ASSIGNMENT #2.

19K-0218
SAAD-UR-REHMAN
SEC : C

Q.1

a) let p: Alice is a mathematics major.
q: computer science major.

can be write as $\dfrac{p}{p \vee q}$

→ This argument use rule of addition

b) p: Jerry is a math major
q: Jerry is computer science major.

$\dfrac{p \wedge q}{p}$

→ This argument uses rule of simplification

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(c) Let p = It is raining
q = pool is closed.

$$\begin{array}{l} p \to q \\ p \\ \hline q \end{array}$$

⇒ This argument use rule of modus ponens.

e) p = I go swimming
q = I will stay in sun too long.
r = I will sunburn

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

This argument uses rule of hypothetical syllogism.

d). ~~p = It snows.~~
~~q =~~

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \neg p \end{array}$$

This argument uses modus tollens which says:

$$\begin{array}{l} \neg q \\ p \to q \\ \hline \therefore \neg p. \end{array}$$

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Q:2.

b) p = I eat spicy food.
q = I have strange dream.
r = There is a thunder while I sleep

1) $p \to q$
2) $r \to q$
3) $\neg q$

4) $\neg p$ Modus Tollen from ① & ③
5) $\neg r$ from ② & ③ = $p \to q$

$$\frac{\neg q}{\neg p}$$

The two conclusions are "I didn't eat spicy food" & "There is no thunder while I sleep"

(c)

p = I am clever
q = I am lucky.
r = I will win lottery.

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1) $p \vee q$
2) $\neg q$
3) $q \to r$.

4) p (Disjunctive syllogism) from ① & ②

$$\begin{array}{l} p \vee q \\ \neg q \\ \hline p. \end{array}$$

Conclusion is "I am clever".

e). p = good for corporation
q = good for Us
r = Good for you.
s = You Buy lots of stuff.

1) $p \to q$
2) $q \to r$.
3) $s \to p$

$$\begin{array}{l} q \to r \\ p \to q. \\ \hline \end{array}$$

$$\begin{array}{l} p \to q \\ q \to r. \\ \hline p \to r \end{array}$$

4) $p \to r$ from ① & ② hypothetical.
5) $s \to q$ " "
6)
$$\begin{array}{l} q \to r \\ s \to q. \\ \hline s \to r \end{array}$$
from ② & ⑤ " "

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(d)

$p(x)$ = x is a computer science major
$q(x)$ = x has a personal computer.
q(Ralph) = Ralph " " "
q(Ann) = Ann " " "

1) $\forall x(p(x) \rightarrow q(x))$ Premise
2) $\neg q$(Ralph) Premise
3) q(Ann) Premise
4) p(Ralph) $\rightarrow q$(Ralph) from ① U.I
5) $\neg p$(Ralph) from ② & ④ M.T

we can't plug q(Ann) because this is incorrect.

Conclusion is "Ralph is not a cs major".

(f)

$P(x)$ = x is a rodents.
$q(x)$ = x gnaws their food.
P(mice) = mice are rodents.

1) $\forall x(P(x) \rightarrow q(x))$ Premise
2) P(mice) Premise
3) P(mice) $\rightarrow q$(mice) UI from ①.
4) q(mice) MP from ② & ③

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The conclusion is "Mice gnaw their food".

Q.3

1. $(p \wedge t) \rightarrow (r \vee s)$
2. $q \rightarrow (u \wedge t)$
3. $u \rightarrow p$
4. $\neg s$

$\therefore q \rightarrow r$

(p∧t→ Premises)

5. $q \rightarrow u$. 2 simplification
3. $u \rightarrow p$
6. $q \rightarrow p$. H.S from 5 & 3.
7. $p \rightarrow r \vee s$ 1 simplifi.
8. $q \rightarrow (r \vee s)$. from 6 & 7. H.S

$\neg s$

NO conclusion (INVALID).

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Q.4 (2).

#1 p = Today is Tuesday
q = I have a test in math
r = I have a test in Economics
s = My Economic Professor is sick

$$p \to (q \vee r)$$
$$s \to \neg r$$
$$\underline{p \wedge s}$$
$$\therefore q$$

1) $p \to (q \vee r)$ premise
2) $s \to \neg r$ premise
3) $p \wedge s$ premise
~~4) ...~~
4) p from (3) using simplification law
5) s " " "
6) $q \vee r$ from (1) & (4) MP.
7) $\neg r$ from (2) & (5) MP.
8) $r \vee q$ from (6) commutative law
9) q from (7) & (8) using DS

g). p = x is a lawyer.
q = x is ambition
r = x is a early riser.
s = x like chocalet.

$\therefore$ ~~$p \to q$~~

$$\begin{array}{l} p \to q \\ r \to \neg s \\ \underline{q \to r} \\ \therefore p \to \neg s \end{array}$$

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$$ hypothetical

$$\begin{array}{l} p \to r \\ \underline{r \to \neg s} \\ p \to \neg s \end{array}$$ H.S.

$\boxed{\text{PROVED}}$

a) $(A-(A\cap B))\cap(B-(A\cap B)) = \emptyset$

Q.5

a) $(A-(A\cap B))\cap(B-(A\cap B)) = \emptyset$.

Sol:-

$\because A-B = A\cap\bar{B}$

$\Rightarrow A\cap\overline{(A\cap B)} \;\cap\; (B\cap\overline{(A\cap B)})$

$A\cap\bar{A}\cup\bar{B} \;\cap\; B\cap\bar{A}\cup\bar{B}$.

$(A\cap\bar{A})\cup\bar{B} \;\cap\; (B\cap\bar{B})\cup\bar{A}$

$\emptyset\cup\bar{B} \;\cap\; (\emptyset\cup\bar{A})$

$= \bar{B}\cap\bar{A}$

$= \emptyset$

(b) $(A-B) \cup (A \cap B) = A$

$= (A \cap \bar{B}) \cup (A \cap B)$

$= (A \cap \bar{B} \cup A) \cap (A \cap \bar{B} \cup B)$.

$= (A \cap \bar{B}) \cap (A \cap \phi)$

$(A \cap B) \cap \phi$

$(A \cap B') \cup (A \cap B)$

$= A \cap (B' \cup B)$.

$= A \cap U$

$= A$ Proven

(c) $(A-B) - C = (A-C) - B$

Taking L.H.S.

$(A \cap B') - C$.

$(A \cap B') \cap C'$

$A \cap (B' \cap C')$

Taking R.H.S

$(A \cap C') - B$.

$(A \cap C') \cap B'$

$A \cap (B' \cap C')$. R

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Q.6

$U = \{1,2,3,4,5,6,7,8\}$
$A = \{1,2,4,5\}$, $B = \{2,3,5,6\}$ $C = \{4,5,6,7\}$

a) $(A \cap B) \cap \bar{C}$

$\bar{C} = U - C$.

$= \{1,2,3,\cancel{4},\cancel{5},\cancel{6},\cancel{7},8\} - \{4,5,6,7\}$

$\bar{C} = \{1,2,3,8\}$

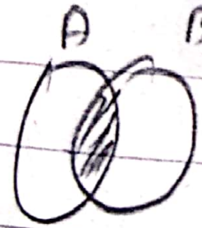
$A \cap B = \{1,2,4,5\} \cap \{2,3,5,6\}$

$= \{2,5\}$

$(A \cap B) \cap \bar{C} = \{2,5\} \cap \{1,2,3,8\}$

$= \{2\}$.

$A \cap B$



$(A \cap B) \cap \bar{C}$

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(b) $\bar{A} \cup (B \cup C)$.

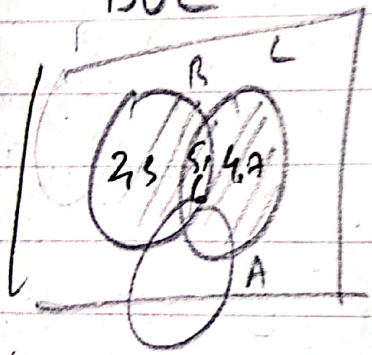
$\bar{A} = U - A = \{\cancel{1},\cancel{2},3,\cancel{4},\cancel{5},6,7,8\} - \{\cancel{1},\cancel{2},\cancel{4},\cancel{5}\}$

$\bar{A} = \{3,6,7,8\}$

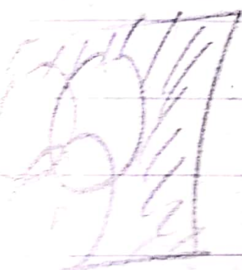
$B \cup C = \{2,3,5,6\} \cup \{4,5,6,7\}$
$= \{2,3,4,5,6,7\}$.

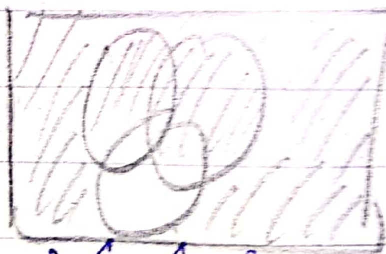
$\bar{A} \cup (B \cup C) = \{2,3,4,5,6,7,8\}$.

BUC



A'





A'U(BUC)

(c) $(A-B) \cap C$.

$A - B = \{1,\cancel{2},4,\cancel{5}\} - \{\cancel{2},3,\cancel{5},6\}$
$= \{1,4\}$

$(A-B) \cap C = \{4\}$

A B

A B

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(d) $(A \cap \bar{B}) \cup \bar{C}$

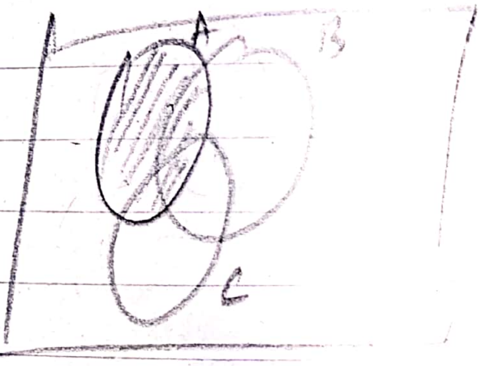
$\bar{B} = U - B$.

$= \{1, 4, 6, 7, 8\}$.

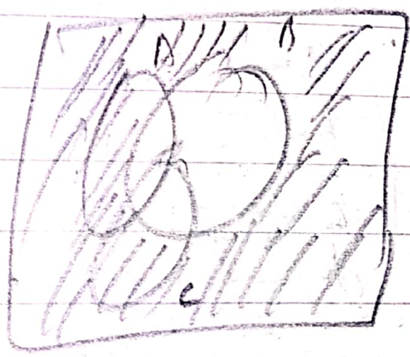
$(A \cap \bar{B}) = \{4\}$ $\bar{C} = \{1, 2, 3, 8\}$

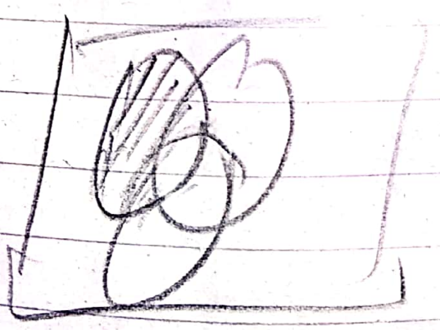
$(A \cap \bar{B}) \cup \bar{C} = \{4\} \cup \{1, 2, 3, 8\}$

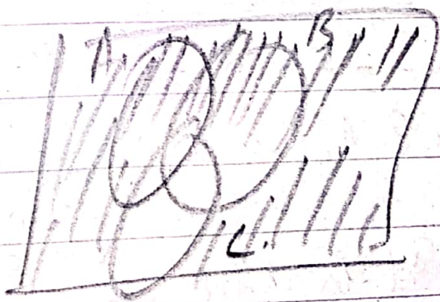
$= \{1, 2, 3, 4, 8\}$.



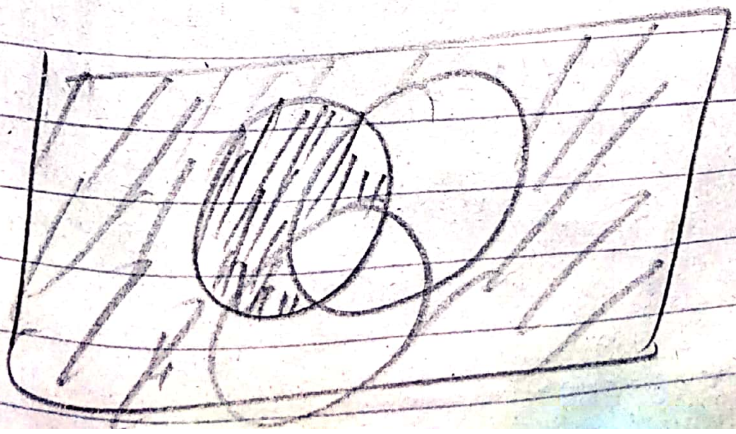
B'







$(A \cap \bar{B})$ $\cup$ $\bar{C}$.



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Q-4.

(a)

$p(x)$ = x is student in class

$q(x)$ = x owns a red convertible.

$r(x)$ = x has got a speeding ticket.

① $\forall x\,(q(x) \rightarrow r(x))$

② $P(Linda)$.

③ $q(Linda)$

$\exists x\,(P(x) \wedge r(x).)$

④ $q(Linda) \rightarrow r(Linda)$ from ① U.I.

⑤ $r(Linda)$ from ④ & ③ MP.

⑥ $P(Lind) \wedge r(Lind)$ from conjunct.

⑦ $\exists x\, P(x) \wedge r(x)$ E.G.

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(b). $P(x)$ = x is one of five roomates.
$q(x)$ = x has taken course in D.M.
$r(x)$ = x can take course in algorithm.

$\forall x\,(q(x) \rightarrow r(x))$ Premise.
$\forall x\,(P(x) \rightarrow q(x))$ Premise.

$\therefore \forall x\,(P(x) \rightarrow r(x))$

$q(a) \rightarrow r(a)$ U.I.
$p(a) \rightarrow q(a)$ U.I.

$p(a) \rightarrow r(a)$ H.S.

$\forall x\,(P(x) \rightarrow r(x))$ U.G.

(c)

$P(x)$ = x movies produced by John Sayles.
$W(x)$ = x are wonderful.
$C(x)$ = x produce movie about coal mi.

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$J(x) \to W(x)$.

$\forall x\,(P(x) \to Q(x))$

$\exists x\,(J(x) \wedge C(x))$

$\therefore \exists x\,(W(x) \wedge C(x))$

$J(a) \to W(a)$	U.I.
$J(a) \wedge C(a)$.	E.I.
$J(a)$	simplfi.
$W(a)$	M.P.
$C(a)$.	
$W(a) \wedge C(a)$	conjunction
$\exists x\,(W(a) \wedge C(a))$	E.G.

(d).

$P(x)$ = x is in class.
$q(x)$ = x has been to France.
$r(x)$ = x visits Louvre.

- $\exists x\,(P(x) \wedge q(x))$
- $\forall x\,(q(x) \to r(x))$.

$\therefore \exists x\,(P(x) \wedge r(x))$.

$P(a) \wedge q(a)$	E.I.
$q(a) \to r(a)$	U.G.

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$q(a)$ — from simplification

$p(a)$ — M.P.

$r(a)$ — M.P.

$p(a) \wedge r(a)$ — Simplification

$\exists x\,(P(x) \wedge r(x))$ — E.G.

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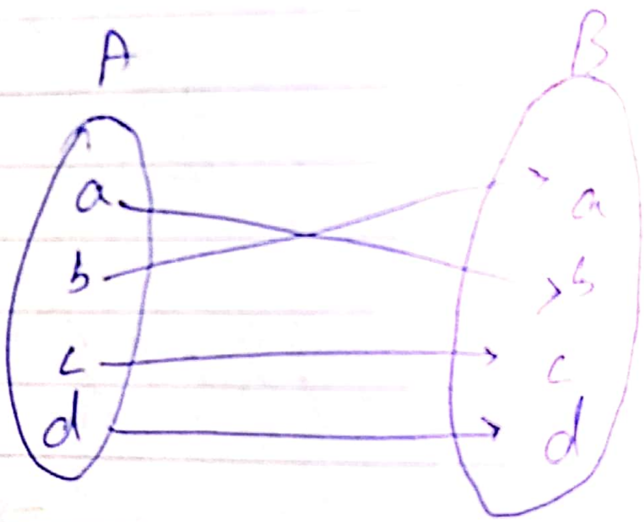
Q.7

A = {a, b, c, d}   B = {a, b, c, d}

(a) f(a) = b, f(b) = a, f(c) = c, f(d) = d.



Domain = {a, b, c, d}
Co-domain = {a, b, c, d}
Range = {b, a, c, d}.

→ It is bijective
→ Yes, inverse exist as it is bijective.

$f^{-1}(a) = b$, $f^{-1}(b) = a$, $f^{-1}(c) = c$, $f^{-1}(d) = d$

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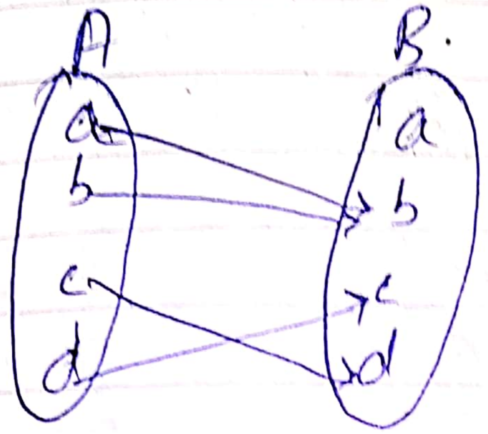
(b) $f(a)=b,\ f(b)=b,\ f(c)=d,\ f(d)=c.$

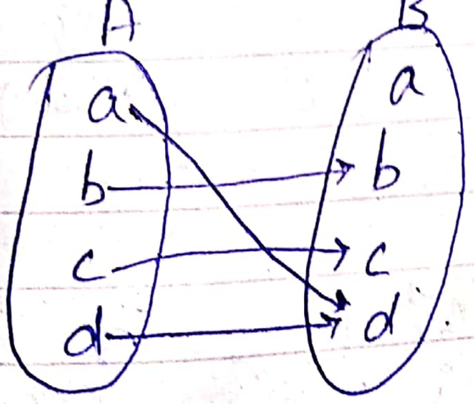


Nor one-to-one, Neither onto.

Domain = {a, b, c, d}
Codomain = {a, b, c, d}
Range = {b, d, c}

⇒ Inverse Doesn't exist.

(c) $f(a)=d,\ f(b)=b,\ f(c)=c,\ f(d)=d.$



Domain = {a, b, c, d}
Codomain {a, b, c, d}
Range = {b, c, d}.

Nor Injective neither surjective.

⇒ Inverse not exist.

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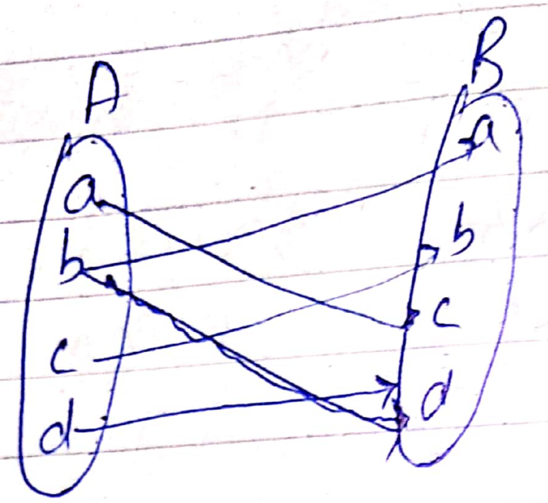
d) $f(a)=c,\quad f(b)=a,\quad f(c)=b,\quad f(d)=d$



Domain = {a, b, c, d}
Codomain = {a, b, c, d}
Range = {a, b, d}.

$f^{-1}(a)=b,\quad f^{-1}(b)=c,\quad f^{-1}(c)$
$f^{-1}(d)=$

Q.8 $f(a)=2a+3$ & $g(a)=3a+2$.

$fog=f(3a+2)$
$=2(3a+2)+3$
$=6a+7.$

$gof=g(2a+3)$
$=3(2a+3)+2$
$=6a+11$

for injective $f(x_1)=f(x_2)$

$f(x_1)=2x_1+3,\quad f(x_2)=2x_2+3$

$2x_1+\cancel{3}=2x_2+\cancel{3}$

$\cancel{2}x_1=\cancel{2}x_2$

$\boxed{x_1=x_2}$

$2a+3 \quad . \quad 3a+2$

$2u+3 = 2v+3$

$f(x)-g(x)f \quad u=v$

$2a$

$f(x_1) = 3x_1+2, \qquad f(x_2) = 3x_2+2.$

$3x_1 + \cancel{2} = 3x_2 + \cancel{2}$

$\cancel{3}x_1 = \cancel{3}x_2$

$\boxed{x_1 = x_2}$

~~for INVERTIBILITY :-~~

~~$f(x) = 2x+3$:~~

~~$f(a) = f(b)$.~~

~~$f(a) = 7 \quad f(b) = 7$.~~

~~$\boxed{f(a) = 17} \quad \boxed{f(b) = 17}$~~

~~$f(a) = f(b) \quad |17 = 17|$~~

~~So, it is invertible. (one to one correspond.)~~

~~Q~~ for SURJECTIVE :-

$f : Z \to Z$

① $y = f(x)$

$y = 2x+3$

$\frac{y-3}{2} = x$.

(i) $y = 5$; $x = 1$ ✓

(ii) $y = 2$; ; $x = \frac{-1}{2}$

$f(x_1) = f(x_2)$

Not surjective.

x^2.

② $y = f(x)$

$y = 3x+2$

$\frac{y-2}{3} = x$

$y = 5$, $x = 1$

$y = 2$, $x = 0$

$y = 1$, $x = \frac{-1}{3}$

$f_1,($

$-x_1^2 = x_2^2$

$x = 3$

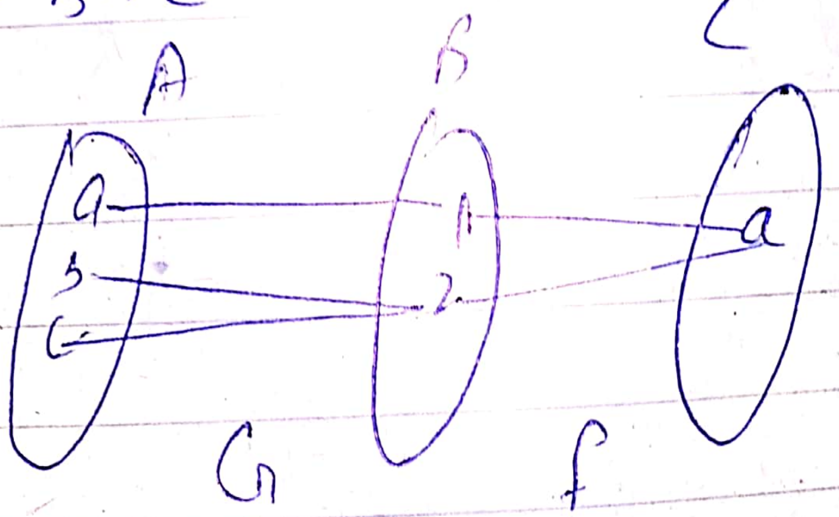
Not surjective

→ It is not invertible.

3

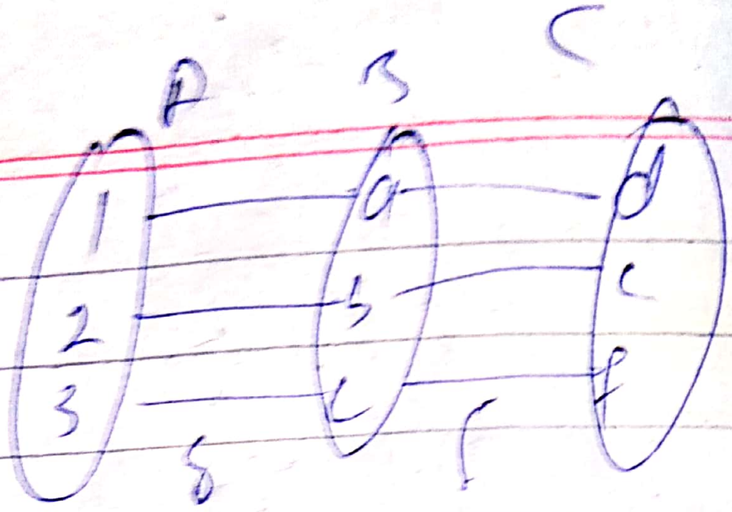
Q. 9

$g : A \to B$
$f : B \to C$



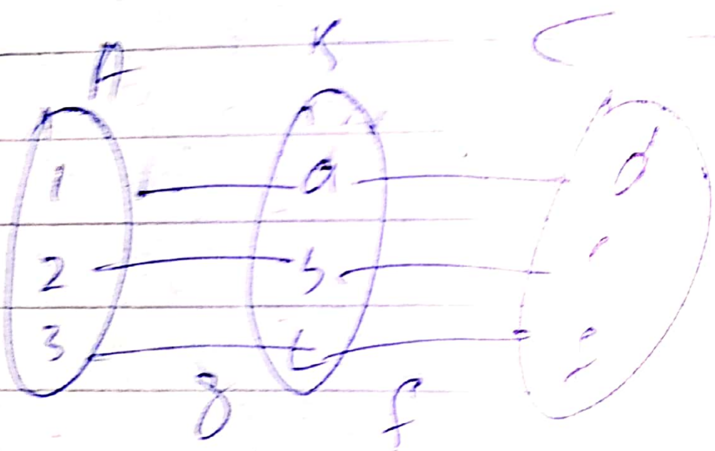
F must be an onto function for fog to be onto function.

b)



gof is one-to-one then fog is one to one.

c)



If fog is bijective then g is onto if and only if f is one to one.

(10)

a) Onto
b) Not
c) Onto
d) Onto
e) Not