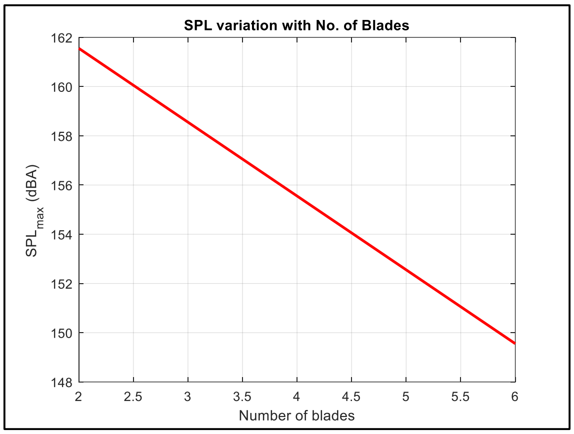
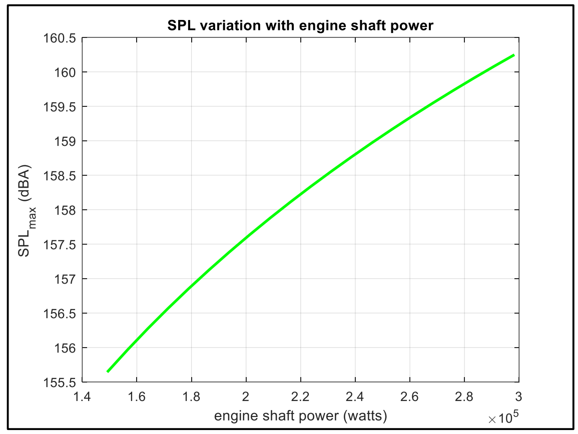
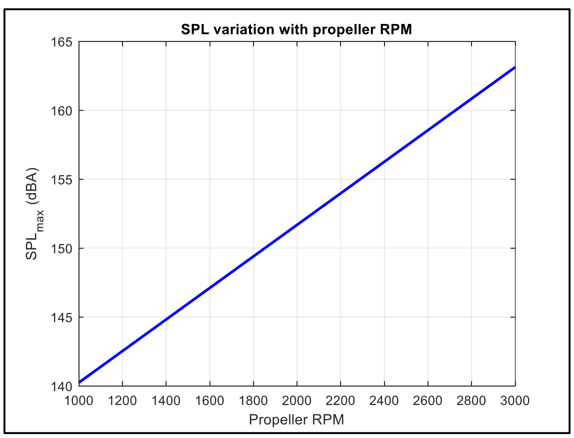
1. Sensitivity Analysis of the Empirical Model

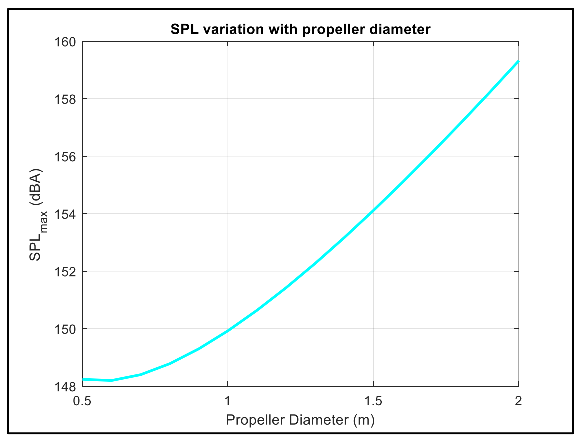
The following is a sensitivity analysis for the empirical model for propeller noise taken from Chapter 12 of the textbook "Elements of Aviation Acoustics" by G.J.J Ruijgrok. The propeller noise equation is given as:

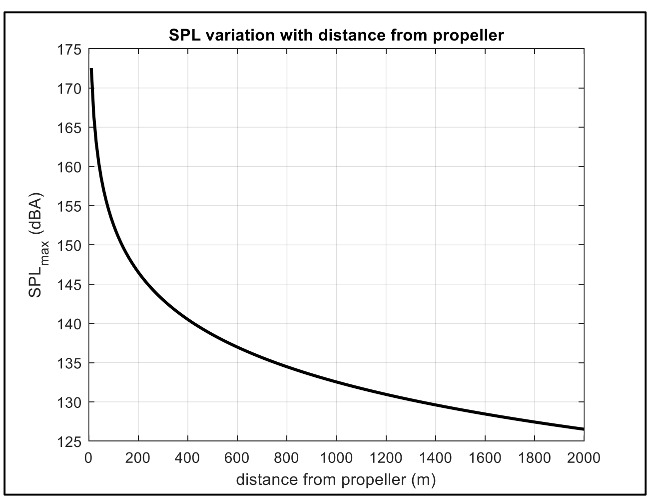
$$SPL_{max}\,(r) = 83.4 + 15.3\log P_{br} - 20\log D + 38.5\,M_t - 3(B-2) + 10\log N - 20\log r$$









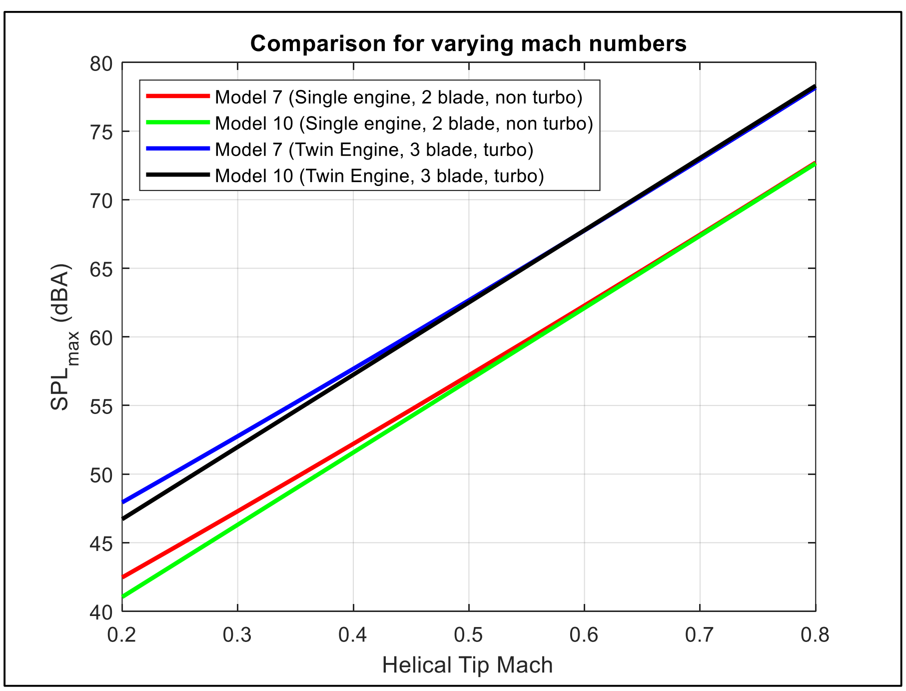


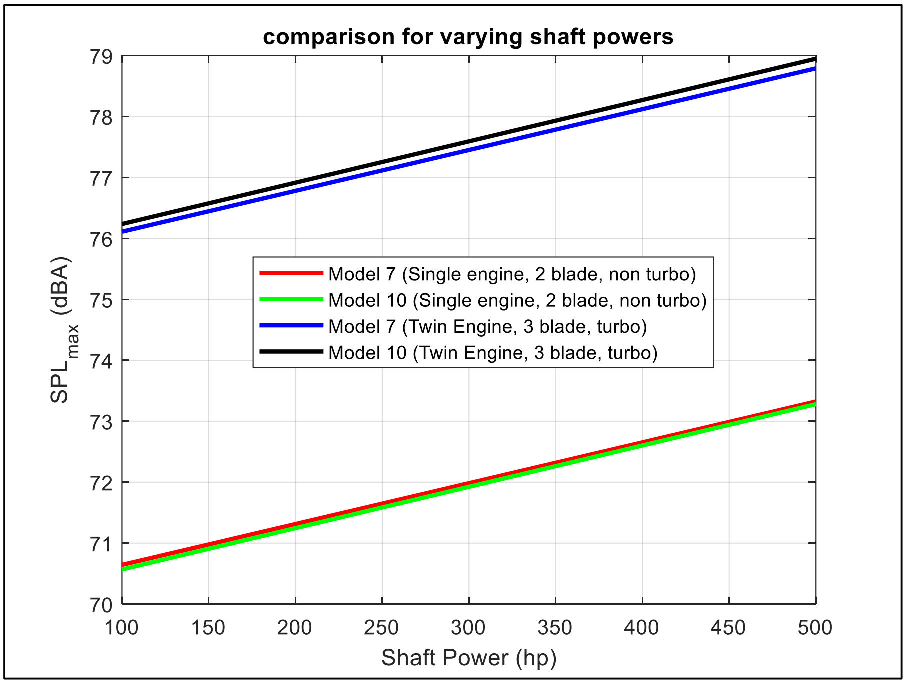
2. Input Identification of Regression Analysis Based Sound Models

