National University of Computer and Emerging Sciences

Assignment No. 0

Design and Analysis of Algorithms

CS2009

Fall 2024

Deadline: September 03, 2024

Submission Instructions:

- All problems must be solved and submitted on Google Classroom.
- Use A4 size papers with assignment front paper.
- Submit the scanned Assignment in PDF form on Google Classroom.
- This is an individual assignment.
- **Plagiarism is strictly prohibited.**
- **Please do not use any AI tool and do not copy from others.**
- **Just analyze the problem and then brainstorm the solution.**

Question 01: (10 Marks)

Design an algorithm for computing $\lfloor\sqrt{n}\rfloor$ for any positive integer n. Besides assignment and comparison, your algorithm may only use the four basic arithmetical operations.

Question 02: (10 Marks)

Design an algorithm to find all the common elements in two sorted lists of numbers. For example, for the lists 2, 5, 5, 5 and 2, 2, 3, 5, 5, 7, the output should be 2, 5, 5. What is the maximum number of comparisons your algorithm makes if the lengths of the two given lists are m and n, respectively?

Question 03: (10 Marks)

Find gcd(31415, 14142) by applying Euclid's algorithm.

a. Estimate how many times faster it will be to find gcd(31415, 14142) by Euclid's algorithm compared with the algorithm based on checking consecutive integers from min{m, n} down to gcd(m, n).
b. Prove the equality gcd(m, n) = gcd(n, m mod n) for every pair of positive integers m and n.
c. What does Euclid's algorithm do for a pair of integers in which the first is smaller than the second?
d. What is the maximum number of times this can happen during the algorithm's execution on such an input?

Question 04: (10 Marks)

Locker doors There are n lockers in a hallway, numbered sequentially from 1 to n. Initially, all the locker doors are closed. You make n passes by the lockers, each time starting with locker #1. On the ith pass, i = 1, 2,...,n, you toggle the door of every ith locker: if the door is closed, you open it; if it is open, you close it. After the last pass, which locker doors are open and which are closed? How many of them are open?

Question 05: (10 Marks)

A peasant finds himself on a riverbank with a wolf, a goat, and a head of cabbage. He needs to transport all three to the other side of the river in his boat. However, the boat has room for only the peasant himself and one other item (either the wolf, the goat, or the cabbage). In his absence, the wolf would eat the goat, and the goat would eat the cabbage. Solve this problem for the peasant or prove it has no solution. (Note: The peasant is a vegetarian

but does not like cabbage and hence can eat neither the goat nor the cabbage to help him solve the problem. And it goes without saying that the wolf is a protected species.)

Question 06: (10 Marks)

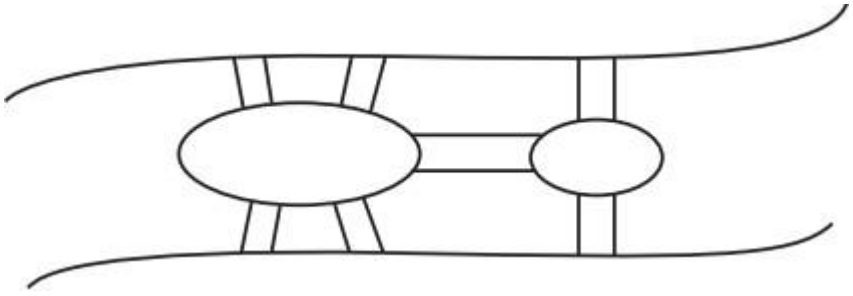
There are four people who want to cross a rickety bridge; they all begin on the same side. You have 17 minutes to get them all across to the other side. It is night, and they have one flashlight. A maximum of two people can cross the bridge at one time. Any party that crosses, either one or two people, must have the flashlight with them. The flashlight must be walked back and forth; it cannot be thrown, for example. Person 1 takes 1 minute to cross the bridge, person 2 takes 2 minutes, person 3 takes 5 minutes, and person 4 takes 10 minutes. A pair must walk together at the rate of the slower person's pace.

Question 07: (10 Marks)

Write pseudocode for an algorithm for finding real roots of equation ax2 + bx + c = 0 for arbitrary real coefficients a, b, and c. (You may assume the availability of the square root function sqrt (x).)

