

# **Lahore University of Management Sciences MATH 309 –Introduction to Analysis II**

Fall 2023-2024

| Instructor      | Shaheen Nazir                 |
|-----------------|-------------------------------|
| Room No.        | 9-153A                        |
| Office Hours    | TBA                           |
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| Secretary/TA    | Shazia Zafar & Noreen Sohail/ |
| TA Office Hours |                               |
| Course URL (if  | LMS website                   |
| any)            |                               |

#### **Course Teaching Methodology**

- Teaching Methodology: Lectures will be delivered on time scheduled by the Ro.
- Lecture Details: Lecture notes and recorded zoom lectures will be uploaded on LMS.

# Course Basics

| Credit Hours              | 3                      |     |          |        |  |  |
|---------------------------|------------------------|-----|----------|--------|--|--|
| Lecture(s)                | Nbr of Lec(s) Per Week | 2   | Duration | 75 min |  |  |
| Recitation/Lab (per week) | Nbr of Lec(s) Per Week |     | Duration |        |  |  |
| Tutorial (per week)       | Nbr of Lec(s) Per Week | 1   | Duration | 50 min |  |  |
| Meeting Times             |                        | TBA | Location | TBA    |  |  |

#### **Course Distribution**

| Core              | For Math Major |
|-------------------|----------------|
| Elective          |                |
| Open for Student  | All students   |
| Category          |                |
| Close for Student | None           |
| Category          |                |

#### **COURSE PREREQUISITE(S)**

Math 205 (Introduction to Analysis-I)

#### **COURSE DESCRIPTION**

Introduction to Analysis II is the sequel to Introduction to Analysis I, and together these two courses constitute the foundations of real analysis in mathematics. This course is designed to prepare students for future advanced analysis and functional analysis courses. It lays the foundation for several other areas, such as complex analysis, topology, dynamical systems, quantum mechanics, and mathematical statistics. The rigorous treatment of the subject in terms of theory and examples gives students the flavor of mathematical reasoning and intuition for other advanced topics in mathematics. Differentiability and Riemann integrals are introduced and their properties are studied in details. Topics covered are open, closed, and compact sets of real numbers. Sequences and series of functions, point wise and uniform convergence. Power series and Taylor series. Metric spaces: basic notions generalized from the setting of the real numbers.



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#### **COURSE GOAL**

In this course, we will

- construct the Riemann Integral and state its properties
- state the Fundamental Theorem of Calculus and use it in proofs
- define pointwise and uniform convergence of series of functions
- use the Weierstrass M-Test to check for uniform convergence of series
- construct Taylor Series and state Taylor's Theorem;
- identify necessary and sufficient conditions for term-by-term differentiation of power series
- learn improper integrals
- differentiate Riemann and Lebesgue Integrable functions
- define metric space; open, closed sets
- learn complete and compact spaces

#### **Learning Outcomes**

At the end of the course, students will:

- present an overview of the basic properties of metric spaces;
- construct rigorous mathematical proofs of basic results in integration;
- appreciate how abstract ideas and rigorous methods in mathematical analysis can be applied to important practical problems,
- recognize the power of abstraction and generalization, and apply logical reasoning to investigate mathematical work with independent judgment,
- apply rigorous deductive reasoning in conjunction with quantative methods to analyze and solve problems related to math profession

#### Grading Breakup and Policy (tentative)

Assignment(s): 15%

Due dates: 20<sup>th</sup> Sept for A1, 4<sup>th</sup> Oct for A2, 18<sup>th</sup> Oct for A3, 8<sup>th</sup> Nov for A4, 22<sup>nd</sup> Nov for A5

Quiz(s): 20%

27<sup>th</sup> Sep for Q1, 11<sup>th</sup> Oct for Q2, 15<sup>th</sup> Nov for Q3, 29<sup>th</sup> Nov for Q4

Project: 10% (dates for presentations will be announced later)

Every student will be assigned a topic/Theorem. Your task is to write a short note and a presentation on your assigned topic. The write-up should contain your original explanation with elaborated examples. Topics will be announced later.

Midterm Examination: 20% (25th Oct class time) Final Examination (Written and Oral): 35 % (TBA)

### Dates for graded components:

The date for assignments, quizzes and exams will be strictly adhered to and late submissions will not be entertained, unless faced with some unforeseen circumstances.

If you have accessibility issues or any other emergency, please email me regarding graded instruments ASAP.



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| <b>Examination Detail</b> |             |                                    |
|---------------------------|-------------|------------------------------------|
|                           | Yes/No: Yes |                                    |
| Midterm Exam              | Date:       |                                    |
|                           | Yes/No: Yes | Exam Specifications: Comprehensive |
| Final Exam                | Date: TBA   |                                    |
|                           |             |                                    |

#### **Academic Honesty**

The principles of truth and honesty are recognized as fundamental to a community of teachers and students. This means that all academic work will be done by the student to whom it is assigned without unauthorized aid of any kind. Plagiarism, cheating and other forms of academic dishonesty in this course (intentional or unintentional) will be dealt with swiftly and severely. Potential penalties include receiving a failing grade on the assignment in question or in the course overall. For further information, students should make themselves familiar with the relevant section of the LUMS student handbook.

