.ones((x.shape[0], 1)), basis(x)], axis=1
)
        else:
            x.apply(lambda row: basis(row, basis_params[row.name]), axis=1)
            return np.concatenate(
                [
                    np.ones((x.shape[0], 1)),
                    x.to_numpy()
                ], axis=1)

    phi = design_mat(x, basis)
    # print("Design mat", phi)

    m_n, cov_n = posterior(phi, t)
    y, var = predict_dist(phi, m_n, cov_n, beta)

    # Frequentist approach uses penrose pseudo-inverse
    # weights = np.matmul(np.linalg.pinv(phi), t)

    # show_weights(["B0"] + pred_names, m_n)
    # print(np.diag(cov_n))
    _, ax_weights = plt.subplots()
    weight_x = np.linspace(1, len(m_n), len(m_n))
    ax_weights.errorbar(x=weight_x, y=m_n, yerr=2*np.sqrt(np.diag(cov_n)), ls=
"none", capsize=8)
    ax_weights.scatter(x=weight_x, y=m_n)
    ax_weights.set_title(f"Weight Uncertainty, $\lambda$ = {lambda_reg}")
    ax_weights.set_xlabel("Weight #")
    ax_weights.set_ylabel("Value")

    cols = min(3, x.shape[1])
    rows = x.shape[1] // cols
    fig_bay, ax_bay = plt.subplots(rows, cols)
    for i, ax in enumerate(ax_bay.ravel()):
        names = list(x.keys())
        data = x[names[i]]
```

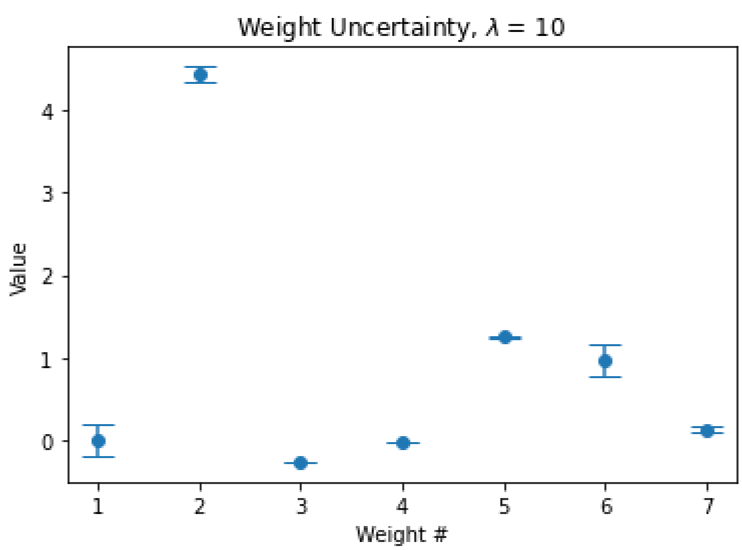
```
        ax.scatter(data, dfr["Y"])
        x_test = np.linspace(min(data), max(data), 200)
        ax.plot(x_test, (m_n[i + 1] * x_test) + m_n[0], "r")
        ax.set_title(f"X{i + 1}, Y Plot")
        ax.set_xlabel(f"X{i + 1}")
        ax.set_ylabel("Y")
    fig_bay.suptitle(f"Predictors + respective regression coeff | $\lambda={lambda_reg}$")
    fig_bay.tight_layout()



bayesian_reg(predictors, dfr["Y"], lambda_reg=1)
bayesian_reg(predictors, dfr["Y"], lambda_reg=10)
bayesian_reg(predictors, dfr["Y"], lambda_reg=100)
```



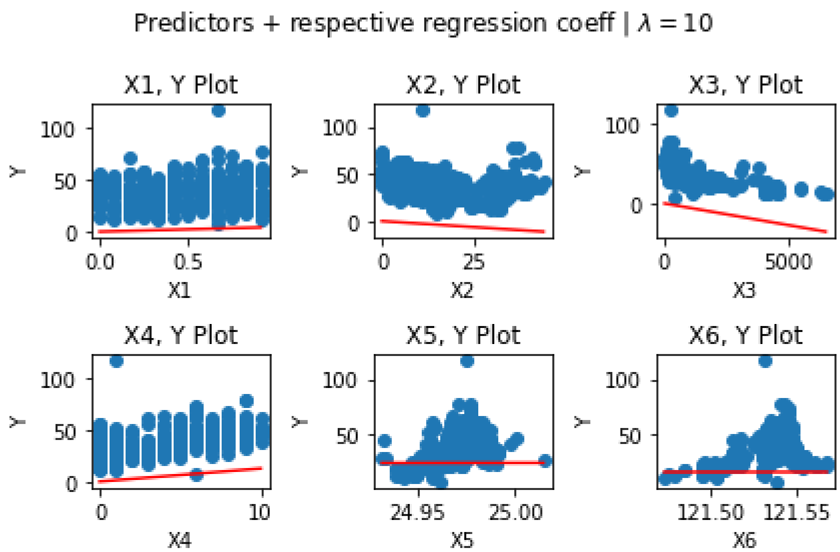
Weight Uncertainty, λ = 1
Value
Weight #

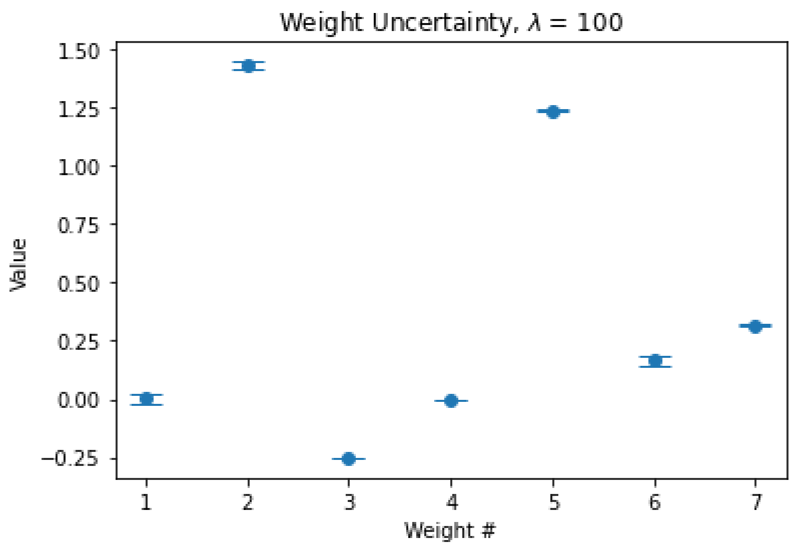


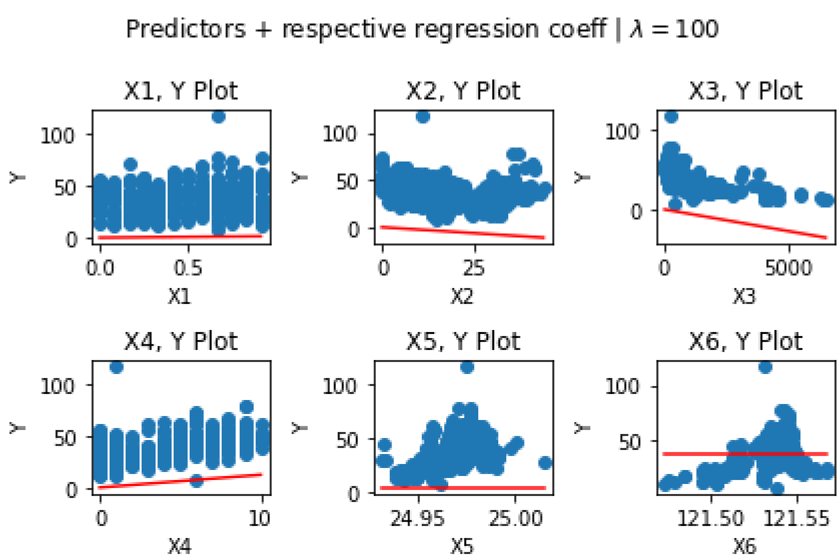
Predictors + respective regression coeff | $\lambda = 1$
X1, Y Plot
X2, Y Plot
X3, Y Plot
X4, Y Plot
X5, Y Plot
X6, Y Plot
X1
X2
X3
X4
X5
X6
Y



Weight Uncertainty, λ = 10
Value
Weight #


Predictors + respective regression coeff | $\lambda = 10$
X1, Y Plot
X2, Y Plot
X3, Y Plot
X4, Y Plot
X5, Y Plot
X6, Y Plot
Y
X1
X2
X3
X4
X5
X6
0
50
100
0.0
0.5
25
5000
10
24.95
25.00
121.50
121.55


Weight Uncertainty, λ = 100
Value
Weight #
1.50
1.25
1.00
0.75
0.50
0.25
0.00
−0.25
1
2
3
4
5
6
7


Predictors + respective regression coeff | $\lambda = 100$
X1, Y Plot
X2, Y Plot
X3, Y Plot
X4, Y Plot
X5, Y Plot
X6, Y Plot
Y
X1
X2
X3
X4
X5
X6
0
50
100
0.0
0.5
25
5000
10
24.95
25.00
121.50
121.55

In [ ]:

```python
# 3. Bayesian Linear Regression, SKLearn
# I attempted to perform this problem with a custom made solution, but the res
ults were poor
def bay_ridge(x, t, lambda_reg, basis=lambda v: v, basis_params=None):
    model = BayesianRidge(lambda_1=lambda_reg)
    design_mat = x.copy()

    if basis_params is not None:
        for c in x.columns:
            for i in basis_params:
                design_mat[f"{c}.mu{i}"] = basis(x[c], i)
    # print(design_mat.shape)
    model.fit(X=design_mat, y=t)
    predicted, std = model.predict(X=design_mat, return_std=True)
    print("Predicted vals", predicted)
    print("Uncertainties", std)
    weights = model.coef_
    weight_std = np.diag(model.sigma_)
    intercept = model.intercept_
    print("Intercept", intercept)

    _, ax_weights = plt.subplots(constrained_layout=True)
    weight_x = np.linspace(1, len(weights), len(weights))
    ax_weights.errorbar(x=weight_x, y=weights, yerr=2*weight_std, ls="none", c
apsize=8)
    ax_weights.scatter(x=weight_x, y=weights)
    ax_weights.set_title(f"Weight Uncertainty, $\lambda$ = {lambda_reg}")
    ax_weights.set_xlabel("Weight #")
    ax_weights.set_ylabel("Value")

    cols = min(3, design_mat.shape[1])
    rows = design_mat.shape[1] // cols
    fig_bay, ax_bay = plt.subplots(rows, cols, constrained_layout=True)
    for i, ax in enumerate(ax_bay.ravel()):
        names = list(design_mat.keys())
        data = design_mat[names[i]]
        ax.scatter(data, dfr["Y"])
        x_test = np.linspace(min(data), max(data), 200)
        ax.plot(x_test, (weights[i] * x_test) + intercept, "r")
        ax.set_title(f"{names[i]}, Y Plot")
        ax.set_xlabel(names[i])
        ax.set_ylabel("Y")
    fig_bay.suptitle(f"Predictors + respective regression coeff | $\lambda={la
mbda_reg}$")
    # fig_bay.tight_layout()
    del design_mat
    return model


bay_1 = bay_ridge(x=predictors, t=dfr["Y"], lambda_reg=1)
bay_2 = bay_ridge(x=predictors, t=dfr["Y"], lambda_reg=10)
bay_3 = bay_ridge(x=predictors, t=dfr["Y"], lambda_reg=100)
```

```
Predicted vals [46.7630763  47.4763383  43.41025225 43.3421758  45.82610157 3
2.89297689
 39.38533815 44.22960541  6.53244148 32.97500145 33.74046204 52.3202548
 43.32659262 29.11549027 38.83715658 34.14562837 49.14619003 38.05136342
 47.00708079 50.97572461 33.59735344 47.91659968 33.33963169 47.74444785
 35.61204881 30.29982552 46.20293543 45.44622948 40.70256166 45.16778658
 12.3015666  39.8711181  32.2296225  44.8448585  46.55489837 17.75111375
 31.33879059 34.21990148 46.31269166 43.85573832 17.39993207 17.09279735
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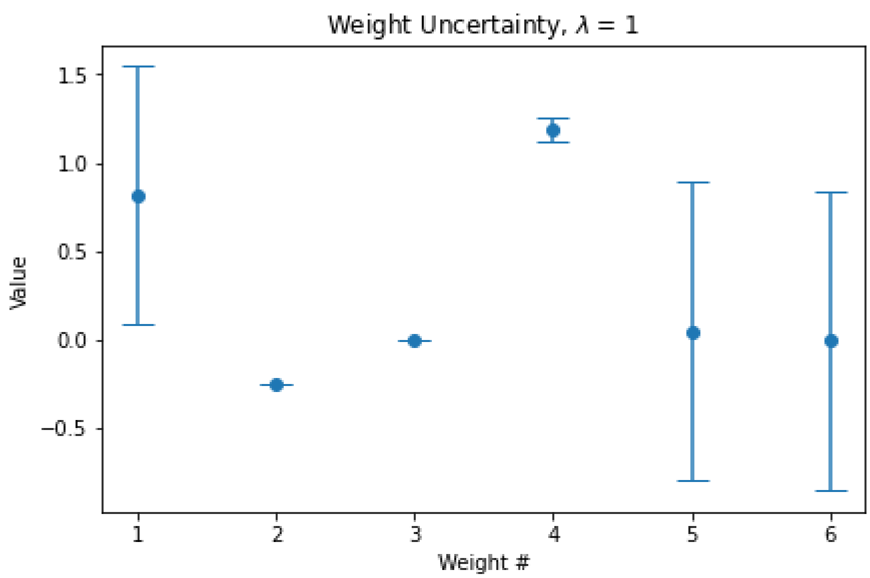

