ASSIGNMENT 1

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SEC: C

DISCRETE MATH STRUCTURES

Q.1

SMARTPHONE A: 256 MB RAM, 32 GB ROM, 8 MP.
SMARTPHONE B: 288 MB RAM, 64 " ", 4 "
" " C: 128 " ", 32 " ", 5 ".

a) TRUE
b) ~~FALSE~~ TRUE
c) FALSE
d) FALSE
e) FALSE

Q.2

a) $\neg p$: I've not bought a lottery ticket this week
b) $p \vee q$: I bought a lottery ticket this week or I won the million dollar jackpot.
c) $p \to q$: If I bought a lottery ticket this week then I will won the million dollar jackpot.
d) $p \wedge q$: I bought a lottery ticket this week and I won million dollar jackpot.

J.D 19k-0218 Page ② [signature]

c) $p \leftrightarrow q$: I bought a lottery ticket this week if & only if I won million dollar jackpot

f) $\neg p \rightarrow \neg q$: If i ~~did~~ ~~does~~ not ~~bought~~ buy a lottery ticket this week then I'll not win million dollar jackpot.

g) $\neg p \wedge \neg q$: I does not ~~bought~~ buy a lottery ticket this week and I have not won million dollar jackpot.

h) $\neg p \vee (p \wedge q)$:

I've not bought a lottery ticket this week or I bought a lottery ticket this week & I won million dollar jackpot.

Q.3

a) $p \rightarrow q$: If you have flu then you'll miss the final examination

b) $\neg q \leftrightarrow r$: You will not miss the final examination if & only if you pass the course.

ID = 19K-0218 Pg ③ [signature]

c) $q \to \neg r$

⇒ If You miss the final exam then you will not pass the course.

d) $p \vee q \vee r$

⇒ You have the flu or you miss the final exam
Or you pass the course.

e) $(p \to \neg r) \vee (q \to \neg r)$

⇒ If you have the flu, then you'll not pass the course Or, If you miss the final exam then you'll not pass the course.

f) $(p \wedge q) \vee (\neg q \wedge r)$

⇒ You have the flu and you miss the final exam Or you didn't miss the final exam and you pass the course.

ID = 191K-0218 Pg #4 [signature]

Q# 4

a) It is inclusive or, because looking at the sentence, means that we have a experience of at least one, but both will also be appreciated.

b) It is exclusive or because, the what the sentence means is that lunch comes with any one of variety not both.

NOTE :- It is same as the case of relatives, we offer them with any one of variety, Either Tea or Coffee, not both

c) It is inclusive or because at least one is necessary, having both in your pocket will be great.

d) It is Exclusive or (You can either publish or perish).

Pg # 5   ID = 19K-0218

Q-5

a) CONVERSE :-

If i will stay at home, then it snows tonight

INVERSE :-

If it doesn't snow tonight, then i will not stay at home.

CONTRAPOSITIVE :-

If i will not stay at home, then it will not snow tonight.

b)

CONVERSE :-

It is sunny summerday whenever I go to the beach.

INVERSE :-

I don't go to the beach whenever it is not a sunny summer day.

CONTRAPOSITIVE :-

It is not a sunny summerday whenever I don't go to the beach.

ID: 19K-0218 Pg # 6

(c) CONVERSE:- When i sleep until noon, it is necessary that i stay up late.

INVERSE:-
When i don't sleep up late, it is necessary that i don't sleep until noon.

CONTRAPOSITIVE:- When i don't sleep until noon, it is necessary that i don't stay up late.

Q6.

→ The equation will always be true because of the disjunction (∨) means that one value has to be true in order to give the truth value.