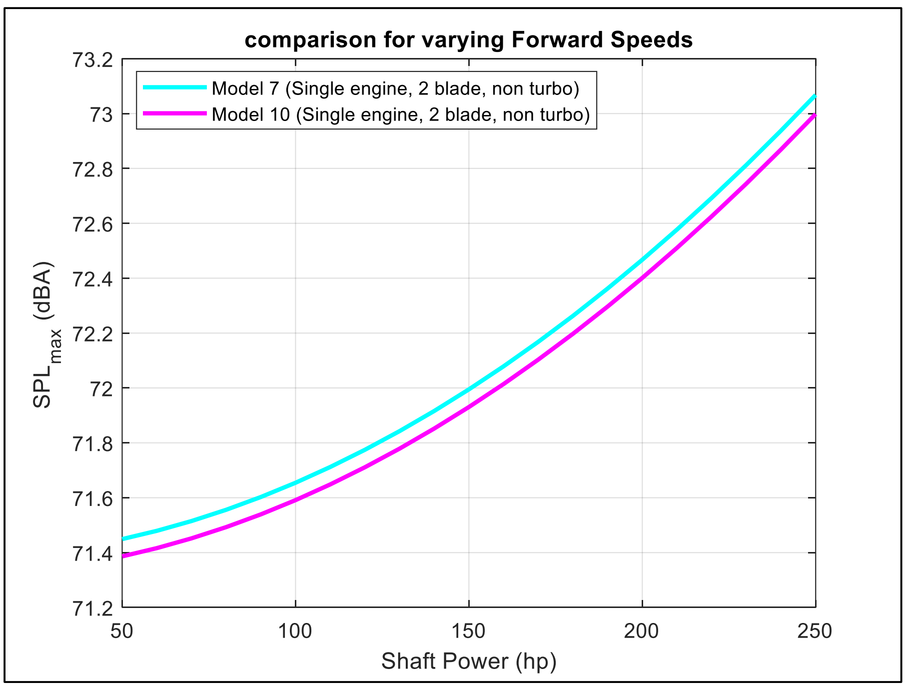
Following table identifies the different inputs to the several flyover sound models delineated in the technical study "The Development of a Flyover Noise Prediction Technique Using Multiple Linear Regression Analysis" by Cessna Aircraft Co.

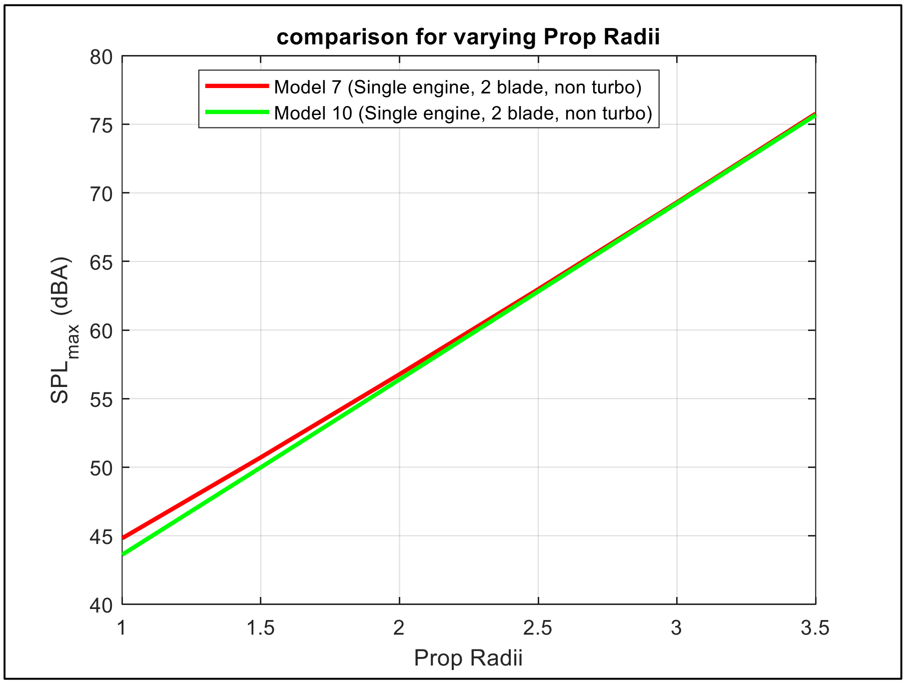
Model Number:		**1**	**2**	**3**	**4**	**5**	**7**	**8**	**9**	**10**
Parameter	**Unit**									
Helical Mach	-	✓	✓	✓	✓	✓	✓	✓	✓	✓
Shaft HP	Hp						✓	✓	✓	✓
Helical Mach Squared	-						✓			
Two or Three Blades ?	-			✓			✓	✓		✓
Turbocharger Enabled?	-						✓	✓		✓
Single or Twin engine?	-						✓			✓