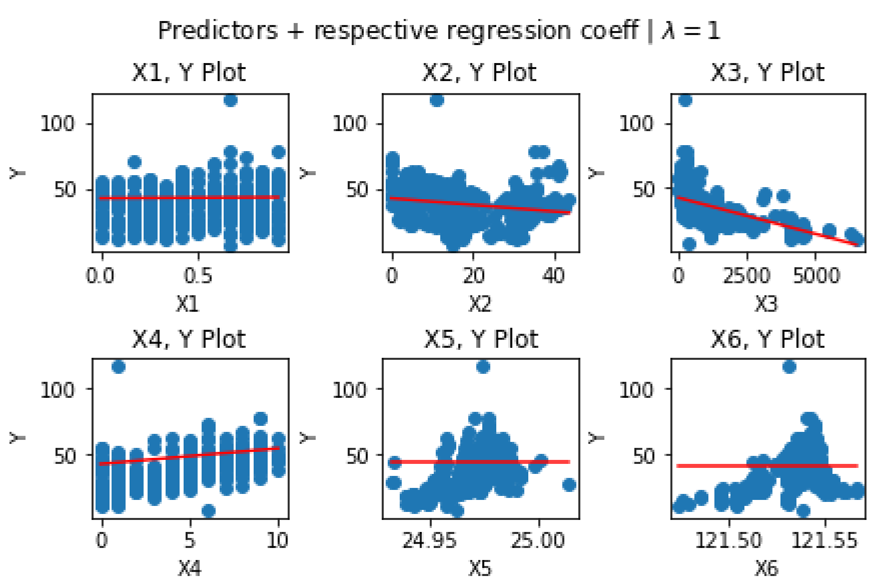

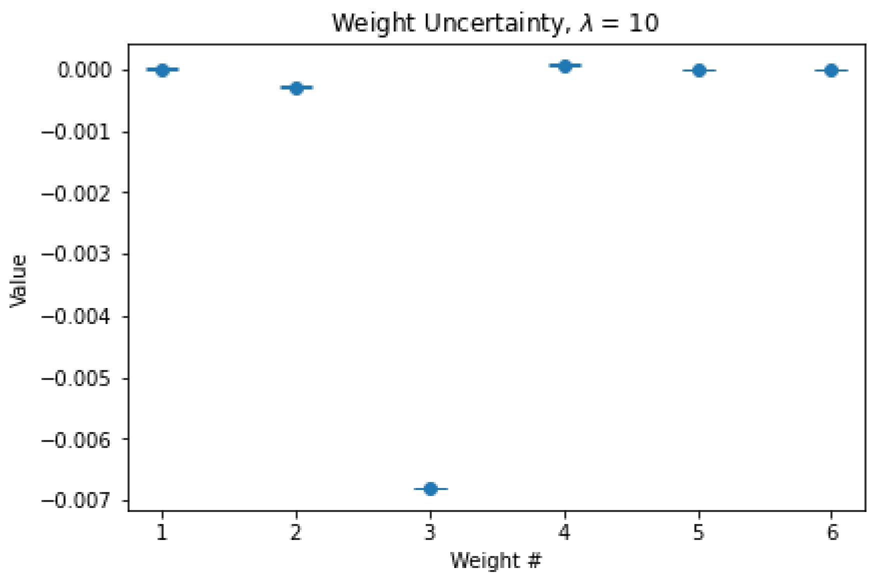

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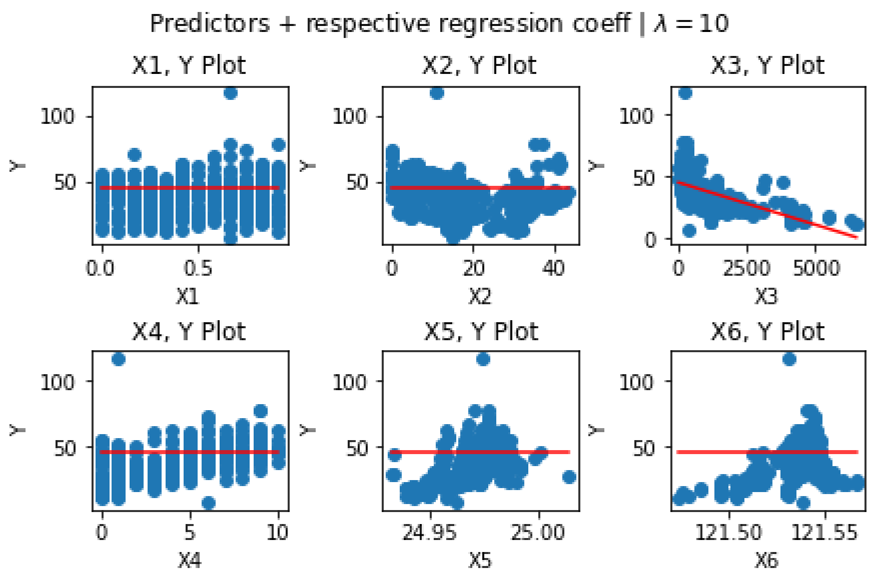
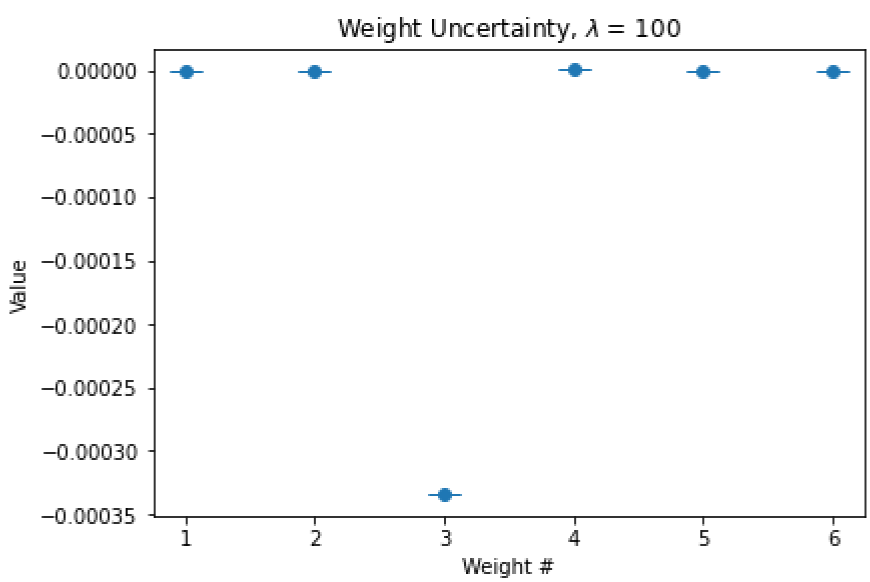
2
3
4
5
6



Predictors + respective regression coeff | $\lambda = 10$
X1, Y Plot
X2, Y Plot
X3, Y Plot
X4, Y Plot
X5, Y Plot
X6, Y Plot
Y
X1
X2
X3
X4
X5
X6



Weight Uncertainty, λ = 100
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−0.00015
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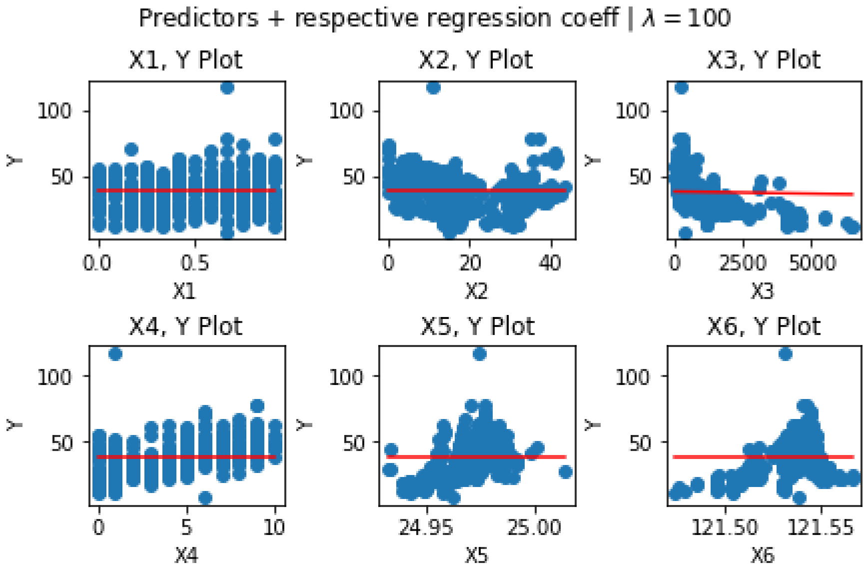
−0.00025
−0.00030
−0.00035
1
2
3
4
5
6