For Ex: if I place "1" to p, q, r the

$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$

$\Rightarrow (1 \vee 0) \wedge (1 \vee 0) \wedge (1 \vee 0)$

$1 \wedge 1 \wedge 1 = \text{TRUE}$

PROVED

19K-0218 Pg#(7) 8

Q.7

a) "User has paid (r) subscription fee, but does not enter a valid pass ($\neg p$)."

b) "Access is granted (q) whenever the user has paid (r) the subscription fee & enters a valid pass (p)." [if $p \to q$].

c) "Access is denied ($\neg q$) if the user has not ($\neg r$) paid the subscription fee."

d) "If the user has not entered a valid pass ($\neg p$), but has paid subscription fee, the access is granted (q)"

$T \wedge T = T$

1. $r \wedge \neg p = T$ $r = T,\ p = f$
2. $(r \wedge p) \to q = T$ $q = T,\ r = T,\ p = f$
3. $\neg r \to \neg q = T$ $r = T,\ q = T$
4. $(\neg p \wedge r) \to q = T$ $q = T,\ p = f,\ r = T$

System is consistent.

ID= 19K-0218 Pg #(9)

Q.9.

DEMORGAN'S LAW:-

(1) $\neg(p \land q) \equiv \neg p \lor \neg q$

(2) $\neg(p \lor q) \equiv \neg p \land \neg q$

a) Jan is not rich ~~and~~ or not happy
) Carlos will not bicycle and not run tommorow
) Mei doesn't walk and doesn't take the bus to class.
) Ibrahim is ~~sm~~ not smart or not hard-working

⇒ Continue

19K-0218 Pg #10

Q.10

Let p = Butler tells the truth.
q = Cook tells the truth
r = Gardener tells the truth.
s = Handyman tells the truth.

i) $p \to q$ = TRUE
ii) $\neg(p \wedge q)$ = TRUE
iii) $\neg(\neg r \wedge \neg s)$ = TRUE $r = T$
iv) $(s \to \neg q)$ = TRUE $q = f$

System is consistent if

a) Butler and cook tell lie
b) if gardener tells truth
c) if handyman tells truth or lie

19K-0218 Pg # (11) 8

Q.11

p	q	$\neg p$	$\neg q$	$p \to q$	$(\neg p \wedge (p \to q))$ A	$A \to B$
T	T	f	f	T	f	T
T	f	f	T	f	f	T
F	T	T	f	T	T	f
f	f	T	T	T	T	T

Not a tautology bcz it says value will always be TRUE!!

Q.13 $(p \to q) \wedge (p \to r)$ & $p \to (q \wedge r)$ are equivalent

Sol:-

Taking R.H.S;

$$p \to (q \wedge r)$$

$\because$ We know that;

$$p \to q \equiv \neg p \vee q$$

$$\neg p \vee (q \wedge r)$$

Using distributive law:

$$(\neg p \vee q) \wedge (\neg p \wedge r)$$

$\because$ We know;

$$p \to q \equiv \neg p \vee q$$

$$\boxed{(p \to q) \wedge (p \to r)}$$

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By Truth Table.

p	q	r	p→q	p→r	(p→q)∧(p→r) A	(q∧r) B	A→[illegible]
T	T	T	T	T	T	T	[illegible]
T	T	f	T	f	f	f	f
T	f	T	f	T	f	f	f
T	f	f	f	f	f	f	f
f	T	T	T	T	T	T	T
f	T	f	T	T	T	f	T
f	f	T	T	T	T	f	T
f	f	f	T	T	T	f	T

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Q.12 Show $\neg(p \oplus q)$ & $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$	$p \oplus q$	$\neg(p \oplus q)$
T	T	T	f	T
T	f	f	T	f
f	T	f	T	f
f	f	T	f	T

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19K-0218 Pg # 13 8

Q.14

$\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

p	q	r	$\neg p$	$p \vee r$	$q \rightarrow r$	$\neg p \rightarrow (q \rightarrow r)$	$q \rightarrow (p \vee r)$
T	T	T	f	T	T	T	T
T	T	f	f	T	f	T	T
T	f	T	f	T	T	T	T
T	f	f	f	T	T	T	T
f	T	T	T	T	T	T	T
f	T	f	T	f	f	F	F
f	f	T	T	T	T	T	T
f	f	f	T	f	f	F	f

PROVED!

Law:

Taking L.H.S $\Rightarrow \neg p \rightarrow (q \rightarrow r)$

$\Rightarrow \because \neg p \rightarrow q = p \vee q$.

