Discrete Structures

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Text book

Discrete Mathematics and Its Application, 7th Edition

Kenneth H. Rosen

References

Chapter 9

Discrete Mathematics and Its Application, 7th Edition

by Kenneth H. Rose

These slides contain material from the above resource.

Determining which Properties a Relation has from its Digraph

- 1. A relation is **reflexive** if and only if there is **a loop at every vertex** of the **directed graph**, so that every ordered pair of the form *(x, x)* occurs in the relation.

- 2. A relation is **symmetric** if and only if for **every edge between distinct vertices** in its digraph there is an **edge in the opposite direction**, so that ***(y, x)*** is in the relation **whenever *(x, y)*** is in the relation.

- 3. Similarly, a relation is **antisymmetric** if and only if there are never **two edges in opposite directions** between distinct vertices.

Determining which Properties a Relation has from its Digraph

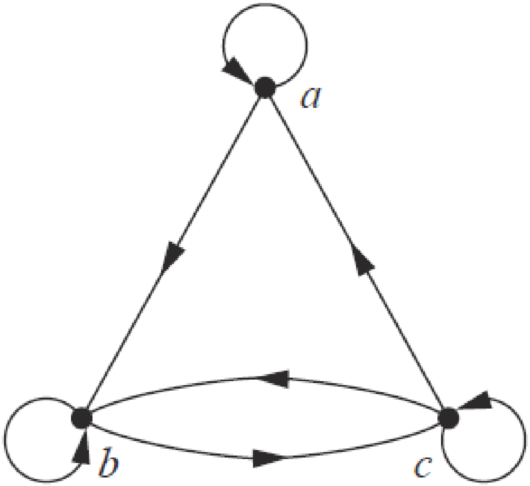
- 4. Finally, a relation is transitive if and only if whenever there is an **edge from a vertex *x* to a vertex *y*** and an edge from a **vertex *y* to a vertex *z***, there is an edge from ***x* to *z*** (completing a triangle where each side is a directed edge with the correct direction).

Remark: Note that a symmetric relation can be represented by an undirected graph, which is a graph where edges do not have directions.

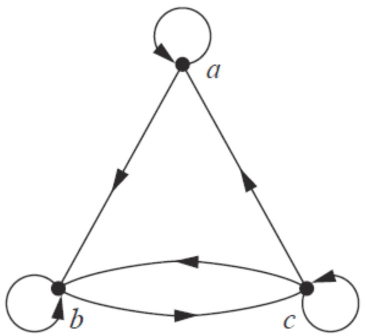
Determining which Properties a Relation has from its Digraph

- ***Reflexivity***: A loop must be present at all vertices in the graph.
- ***Symmetry***: If (x, y) is an edge, then so is (y, x).
- ***Antisymmetry***: If (x, y) with $x \neq y$ is an edge, then (y, x) is not an edge.
- ***Transitivity***: If (x, y) and (y, z) are edges, then so is (x, z).

Example Determine whether the relations for the directed graphs shown in Figure below is reflexive, symmetric, antisymmetric, and/or transitive.



(a) Directed graph of R



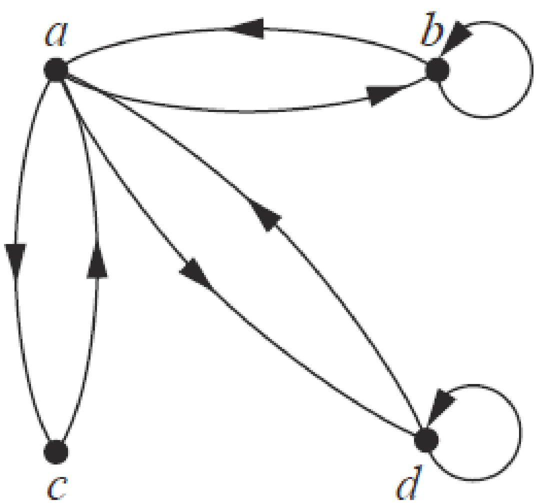
(a) Directed graph of R

Solution: Because there are **loops** at every **vertex** of the directed graph of *R*, it is **reflexive**.

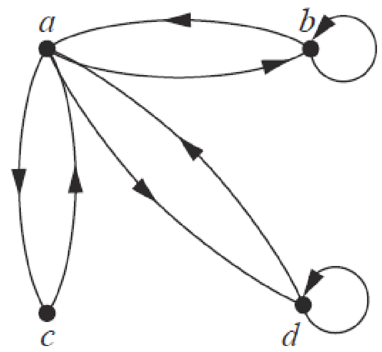
R is neither symmetric nor antisymmetric because there is an edge from ***a* to *b*** but not one from ***b* to *a***, but there are edges in **both directions** connecting *b* and *c*.

Finally, *R* is **not transitive** because there is an edge from ***a* to *b*** and an edge from ***b* to *c***, but **no edge from *a* to *c***.

Example Determine whether the relations for the directed graphs shown in Figure below is reflexive, symmetric, antisymmetric, and/or transitive.



(b) Directed graph of S

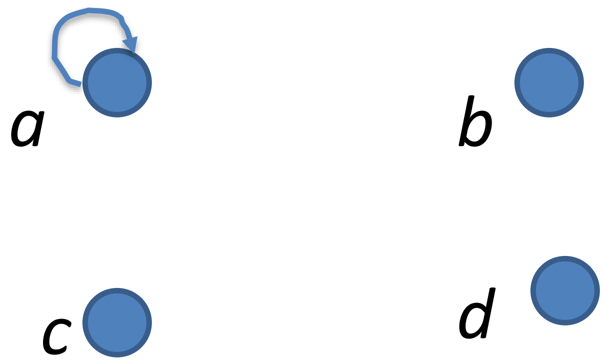


(b) Directed graph of S

Solution:

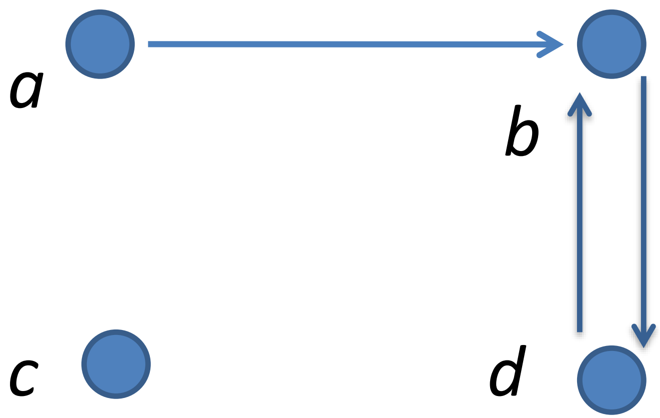
- Because **loops are not present** at all the vertices of the directed graph of *S*, this relation is **not reflexive**.
- It is **symmetric and not antisymmetric**, because every edge **between distinct vertices is accompanied by an edge** in the opposite direction.
- The directed graph that *S* is **not transitive**, because ***(c, a)*** and ***(a, b)*** belong to *S*, but ***(c, b)*** does not belong to *S*.

Determining which Properties a Relation has from its Digraph – Example 1



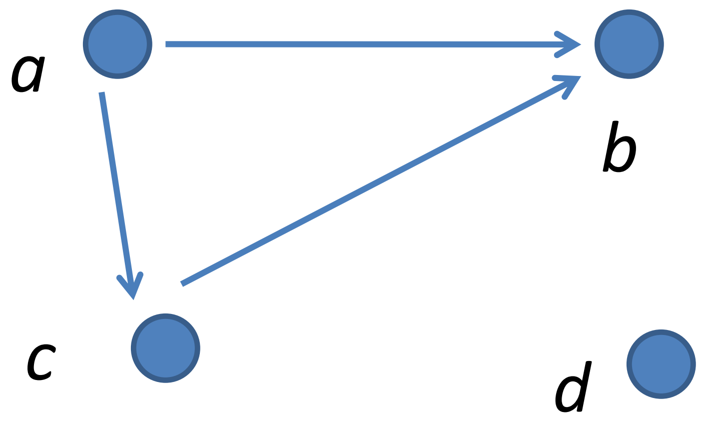
- ❑ ***Reflexive?*** No, there are no loops
- ❑ ***Symmetric?*** No, for example, there is no edge from *d* to *a*
- ❑ ***Antisymmetric?*** Yes, whenever there is an edge from one vertex to another, there is not one going back
- ❑ ***Transitive?*** Yes (trivially), there are no two edges where the first edge ends at the vertex where the second edge begins

Determining which Properties a Relation has from its Digraph – Example 2



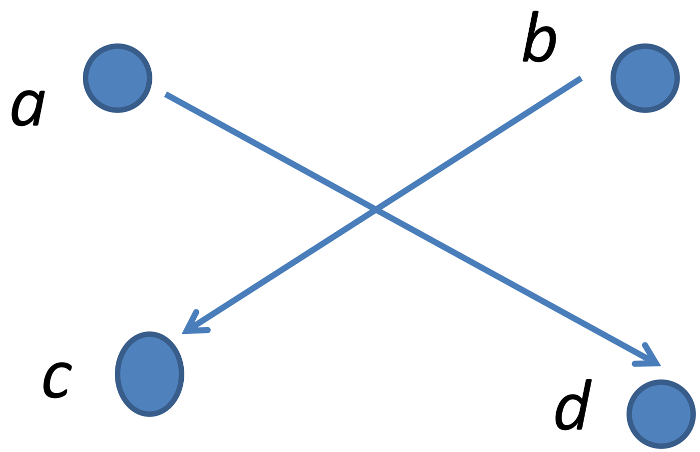
- ***Reflexive?*** No, there are no loops
- ***Symmetric?*** No, there is an edge from *a* to *b*, but not from *b* to *a*
- ***Antisymmetric?*** No, there is an edge from *d* to *b* and *b* to *d*
- ***Transitive?*** No, there are edges from *a* to *c* and from *c* to *b*, but there is no edge from *a* to *d*

Determining which Properties a Relation has from its Digraph – Example 3



- ***Reflexive?*** No, there are no loops
- ***Symmetric?*** No, for example, there is no edge from *c* to *a*
- ***Antisymmetric?*** Yes, whenever there is an edge from one vertex to another, there is not one going back
- ***Transitive?*** No, there is no edge from *a* to *b*

Determining which Properties a Relation has from its Digraph – Example 4



- ***Reflexive?*** No, there are no loops
- ***Symmetric?*** No, for example, there is no edge from d to a
- ***Antisymmetric?*** Yes, whenever there is an edge from one vertex to another, there is not one going back
- ***Transitive?*** Yes (trivially), there are no two edges where the first edge ends at the vertex where the second edge begins

Suggested Readings

9.3 Representing Relations