Predictors + respective regression coeff | $\lambda = 100$
X1, Y Plot
X2, Y Plot
X3, Y Plot
X4, Y Plot
X5, Y Plot
X6, Y Plot
Y
X1
X2
X3
X4
X5
X6
100
50
0.0
0.5
0
20
40
2500
5000
5
10
24.95
25.00
121.50
121.55

In [ ]:

```
# 4. Standardization + Gaussian Basis
standardized_pred = preprocessing.StandardScaler().fit_transform(dfr[["X2", "X
3"]])
standardized_pred = pd.DataFrame(standardized_pred, columns=["X2", "X3"])
gauss = lambda x_i, mu: np.exp(-0.5 * (x_i - mu) ** 2)

mu = [-3, -2, -1, 1, 2, 3]

stan_1 = bay_ridge(standardized_pred, dfr["Y"], lambda_reg=1, basis=gauss, bas
is_params=mu)
stan_2 = bay_ridge(standardized_pred, dfr["Y"], lambda_reg=10, basis=gauss, ba
sis_params=mu)
stan_3 = bay_ridge(standardized_pred, dfr["Y"], lambda_reg=100, basis=gauss, b
asis_params=mu)
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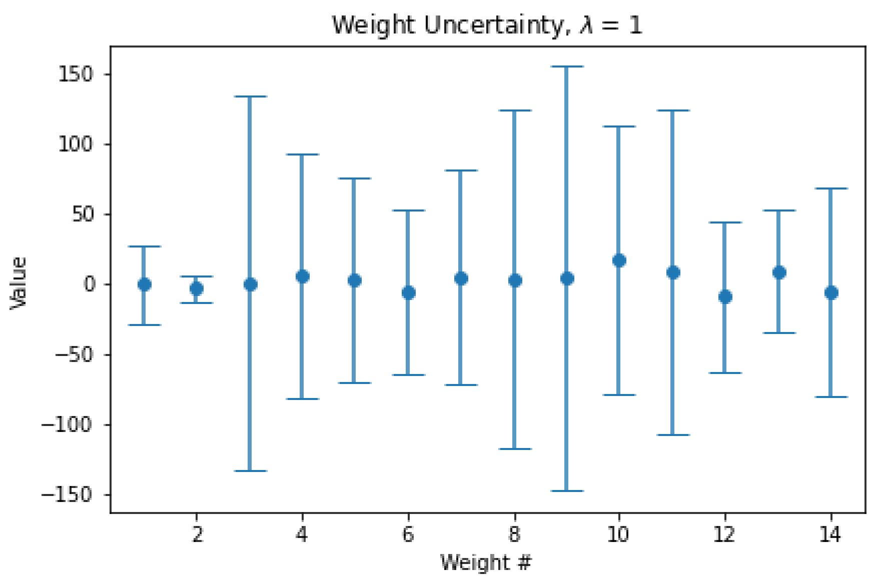
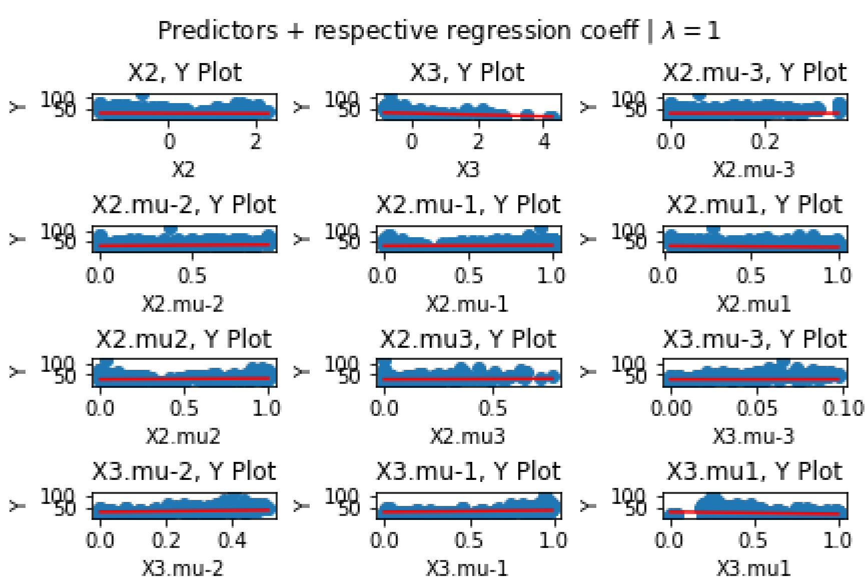
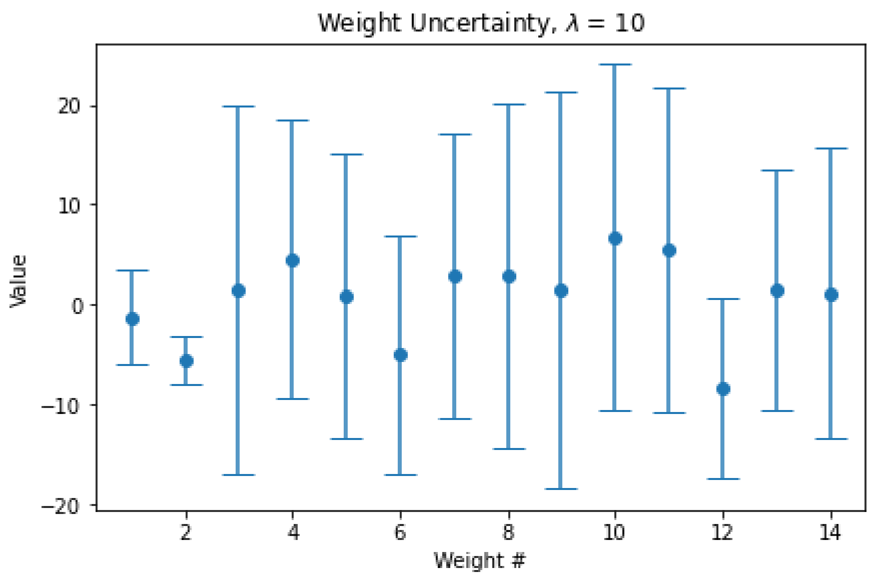
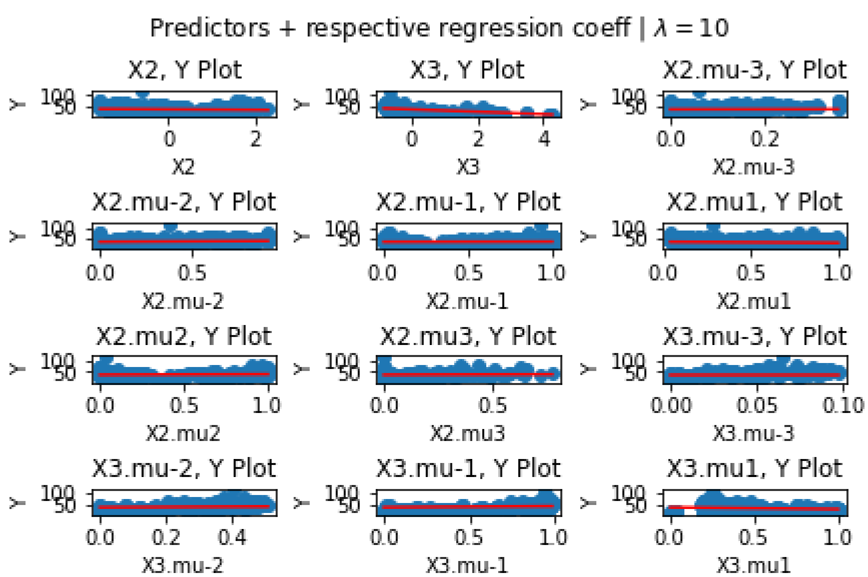
37.98019355	37.98019322	37.98019266	37.98019358	37.98019311	37.9801923
37.98019353	37.98019353	37.9801934	37.98019309	37.98019286	37.9801931
37.98019321	37.9801935	37.98019349	37.98019354	37.98019357	37.98019303
37.98019333	37.98019325	37.98019325	37.98019333	37.9801932	37.98019295

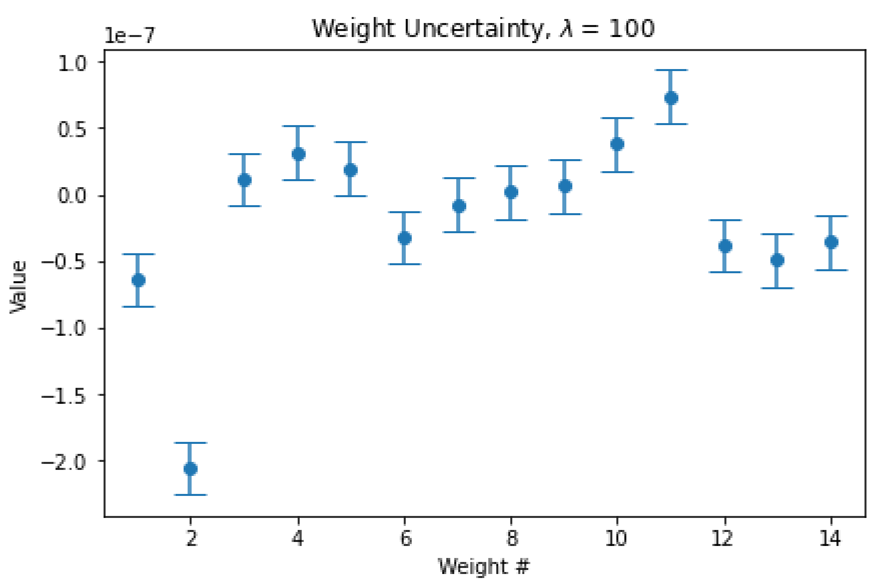
```
 37.98019308 37.98019307 37.98019338 37.98019294 37.98019342 37.98019352
 37.98019331 37.98019356 37.98019349 37.98019317 37.9801931  37.98019357
 37.9801932  37.98019355 37.98019343 37.98019352 37.98019272 37.98019326
 37.98019264 37.98019344 37.98019358 37.980193   37.98019302 37.98019328
 37.98019327 37.98019312 37.98019321 37.98019325 37.9801925  37.98019332
 37.98019312 37.98019324 37.9801931  37.98019347 37.98019328 37.98019306
