



Department of Artificial Intelligence and Mathematics

Calculus and Analytical Geometry

PROJECT NO # 01

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Degree: BS-Artificial Intelligence

Section: AI-1(A)

Course: Calculus & Analytical Geometry

Course Code: MAT-101

Calculus Project

Q 1: Write down the application of Functions, Absolute Value function, Limit of a function, Derivative in computer science, Physics and in Engineering or in daily life.

Application of Calculus

Function

A function is a rule of Relationship that Assign or give exactly give one output for each input.

Functions are broadly used in engineering where engineers used it to model and analyze systems in various engineering fields, such as electrical engineering, mechanical engineering, and civil engineering.

Computer Science

Functions are persuasive and very important in almost every aspect of computer science and software engineering. In programming languages, functions are essential for modularizing code, improving the code readability, and promoting code reuse.

- Function are used to separate code into different part to make readable code
- Hash function are used in password and security
- In Function used Algorithm (Sorting, Searching)
- Function are also Used in Machine learning for mathematical function

Physics

Functions are widely used in physics to plot various phenomena and relationships between physical quantities. In mechanics, functions describe the motion of objects, such as position as a function of time, velocity as a function of displacement, and acceleration as a function of velocity. All these various functions are used in physics to predict and analyze the behavior of systems.

- Law of Physics Like $F=ma$, $V=IR$ are function like variable
- It Predict Like Future of Function Motion over time.
- Function Allow us Graph to Analysis Real Time-world Physical behavior
- The more you pull, the more it stretches (stretches = Function of Force)

Engineering

Functions are broadly used in engineering where engineers used it to model and analyze systems in various engineering fields, such as electrical engineering, mechanical engineering, and civil engineering.

- Circuit current used as function of Voltage
- Control Circuit Use as Output and Input (on/off) function
- Function Help Explain System How System Behave like How Material Stretches when Force is Applied
- Function Also Describe the Electrical relationship between the Voltage-Current Curves

Daily Life

- Mobile balance of Sim or Mobile decrease as a function of usage
- Distance of travel depend upon speed and time(function)
- Battery goes down as a increase a usage of time (battery = Function of Time)
- Temperature Change During The day (Temperature = Function of time)

Absolute Value Function

An Absolute Value Function is a function that a give the distance of number form zero, and answer always be a positives

Computer Science

In Absolute value function is `abs()` is used to remove negative sign form number and return it distance from zero

In python we use in code like

```
x = -15
```

```
print(abs(x))
```

Output

```
15
```

- Absolute Value Function used in Machine learning (difference between predicted value and true value)
- In Graphic designing pixel rounding error
- Sensor detection error(difference from actual reading)
- In Electricity, its measure the size of signal without knowing the direction.

Physics

The absolute value function is often used to represent physical quantities that are direction independent or to calculate magnitudes. In mechanics, the displacement of an object represents the change in its position from an initial point to final point.

- Speed is Example of Absolute Value of Velocity
- Distance or Velocity is Always Positive(Absolute Displacement)
- Show how far a measured value is from the actual absolute value
- Absolute Value is used to represent the magnitude of positive or negative value

Engineering

In engineering, the absolute value of function is used in situations where on the magnitude matters, such as when calculating distances, displacements, or errors. In measurement systems, the absolute value function is applied to ensure that physical quantities are represented in a positive manner, without their direction or polarity. This is critical in application such a sensor data processing and instrumentation.

- Civil Engineer used Absolute Value of Function in manufacturing building
- Signal deviation by Absolute maximum value
- Measure Vibration Amplitude without Carry which way its move
- Magnitude of force is also taken by Absolute value

Daily Life

- Difference in temperature(Today-Yesterday)
- Penalty charge based on absolute difference in time(Cancelation in Rail Ticket)
- How Far you are from home , Ignoring the direction(Google Map)
- Show how something move fast. No matter its forward or backward

Limit of Function

The limit function of value that the function approaches as a input(x) get closer to a certain point The limit of a function describe the behavior of the function as its input approaches a certain value or as it approaches Infinity or negative infinity.

Computer Science

The concept of limit of a function is fundamental in computer but it is not directly used in most programming tasks, particularly in the analysis of algorithms and study of computational complexity.

- Used in Numerical to calculate an approximate value
- Limit are also used in machine learning to optimize the maximum and minimum value of function
- It is also use in network to analysis traffic behavior
- Also limit help to compute a smooth motion by calculation small time change over

Physics

In physics, the limit of a function of an describe the behavior of a variable as it approaches a certain value. This calculus concept is used to describe many physical phenomena.

- Limit found exact rate of change of velocity
- It Analyze the small changes in waves like amplitude, crest , trough
- It also show how position change at a certain time
- The Speed of object at a specific movement

Engineering

Limits play a key role in engineering, it is used to analysis and design of systems where continuous variation is involved.

- Determine flow rate of water at a specific point
- Calculate a instantaneous current or voltage in changing circuit
- Determine Signal Change as they were change continuously over a time
- Find the specific point at vibration of Bridge

Daily Life

- Limit help to check growth rate specially in Trading
- Calculating Instantaneous load of Processor and Ram
- Numerical method use limit to approximate Values
- They predict it position and velocity in small time interval

Derivative

The Derivative of a function at a certain point the function value change with respect to its variable. Derivative is written in the form $\frac{dy}{dx}$ It tell us how fast something is Change.

