Simulations QR

1. Round-Trip Time

We plot the distribution of inter-event times Δt for multiple stocks in $\log_{10}$ space. Across all tickers, we observe a consistent mode around $\log_{10}(\Delta t) \approx 4.47$, corresponding to approximately **29 microseconds**.

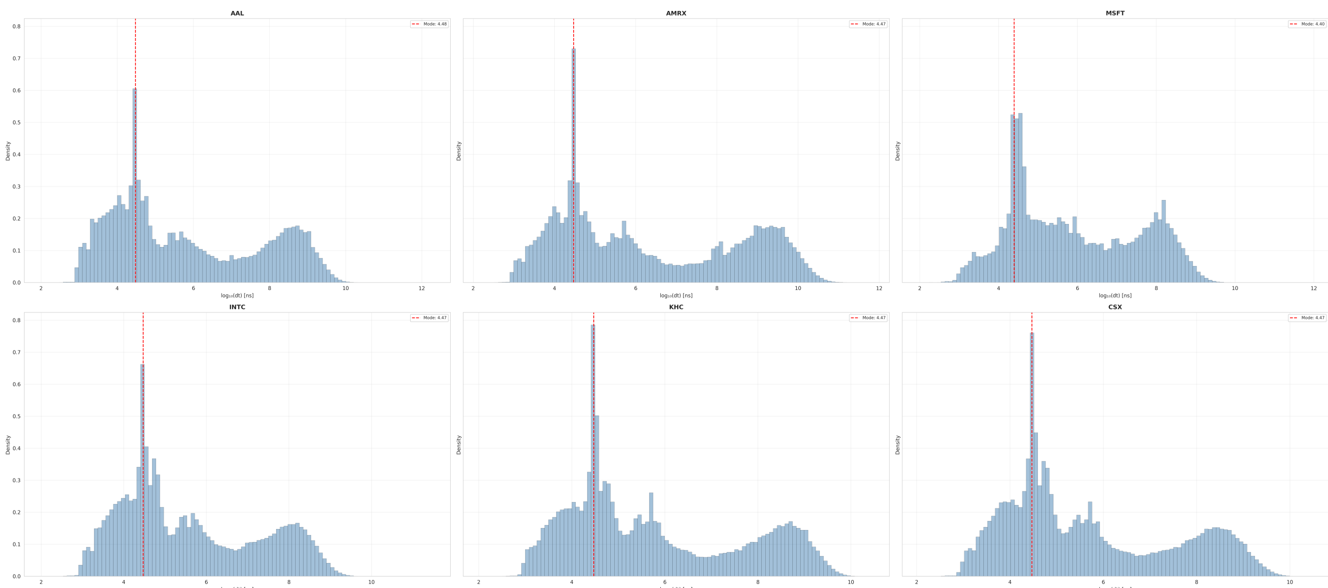

Figure 1: Distribution of inter-event times across stocks. The mode is consistent at ~29 s.

Zooming into the peak region, the interval where density drops to 50% of the maximum is remarkably tight: $[4.39, 4.55]$ in $\log_{10}$ space, or roughly $[24\mu s, 35\mu s]$.

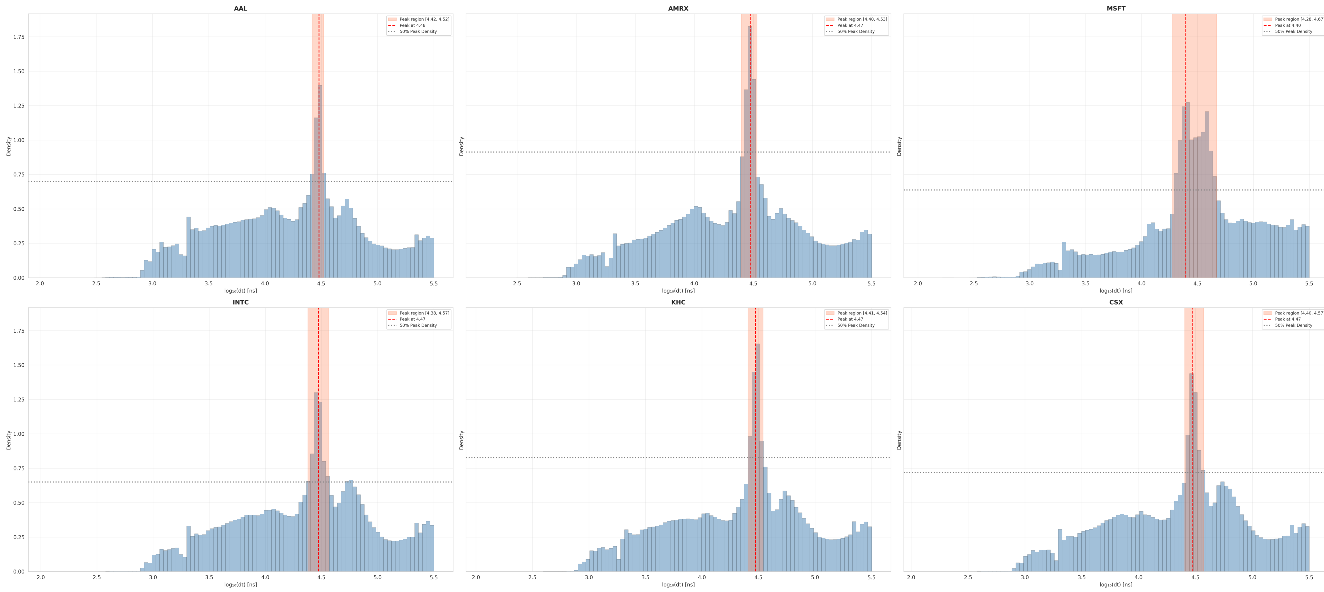

Figure 2: Peak region with 50% density bounds.

A plausible explanation for this is that it represents the **round-trip time**: the time for an order to reach the exchange, be processed by the matching engine, and propagate back to the public data feed. We denote this latency δ and model it as a random delay that every order is subject to.

Fitting a Gaussian to the peak region yields:

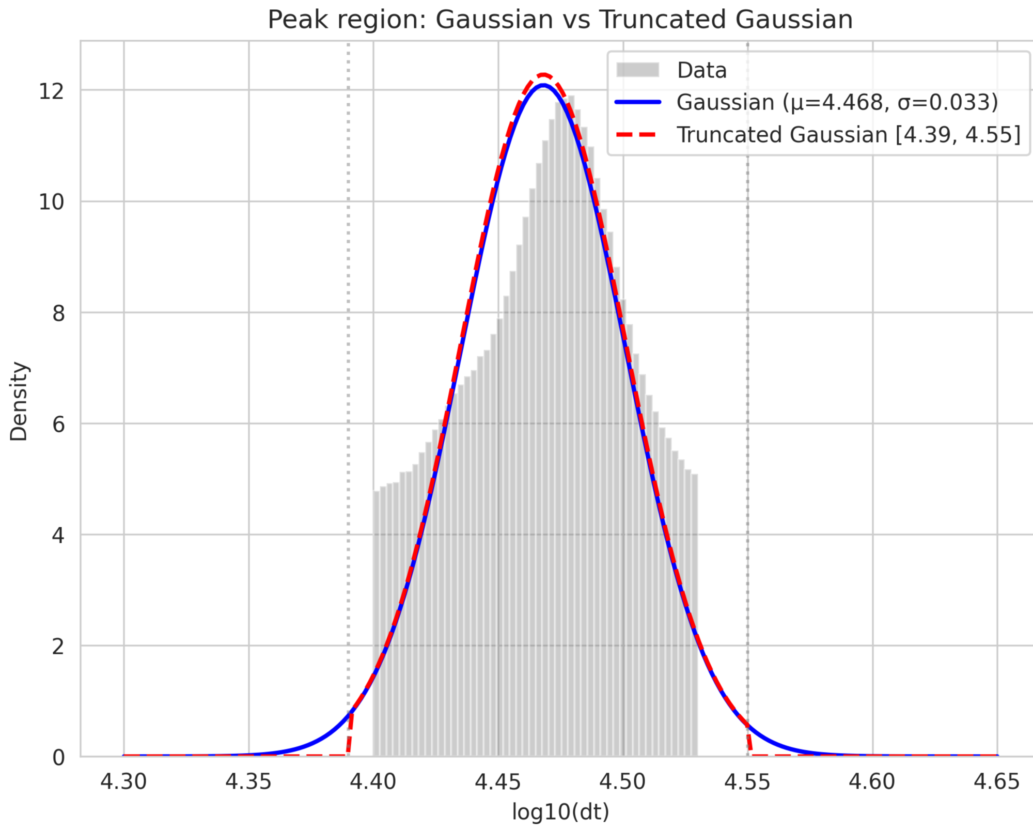

Figure 3: Gaussian fit to the round-trip time peak.

Parameter	Value
μ	4.471
σ	0.040
Bounds	$[4.39, 4.55]$

2. QR Inter-Event Times

Queue-reactive events are reactions to the current state of the order book. An agent observes the book, decides to act, and sends an order. Thus the inter-event time should be at least δ (round-trip) plus some reaction time. This justifies fitting only the peak region and beyond—excluding the fast left tail.

We fit a 3-component Gaussian Mixture Model (GMM) conditioned on imbalance and spread:

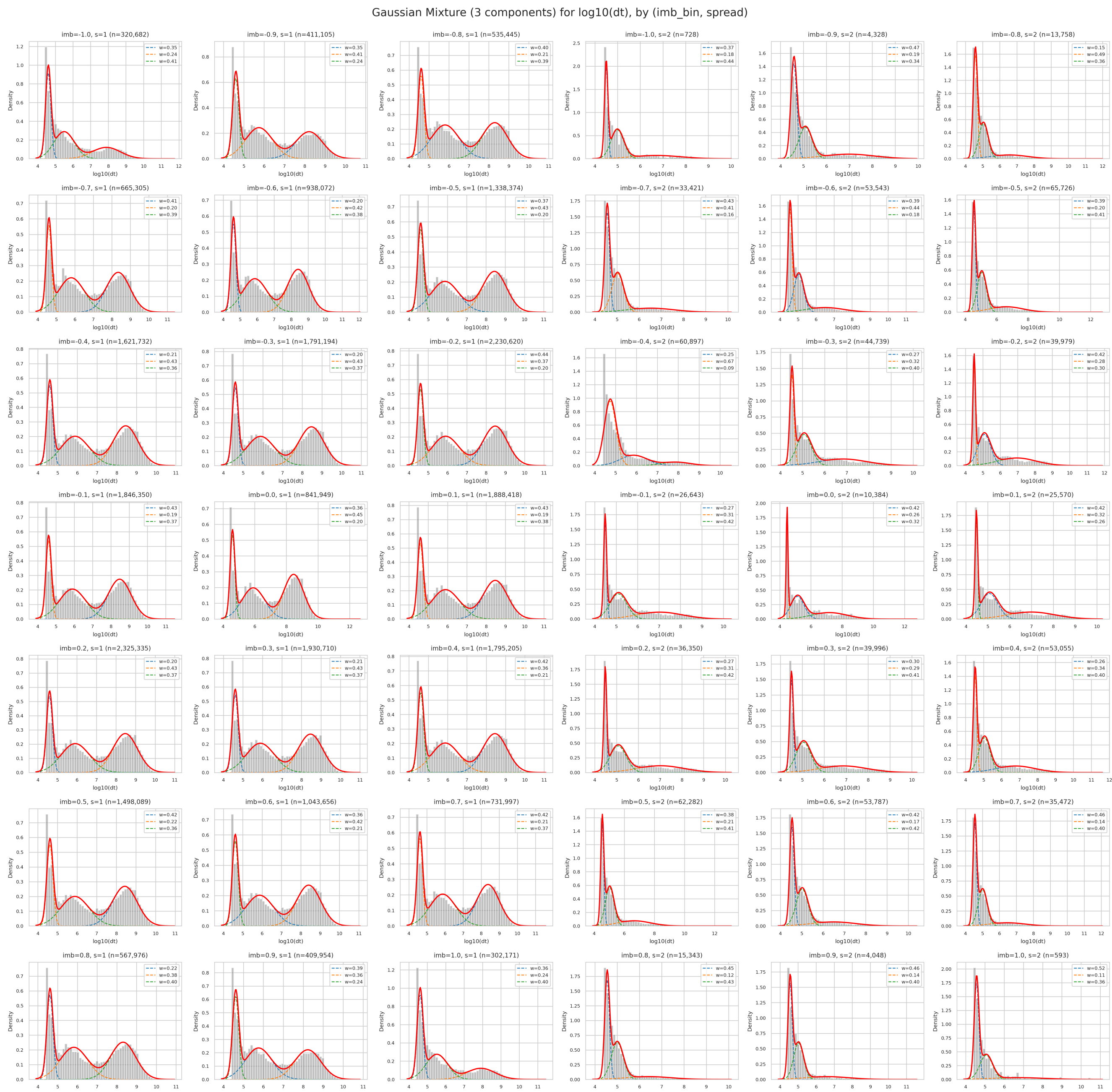

Figure 4: GMM fit to QR inter-event times by (imbalance, spread).

3. Race Mechanism

The fast left tail of the Δt distribution represents race events—bursts of aggressive orders in the direction of an exogenous signal α_t. When $\alpha > 0$, racers target the ask side; when $\alpha < 0$, they target the bid side.

3.1 Race Triggering

Races are triggered with probability depending on $|\alpha|$:

$$P(\text{race}|\alpha) = \begin{cases} 0 & \text{if } |\alpha| < 0.8 \\ \frac{1}{1+e^{-8(|\alpha|-0.7)}} & \text{otherwise} \end{cases}$$

The number of racers follows a geometric distribution with mean scaling with $|\alpha|$:

$$\mathbb{E}[N|\alpha] = 4 + 2.5 \cdot (|\alpha| - 0.7)$$

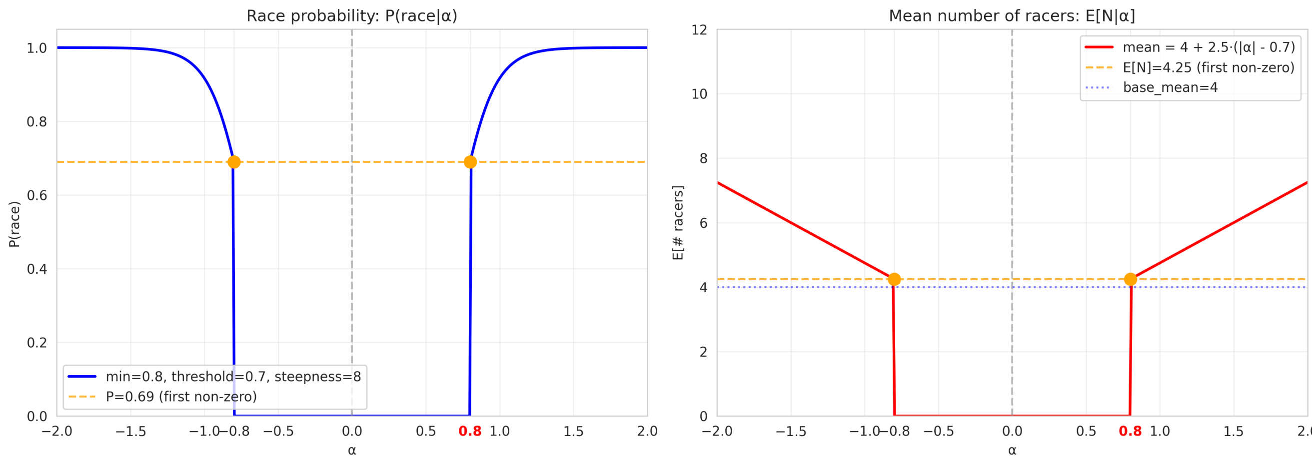

Figure 5: Race probability and expected number of racers vs α.

3.2 Race Composition

Race orders are composed of:

- **30% trades**: aggressive orders consuming liquidity
- **70% cancels**: defensive orders pulling liquidity before adverse selection

3.3 Inter-Arrival Times

We model the timing as:

- The first racer arrives at time $t_0 + \delta$
- Subsequent racers arrive with inter-arrival delays γ_i drawn from the left tail
- The i-th racer arrives at: $t_0 + \delta + \sum_{j=1}^{i-1} \gamma_j$

We fit both Gamma and Weibull distributions to the left tail ($\log_{10}(\Delta t) < 4.39$):

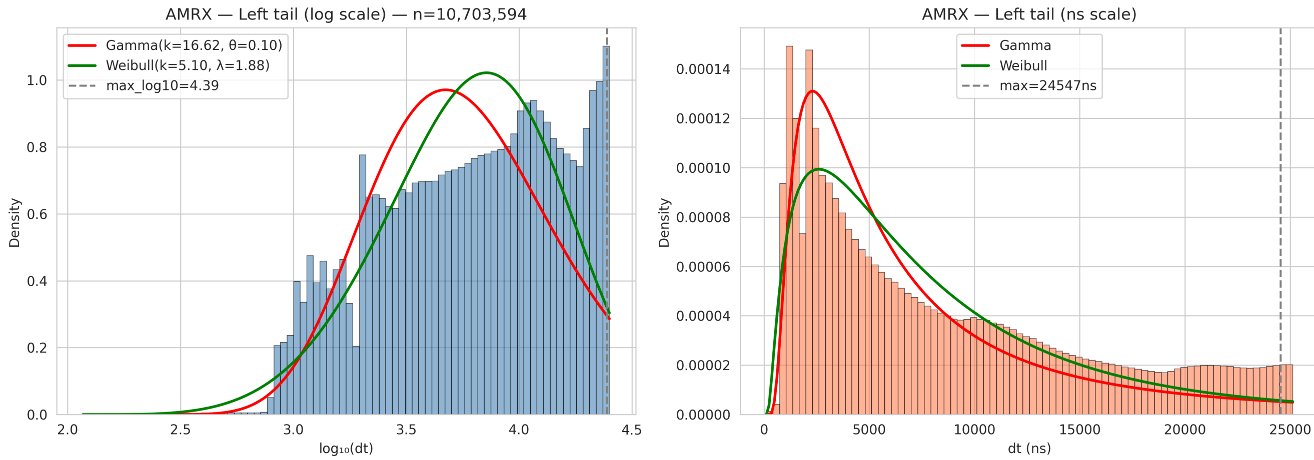

Figure 6: Gamma and Weibull fits to the left tail.

Both provide good fits; Weibull is used by default.

With some tweaking we obtain this fit for the distribution of Δt:

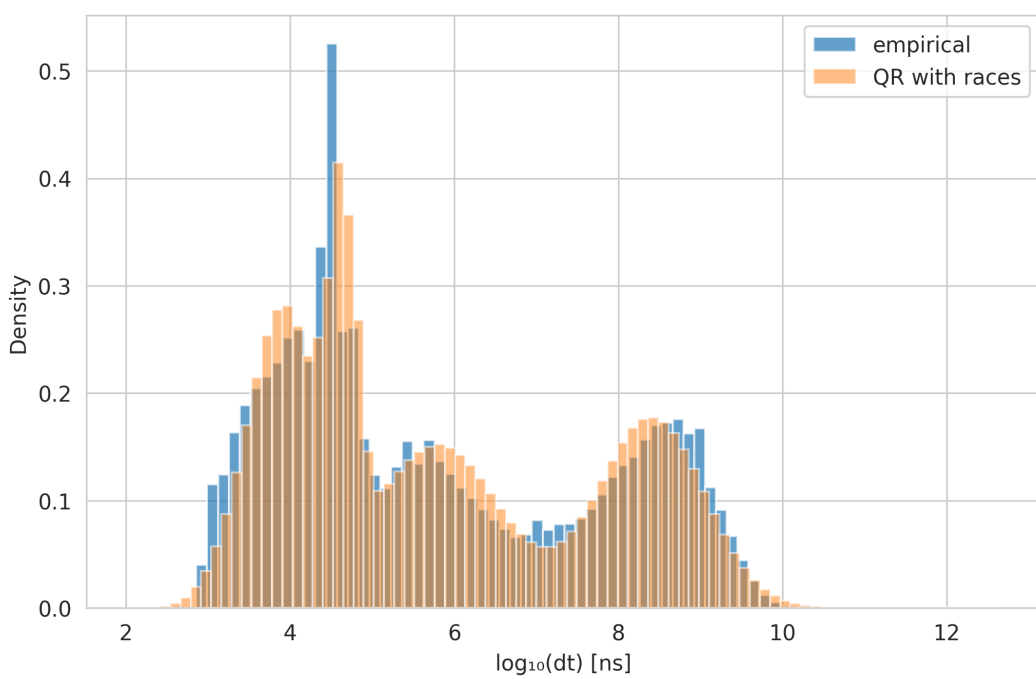

Figure 7: Empirical vs model Δt distribution with races.

But we loose the fit of some other statistics in the process.

4. Alpha Predictivity

4.1 Biasing Mechanism

The bid and ask trade probabilities are biased by α and market impact:

$$P(\text{bid}) \propto e^{\text{impact}} \cdot e^{-\alpha}, \quad P(\text{ask}) \propto e^{-\text{impact}} \cdot e^{\alpha}$$

Thus $\alpha > 0$ increases ask trades (buys), pushing price up toward the signal. We measure predictivity via:

$$\mathbb{E}[\alpha_t \cdot (P_{t+\Delta t} - P_t)]$$

4.2 Without Races

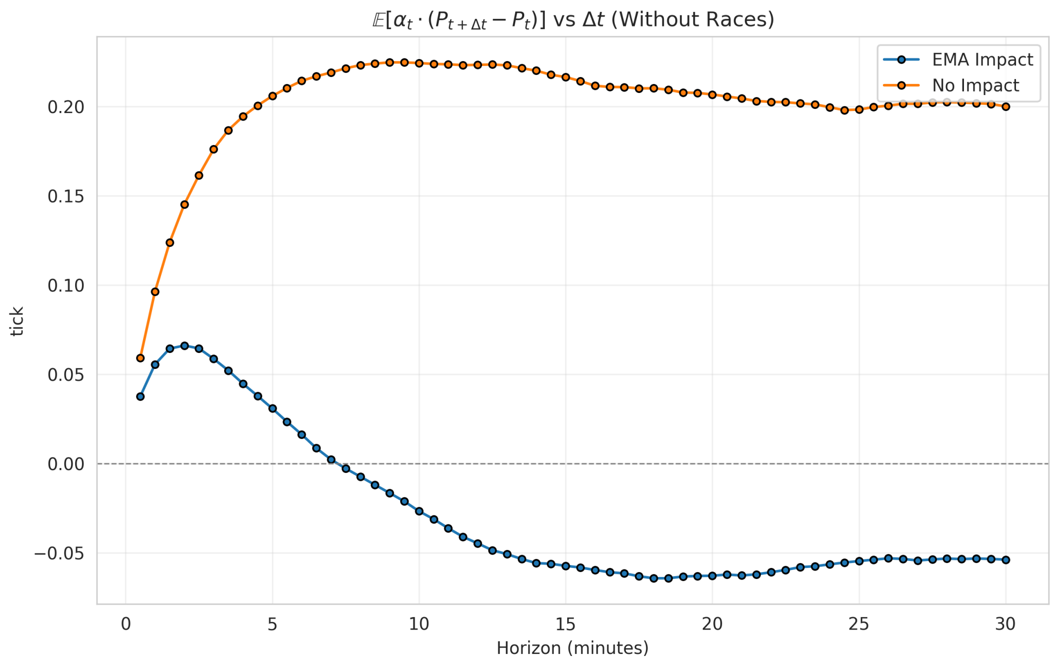

Figure 8: Alpha predictivity without races.

Without races, when we introduce impact it compensates the α bias—predictivity vanishes as the two effects cancel out.

4.3 With Races

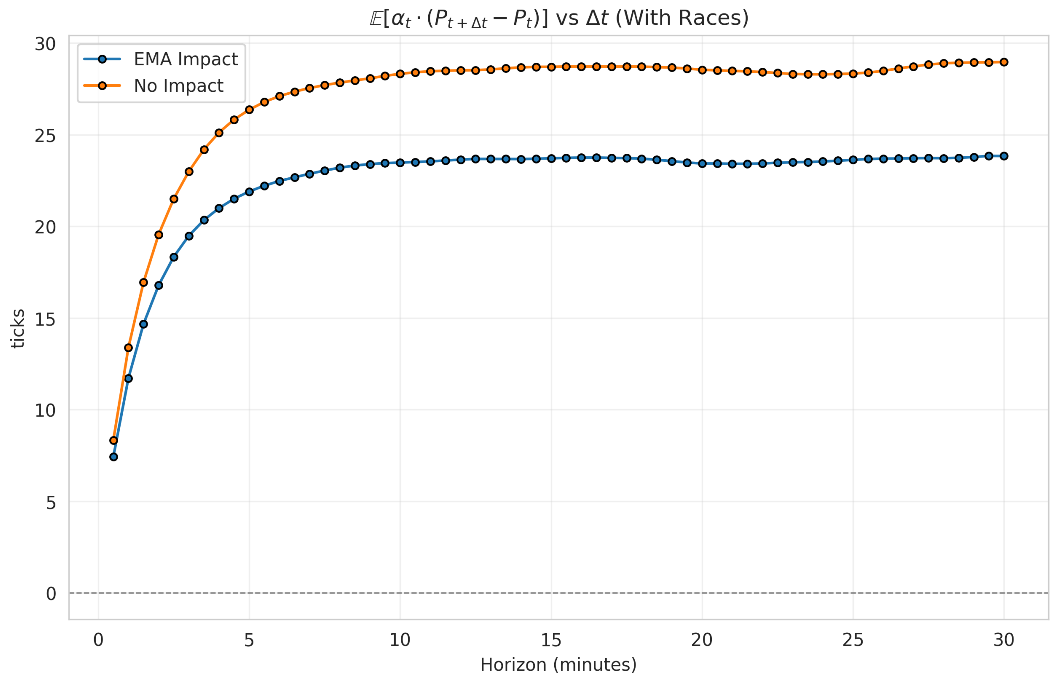

Figure 9: Alpha predictivity with races.

With races, impact dampens predictivity but it remains strong. The race mechanism consumes part of the signal, yet α retains significant predictive power over future price moves.