 37.98019347 37.98019332 37.98019341 37.98019339 37.98019352 37.98019303
 37.98019296 37.98019266 37.98019355 37.98019337 37.98019353 37.98019355]
Uncertainties 0      13.590045
1       13.590045
2       13.590045
3       13.590045
4       13.590045
          ...
409     13.590045
410     13.590045
411     13.590045
412     13.590045
413     13.590045
Length: 414, dtype: float64
Intercept 37.980193203288984
```

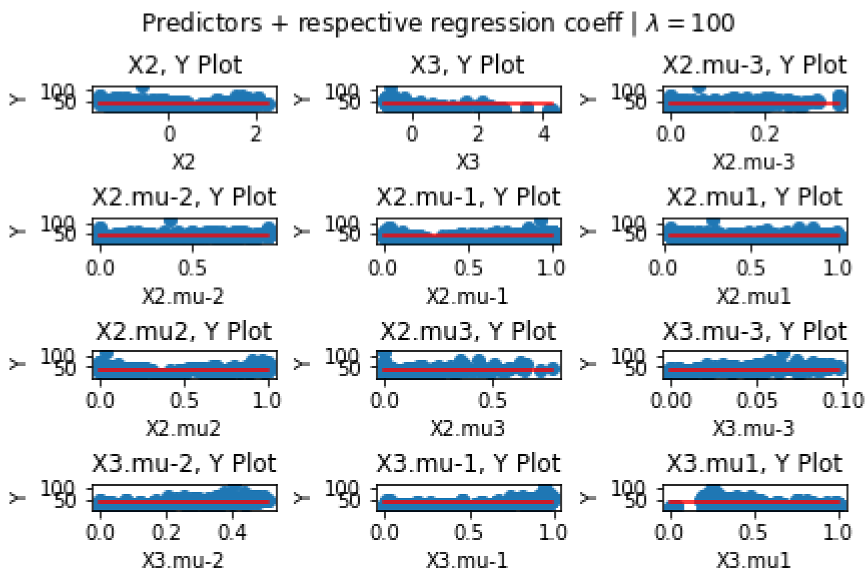





Weight Uncertainty, λ = 10
Value
Weight #
20
10
0
−10
−20
2
4
6
8
10
12
14


Predictors + respective regression coeff | λ = 10
X2, Y Plot
X3, Y Plot
X2.mu-3, Y Plot
X2.mu-2, Y Plot
X2.mu-1, Y Plot
X2.mu1, Y Plot
X2.mu2, Y Plot
X2.mu3, Y Plot
X3.mu-3, Y Plot
X3.mu-2, Y Plot
X3.mu-1, Y Plot
X3.mu1, Y Plot
X2
X3
X2.mu-3
X2.mu-2
X2.mu-1
X2.mu1
X2.mu2
X2.mu3
X3.mu-3
X3.mu-2
X3.mu-1
X3.mu1
Y


Weight Uncertainty, λ = 100
1e−7
Value
Weight #
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
2
4
6
8
10
12
14



In [ ]:
```
# 5. Compare linear regression model, gaussian basis model
BayesianRidge().fit(X=predictors, y=dfr["Y"])
```

In [ ]:
```
# 6. Cross Validation
from sklearn.metrics import make_scorer

rmse = make_scorer(mean_squared_error)
print(cross_val_score(bay_1, X=predictors, y=dfr["Y"], cv=10, scoring=rmse))
print(cross_val_score(bay_2, X=predictors, y=dfr["Y"], cv=10, scoring=rmse))
print(cross_val_score(bay_3, X=predictors, y=dfr["Y"], cv=10, scoring=rmse))

print(cross_val_score(stan_1, X=standardized_pred, y=dfr["Y"], cv=10, scoring=
rmse))
print(cross_val_score(stan_2, X=standardized_pred, y=dfr["Y"], cv=10, scoring=
rmse))
print(cross_val_score(stan_3, X=standardized_pred, y=dfr["Y"], cv=10, scoring=
rmse))
```

