Lab 7

Linearity in Simulink MATLAB

7.1 Linearity

Suppose that when one inputs x_1 to a system, the output of the system is y_1, and when one inputs x_2 to a system, the output of the system is y_2. A system is said to be linear if from these two facts one can conclude that if the input to the system is $\alpha_1 x_1 + \alpha_2 x_2$, then the output of the system will be $\alpha_1 y_1 + \alpha_2 y_2$. Linear systems are said to satisfy *the principle of superposition.*

7.2 Simulink

To "see" when a system is linear and when it is not, we make use of Simulink—a MATLAB "add-on." Simulink is, essentially, a graphical user interface (GUI) to MATLAB. It allows one to drag-and-drop blocks to build up a system.

To open Simulink, one can type "simulink" at the MATLAB command prompt. Alternatively one can click on the Simulink icon on the toolbar in the main MATLAB window.

After performing either of these actions, the Simulink Library Browser will open. After it has opened, click on the blank page in this window's toolbar to open a new Simulink "page." To start working, one drags and drops items from the library browser into the worksheet, one connects the items, and then one runs the simulation.

7.3 A Simple Example

We start by building a system that amplifies a sine wave by a factor of two. To do this, go to the browser, click on the "sources" tab (in the Simulink "blockset"), and then drag a "sine wave" over to the untitled worksheet.

Having actually put something in the worksheet, it is probably best to save the worksheet; do so. Note that Simulink saves its worksheets with a `.mdl` extension. (Make sure to save worksheets regularly while working on them.)

Next, go to the "Math Operations" tab, click on it and drag a gain block to the worksheet. To connect the sine wave to the gain block select the sine wave block, hold down the control key and left-click on the gain block. (This is the general procedure for connecting blocks.)

Go to the "Sinks" tab, click on it, and drag a Scope block from the right-hand panel to the worksheet. Then connect the gain block to the scope. Double click on the gain block, and use the dialog box that opens to change the gain of the gain block from 1 to 2. Double click on the sine wave block, and change its frequency from 1 radian per second to 2π radians per second. (Enter `2 * pi` as the frequency.) Double click on the scope to actually open a scope window. Finally hit the "play" button on top of the worksheet window, and a sine wave should appear in the scope window.

The sine wave may not be very pretty. Simulink is numerical software, and it is not always good at "guessing" how many samples of a function the user needs. In our case, in order to improve the quality of the sine wave, one can click on the sine wave block again and change the sample time from zero—which is "continuous-time"—to 0.001 s—which gives lots of samples in each period of the sine wave. Make the change and hit play again. How does the sine wave look now?

7.4 Testing Linearity

We now test the linearity of the gain block. To do this, build the system of Figure 7.1. The following tips should prove helpful.

- To copy a block, hold down the control key, left-click on the block and drag the copy to wherever it is needed.

- The summing block (the circle with the pluses inside) is located in the "Math Operations" library.

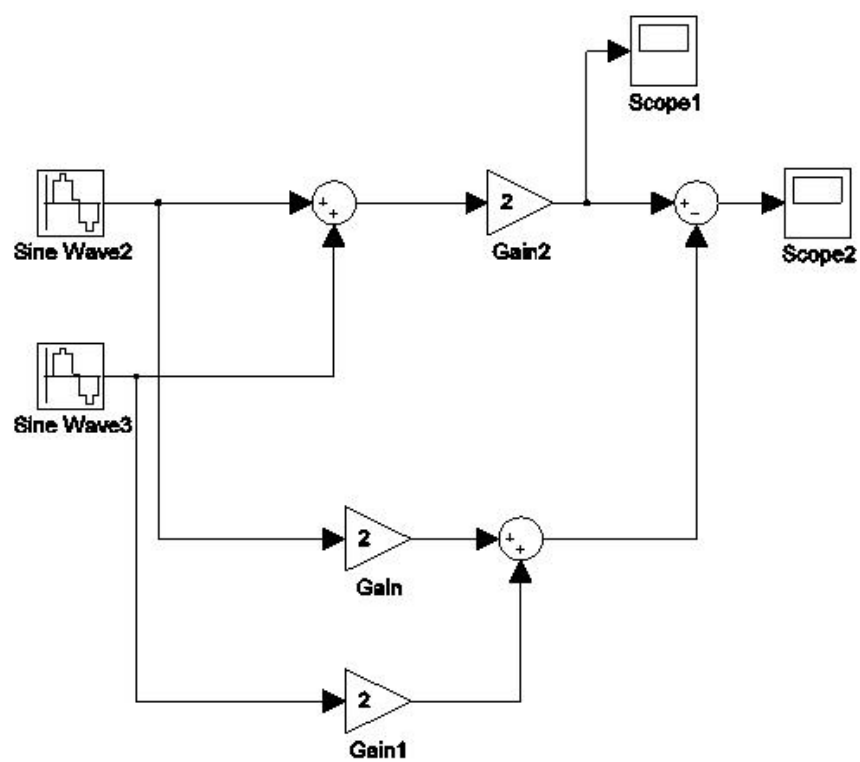- Double clicking on the summing block opens up a dialog box that allows

Figure 7.1: A system to test the linearity of the gain block

one to change the sums to differences. That is how one produces a differencing block.

- To add a connection to a "wire," hold down the control key, click on the spot on the wire to which you would like to add a connection, and drag the cursor to the input of the item to which you would like to connect. Release the cursor when a double cross-hair is shown over the block's input.

- In order to open a scope window, double click on the scope of interest.

- If one does not see the whole signal on a scope, the problem is probably that the window is limiting the number of samples that it saves. To remove this restriction, go to the scope window. Go to the "parameters" tab (the second tab from the left) and click on it. A dialog box will open. Click on the "data history" tab, and unclick the "limit data points to last" box.

- Make the frequencies of the two sine waves different.

7.5 A Nonlinear System

Save the model that you have build for the linear system. Pick another block that *should* be nonlinear—perhaps the "sign" block in the "Math Operations" sub-library—and replace all the gains with this block. Run the simulation again. What is the output of the differencing block? How do your results show that the new block is not linear—that it is non-linear?

7.6 Exercise

1. Use Simulink to design a non-linear system, and show that the system is non-linear by showing that the principle of superposition does not hold.

2. Use the symbolic toolbox to show that the squaring operation is not linear. That is, use the symbolic toolbox to show that

$$(\alpha x + \beta y)^2 \neq \alpha x^2 + \beta y^2.$$