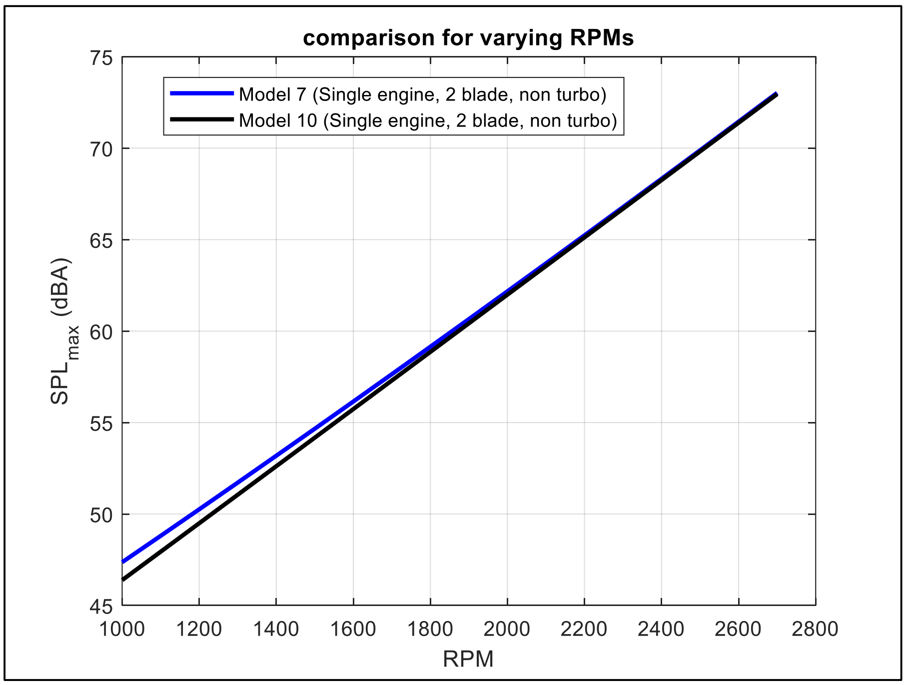
2.1 Comparison between Models 7 and 10




comparison for varying shaft powers
79
78
77
76
75
74
73
72
71
70
SPLmax (dBA)
100
150
200
250
300
350
400
450
500
Shaft Power (hp)
Model 7 (Single engine, 2 blade, non turbo)
Model 10 (Single engine, 2 blade, non turbo)
Model 7 (Twin Engine, 3 blade, turbo)
Model 10 (Twin Engine, 3 blade, turbo)