```
[ 63.89114084  56.89993407  98.38583199  94.33255073  36.11357478
  83.65535406 213.3551035   82.54990061  59.33830889  64.5932842 ]
[ 77.88860063  87.40388346 124.81751986  99.90442721  54.60008824
 102.32613294 222.84041319 107.13168165  59.16076487  81.85398287]
[159.2164679  180.81317535 212.84521591 168.61501452 144.33281553
 176.98253728 281.3658193  186.24846372 116.48008443 169.07217252]
[ 68.87284554  69.19693126 110.71281546  96.49990031  46.60044329
 104.61997928 210.19846791 101.07330672  65.72946081  74.38563569]
[ 68.54828677  71.12478252 110.89243952  97.23924919  47.08919486
 103.89155106 210.71163211 101.91304775  64.47974023  75.90370003]
[165.46161974 187.04141041 219.67453442 173.68079031 150.93929919
 183.30308791 285.77624639 192.15005657 121.26635377 174.9351637 ]
```

Classification Problem (25 pts)

In [ ]:
```
# Dataset Init
dfc = pd.read_excel("ENB2012_data.xlsx")
dfc["Y1-Y2"] = np.where(dfc["Y1"] - dfc["Y2"] > 0, 1, 0)
dfc.head()
```

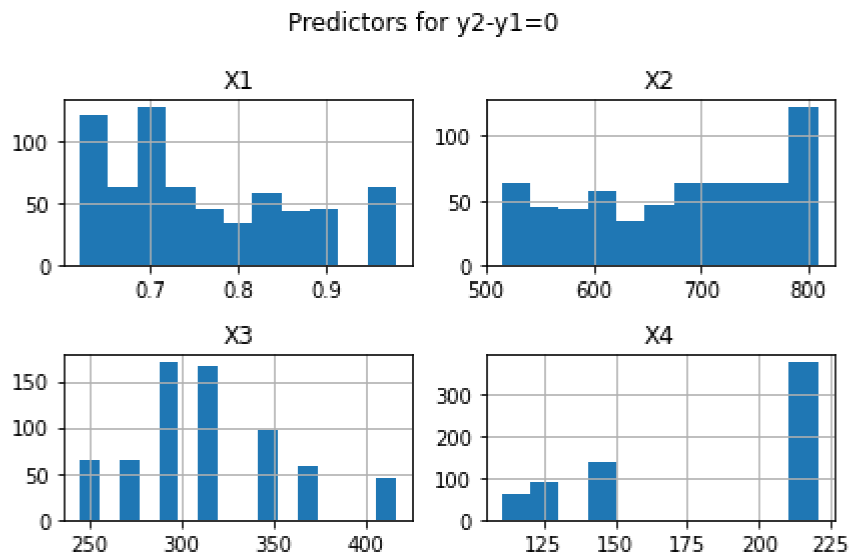
Out[ ]:

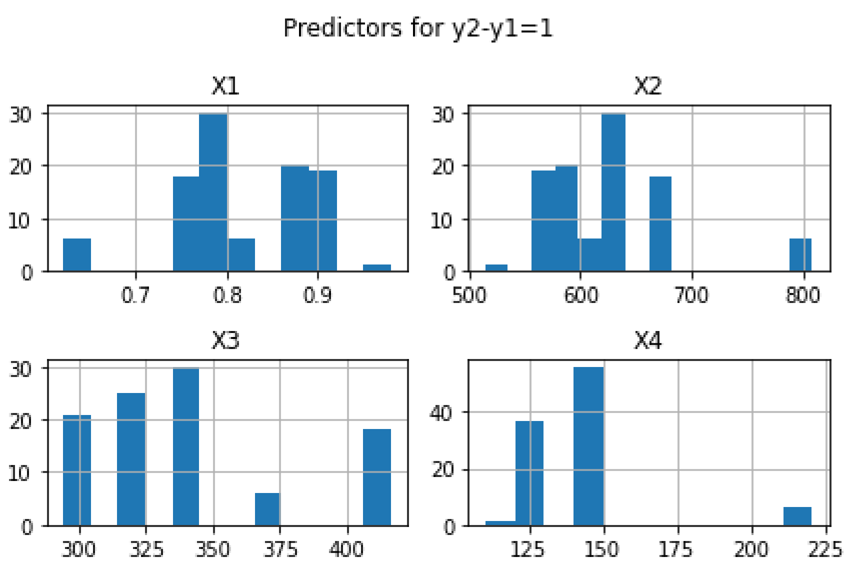
	X1	X2	X3	X4	X5	X6	X7	X8	Y1	Y2	Y1-Y2
0	0.98	514.5	294.0	110.25	7.0	2	0.0	0	15.55	21.33	0
1	0.98	514.5	294.0	110.25	7.0	3	0.0	0	15.55	21.33	0
2	0.98	514.5	294.0	110.25	7.0	4	0.0	0	15.55	21.33	0
3	0.98	514.5	294.0	110.25	7.0	5	0.0	0	15.55	21.33	0
4	0.90	563.5	318.5	122.50	7.0	2	0.0	0	20.84	28.28	0

In [ ]:
```
# 1. Visualization

excl_hist = ["Y1", "Y2", "Y1-Y2", "X5", "X6", "X7", "X8"]

# Histogram of predictors X1, X2, X3, X4 that have label 0
fig_hist_0, ax_hist_0 = plt.subplots()
dfc[dfc["Y1-Y2"] == 0].drop(excl_hist, axis=1).hist(bins="auto", ax=ax_hist_0)
fig_hist_0.suptitle("Predictors for y2-y1=0")
fig_hist_0.tight_layout()

# Histogram of predictors X1, X2, X3, X4 that have label 1
fig_hist_1, ax_hist_1 = plt.subplots()
dfc[dfc["Y1-Y2"] == 1].drop(excl_hist, axis=1).hist(bins="auto", ax=ax_hist_1)
fig_hist_1.suptitle("Predictors for y2-y1=1")
fig_hist_1.tight_layout()

# Construct conditional PMFs
x_discrete = ["X5", "X6", "X7", "X8"]

# P(Xn | Y=0)
fig_pmf_0, ax_pmf_0 = plt.subplots(2, 2)
for i in range(len(x_discrete)):
    data = dfc[dfc["Y1-Y2"] == 0][x_discrete[i]]
    data = data.value_counts().sort_index()
    data = data.divide(sum(data.values))
    # print(data)
    x = data.index
    p = data.values
    ax_pmf_0[lin_to_mat(2, i)].plot(x, p, "bo", ms=8, mec="b")
    ax_pmf_0[lin_to_mat(2, i)].vlines(x, 0, p, linestyles="-", lw=2)
    ax_pmf_0[lin_to_mat(2, i)].set_title(x_discrete[i])
    ax_pmf_0[lin_to_mat(2, i)].set_ylim([0, 0.6])
    ax_pmf_0[lin_to_mat(2, i)].set_xticks(x)
fig_pmf_0.suptitle("Conditional PMFs for Y=0")

fig_pmf_0.tight_layout()


