



Assignment 1

Deadline 30 September 2020 11:55 pm

Course Code: CS211

Course Name: Discrete Structures

Q1: Suppose that Smartphone A has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

- a) Smartphone B has the most RAM of these three smartphones.
- b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.
- c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
- d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
- e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

Q2: 10. Let p and q be the propositions

p: I bought a lottery ticket this week.

q: I won the million dollar jackpot.

Express each of these propositions as an English sentence.

- a) $\neg p$ b) $p \vee q$ c) $p \rightarrow q$
- d) $p \wedge q$ e) $p \leftrightarrow q$ f) $\neg p \rightarrow \neg q$
- g) $\neg p \wedge \neg q$ h) $\neg p \vee (p \wedge q)$

Q3: Let p , q , and r be the propositions

p : You have the flu.

q : You miss the final examination.

r : You pass the course.

Express each of these propositions as an English sentence

a) $p \rightarrow q$ b) $\neg q \leftrightarrow r$ c) $q \rightarrow \neg r$

d) $p \vee q \vee r$ e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$

f) $(p \wedge q) \vee (\neg q \wedge r)$

Q4. For each of these sentences, determine whether an inclusive or, or an exclusive or, is intended. Explain your answer.

a) Experience with C++ or Java is required.

b) Lunch includes soup or salad.

c) To enter the country you need a passport or a voter registration card.

d) Publish or perish.

Q5: State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows tonight, then I will stay at home.

b) I go to the beach whenever it is a sunny summer day.

c) When I stay up late, it is necessary that I sleep until noon.

Q6: Explain, without using a truth table, why $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is true when p , q , and r have the same truth value and it is false otherwise.

Q7: Express these system specifications using the propositions p : "The user enters a valid password,"

q : "Access is granted," and r : "The user has paid the subscription fee" and logical connectives (including negations).

a) "The user has paid the subscription fee, but does not enter a valid password."

b) "Access is granted whenever the user has paid the subscription fee and enters a valid password."

c) "Access is denied if the user has not paid the subscription fee."

d) "If the user has not entered a valid password but has paid the subscription fee, then access is granted."

Q8: Suppose that in Example 7, the inscriptions on Trunks 1, 2, and 3 are "The treasure is in Trunk 3," "The treasure is in Trunk 1," and "This trunk is empty." For each of these statements, determine whether the Queen who never lies could state this, and if so, which trunk the treasure is in.

a) "All the inscriptions are false."

b) "Exactly one of the inscriptions is true."

c) "Exactly two of the inscriptions are true."

d) "All three inscriptions are true."

Q9. Use De Morgan's laws to find the negation of each of the Following statements.

a) Jan is rich and happy.

b) Carlos will bicycle or run tomorrow.

c) Mei walks or takes the bus to class.

d) Ibrahim is smart and hard working.

Q10: A detective has interviewed four witnesses to a crime .From the stories of the witnesses the detective has concluded that if the butler is telling the truth then so is the cook; the cook and the gardener cannot both be telling the truth; the gardener and the handyman are not both lying; and if the handyman is telling the truth then the cook is lying. For each of the four witnesses, can the detective determine whether that person is telling the truth or lying?

Q11: Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology.

Q12: Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

Q13: Show that $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logically equivalent.

Q14: Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.

Q15: Show that $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$ are logically equivalent.

Q16: Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.

Q17: Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically Equivalent.

Q18: Let $N(x)$ be the statement “x has visited North Dakota,” where the domain consists of the students in your school. Express each of these quantifications in English.

a) $\exists x N(x)$ b) $\forall x N(x)$ c) $\neg \exists x N(x)$ d) $\exists x \neg N(x)$

e) $\neg \forall x N(x)$ f) $\forall x \neg N(x)$

Q19: Translate these statements into English, where $R(x)$ is “ x is a rabbit” and $H(x)$ is “ x hops” and the domain consists of all animals.

- a) $\forall x(R(x) \rightarrow H(x))$
- b) $\forall x(R(x) \wedge H(x))$
- c) $\exists x(R(x) \rightarrow H(x))$
- d) $\exists x(R(x) \wedge H(x))$

Q20: Let $C(x)$ be the statement “ x has a cat,” let $D(x)$ be the statement “ x has a dog,” and let $F(x)$ be the statement “ x has a ferret.” Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

- a) A student in your class has a cat, a dog, and a ferret.
- b) All students in your class have a cat, a dog, or a ferret.
- c) Some student in your class has a cat and a ferret, but not a dog.
- d) No student in your class has a cat, a dog, and a ferret.
- e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet

Q21: Let $Q(x)$ be the statement “ $x + 1 > 2x$.” If the domain consists of all integers, what are these truth values?

- a) $Q(0)$
- b) $Q(-1)$
- c) $Q(1)$
- d) $\exists x Q(x)$
- e) $\forall x Q(x)$
- f) $\exists x \neg Q(x)$
- g) $\forall x \neg Q(x)$

Q22: Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

- a) $\exists x(x^2 = 2)$
- b) $\exists x(x^2 = -1)$
- c) $\forall x(x^2 + 2 \geq 1)$
- d) $\forall x(x^2 \neq x)$

Q23: Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

- a) Everyone in your class has a cellular phone.**
- b) Somebody in your class has seen a foreign movie.**
- c) There is a person in your class who cannot swim.**
- d) All students in your class can solve quadratic equations.**
- e) Some student in your class does not want to be rich.**

Q24: Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

- a) Something is not in the correct place.**
- b) All tools are in the correct place and are in excellent condition.**
- c) Everything is in the correct place and in excellent condition.**
- d) Nothing is in the correct place and is in excellent condition.**
- e) One of your tools is not in the correct place, but it is in excellent condition.**

Q25: Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase “It is not the case that.”)

- a) All dogs have fleas.**
- b) There is a horse that can add.**
- c) Every koala can climb.**
- d) No monkey can speak French.**
- e) There exists a pig that can swim and catch fish.**

Q26: Express the negation of these propositions using quantifiers, and then express the negation in English.

- a) Some drivers do not obey the speed limit.**
- b) All Swedish movies are serious.**
- c) No one can keep a secret.**
- d) There is someone in this class who does not have a good attitude.**

Q27: Translate these system specifications into English, where the predicate $S(x, y)$ is “ x is in state y ” and where the domain for x and y consists of all systems and all possible states, respectively.

- a) $\exists x S(x, \text{open})$**
- b) $\forall x (S(x, \text{malfunctioning}) \vee S(x, \text{diagnostic}))$**
- c) $\exists x S(x, \text{open}) \vee \exists x S(x, \text{diagnostic})$**
- d) $\exists x \neg S(x, \text{available})$**
- e) $\forall x \neg S(x, \text{working})$**

Q28: Express each of these system specifications using predicates, quantifiers, and logical connectives.

- a) Every user has access to an electronic mailbox.**
- b) The system mailbox can be accessed by everyone in the group if the file system is locked.**
- c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.**
- d) At least one router is functioning normally if the throughput is between 100 kbps and 500 kbps and the proxy server is not in diagnostic mode.**