#### Harassment Policy

SSE, LUMS and particularly this class, is a harassment free zone. There is absolutely zero tolerance for any behavior that is intended, or has the expected result of making anyone uncomfortable and negatively impacts the class environment, or any individual's ability to work to the best of their potential.

In case a differently-abled student requires accommodations for fully participating in the course, students are advised to contact the instructor so that they can be facilitated accordingly.

If you think that you may be a victim of harassment or if you have observed any harassment occurring in the purview of this class, please reach out and speak to me. If you are a victim, I strongly encourage you to reach out to the Office of Accessibility and Inclusion at oai@lums.edu.pk or the sexual harassment inquiry committee at harassment@lums.edu.pk for any queries, clarifications, or advice. You may choose to file an informal or a formal complaint to put an end of offending behavior. You can find more details regarding the LUMS sexual harassment policy here.

To file a complaint, please write to <a href="mailto:harassment@lums.edu.pk">harassment@lums.edu.pk</a>.

#### SSE Council on Equity and Belonging

In addition to LUMS resources, SSE's Council on Belonging and Equity is committed to devising ways to provide a safe, inclusive and respectful learning environment for students, faculty and staff. To seek counsel related to any issues, please feel free to approach either a member of the council or email at cbe.sse@lums.edu.pk

#### Mental Health Support at LUMS

For matters relating to counselling, kindly email <a href="mailto:student.counselling@lums.edu.pk">student.counselling@lums.edu.pk</a>, or visit <a href="https://osa.lums.edu.pk/content/student-counselling-office">https://osa.lums.edu.pk/content/student-counselling-office</a> for more information.

You are welcome to write to me or speak to me if you find that your mental health is impacting your ability to participate in the course. However, should you choose not to do so, please contact the Counselling Unit and speak to a counsellor or speak to the OSA team and ask them to write to me so that any necessary accommodations can be made.



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| COURSE OVERVIEW             |   |   |  |  |  |
|-----------------------------|---|---|--|--|--|
| Week/<br>Lecture/<br>Module | Topics  | Recommended<br>Readings                         | Objectives/ Application  |  |  |
| 1-4                         | Review  Real numbers, sequences and series of real numbers, Limits, Continuity, Uniform Continuity and differentiability  | Lecture Notes                                   | Recall and review the contents<br>learnt in Math 205   |  |  |
| 5-9                         | The Riemann Integral  The Riemann sum, Darboux integration, Riemann Integrable Functions, The Fundamental Theorems  | Chapter 7[BS] Chapter 11[T1]                    | Rigorous definition of integration<br>and its properties; Formal proofs<br>of all forms of Fundamental<br>theorem of Calculus  |  |  |
| 10-16                       | Metric Spaces  Examples of Metric Spaces, Open Sets, Closed Sets, Convergent Sequences, Continuous Mappings Between Metric Spaces, Complete Metric Spaces, Compact Metric Spaces  | Lecture notes & [Ru] Chapter 1&2[T2]            | Basic definition, properties of metric spaces are given. It generalize the idea of real line with distance function to an abstract space with some metric/distance function  |  |  |
| 17-21                       | Sequences and Series of Functions  Pointwise and Uniform Convergence, Continuity and Uniform Convergence, Integration and Uniform Convergence, Uniform Convergence and Differentiation, Series of Functions, Power Series | Chapter 8 & 9(Section 9.4) [BS] Chapter 4&4[T2] | In Math 205, sequences and series of real numbers are studied. In this part, we describe these notions for functions; also describe some important functions as power series |  |  |
| 22-28                       | Improper and Lebesgue Integrals  Improper Integrals, Infinite Intervals, step functions on intervals, Lebesgue Integrable functions on intervals, Relation with Riemann integrals   | Lecture notes & [Tr]                            | Convergence or divergence of improper integrals will be discussed. Also, we will introduce and learn the Lebesgue integrals.   |  |  |

#### Textbook(s)/Supplementary Readings

- [BS] R. G. Bartle, D. R. Sherbert, Introduction to Real Analysis, 3<sup>rd</sup> edition, John Wiley & Sons, Inc 2000
- [T1] Terence Tao, Analysis I, Third Edition, Springer, 2016
- [T2] Terence Tao, Analysis II, Third Edition, Springer, 2016
- [Tr] W. Trench, Introduction to Real Analysis
- [TB2] B S. Thomson B. Bruckner, A M. Bruckner, Elementary Real Analysis Prentice Hall (Pearson) 2001
- [L] Notes on Real Analysis by Lee Larson. Available at <a href="http://www.math.louisville.edu/~lee/ira/IntroRealAnal.pdf">http://www.math.louisville.edu/~lee/ira/IntroRealAnal.pdf</a>
- [Ru] W. Rudin,, Principles of Mathematical Analysis, 3<sup>rd</sup> edition

Course material and any info about the course will be uploaded on LMS. It is your responsibility to visit LMS regularly.