# P(Xn | Y=1)
fig_pmf_1, ax_pmf_1 = plt.subplots(2, 2)
for i in range(len(x_discrete)):
    data = dfc[dfc["Y1-Y2"] == 1][x_discrete[i]]
    data = data.value_counts().sort_index()
    data = data.divide(sum(data.values))
    # print(data)
    x = data.index
    p = data.values
    ax_pmf_1[lin_to_mat(2, i)].plot(x, p, "bo", ms=8, mec="b")
    ax_pmf_1[lin_to_mat(2, i)].vlines(x, 0, p, linestyles="-", lw=2)
    ax_pmf_1[lin_to_mat(2, i)].set_title(x_discrete[i])
    ax_pmf_1[lin_to_mat(2, i)].set_ylim([0, max(0.6, max(p))])
    ax_pmf_1[lin_to_mat(2, i)].set_xticks(x)
fig_pmf_1.suptitle("Conditional PMFs for Y=1")
fig_pmf_1.tight_layout()

_="""
```

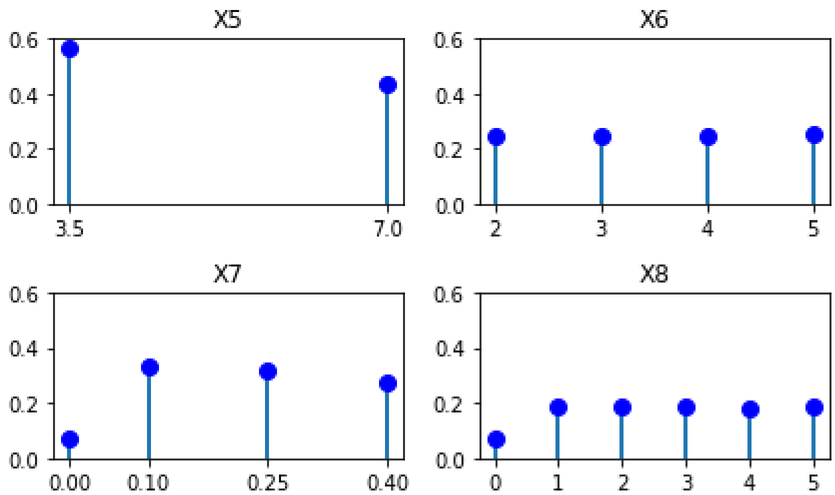
```
X5 and X7 appear to be very good predictors of the class labels, looking at th
eir conditional distributions.
"""
```

```
C:\Users\SAADMU~1\AppData\Local\Temp/ipykernel_28320/1939464138.py:7: UserWar
ning: To output multiple subplots, the figure containing the passed axes is b
eing cleared
  dfc[dfc["Y1-Y2"] == 0].drop(excl_hist, axis=1).hist(bins="auto", ax=ax_hist
_0)
C:\Users\SAADMU~1\AppData\Local\Temp/ipykernel_28320/1939464138.py:13: UserWa
rning: To output multiple subplots, the figure containing the passed axes is
being cleared
  dfc[dfc["Y1-Y2"] == 1].drop(excl_hist, axis=1).hist(bins="auto", ax=ax_hist
_1)
```

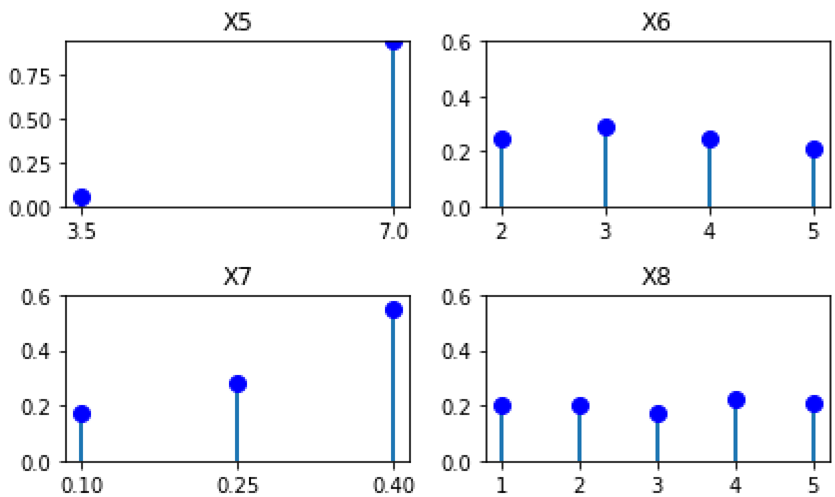




Conditional PMFs for Y=0



Conditional PMFs for Y=1



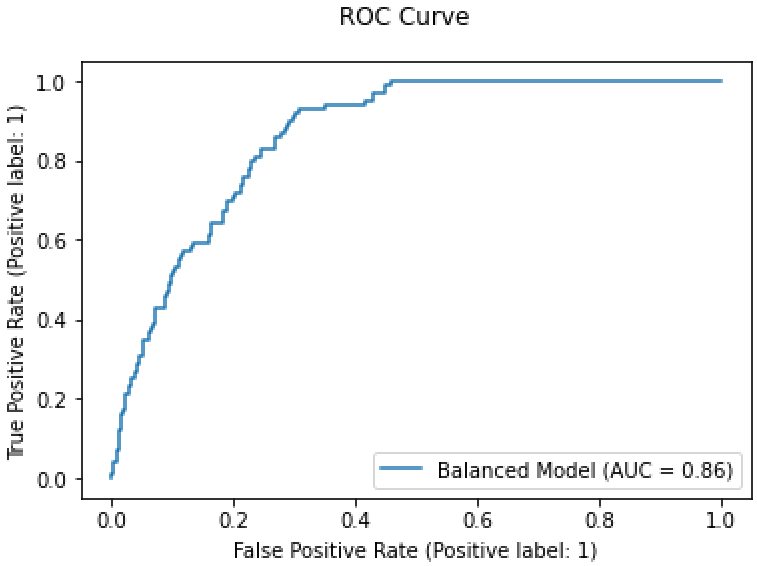
In [ ]:
```python
# 2. Logistic Regression
pred_c_names = ["X1", "X2", "X3", "X4", "X5", "X6", "X7", "X8"]
pred_c_data = dfc[pred_c_names]
logistic_model = LogisticRegression(max_iter=500).fit(X=pred_c_data, y=dfc["Y1
-Y2"])

prob_preds = logistic_model.predict_proba(X=pred_c_data)[:,1]
show_weights(pred_c_names, logistic_model.coef_[0])

