

Date

⇒ Discrete Probability Distribution function / p.m.f. / p.d.f.

discrete s.v. $X \rightsquigarrow P(x)$
follows

$$0 \leq P(x) \leq 1$$

$$\sum_x P(x) = 1$$

— if a function holds the two properties then its discrete Probability dis. function.

⇒ Expected Values / $E(x)$:

Using the available data we predict the probability.

- Average is for past data.
- Expected value is for future data.

$$E(x) = \sum x \cdot P(x)$$

$$\Rightarrow \cancel{E(x)} \cdot E(g(x)) = \sum g(x) \cdot P(x)$$

\downarrow
function of x .

⇒ Variance of Expected value:

$$V(x) = E(x - E(x))^2$$
$$= \sum x^2 P(x) - [\sum x P(x)]^2$$

$$\text{Standard Deviation} = \sqrt{V(x)}$$

$$\left\{ \begin{aligned} V(x) &= \sum (x - \bar{x})^2 \\ &= E(x - \bar{x})^2 \\ &= E(x) - (E(x))^2 \end{aligned} \right.$$

⇒ Continuous Probability Distribution function, expected value
continuous s.v. $X \rightsquigarrow P(x)$ is mean. $\frac{\sum x}{n}$

$$0 \leq P(x) \leq 1$$



$$\int_{-\infty}^{+\infty} P(x) dx = 1$$

— at specific point probability will be zero. so we find area between two points.

Date

$$E(x) = \int x P(x) dx$$

$$V(x) = \int E(x - E(x))^2 dx$$

$$= \int x^2 P(x) - \int [x P(x)]^2 dx$$

R-language.

$x = c(1:6)$

↓ vector

$px = rep(1/6, 6)$

mean(x)

$Ex = sum(x * px)$

$Vx = sum(x^2 * px) - Ex^2$

$sdx = sqrt(vx)$

$c(round(Ex - sdx), round(Ex + sdx))$

$c(2, 5)$

⇒ Discrete Probability Distribution:-

1) ⇒ Binomial Probability distribution:-

* where we have two possibilities e.g. pass & fail.

* $X \sim b(n, p)$

* $P(\text{success}) = P$

* $P(\text{failure}) = 1 - P$

n :- no. of time exp. repeated.

x :- no. of success. where $x = 0, 1, \dots, n$

$$P(X=x) = {}^n C_x P^x (1-P)^{n-x}$$

r :- r binom \Rightarrow random no generator.

p :- p binom \Rightarrow probability

d :- density:-

q :- quantiles:-

$n = 10, P = 0.7,$

$P(X=1)$

$$P(X=1) = {}^{10} C_1 (0.7)^1 (1-0.7)^{10-1} = 0.000137$$

Date

⇒ 4 properties/characteristics to follow Binomial:-

- 1) Two Possible outcomes either success or failure.
- 2) $P(\text{Success}) = \text{Constant}$ in each trial (full-time repetition).
- 3) If trial is constant, then independent.
- 4) Repeated fixed no. of time "n"

— e.g. Coin Tossing follows Binomial distribution.

— in a deck of cards it is not constant like $\frac{1}{52}, \frac{1}{51}, \frac{1}{50}$

Example:- Coin is Tossed 10 times. and to get exactly 2 heads.

$$n=10 \quad P=\frac{1}{2} \quad P(X=2) = {}^{10}C_2 \left(\frac{1}{2}\right)^2 \left(1-\frac{1}{2}\right)^{10-2} \\ = 0.0439.$$

⇒ for at least 1 Head $\Rightarrow P(X \leq 1) \Rightarrow 1 - [P(X=0) + P(X=1)]$

⇒ for ~~at~~ x is greater than 2 $\Rightarrow P(X > 2) \Rightarrow 1 - [P(X=0) + P(X=1)]$

⇒ for $P(X > 1)$ ya tu saari $n=2, 3, 4, 5, \dots, 10$ karke add karlein ya $1 - P(X=0) + P(X=1)$
0, 1, 2, 3, \dots, 10

Example:- rolls a dice 10 times what is probability of at least getting 6 for one time?

$$n=10 \quad P=\frac{1}{6} \quad (1-P)=\frac{5}{6}$$

$$P(X \leq 1) = 1 - \left[{}^{10}C_0 \left(\frac{1}{6}\right)^0 \left(1-\frac{1}{6}\right)^{10-0} \right] + \left[{}^{10}C_1 \left(\frac{1}{6}\right)^1 \left(1-\frac{1}{6}\right)^{10-1} \right] \\ = 0.8384$$

Date

\Rightarrow 2 ya 2 se zyada tu uska liye 0 or 1 ka hi nikalain gay. 0, 1, 2, 3, 4, ..., 10

\Rightarrow Kam se Kam ek theek hoga ($x > 1$)

$$\begin{aligned} E(x) &= \sum x \cdot P(x) \\ &= \sum x \binom{n}{x} p^x (1-p)^{n-x} \\ &= n \cdot p. \end{aligned}$$

$$\begin{aligned} V(x) &= E(x^2) - [E(x)]^2 \\ &= \sum x^2 \binom{n}{x} p^x (1-p)^{n-x} - \left(\sum x p^x \right)^2 \\ &= n \cdot p (1-p). \end{aligned}$$

$$\text{Stdev.} = \sqrt{V(x)}$$

\Rightarrow What is the ^{expected value} ~~probability~~ of getting head if we toss 10 coins.

$$\begin{aligned} E(x) &= n \cdot p \\ &= 10 \cdot \frac{1}{2} = 5 \text{ times.} \end{aligned}$$

$$V(x) = 5 \left(1 - \frac{1}{2} \right) = 2.5$$

$$\text{St dev} = \sqrt{2.5}$$

$$\text{Measure of dispersion} = 5 \pm \sqrt{2.5}$$