$\Rightarrow p \vee (q \rightarrow r)$

$\Rightarrow \because p \rightarrow q = \neg p \vee q$

~~$p \vee (\neg p \vee q)$~~

$p \vee (\neg q \vee r)$

$\neg q \vee (p \vee r)$

$q \rightarrow (p \vee r)$ PROVED

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19K-0218 Pg # 14 8

Q.15 $p \leftrightarrow q$ & $\neg p \leftrightarrow \neg q$ are equal

Sol

Taking R.H.S

$\Rightarrow \neg p \leftrightarrow \neg q.$

$\neg (p \to q) \wedge \neg (q \neg p)$

$\neg (\neg p \vee q) \wedge \neg (\neg q \to p)$

$\because$ We know;

$p \vee q = \neg p \to \neg q.$

$\neg (\neg (\neg p \to \neg q) \wedge \neg (\neg (\neg q \to \neg p))$

$(\neg p \to \neg q) \wedge (\neg q \to \neg p)$

$(p \to q) \wedge (q \to p)$

$p \leftrightarrow q$ PROVED

By Truth Table.

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$	$\neg p \leftrightarrow \neg q$
T	T	f	f	T	T
T	f	f	T	f	f
f	T	T	f	f	f
f	f	T	T	T	T

Q.16

By law:- $\overset{p}{(p \vee q) \wedge (\neg p \vee r)} \rightarrow \overset{q}{(q \vee r)}$

$\because p \rightarrow q = \neg p \vee q$

$\Rightarrow \neg [(p \vee q) \wedge (\neg p \vee r)] \vee (q \vee r)$

using demorgan's law

$\neg (p \vee q) \vee \neg (\neg p \vee r) \vee (q \vee r)$

$(\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee (q \vee r)$

$\Rightarrow (\neg p \wedge \neg q) \vee (q \vee r) \vee (p \wedge \neg r)$

$(\neg p \wedge \neg q) \vee q \vee r \vee (p \wedge \neg r)$

$(\neg p \vee q) \wedge (\neg q \vee q) \vee r \vee (p \wedge \neg r)$

$(\neg p \vee q) \wedge T \vee r \vee (p \wedge \neg r)$

using identity law;

$(\neg p \vee q) \vee (r \vee p) \wedge (r \vee \neg r)$

$(\neg p \vee q) \vee (r \vee p) \wedge T$

$(\neg p \vee q) \vee (r \vee p)$

$\neg p \vee q \vee r \vee p$

$(\neg p \vee p) \vee q \vee r$

$T \vee (q \vee r) = \boxed{T}$ it is a tautology.

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19K-0218 Pg #16 Date

Q.16 Show $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.

p	q	r	¬p	(p∨q) A	(¬p∨r) B	(q∨r) C	A∧B	A∧B→C
T	T	f	T	T	T	T	T	T
T	f	f	T	f	T	f	T	T
f	T	f	T	T	T	T	T	T
f	f	f	T	f	f	f	T	T
T	T	T	T	T	T	T	T	T
T	f	T	T	T	T	T	T	T
f	T	T	f	T	T	f	T	T
f	f	T	f	T	f	f	T	T

Yes, it is a tautology!

Q.17 Show $(p \wedge q) \rightarrow r$ & $(p \rightarrow r) \wedge (q \rightarrow r)$

p	q	r	(p∧q)	(p→r) A	(q→r) B	(p∧q)→r	A∧B
				T	T	T	T
T	T		T	f	f	f	f
T	f		T	T	T	T	T
f	T		f	f	T	T	f
f	f		f	T	T	T	T
T	T		f	T	f	T	f
T	f		f	T	T	T	T
f	T		f	T	T	T	T
f	f		f	T			

Not equal

19K-0218 Pg # 17 Date 8

Q.18

(a) $\exists x\, N(x)$

→ Some of the student in your school who has visited North Dakota OR There exist a student who has visited North Dakota.

b) $\forall x\, N(x)$

→ All of the student in your school has visited North Dakota.

c) $\exists x \neg N(x)$

→ Some of the student in your school has not visited North Dakota.

d) $\neg \exists x\, N(x)$

Not some of the student has visited North Dakota OR there does not exist a student in your school who has visited North Dakota.

e) $\neg \forall x\, N(x)$

Not all student in your school has visited North Dakota.