# Assuming Y=1 is "positive", Y=0 is "negative"
preds = logistic_model.predict(pred_c_data)
preds_df = pd.DataFrame(preds.transpose(), columns=["preds"])
true_pos = len(dfc[(dfc["Y1-Y2"] == 1) & (preds_df["preds"] == 1)])

print("\nAccuracy\nRecall: ", true_pos / len([i for i in preds if i == 1]))
print("Precision: ", true_pos / len(dfc[dfc[pred_c_names] == 1]))
print("Model score:", logistic_model.score(X=pred_c_data, y=dfc["Y1-Y2"]))
```

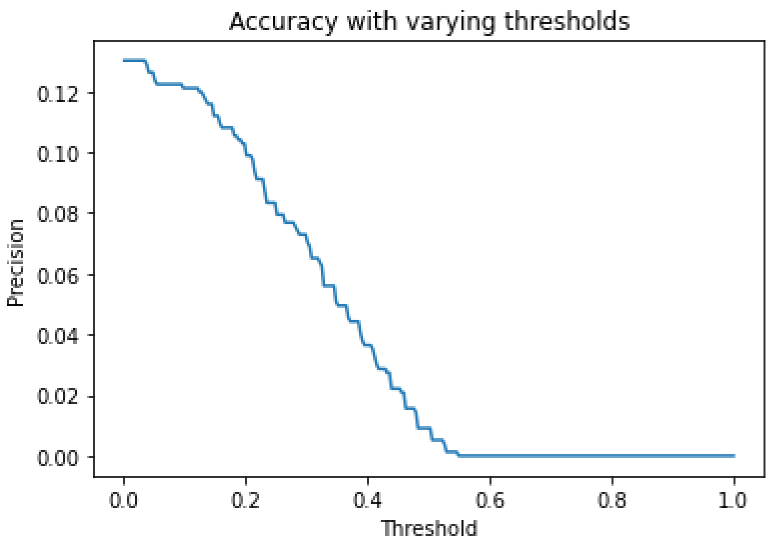
```
X1 weight: 0.1314098373909968
X2 weight: 0.01376272663718662
X3 weight: -0.0019824349199593157
X4 weight: 0.007872580757269867
X5 weight: 1.6519035109073805
X6 weight: -0.08958701981032652
X7 weight: 3.1448815458541177
X8 weight: 0.08805294156710261

Accuracy
Recall:  0.5833333333333334
Precision:  0.009114583333333334
Model score: 0.8723958333333334
```

In [ ]:

```
# 3. ROC Curve
fig_roc, ax_roc = plt.subplots()
fig_roc.suptitle("ROC Curve")
RocCurveDisplay.from_predictions(y_true=dfc["Y1-Y2"], y_pred=prob_preds, ax=ax
_roc, name="Balanced Model")

fig_thresh, ax_thresh = plt.subplots()

threshs = np.linspace(0.001, 1, 300)
prec_data = []
for thresh in threshs:
    preds_df["thresh"] = [1 if i > thresh else 0 for i in prob_preds]
    true_pos = len(dfc[(dfc["Y1-Y2"] == 1) & (preds_df["thresh"] == 1)])
    prec_data.append(true_pos / len(dfc[dfc[pred_c_names] == 1]))


len(dfc[(dfc["Y1-Y2"] == 1) & (preds_df["preds"] == 1)])
ax_thresh.plot(threshs, prec_data)
ax_thresh.set_title("Accuracy with varying thresholds")
ax_thresh.set_ylabel("Precision")
ax_thresh.set_xlabel("Threshold")
```

```
Out[ ]: Text(0.5, 0, 'Threshold')
```





In [ ]:
```python
# 4. Bayesian Regression
def bay_logistic(alpha):
    model = BernoulliNB(alpha=alpha, fit_prior=True)
    model.fit(X=pred_c_data, y=dfc["Y1-Y2"])
    model.predict_proba(pred_c_data)
    weights = model.coef_
    print("Weights", weights)
    # print("Weight uncertainties", )
    return model, weights


a_1, w1 = bay_logistic(0.1)
a_2, w2 = bay_logistic(1)
a_3, w3 = bay_logistic(10)
a_4, w4 = bay_logistic(100)
```

```
Weights [[-0.0009985 -0.0009985 -0.0009985 -0.0009985 -0.0009985 -0.0009985
  -0.0009985 -0.0009985]]
Weights [[-0.0098523 -0.0098523 -0.0098523 -0.0098523 -0.0098523 -0.0098523
  -0.0098523 -0.0098523]]
Weights [[-0.08701138 -0.08701138 -0.08701138 -0.08701138 -0.08701138 -0.0870
1138
  -0.08701138 -0.08701138]]
Weights [[-0.40546511 -0.40546511 -0.40546511 -0.40546511 -0.40546511 -0.4054
6511
  -0.40546511 -0.40546511]]

C:\Python39\lib\site-packages\sklearn\utils\deprecation.py:103: FutureWarnin
g: Attribute `coef_` was deprecated in version 0.24 and will be removed in 1.
1 (renaming of 0.26).
  warnings.warn(msg, category=FutureWarning)
C:\Python39\lib\site-packages\sklearn\utils\deprecation.py:103: FutureWarnin
g: Attribute `coef_` was deprecated in version 0.24 and will be removed in 1.
1 (renaming of 0.26).
  warnings.warn(msg, category=FutureWarning)
C:\Python39\lib\site-packages\sklearn\utils\deprecation.py:103: FutureWarnin
g: Attribute `coef_` was deprecated in version 0.24 and will be removed in 1.
1 (renaming of 0.26).
  warnings.warn(msg, category=FutureWarning)
C:\Python39\lib\site-packages\sklearn\utils\deprecation.py:103: FutureWarnin
g: Attribute `coef_` was deprecated in version 0.24 and will be removed in 1.
1 (renaming of 0.26).
  warnings.warn(msg, category=FutureWarning)
```

In [ ]:
```python
# 5. Cross validation
print(cross_val_score(estimator=a_1, X=pred_c_data, y=dfc["Y1-Y2"], cv=10))
print(cross_val_score(estimator=a_2, X=pred_c_data, y=dfc["Y1-Y2"], cv=10))
print(cross_val_score(estimator=a_3, X=pred_c_data, y=dfc["Y1-Y2"], cv=10))
print(cross_val_score(estimator=a_4, X=pred_c_data, y=dfc["Y1-Y2"], cv=10))

_="""
The model accuracy does not appear to change with varying values of alpha
"""
```

```
[0.24675325 0.87012987 0.87012987 0.87012987 0.87012987 0.87012987
 0.87012987 0.87012987 0.86842105 0.86842105]
[0.24675325 0.87012987 0.87012987 0.87012987 0.87012987 0.87012987
 0.87012987 0.87012987 0.86842105 0.86842105]
[0.24675325 0.87012987 0.87012987 0.87012987 0.87012987 0.87012987
 0.87012987 0.87012987 0.86842105 0.86842105]
[0.87012987 0.87012987 0.87012987 0.87012987 0.87012987 0.87012987
 0.87012987 0.87012987 0.86842105 0.86842105]
```

In [ ]:
```python
# 6.

x1=np.array([0.8,600.0,286.0,138.1,5,4,0.25,0]).reshape((1, 8))
x2=np.array([0.67,630.0,296.0,238.1,2,6,0.5,3]).reshape((1, 8))

for w in [w1, w2, w3, w4]:
    print((w * x1 + w * x2) / 2)
    # print(m.predict_proba(x2))
```

```
[[-7.33899212e-04 -6.14078933e-01 -2.90564178e-01 -1.87818288e-01
  -3.49475815e-03 -4.99251165e-03 -3.74438374e-04 -1.49775349e-03]]
[[-7.24143789e-03 -6.05916231e+00 -2.86701826e+00 -1.85321696e+00
  -3.44830376e-02 -4.92614822e-02 -3.69461117e-03 -1.47784447e-02]]
[[-6.39533621e-02 -5.35119968e+01 -2.53203107e+01 -1.63668400e+01
  -3.04539819e-01 -4.35056885e-01 -3.26292664e-02 -1.30517065e-01]]
[[-2.98016854e-01 -2.49361041e+02 -1.17990346e+02 -7.62679868e+01
  -1.41912788e+00 -2.02732554e+00 -1.52049416e-01 -6.08197662e-01]]
```

In [ ]: