ASSIGNMENT 5.

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Proof Q.I :-

Sol:-

$a = \frac{x}{y}$, $b = \frac{z}{v}$ So, a^b

$\left(\frac{x}{y}\right)^{z/v}$

$\Rightarrow \frac{x^{z/v}}{y^{z/v}}$ to disapprove, bcz if any no i.e. x or y is negative & we had to take root, it will give imaginary value.
Hence rationals are not imaginary numbers.

Q.2:

$n = (k-2) + (k+3)$

$n = [(2k+1)-2] + [(2k+1)+3]$

$n = (2k-1) + (2k+4)$

$n = 4k+3$.

$n = 4k+2+1$

$n = 2(2k+1)+1$

$n = 2r+1$, Hence, it is odd.

INDUCTION :-

Q.3 $1^2+2^2+\dots n^2 = n(n+1)(2n+1)/6.$

(a) for P(1) :-

$1 = 1(1+1)(2(1)+1)/6$

$1 = 2(3)/6$

$1 = 1.$

b) 1) $P(k) = 1^2+2^2+\dots k^2 = \frac{k(k+1)(2k+1)}{6}$ — ①

2) for k+1 ;

$1^2+2^2+\dots k^2+(k+1)^2 = \frac{(k+1)(k+2)(2[k+1]+1)}{6}$

" " $\Rightarrow \frac{(k+1)(k+2)(2k+3)}{6}$ — ②

Taking L.H.S of eq ②.

$\Rightarrow 1^2+2^2+\dots k^2+(k+1)^2$

$\Rightarrow \frac{k(k+1)(2k+1)}{6}+(k+1)^2$

$\Rightarrow \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$

$\Rightarrow \frac{(k+1)[k(2k+1)+6(k+1)]}{6.}$

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$$= (k+1)\frac{(2k^2+k+6k+6)}{6} \Rightarrow (k+1)\frac{(2k^2+7k+6)}{6}$$

$$\Rightarrow \frac{(k+1)(k+2)(2k+3)}{6} \Rightarrow \text{R.H.S of eq } ②$$

Hence proved! stat. is true for evy Z^+ n.

Q.4 $1^2+3^2+5^2+ \dots (2n+1)^2 = (n+1)(2n+1)(2n+3)/3$

Sol:-

Basic Step n=0.

$$1^2 = (0+1)(0+1)(0+3)/3$$

$$1 = 1. \checkmark$$

Induction Step:-

let P(k) be true.

$$1^2+3^2+5^2+ \dots (2k+1)^2 = (k+1)(2k+1)(2k+3)/3 \quad ①$$

k+1 to be prove:-

$$1^2+3^2+5^2+\dots(2k+1)^2+(2(k+1)+1)^2 = (k+2)(2(k+1)+1)(2(k+1)+3)/3$$

$$1^2+3^2+5^2 \dots (2k+1)^2+(2k+3)^2 = \frac{(k+2)(2k+3)(2k+5)}{3} \quad ②$$

25.

$3\frac{(5)}{3}(4)$

Using R.H.S of eq ③.

$\Rightarrow 1^2+3^2+5^2 \ldots (2k+1)^2 + (2k+3)^2$

$$\Rightarrow \frac{(k+1)(2k+1)(2k+3)}{3} + (2k+3)^2$$

$$\Rightarrow \frac{(k+1)(2k+1)(2k+3) + 3(2k+3)^2}{3}$$

$$\Rightarrow \frac{2k+3\left[(k+1)(2k+1) + 3(2k+3)\right]}{3}$$

$$= \frac{2k+3\left[(k+1)(2k+1) + (6k+9)\right]}{3}$$

$$\Rightarrow \frac{2k+3\left[2k^2+k+2k+1 + 6k+9\right]}{3}$$

$$\Rightarrow \frac{2k+3\left[2k^2+9k+10\right]}{3}$$

$$\Rightarrow \frac{(2k+3)(2k+5)(k+2)}{3} \rightarrow \text{R.H.S of eq ②}$$

Proved!

Q.5 $n, \sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2.$

for $n=1$:

$1 \cdot 2^1 = (1-1)2^{1+1} + 2$

$2 = 2.$ ✓

Inductive step:

① $k2^k = (k-1)2^{k+1} + 2.$

② $k2^k + (k+1)2^{k+1} = [(k+1)-1]2^{k+2} + 2.$

$k2^k + (k+1)2^{k+1} = k2^{k+2} + 2. \quad$ — ②.

Taking L.H.S of eq ②.

$\Rightarrow k2^k + (k+1)2^{k+1}$

$\Rightarrow (k-1)2^{k+1} + 2 + (k+1)2^{k+1}$

$\Rightarrow \cancel{2^{k+1}[(k-1)(k+1)] + 2}$

$= k2^{k+1} - \cancel{2^{k+1}} + 2 + k2^{k+1} + \cancel{2^{k+1}}$

$2k2^{k+1} + 2.$

$\Rightarrow \boxed{k2^{k+2} + 2}$ Proved

$k + \frac{5}{2}$

$2k+5$

COUNTING:-

Sol:

Q.6

have to use product rule;

a). As there are 10 questions and each each question has 4 possibilities then

1- _ _ _ _ = 4

2- _ _ _ _ = 4

⋮ ⋮

10- _ _ _ _ = 4.

$\Rightarrow 4\times4\times4\times4\times4\times4\times4\times4\times4\times4 = 4^{10} = 1048576$.

b) Student can leave question blank is also a option = 4+1=5

$\Rightarrow 5^{10} = \boxed{9765625}$

Q.7

Colors = 12. M L

Gender = 2.

Size = 3.

$12\times2\times3 = 72$. diff. type of shirts

Q.8 :

$$\not{} S_4$$

$$S_4 + S_3 + S_2 + S_1 6.$$

$\Rightarrow \underline{26} \times \underline{26} \times \underline{26} \times \underline{26} \quad + \underline{26} \times \underline{26} \times \underline{26} \quad + \frac{26 \times 26}{1.} \quad + \underline{26}$

$\Rightarrow \quad 26^4 + 26^3 + 26^2 + 26^1.$

$\Rightarrow \quad 456976 + 17576 + 676 + 26 = 475254$

Q.9:

RNA bases $\Rightarrow$ A, C, G, U

a) 6 elements $\Rightarrow$ 1 = 3, 2 = 3, 3 = 3, 4 = 3, 5 = 3, 6 = 3

all neglecting U.

$3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6 = 729.$

b). First base = 4 ways.
Second base = 4 ways.
Third bass = 4 ways
fourth " = 4 ways
fifth " = 1 w"
Sixth " = 1 ways

1 2 3 4 1 1.

$\Rightarrow 4 \times 4 \times 4 \times 4 \times G \times U$

$\Rightarrow 4 \times 4 \times 4 \times 4 \times 1 \times 1$

$= 256.$

A C G U

c) $C \times 4 \times 4 \times 4 \times 4 \times 4$

$1 \times 4^5 = 1024$.

d) $2 \times 2 \times 2 \times 2 \times 2 \times 2$, Since it has to be A or G

$= 64$.

Q.10

__ __ __ __ __ __ + __ __ __ __ __ __

26 26 26 10 10 10. 26 26 26 26 10 10.

$\Rightarrow (26)^3 \times (10)^3 + (26)^4 \times (10)^2$

$\Rightarrow$ 63,273,600

Q.11

__ __ __ __ __ __ __ __

a) $26 \times 26 \times 26 \times 26 \times 26 \times 26 \times 26 \times 26 = 26^8$ Ans

b) $26 \times 25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19 = 62990928000$

c) $X \times 26 \times 26 \times 26 \times 26 \times 26 \times 26 \times 26 = 26^7 \times 1$.

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d) $X \cdot \cancel{26} \times 25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19 \Rightarrow 1 \times 25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19$

$= 2422728000$ $\Rightarrow$

e) $\cancel{X}, 1 \times 26 \times 26 \times 26 \times 26 \times 26 \times 26 \times 1$

$\Rightarrow 26^6 = 308,915,776$

B O

f) $1 \times 1 \times 26^6 = 308,915,776.$

B O $\qquad$ B O

g) $1 \times 1 \quad \underline{26^4} \quad 1 \times 1$

$\Rightarrow 456,976.$

h). letter start with BO $= 26^6$.

letter end with BO $= 26^6$.

B O $\underbrace{26\ 26\ 26\ 26}\ 26\ 26$

$26\ 26\ \underbrace{26\ 26\ 26\ 26}_{26^4}$ B O

$\Rightarrow 26^6 + 26^6 - 26^4 = 617,374,576.$

x

434 f,
883 S,] → CS.
43 J

Q.12

⇒ 434 + 883 + 43 = 1360

each section = 34.

$\Rightarrow \frac{1360}{34}$

= 40 section need to accomodate

PIGEON HOLE PRINCIPLE:-

$\frac{N}{k}$ = objects.

Q.13

$\frac{x}{2} = 2. \Rightarrow x = 3$

12 → brown socks, 12 → black sock

a) 3 because if he picks one extra, there is a possibility of atleast 2 of same color

b) 14 because a person can choose all 12 brown socks, so in order to get atleast 2 black, he has to take out 14 socks

Q.14

10 - red balls

10 - blue ball.

a) 2 blue + 3 red = 5

or

3 blue + 2 red = 5 atleast 5.

$\boxed{\frac{N}{k} = \frac{x}{2} = 5}$

b) - If she takes out all 10 from red balls, the she had to take out 3 more to possible outcomes.

$10+3=13.$

Q.15 Pigeons are $\{1,3,5,7,9,11,13,15\}$ & pigeon holes are $\{1,15\},\{3,13\},\{5,11\},\{7,9\}$. So

$\Rightarrow \left\lceil \frac{x}{4} \right\rceil = 2 \Rightarrow x$ should be 5.

Q.16

9 → students

a) as there are two pigeon hole, male, female = 2.

$\left\lceil \frac{9}{2} \right\rceil = 5.$

b) $\left\lceil \frac{9}{2} \right\rceil$ for 3 male students.

$3 = 9 -$ female.

$9-6 = 3$ male.

for 7 female stud

$7 = 9 -$ male.

$9-2 = 7$ female.

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Permutation & Combination :-

Q.A:

Seven → women

9 → men.

sol $P(n,r) = \frac{n!}{(n-r)!}$, $C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$C(n,n) = n$

a) $^{16}C_5$

⇒ 4368 .

as we have to select atleast one women .

men. $^{9}C_5$

⇒ 126 .

⇒ 4368 − 126 = 4242 ways .

b) $^{16}C_5$

⇒ 4368 .

at least one women

⇒ 4242

for at least one man:- $^7C_5$

$\Rightarrow 21.$

$4242 - 21$

$\Rightarrow \boxed{4221 \text{ ways}}$

BINOMIAL THEOREM:-

Q.18:-

We know: $(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 - {}^nC_n a^0 b^n$

$(x+y)^4 \Rightarrow {}^4C_0 x^4 y^0 + {}^4C_1 x^3 y^1 + {}^4C_2 x^2 y^2 + {}^4C_3 x y^3 + {}^4C_4 x^0 y^4$

$= x^4 + 4x^3y^1 + 6x^2y^2 + 4xy^3 + y^4.$

Q.19:-

$x^5 y^{8 \to r}$ in $(x+y)^{13 \to n}$

$^{13}C_8 \, x^5 y^8 = 1287 x^5 y^8$

$\Rightarrow 1287$ coefficient.

Q.20: $x^{101} y^{99}$ in $(2x-3y)^{200}$.

${}^nC_r \, x^{n-r} y^r \Rightarrow {}^{200}C_{99} \, 2^{101} \cdot (-3)^{99}$

$= \dfrac{200!}{99!\,(100!)} \, 2^{101} \cdot (-3)^{99}$ Ans