

## MATH 210 – Introduction to Differential Equations Fall 2023-2024

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Course URL (if any)	www.lms.lums.edu.pk

Course Basics				
Credit Hours	3			
Lecture(s)	Nbr of Lec(s) Per Week	2	Dur atio n	75 min
Recitation/Lab (per week)	Nbr of Lec(s) Per Week		Dur atio n	
Tutorial (per week)	Nbr of Lec(s) Per Week		Dur atio n	

Course Distribution		
Core	Core for math majors	
Elective		
Open for Student Category	All students	
Close for Student Category	None	

#### **COURSE DESCRIPTION**

Differential equations (DEs) involve derivatives of functions. Differential equations have remained at the forefront of various disciplines including applied mathematics, physics, engineering, chemistry, biology, economics and social sciences. These equations enable a natural, physical and social phenomena to be expressed in the language of mathematics and provide useful information about the phenomena. The study of these equations and their solutions is therefore important. This course deals with: First order differential equations; modeling; second order linear equations; Damped motion in mechanical and electrical systems; System of first order linear equations; Eigenvalues and Eigenvectors; Series solutions; Introduction to special functions; Fourier series; Partial differential equations; separation of variables and Sturm-Liouville problems.



COURSE PREREQUISITE(S)		
•	MATH 120 & Anti-reg: EE 203	
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COURSE OBJE	COURSE OBJECTIVES	
	Students will be trained to:	
•	Solve differential equations by suitable methods Apply differential equations to realworld problems. Analyze solutions of differential equations	

Learning Outcomes		
	Students will learn to:	
•	Solve differential equations using suitable methods.	
•	Apply mathematics to problems in social, physical and engineering sciences etc.	
•	Formulate first order ordinary differential equations for modelling of population dynamics and Mechanics etc.	
•	Formulate second order differential equations to model physical problems involving damped motion in mechanical electrical	
•	system.	

Find Series solutions of higher order linear differential equations. Solve systems of first order linear equations by using matrix algebra. Use method of separation of variables for solutions of PDEs.

### **Grading Breakup and Policy**

Assignments: 10%

Class

Participation: Attendance:

Midterm Examination:

40%Project:

Final Examination: 50%



Examination I	Examination Detail		
Midter m Exa m	Yes/No: Yes Combine/Separat e: Duration: TBA Exam Specifications: No notes/No books/No calculators		
Final Exam	Yes/No: Yes Combine Separate: Duration: TBA Exam Specifications: No notes/No books/No calculators		

COURSE OVERVIEW			
We ek	T O p i C S	Recommmended	Objectives/Application
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4	Second order linear differential equations Initial value and boundary value problems Homogeneous equations Characteristic equations Complex roots, repeated roots; reduction of order	Solutions of differential equations
5	Non- homogeneous equations Undetermined coefficients Method Wronskian, Particular Solution	Solutions of differential equations
6	Modeling Damped motion in mechanical and electrical	Modeling of real-world systems



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7	System of first order Linear equations Introduction, Review of matrices, linear independence Eigen values, Eigen vectors	Solutions of differential equations
8	Basic theory of systems of first order linear equations, Homogeneous linear system with constant coefficients	Solutions of differential equations
9	Complex Eigen values, repeated Eigen values,Non homogeneous linear system Series Solutions Power series	Solutions of differential equations
10	Series solutions near an ordinary point, Classification of singular points Frobenius theorem	Solutions of differential equations
11	Frobenius theorem (Continue), Introduction tospecial functions Partial Differential equations and Fourier seriesTwo point boundary value problems, Fourier series	Solutions of differential equations
12	The Fourier convergence theorem, even andodd functions Separation of variables and partial differential equations	Solutions of differential equations
13	Separation of variables and partial differentialequations (Continue) Sturm-Liouville problems	Solutions of differential equations
	Sturm-Liouville problems (Continue)	Solutions of differential equations

### Textbook(s)/Supplementary Readings

There is no required text but the following texts will be used for reference.

- 1) Differential equations with boundary-value problems by Dennis G. Zill and Michael R. Cullin (7th Edition Brooks/Cole)
- 2) Elementary differential equations and boundary value problems by William E. Boyce and Richard C. Diprima. (Seventh Edition John Wiley& Sons, Inc.)