Question 08: (10 Marks)

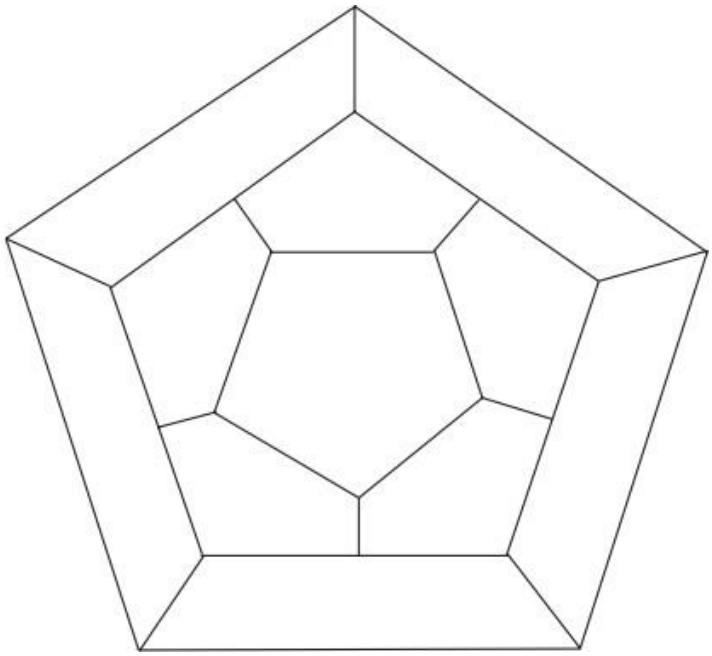
The Konigsberg bridge puzzle is universally accepted ¨ as the problem that gave birth to graph theory. It was solved by the great Swiss-born mathematician Leonhard Euler (1707–1783). The problem asked whether one could, in a single stroll, cross all seven bridges of the city of Konigsberg exactly once and return to a starting point. Following is a sketch ¨ of the river with its two islands and seven bridges



1. State the problem as a graph problem.
2. Does this problem have a solution? If you believe it does, draw such a stroll; if you believe it does not, explain why and indicate the smallest number of new bridges that would be required to make such a stroll possible.

Question 09: (10 Marks)

A century after Euler's discovery (see Problem 8), another famous puzzle—this one invented by the renowned Irish mathematician Sir William Hamilton (1805–1865)—was presented to the world under the name of the Icosian Game. The game's board was a circular wooden board on which the following graph was carved:



a. Find a Hamiltonian circuit—a path that visits all the graph's vertices exactly once before returning to the starting vertex—for this graph

Question 10: (10 Marks)

Design an algorithm for the following problem: Given a set of n points in the Cartesian plane, determine whether all of them lie on the same circumference.

Question 11: (10 Marks)

Write a program that reads as its inputs the (x, y) coordinates of the endpoints of two-line segments P_1Q_1 and P_2Q_2 and determines whether the segments have a common point.

Question 12: (10 Marks)

Describe how one can implement each of the following operations on an array so that the time it takes does not depend on the array's size n. a. Delete the ith element of an array ($1 \leq i \leq n$). b. Delete the ith element of a sorted array (the remaining array has to stay sorted, of course).

Question 13: (10 Marks)

If you have to solve the searching problem for a list of n numbers, how can you take advantage of the fact that the list is known to be sorted? Give separate answers for

a. Lists represented as arrays.

b. Lists represented as linked lists.

Question 14: (10 Marks)

a. Show the stack after each operation of the following sequence that starts with the empty stack:

push(a), push(b), pop, push(c), push(d), pop

b. Show the queue after each operation of the following sequence that starts with the empty queue:

enqueue(a), enqueue(b), dequeue, enqueue(c), enqueue(d), dequeue

Question 15: (10 Marks)

Let A be the adjacency matrix of an undirected graph. Explain what property of the matrix indicates that i. the graph is complete. ii. the graph has a loop, i.e., an edge connecting a vertex to itself. iii. the graph has an isolated vertex, i.e., a vertex with no edges incident to it.

Question 16: (10 Marks)

Indicate how the ADT priority queue can be implemented as

a. an (unsorted) array.
b. a sorted array.
c. a binary search tree

Question 17: (10 Marks)

Design an algorithm for checking whether two given words are anagrams, i.e., whether one word can be obtained by permuting the letters of the other. For example, the words tea and eat are anagrams.