

9.41

Trial and error, $p = 43$, $q = 67$

$$\phi(n) = (43-1) \times (67-1) = 2772$$

$$65 \bmod 2772$$

$$2772 = 65 \times 42 + \underline{42}$$

$$65 = 42 \times 1 + \underline{23}$$

$$42 = 23 \times 1 + \underline{19}$$

$$23 = 19 \times 1 + \underline{4}$$

$$19 = 4 \times 4 + \underline{3}$$

$$4 = 3 \times 1 + \underline{1}$$

Rewrite:

$$42 = 2772 - 65 \times 42$$

$$23 = 65 - 42 \times 1$$

$$19 = 42 - 23 \times 1$$

$$4 = 23 - 19 \times 1$$

$$3 = 19 - 4 \times 4$$

$$1 = 4 - 3 \times 1$$

$$42 = 2772 - 65 \times 42$$

$$23 = 65 - 42 \times 1$$

$$19 = 42 - 23 \times 1$$

$$4 = 28 - 19 \times 1$$

$$3 = 19 - 4 \times 4$$

$$1 = 4 - 3 \times 1$$

$$1 = 4 - (19 - 4 \times 4)$$

$$= 4 - 19 + 4 \times 4$$

$$= 4 \times 5 - 19$$

$$= (23 - 19 \times 1) \times 5 - 19$$

$$= 23 \times 5 - 19 \times 6$$

$$= 23 \times 5 - (42 - 23 \times 1) \times 6$$

$$= 23 \times 5 - 42 \times 6 + 23 \times 6$$

$$= 23 \times 11 - 42 \times 6$$

$$= \cancel{65 \times 11}$$

$$= (65 - 42 \times 1) \times 11 - 42 \times 6$$

$$= 65 \times 11 - 42 \times 17$$

$$= 65 \times 11 - (2772 - 65 \times 42) \times 17$$

$$= 65 \times 4 - 2772 \times 17 + 65 \times 714$$

$$= 65 \times 725 - 2772 \times 17$$

$$= 65 \times (725) + 2772 \times (-17)$$

$$\therefore d = 725$$

$$\therefore PR = \{775, 2881\}$$