

# **MATH 204 - Introduction to Formal Mathematics**

Fall 2023-2024

Shaheen Nazir
9-153A
TBA
shaheen.nazir@lums.edu.pk; waqas.azhar@lums.edu.pk
8250
TBA
LMS website

## **Course Teaching Methodology**

- Teaching Methodology: Lectures will be delivered in person scheduled by the Ro.
- Lecture Details: Lecture notes will be uploaded on LMS.

#### **Course Basics**

Credit Hours	3			
Lecture(s)	Nbr of Lec(s) Per Week	2	Duration	75 min
Recitation/Lab (per week)	Nbr of Lec(s) Per Week		Duration	
Tutorial (per week)	Nbr of Lec(s) Per Week	1	Duration	50 min
Meeting Times	MW 10:00AM-11:50AM		Location	TBA

#### **Course Distribution**

Core	Core for Math Majors
Elective	
Open for Student	All
Category	
Close for Student	None
Category	

### **COURSE PREREQUISITE(S)**

Math 120-Linear Algebra with Differential Equations

#### **COURSE DESCRIPTION**

Mathematics is created by the interplay between formal and informal, proof by logic and discovery by intuition. In this course, the student will see this interplay, learn simple proof techniques and heuristics, and come to appreciate the notion of mathematical structure.

## **COURSE GOAL**

- Develop an understanding of formal mathematics
- Understand the interplay between formal and informal mathematical reasoning
- Understand how to go about learning about mathematical structures



#### **Learning Outcomes**

## "I hear and I forget. I see and I remember. I do and I understand."- Chinese proverb

By the end of the course the student should be able to:

- Learn about the construction of real numbers and the difference between real and rational numbers
- Deal with sequence and series of real numbers, their convergence, divergence; Cauchy sequences
- Learn topology of real line, concepts of open sets and closed sets
- Learn the concepts of limits of functions and continuity of a function; uniform continuity
- Learn the concepts of differentiability and mean value theorem, Taylor's theorem
- Learn to read and write rigorous proofs
- Learn good mathematical writing skills and style; and
- Unravel abstract definitions, create intuition-forming examples or counterexamples.

#### **Grading Breakup and Policy**

#### Assignment(s): 15% (best 5 assignments)

(Tentative submission dates- A1: 20th Sep, A2: 4th Oct, A3: 18th Oct, A4: 8th Nov, A5: 22th Nov, A6: 6th Dec )

The date for assignments will be strictly adhered to and late submissions will not be entertained, unless faced with some unforeseen circumstances. If you have accessibility issues or any other emergency, please email me regarding graded instruments ASAP. Please use whatever resources aid you in learning the material, including me, other students, professors, other

math books, etc. I strongly recommend that you avoid copying from classmates, textbooks, or the internet.

This is known as plagiarism, and it is considered cheating.

Quiz(s): 20% (4 best)

(Q1: 27<sup>th</sup> Sep, Q2: 11<sup>th</sup> Oct, Q3: 15<sup>th</sup> Nov, Q4: 29<sup>th</sup> Nov, Q5: 6<sup>th</sup> Dec)

**Midterm Examination: 25%**(Date: 25<sup>th</sup> October) **Final Examination:40 %** (Date: TBA by RO)

#### **Examination Detail**

Midterm Exam	Yes/No: Yes Date:	
Final Exam	Yes/No: Yes Date:	Exam Specifications: Comprehensive

#### **Academic Honesty**

The principles of truth and honesty are recognized as fundamental to a community of teachers and students. This means that all academic work will be done by the student to whom it is assigned without unauthorized aid of any kind. Plagiarism, cheating and other forms of academic dishonesty are prohibited. Any instances of academic dishonesty in this course (intentional or unintentional) will be dealt with swiftly and severely. Potential penalties include receiving a failing grade on the assignment in question or in the course overall. For further information, students should make themselves familiar with the relevant section of the LUMS student handbook.

#### **Harassment Policy**

SSE, LUMS and particularly this class, is a harassment free zone. There is absolutely zero tolerance for any behavior that is intended or has the expected result of making anyone uncomfortable and negatively impacts the class environment, or any individual's ability to work to the best of their potential.

In case a differently abled student requires accommodation for fully participating in the course, students are advised to contact the instructor so that they can be facilitated accordingly.

If you think that you may be a victim of harassment or if you have observed any harassment occurring in the purview of this class, please reach out and speak to me. If you are a victim, I strongly encourage you to reach out to the Office of Accessibility and Inclusion at



oai@lums.edu.pk or the sexual harassment inquiry committee at harassment@lums.edu.pk for any queries, clarifications, or advice. You may choose to file an informal or a formal complaint to put an end of offending behavior. You can find more details regarding the LUMS sexual harassment policy <u>here</u>.

To file a complaint, please write to <a href="mailto:harassment@lums.edu.pk">harassment@lums.edu.pk</a>.

#### **SSE Council on Equity and Belonging**

In addition to LUMS resources, SSE's Council on Belonging and Equity is committed to devising ways to provide a safe, inclusive and respectful learning environment for students, faculty and staff. To seek counsel related to any issues, please feel free to approach either a member of the council or email at <a href="mailto:cbe.sse@lums.edu.pk">cbe.sse@lums.edu.pk</a>.

## **Mental Health Support at LUMS**

For matters relating to counselling, kindly email <a href="mailto:student.counselling@lums.edu.pk">student.counselling@lums.edu.pk</a>, or visit <a href="https://osa.lums.edu.pk/content/student-counselling-office">https://osa.lums.edu.pk/content/student-counselling-office</a> for more information.

You are welcome to write to me or speak to me if you find that your mental health is impacting your ability to participate in the course. However, should you choose not to do so, please contact the Counselling Unit and speak to a counsellor or speak to the OSA team and ask them to write to me so that any necessary accommodations can be made.

COURSE OVER	VIEW		
Week/ Lecture/ Module	Topics	Recommended Readings	Objectives/ Application
0	Mathematics for Human Flourishing (Francis Su)	YouTube [Su]	
	Communicating Mathematics		
1	Learning Mathematics, What Others Have Said About Writing, Mathematical Writing, Using Symbols, Writing Mathematical Expressions, Common Words and Phrases in Mathematics, Some Closing Comments About Writing	Chapter 0 [CPZ] Chapter 1[D]	
	Sets	Section 3[B]	
	Describing a Set, Subsets, Set Operations, Indexed Collections of	Chapter 1 [CPZ]	
2	Sets, Partitions of Sets, Cartesian Products of Sets		
3-6	Logics  Statements, Negations, Disjunctions and Conjunctions, Implications, More on Implications, Biconditionals, Tautologies and Contradictions, Logical Equivalence, Some Fundamental Properties of Logical Equivalence, Quantified Statements, Characterizations	Section 1 [B] Chapter 2 [CPZ] Chapter 2[D]	
	Direct Proof and Proof by Contrapositive		
7-10	Trivial and Vacuous Proofs, Direct Proofs, Proof by Contrapositive, Proof by Cases, Proof Evaluations, Proofs Involving Real Numbers, Proofs Involving Sets, Fundamental Properties of Set Operations, Proofs Involving Cartesian Products of Sets	Chapter 3 &4 [CPZ] Chapter 3[D] Section 2 &3 [H]	
	Existence and Proof by Contradiction	Chapter 5 [CPZ]	
11-12	Counterexamples, Proof by Contradiction, A Review of Three	Chapter 3[D]	
11-12	Proof Techniques, Existence Proofs, Disproving Existence	Section 2 &3 [H]	
	Statements		
13	Mathematical Induction		



	The Principle of Mathematical Induction, A More General Principle of Mathematical Induction, The Strong Principle of Mathematical Induction, Proof by Minimum Counterexample	Chapter 6[CPZ] Chapter 3[D]	
14	Equivalence Relations  Relations, Properties of Relations, Equivalence Relations,  Properties of Equivalence Classes, Congruence Modulo n, The Integers Modulo n	Chapter 4[B] Chapter 9[CPZ]	
15-16	Functions  The Definition of Function, One-to-one and Onto Functions, Bijective Functions, Composition of Functions, Inverse Functions	Chapter 5[B] Chapter 10[CPZ]	
17-20	Cardinalities of Sets  Numerically Equivalent Sets, Denumerable Sets, Uncountable Sets, Comparing Cardinalities of Sets, The Schroeder-Bernstein Theorem	Chapter 11[CPZ]	
21-25	Proofs in Number theory  Divisibility Properties of Integers and rational numbers, The Division Algorithm, Greatest Common Divisors, The Euclidean Algorithm, Relatively Prime Integers, The Fundamental Theorem of Arithmetic, Concepts Involving Sums of Divisors	Chapter 12[CPZ] Chapter1&2 [HW] [AZ]	
25-28	Proofs in Combinatorics  The Multiplication and Addition Principles, The Principle of Inclusion-Exclusion, The Pigeonhole Principle, Permutations and Combinations, The Pascal Triangle, The Binomial Theorem, Permutations and Combinations with Repetition	Chapter 13[CPZ] Chapter 2-4[M] [AZ]	

### **Textbook(s)/Supplementary Readings**

- [AZ] Martin Aigner, Günter M. Ziegler, Proofs from THE BOOK, Springer-Verlag Berlin Heidelberg 2010.
- [B] B. J. Baker, Introduction to Advanced Mathematics, JOURNAL OF INQUIRYBASED LEARNING No. 42, (May 2016)
- [BR] M. Beck & R. Geoghegan, The Art of Proof- Basic Training for Deeper Mathematics, 2011-Springer
- [CPZ] Gary Chartrand, Albert D. Polimeni, Ping Zhang, Mathematical Proofs: A Transition to Advanced Mathematics,
- [D] K. Devlin, Introduction to Mathematical Thinking
- [H] M. Hutchings Introduction to mathematical arguments (background handout for courses requiring proofs)
- [HW] G.H. Hardy, E. M. Wright, An introduction to the theory of numbers, Oxford.
- [M] J. Morris, Combinatorics,
- [S] A. Stefanowicz, Proofs and Mathematical Reasoning,
- [Su] F. Su Mathematics for Human Flourishing, Yale University press 2020.

Note: Course material and any info about the course will be uploaded on LMS. It is your responsibility to visit LMS regularly.