Computer Science

In Programing we use derivative in programing machine learning and deep learning also it is use in making game it how change a value with respect to other

- In Machine learning Derivative help to find the best modal By Minimizing Error
- It Also detect the change of signal over a time
- Calculate a Velocity and acceleration of robot at specific movement
- Also help in algorithm to find the Maximum and minimum value

Physics

In physics it how the quantities change with respect to other it use to describe motion and change

- Derivative are used to measure force when energy change
- Derivative also describe how velocity , displacement, and acceleration change over time
- The Derivative of Velocity with respect to time give acceleration
- The derivative of position with respect to time give instantaneous velocity

Engineering

In engineering it tell how the quantities change with respect to other it is used in mechanical engineering, electrical engineering circuit

- Derivative also help to find the deformation of material
- Velocity and Acceleration of machine used are calculating using derivative
- Also Help to design that Responded correctly to change of signal
- Derivative are used to change current in circuit

Daily Life

- Also Calculate the Temperature Change over a time
- Analyze How Signal Change or Enhance
- It also compute speed of robots at specific time
- Calculate how pressure or air lift change over a time

Calculus Project

Q 02: Make a piece wise defined function with its graph for your electricity bill up to 700 units, as the cost of electricity is dependent upon number of units in different interval.

Note

- This slab and tariff system is the new July 2025 tariff for people who use electricity.

Electricity Bill K Electric (Default Tariff)

Price or Slab of Unit

Unprotected Bill

$$Bill(x) = \begin{cases} 22.44 & 1 \leq x \leq 100 \\ 33.10 & 201 \leq x \leq 100 \\ 37.10 & 301 \leq x \leq 100 \\ 40.20 & 401 \leq x \leq 100 \\ 41.62 & 501 \leq x \leq 100 \\ 42.76 & 601 \leq x \leq 100 \\ 47.69 & x > 700 \end{cases}$$

Protected Bill

$$Bill(x) = \begin{cases} 10.54 & 1 \leq x \leq 100 \\ 13.01 & 101 \leq x \leq 200 \end{cases}$$

Note

- If a User can Use Electricity under 200 Unit. 6 month it is Protected User If They Cross 200 Unit They Are not in Protected User

Condition-1

Electricity Bill K Electric (Tariff)

Unprotected Bill

$$Bill(x) = \begin{cases} 22.44(x) & 1 \leq x \leq 100 \\ 33.10(x) & 201 \leq x \leq 100 \\ 37.10(x) & 301 \leq x \leq 100 \\ 40.20(x) & 401 \leq x \leq 100 \\ 41.62(x) & 501 \leq x \leq 100 \\ 42.76(x) & 601 \leq x \leq 100 \\ 47.69(x) & x > 700 \end{cases}$$

Protected Bill

$$Bill(x) = \begin{cases} 10.54(x) & 1 \leq x \leq 100 \\ 13.01(x) & 101 \leq x \leq 200 \end{cases}$$

Note

- If a User can Use Electricity under 200 Unit. 6 month it is Protected User If They Cross 200 Unit They Are not in Protected User

Condition-2

Electricity Bill K Electric (Tariff)

Unprotected User

$$Bill(x) = \begin{cases} 22.44(x) & 1 \leq x \leq 100 \\ 2244 + 28.91(x - 100) & 101 \leq x \leq 200 \\ 2244 + 2891 + 33.10(x - 200) & 201 \leq x \leq 300 \\ 2244 + 2891 + 3340 + 37.10(x - 300) & 301 \leq x \leq 400 \\ 2244 + 2891 + 3340 + 3710 + 40.20(x - 400) & 401 \leq x \leq 500 \\ 2244 + 2891 + 3340 + 3710 + 4020 + 41.62(x - 500) & 501 \leq x \leq 600 \\ 2244 + 2891 + 3340 + 3710 + 4020 + 4162 + 42.76(x - 600) & 601 \leq x \leq 700 \\ 2244 + 2891 + 3340 + 3710 + 4020 + 4162 + 4276 + 47.69(x - 700) & x > 700 \end{cases}$$

Protected Bill

$$Bill(x) = \begin{cases} 10.54(x) & 1 \leq x \leq 100 \\ 1054 + 13.01(x - 100) & 101 \leq x \leq 200 \end{cases}$$

:

Condition-2-Calculation

For Example

Protected User

I Used 170 Unit and I am Protected User

$$Bill(170) = \begin{cases} 10.54(100) & 1 \leq x \leq 100 \\ 1054 + 13.01(170 - 100) & 101 \leq x \leq 200 \end{cases}$$

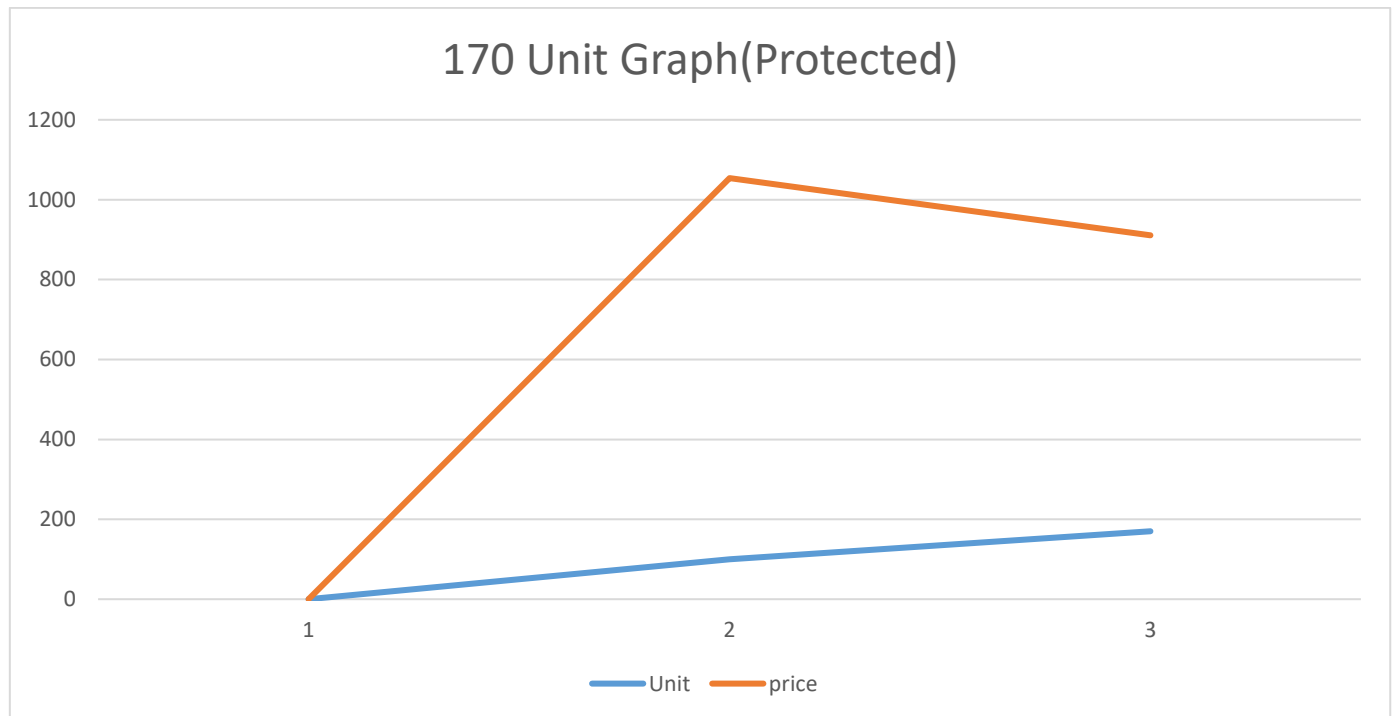
$$Bill(170) = 1054 + 13.01(170 - 100)$$

$$Bill(170) = 1054 + 13.01(70)$$

Under 100 Unit Rs 1054

Up to 101 Unit 70 Rs 910.7

My Bill Rs 1964.7



Graph:

The Graph is Increase because the number of Unit Increase When the Unit of Electricity. The Electricity is also Increase Its Totally Depend upon Billing Slab

Unprotected User

I Used 648 Unit and I am Unprotected User

$$Bill(x) = \begin{cases} 22.44(x) & 1 \leq x \leq 100 \\ 2244 + 28.91(x - 100) & 101 \leq x \leq 200 \\ 2244 + 2891 + 33.10(x - 200) & 201 \leq x \leq 300 \\ 2244 + 2891 + 3340 + 37.10(x - 300) & 301 \leq x \leq 400 \\ 2244 + 2891 + 3340 + 3710 + 40.20(x - 400) & 401 \leq x \leq 500 \\ 2244 + 2891 + 3340 + 3710 + 4020 + 41.62(x - 500) & 501 \leq x \leq 600 \\ 2244 + 2891 + 3340 + 3710 + 4020 + 4162 + 42.76(648 - 600) & 601 \leq x \leq 700 \\ 2244 + 2891 + 3340 + 3710 + 4020 + 4162 + 4276 + 47.69(x - 700) & x > 700 \end{cases}$$

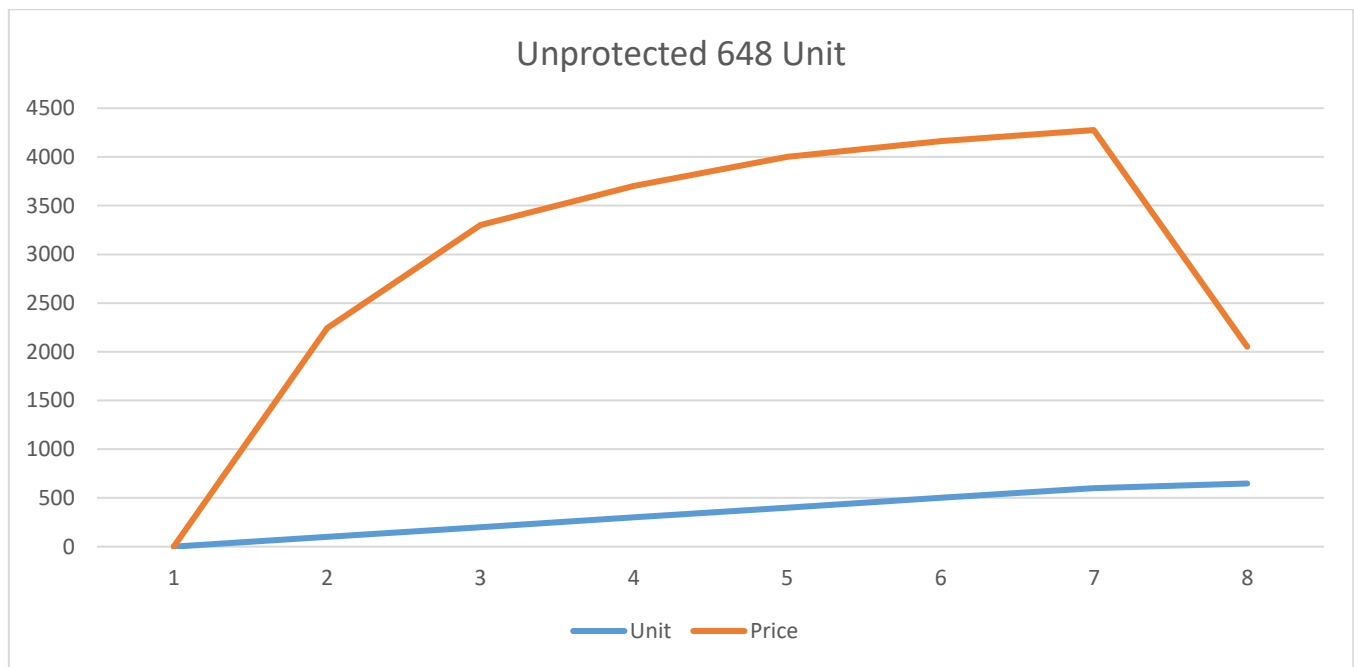
$$Bill(648) = 2244 + 2891 + 3340 + 3710 + 4020 + 4162 + 42.76(648 - 600)$$

$$Bill(648) = 2244 + 2891 + 3340 + 3710 + 4020 + 4162 + 42.76(48)$$

Under 600 Unit Rs 20367

Up to 601 Unit 48 Rs 2052.48

My Bill Rs 22,419.48



Graph:

The Graph is Increase because the number of Unit Increase When the Unit of Electricity. The Electricity is also Increase Its Totally Depend upon Billing Slab

Condition-1 Calculation

Electricity Bill K Electric (Tariff)

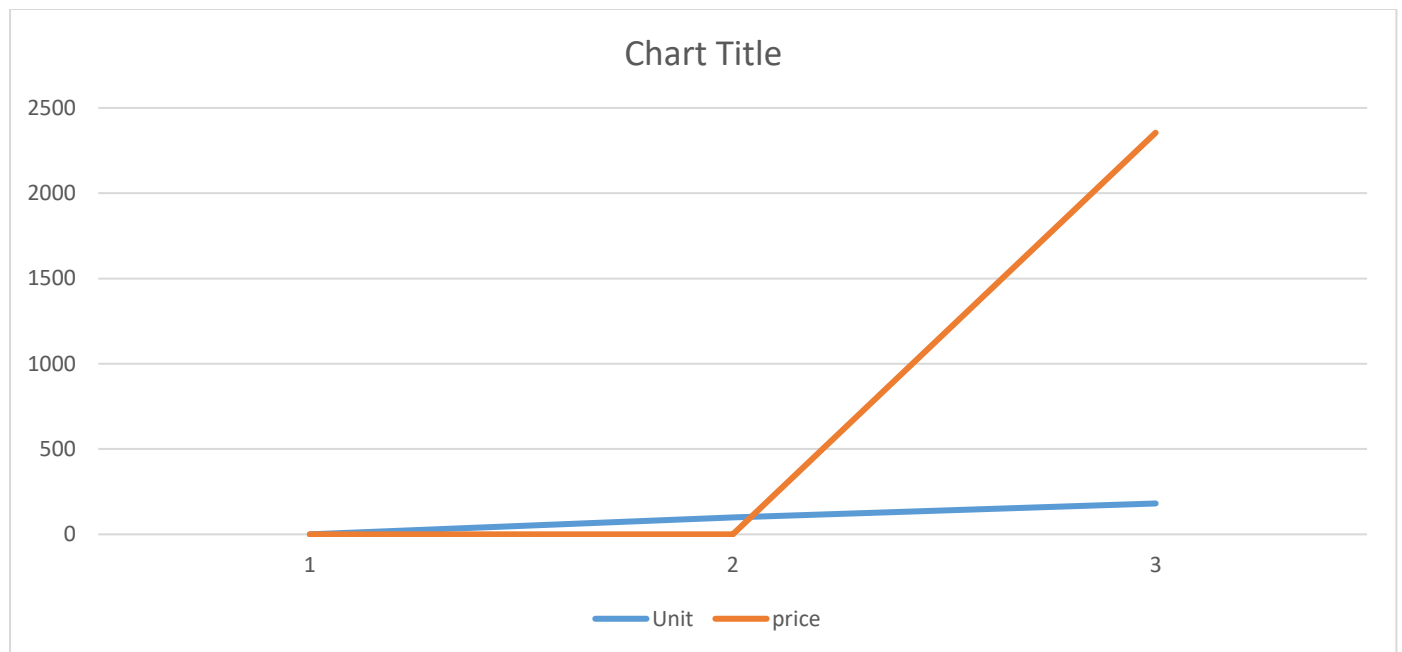
My Relative Used 181 Unit and They are Protected User

$$Bill(181) = \begin{cases} 10.54(x) & 1 \leq x \leq 100 \\ 13.01(181) & 101 \leq x \leq 200 \end{cases}$$

$$Bill(181) = 13.01(181)$$

Unit Bill 181 Unit Rs 2354.68

Total Bill Rs 2354.68



Unprotected User

My Relative Used 693 Unit and They are Unprotected User

$$Bill(693) = \begin{cases} 22.44(x) & 1 \leq x \leq 100 \\ 33.10(x) & 201 \leq x \leq 100 \\ 37.10(x) & 301 \leq x \leq 100 \\ 40.20(x) & 401 \leq x \leq 100 \\ 41.62(x) & 501 \leq x \leq 100 \\ 42.76(693) & 601 \leq x \leq 100 \\ 47.69(x) & x > 700 \end{cases}$$

$$Bill(693) = 42.76(693)$$

Unit Bill 693 Unit Rs 29632.68

Total Bill Rs 29632.68

Unprotected User

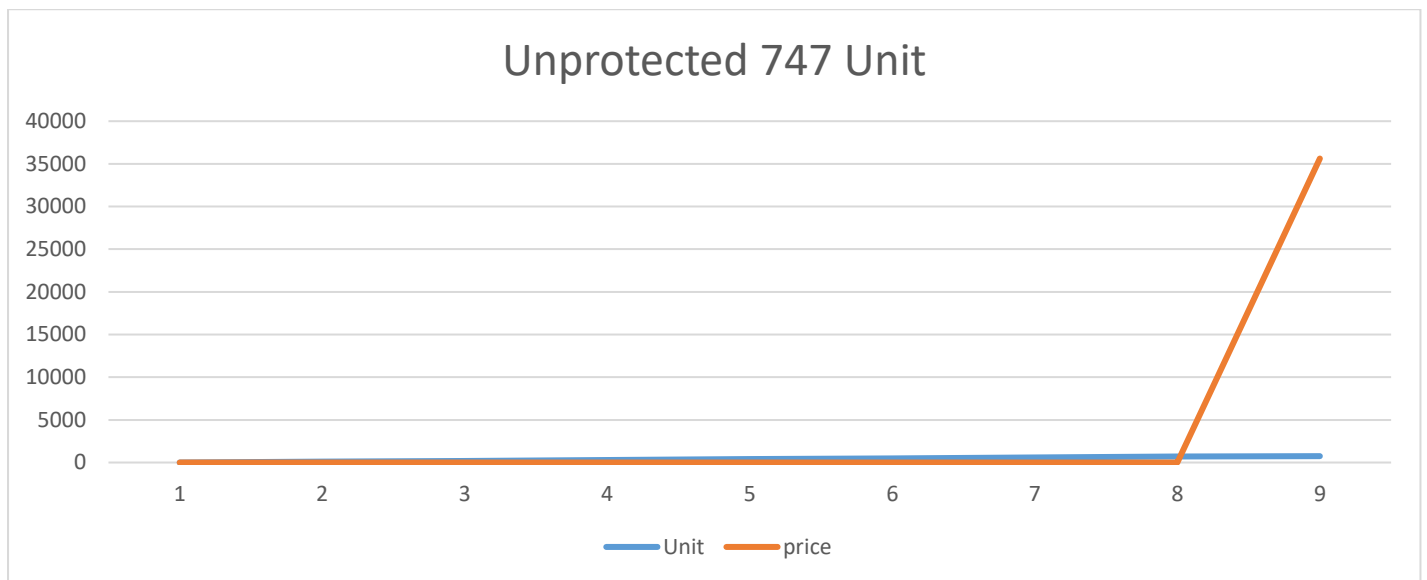
Sir Used 747 Unit and They are Unprotected User

$$Bill(x) = \begin{cases} 22.44(x) & 1 \leq x \leq 100 \\ 33.10(x) & 201 \leq x \leq 100 \\ 37.10(x) & 301 \leq x \leq 100 \\ 40.20(x) & 401 \leq x \leq 100 \\ 41.62(x) & 501 \leq x \leq 100 \\ 42.76(x) & 601 \leq x \leq 100 \\ 47.69(747) & x > 700 \end{cases}$$

$$Bill(747) = 47.69(747)$$

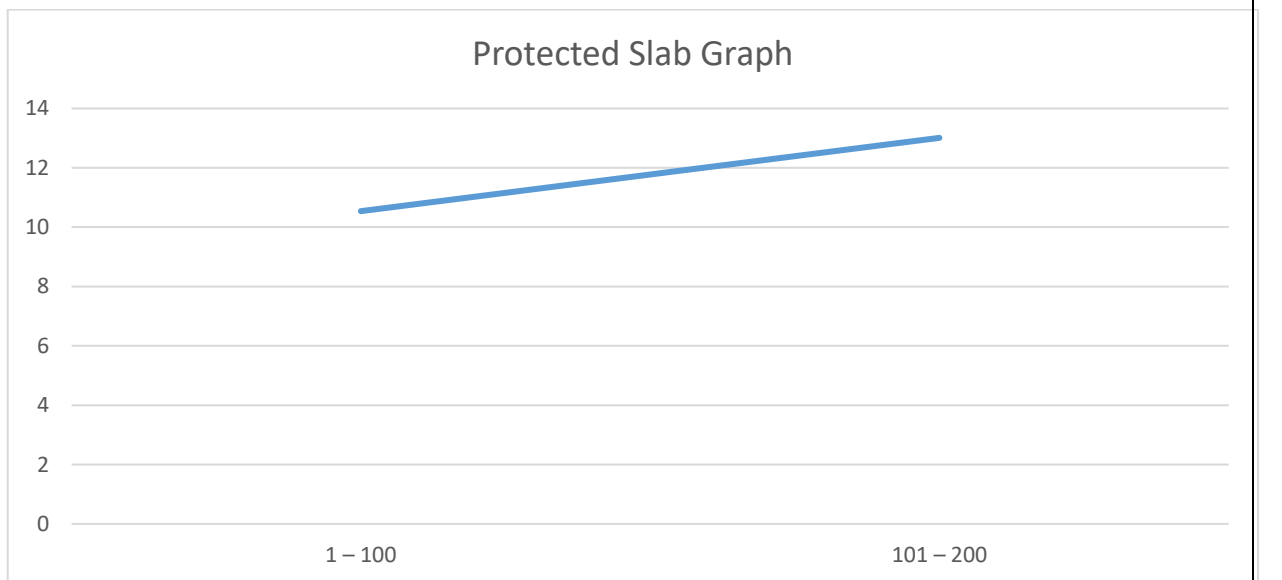
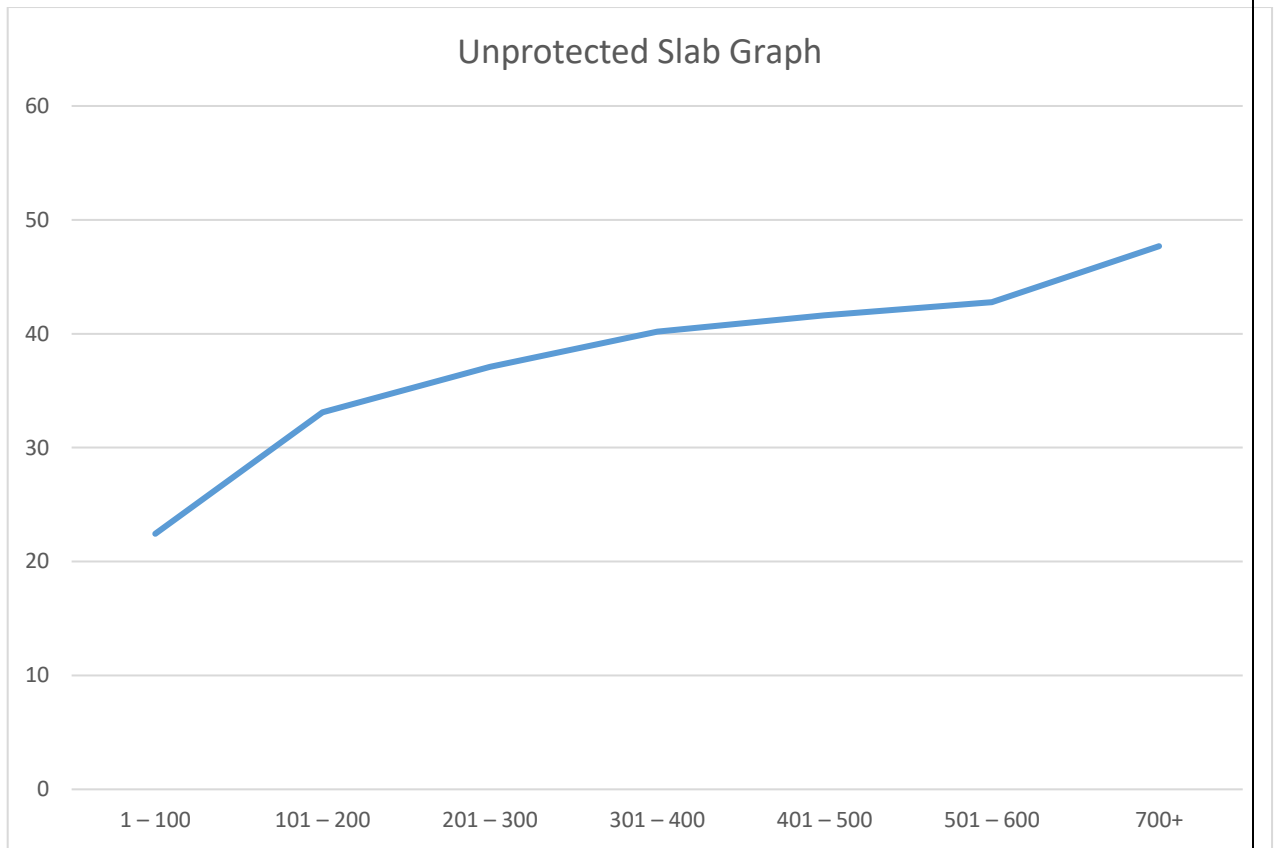
Unit Bill 747 Unit Rs 35624.43

Total Bill Rs 35624.43



Graph:

The Graph is Increase because the number of Unit Increase When the Unit of Electricity. The Electricity is also Increase Its Totally Depend upon Billing Slab

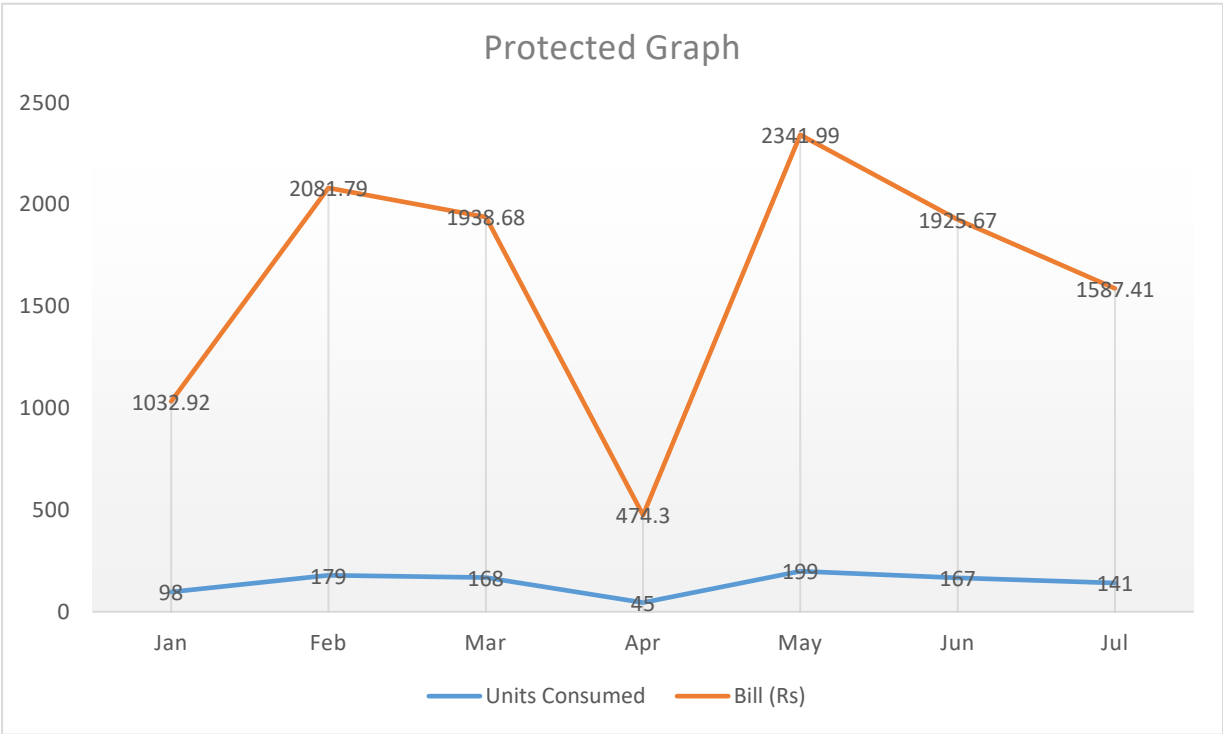


- Without units, the graph of $\text{Bill}(x)$ is a piece-wise linear graph.
- Hence, the bill increases at an increasing rate.

7-Month Unit Point

Protected Graph

Month	Units Consumed	Bill (Rs)
Jan	98	1032.92
Feb	179	2081.79
Mar	168	1938.68
Apr	45	474.3
May	199	2341.99
Jun	167	1925.67
Jul	141	1587.41

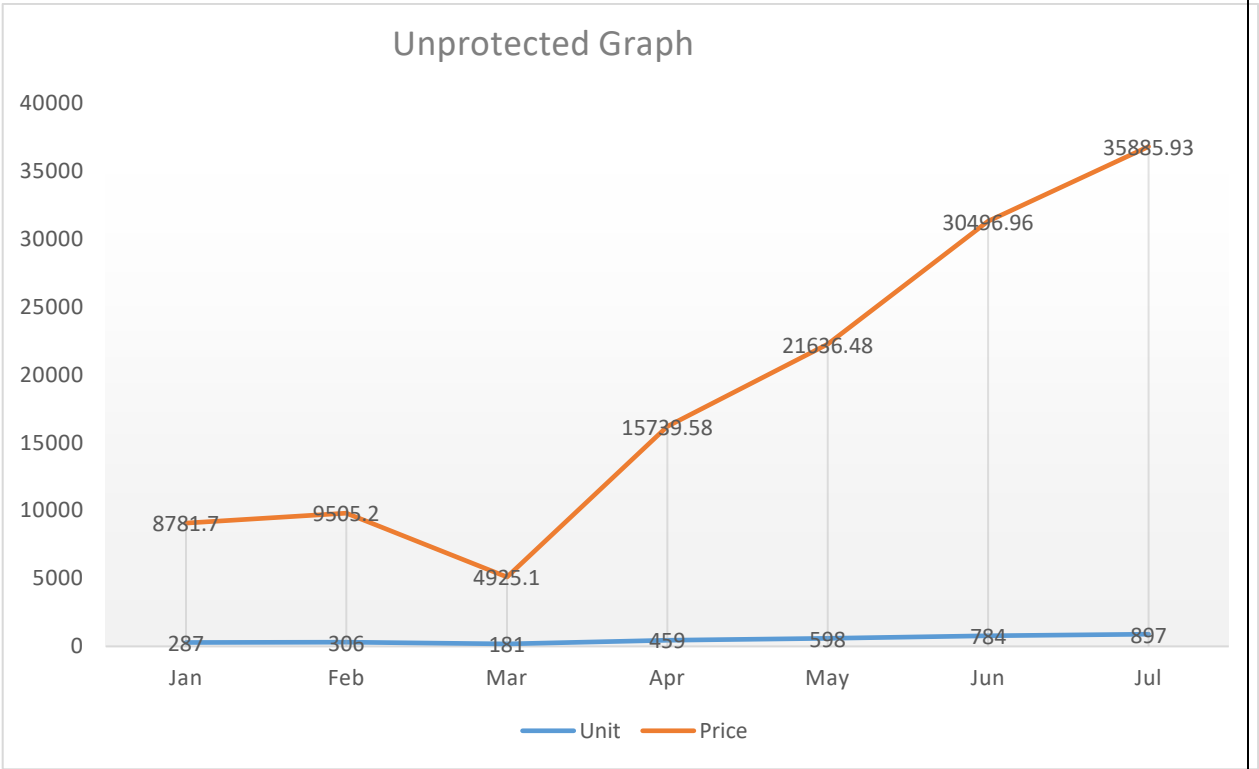


- According to user use unit the graph is increase or decrease with respect to use unit

7-Month Unit Point

Unprotected Graph

Month	Units Consumed	Bill (Rs)
Jan	287	8781.7
Feb	306	9505.2
Mar	181	4925.1
Apr	459	15739.58
May	598	21636.48
Jun	784	30496.96
Jul	897	35885.93



- According to user use unit the graph is increase or decrease with respect to use unit

Calculus Project

Calculus and Analytical Geometry

Q 3: Write down only four pages in your own words very precisely by studying the history of Mathematics, on the topics below.

The Role of Newton and Leibniz in the Development of Calculus

History of Math

History

Mathematics Started with Early Human over 20000 year ago. Math is used for Counting, tracking by Artifact from 20000 year. Math was used first Time To Count like Animal , Money or food Early Human used Stick Stone and Mark on wall to Help in calculate them

Over a time people invented number and symbol The Egypt people used math for make building like pyramid and measuring land. Babylonian used math to study star and planets.

Also Greek make major contribution on They Studied math shape like lines and angle. Famous Greek Mathematician like Pythagoras and Euclid about Geometry

Late in India and Middle East people work on Zero and Algebra. The Decimal System and Number Zero came from Indian and its made math more Easier

In Europe During the period a time of learning and new ideas (14th century to 17 century) many mathematician worked on math like Isaac Newton and Gottfried Wilhelm Leibniz discovered Calculus which helped understand motion and change

Today Math is used in Allover World Math is used in Science Computer Science engineering and everything in life It help us solved problem understand a problem and build thing

Mathematician in Pakistan have good progress Many Pakistani mathematician have contributed to teaching and researching Some Pakistani Scientist are given below:

- Dr. Abdus Salam : Even Though he was a physicist he used calculus and integration in his work He worked in Mathematical modal and equations Calculus helped him to solved problem
- Dr. Muhammad Zafar Iqbal : As mathematicians and Educator He has worked on Higher mathematics, including calculus and integration and mathematical problem solving
- Other Pakistani mathematicians : Many Mathematician and researcher worked on calculus integration in field of computer science and engineering

Mathematics has grown over thousands of year, and it continue to be important for everyone

Scientist (Mathematician History)

Isaac Newton Role (1643–1727)

Isaac Newton was born on January 4, 1643, in England. He became one of the most famous scientists and mathematicians in history. As a child, he was curious about how things worked and liked to study nature and science.

Isaac Newton Worked on Derivatives He Invented a fluxions. Fluxions are like a derivative today. Fluxion mean the rate of change of something. Newton Calculus are very useful in Modern World His Idea became a base of Mathematics. He calculated how fast a planet move. His Worked focused on applied higher derivative to real world problem like physics (Speed, Motion, Electricity, and Energy)

Isaac Newton Approached to calculus was mainly applied he used it to solve real world problem in science. His Idea laid the foundation for modern physics, mathematics and Engineering

Newton used algebra and geometry together with calculus to solve problems.

Algebra History

Algebra is a major or Most Important part of Mathematics without Algebra Math is Uncompleted in Algebra We Use letter and symbol to represent number and problem. Algebra Started from as long time ago around 2000 year ago The word of Algebra come from Arabic book written by Muslim Mathematician Scientist Al-Khwarizmi in the 9th century The Algebra was used to solve problem like trading measuring land used to solve Equation Over a time It is become most Advanced and is major part of Modern Math

Newton invented the major branch of math is Calculus They Used Algebra to work with Algebra Formula and Equation to Create Calculus. Algebra helped him to write and solve problem like equation and motion, growth and speed. They Connect Algebra with Calculus making it easier to solve Real world problem in physics and engineering

Geometry History

Geometry is major branch of mathematics in Geometry we studies about Shape Size and that space around it. It History also come from Thousands of year back become civilization of Egypt and Mesopotamia People use geometry to measure the size land build building and create art The ancient Greek developed many rule in geometry that are still thought today Over A time The Geometry become most important like before thousand year before And it is important part of Modern Math

Newton used Geometry to study shape, curve and space he applied geometry to understand how Planet move around Sun and how object fall. Newton Combined Geometry and calculus to solve problem in physics and mathematics. His worked helped to make geometry most efficient and useful for scientist and engineer in field of Engineering

Calculus History

Calculus is major branch of Math mathematics in we study how time is change. It was invented in 17 years ago By Isaac Newton from England and Gottfried Wilhelm Leibniz from Germany. Newton Used Calculus to study motion, physics gravity. Gottfried Wilhelm Leibniz created a symbol and rules calculus to easier to use over a time Calculus is most important tool in mathematics, science and Engineering.

Isaac Newton play an important role in calculus to study about how thing changes. Calculus used to study understand gravity motion, effect of force and the movement of planet. Newton used Algebra and geometry to create a formula and solve problem. Newton Worked on Calculus its helped scientist and engineer study in the world very accurately and without mistake.

Newton also Worked on Integration helped to find the total amount of something like the area under the curve Newton used Calculus to solve many problem in physics Newton also Explained how planet move and object fall to the ground. His Method of Calculus allow him to develop a law of motion and law of universal gravitation

However, a lot of controversies were made on the discovery of calculus when Newton and Leibniz become rival enemies dispute over who first invented calculus, which has gone for many years and even lead to harsh debates among mathematicians. Despite all this controversy, calculus continue to evolve in the 18th and 19th centuries, with the help of mathematician such is Leonhard Euler and Joseph-Louis Lagrange making significant contributions to its consolidation and expansion.

In 20th century, calculus have become a necessary tool in various scientific and engineering fields. Further development of calculus in its areas like multivariable calculus, differential equations, and functional analysis drive calculus by making it the lead or head of mathematical research and application.

The Role of Newton and Leibniz in the Development of Calculus

Isaac Newton and Gottfried Wilhelm Leibniz both discoveries calculus in 17th century but they are not dependent on other both can work independently Newton crate a basic ideas or rule of calculus which method is fluxions and worked with small quantities Leibniz's also developed calculus on his own and introduced the symbol notation that we used today it is most useful in modern world

Their contributions in the development of calculus revolutionized mathematics in laid the foundation of ground work for modern physics and Engineering. However, Newton and Leibniz were the first to provide a systematic method of carrying out operations, complete with set rules and symbolic representation. They were the ones to trolley found calculus as we recognize it today. Newton and Leibniz used different notation since the developed their theories independently.

Gottfried Wilhelm Leibniz (1646–1716)

Gottfried Wilhelm Leibniz was a German mathematician and philosopher. He lived from 1646 to 1716. Around the same time as Isaac Newton, Leibniz also discovered calculus.

Leibniz played an important role in calculus; it focused on math itself and physics. He created a special way of writing like a symbol that is still used today. For example, like the symbol of derivatives “d” and Symbol of Integration “ $\int(x)$ ” His Symbol made calculus easier to use and learn.

Leibniz worked on finding area under curve, Understanding change and solve equation. Leibniz Worked can helped many mathematician to study Calculus more accurately and Easier.

Even Though Leibniz and Newton are play independently role in calculus Both of them are most Important Newton make Easier while study Calculus and Leibniz make it Easier to write(While Creating a Symbol) and make understand to Mathematician

Leibniz developed derivative separately at the same time of Isaac Newton He Created a notation we used today like “ $\frac{dy}{dx}$ ” to show the rate of change His worked made easier to other mathematician to derivatives in many calculation and mathematics

Leibniz published papers to explain calculus clearly. His symbols, like “ $\frac{dy}{dx}$ ” and “ $\int x$ ”, are still used in schools, research, and engineering. Without his notation, calculus would be harder to write, read, and use.

Leibniz give very important equation in Mathematics world .We use in today world it easier to derivate any nth number derivative like we can easily derivate fifth derivative without derivate first derivative of equation.

$$(f.g)^n = f^{(n)}.g + nf^{(n-1)}.g^{(1)} + n(n-1)/2! . f^{(n-2)}.g^{(2)} + + f.g^{(n)}$$

Conclusion:

In conclusion, the history of calculus light and the remarkable ingenuity and resolution of mathematicians like Newton and Leibniz. Their ground breaking methods and conceptual insights let the foundation for today's modern calculus, demonstrating the transformative impact of collaboration, creativity, and intellectual pursuit on the advancement of human knowledge. Despite their priority dispute, their legacies induced testament to the enduring power of collaborative effort in shaping the mathematical landscape.