Pg # (18) Date ___

f) $\forall x \neg N(x)$

Every student in your school has not visited North Dakota.

Q.19

a) $\forall x (R(x) \rightarrow H(x))$

for every animal x, if x is a rabbit, then x hops.

for every animal, if it is a rabbit then it hops.

b) $\forall x (R(x) \wedge H(x))$

for every animal x, x is a rabbit and x hops.

c) $\exists x (R(x) \rightarrow H(x))$

for some animal x, if x is a rabbit, then x hops

d) $\exists x (R(x) \wedge H(x))$

for some animal x, x is a rabbit and x hops

OR

There exist an animal x, who is a rabbit & it hops.

19K-0218 Pg # 18 Date

Q.20

a) $C(x)$ = x has a cat
$D(x)$ = x has a dog
$F(x)$ = x has a ferret

a) $\exists x\,(C(x) \land D(x) \land F(x))$

b) $\forall x\,(C(x) \land D(x) \lor F(x))$

c) $\exists x\,(C(x) \land F(x) \land \neg D(x))$

d) $\neg \exists x\,(C(x) \land D(x) \land F(x))$

e) $\exists x\,C(x) \land \exists x\,D(x) \land \exists x\,F(x)$

Q.21

a) $Q(0)$ → ~~FALSE~~ TRUE
b) $Q(-1)$ → TRUE
c) $Q(1)$ → FALSE
d) $\exists x\,Q(x)$ → for ~~x = 0, ... TRU~~ $x=0, x=-1$, TRUE
e) $\forall x\,Q(x)$ → FALSE
f) $\exists x\,\neg Q(x)$ → $x+1 < 2x$ for $x=2$, TRUE.
g) $\forall x\,\neg Q(x)$ → $x+1 < 2x$ FALSE.

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19K-0218 Pg # 20 8/

Date

Q.22.

a) $\exists x\ (x^2 = 2)$

if we place $x=1$

$1 = 2$ f.

place $x = \sqrt{2}$ $\Rightarrow$ TRUE.

b) $\exists x\ (x^2 = -1)$

because of square, it always give positive value, so this is FALSE.

e) $\forall x\ (x^2 + 2 \geq 1)$

TRUE!

f) $\forall x\ (x^2 != x)$ f

$x = 0$

$0 != 0$, FALSE.

19/1/2018 Pg # 21 Date

Q.23:-

a) (i) Domain: student in your class

let suppose, "x" has a cellular phone.

$A(x)$

finally, $\forall x\, A(x)$

(ii) Domain :- All people.

we can write the statement as;

for all people x, if x is a student in your class then x has a cellular phone

$A(x)$ $B(x)$

$\Rightarrow \quad \forall x\,(A(x) \rightarrow B(x))$

b) (i) Domain: student in your class

$\Rightarrow \quad \exists x\, A(x)$

(ii) Domain = All people.

Stat. can be written as;

there exist a person x such that x is a student in your class & x has seen a foreign movie.

$A(x)$ $B(x)$

$\exists x\,(A(x) \wedge B(x))$

Pg # 22 Date 8

c) (i) Domain = student in your class.

$\rightarrow \exists x \neg A(x)$

ii) Domain = All people.

stat. can be written as;

There exist a student x such that if x is a person in your class ($A(x)$) and x can't swim ($\neg B(x)$)

$$\exists x (A(x) \rightarrow \neg B(x))$$

d) (i) Domain = student in your class.

$$\forall x \, A(x)$$

ii) Domain = All people

$\rightarrow$ for all people x, if x is a student in your class ($A(x)$), then x can solve quadratic equation ($B(x)$)

$$\forall x (A(x) \rightarrow B(x))$$

e) Domain = student.

$\rightarrow \exists x \neg A(x).$

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19K-0218 Date 8

(ii) Domain = all ~~students~~ people

→ There exist a student in your class x such that x is a student in your class $A(x)$ & x doesn't want to be rich. $\neg B(x)$

$$\exists x\,(A(x) \wedge \neg B(x))$$

Q.24

a) $\exists x\ \neg(A(x))$

(b) $\forall x\,(A(x) \wedge B(x))$

(c) $\exists x\,(A(x) \wedge B(x))$

(d) $\neg\exists x\,(A(x) \wedge B(x))$

(e) $\exists x\,(\neg A(x) \wedge B(x))$

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17K-8218 Pg # 24 Date

Q. 25

P(x) = x have fleas.

$\forall x\ P(x)$

NEGATION:- $\neg(\forall x\ P(x)) \Rightarrow \exists x \neg P(x)$

⇒ There is a dog which does not have fleas.

) P(x) = x can add

$\exists x\ P(x)$

NEGATION: $\neg(\exists x\ P(x))$

$= \forall x \neg P(x)$

⇒ All horses can't add.

) P(x) = x can climb

$\exists x\ P(x)$

NEGATION: $\neg(\exists x\ P(x))$

$= \forall x \neg P(x)$

⇒ There is a koala that can't climb

) Sol:

No monkey can speak french.

It means that all monkey can't speak french

$\Rightarrow \forall x \neg A(x)$

NEGATION:- $\neg(\forall x \neg A(x))$

$= \exists x\ A(x)$

19K-0217 Pg # 28 Date

⇒ Every monkey can speak french.

e) $P(x)$ = x can swim
$Q(x)$ = x catch fish.

$\exists x (P(x) \wedge Q(x))$

NEGATION:

$\neg(\exists x (P(x) \wedge Q(x)))$
$\forall x (\neg P(x) \vee \neg Q(x))$

All fig can't swim or can't catch fish.

Q.26

a) $P(x)$ = x obey the speed limit.

$\exists x \neg P(x)$

NEGATION:

$\neg(\exists x \neg P(x))$
$\forall x P(x)$

⇒ All drivers obey speed limit.

(b) $\forall x P(x)$

NEGATION:

$\neg(\forall x P(x))$
$\exists x \neg P(x)$

⇒ There exist a Swedish movie that are not serious

(c) Stat. can be written as.
Everyone can't keep a secret.

$$\forall x \neg P(x)$$

NEGATION:

$$\neg (\forall x \neg P(x))$$
$$\Rightarrow \exists x\, P(x)$$

Someone can keep a secret
OR

$P(x)$ = x can keep a secret

$$\neg \exists (x)\, P(x)$$

EGATION:- $\neg(\neg \exists x\, P(x))$
$\exists x\, P(x)$

Same!

d)

$$\exists x \neg A(x)$$

NEGATION:- $\neg(\exists x \neg A(x))$
$\forall x\, A(x)$

$\Rightarrow$ Everybody in this class has a good attitude

19/1/2018 Pg # 27 Date

Q.27

a) $\exists x\ S(x, open)$

⇒ There exist a system that is in state open.

b) $\forall x\ (S(x, malfunctioning) \vee S(x, diagnostic))$

Every system is in state malfunctioning ~~and~~ or in state diagonistic.

c) $\exists x\ S(x, open) \vee \exists x\ S(x, diagonistic)$

⇒ There exist a system in state open ~~and~~ or there exist a system in state diagonistic.

d) $\exists x\ \neg S(x, available)$

There exist a system which doesn't in a state available.

e) $\forall x\ \neg S(x, working)$

Every system does not in a state working.

19K-0218 Pg # 28 Date 8/

Q.28

a) $P(x)$ = x has access to an electronic mailbox.

$\forall x\, P(x)$

b) $P(x)$ = system mailbox can be accessed by user
$Q(x)$ = the file system in group is locked

$\forall x\, (P(x) \rightarrow Q(x))$

(c) $\exists x\, (P(x) \rightarrow Q(x))$

(d) $P(x)$ = if the throughput is b/w 100 & 500
$Q(x)$ = monitor is functioning normally.
$R(x)$ = server is in the diagnostic mode

$\exists x\, [(Q(x) \wedge \neg R(x)] \rightarrow P(x))$

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