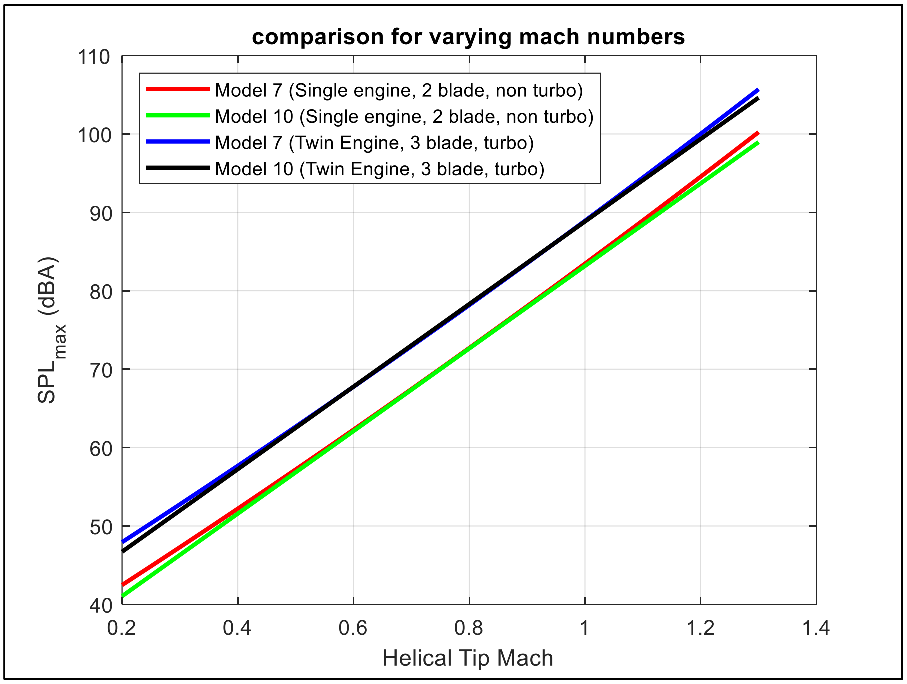

comparison for varying Forward Speeds
73.2
73
72.8
72.6
72.4
72.2
72
71.8
71.6
71.4
71.2
SPLmax (dBA)
50
100
150
200
250
Shaft Power (hp)
Model 7 (Single engine, 2 blade, non turbo)
Model 10 (Single engine, 2 blade, non turbo)

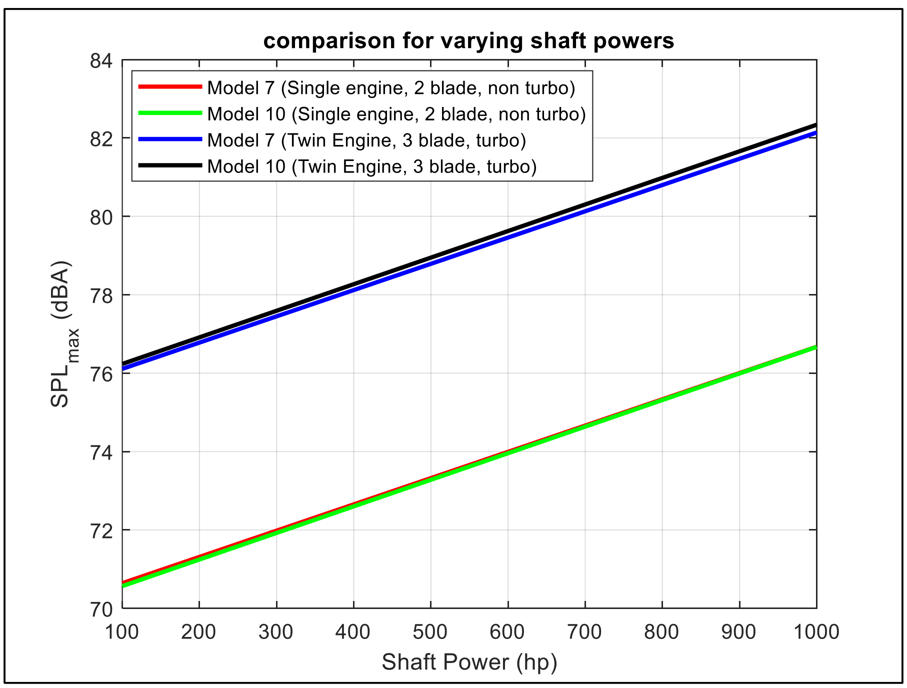




2.2 Extended Limits Comparison Between Models 7 and 10

Following two figures show an extended limits test for the two models applied to the two relevant inputs i.e. Helical Tip Mach and Shaft Horse Power. Increasing both input sets to twice their original sizes has produced the following figures and it clearly shows that not much of an appreciable difference can be spotted even if the input values are increased past their expected values.

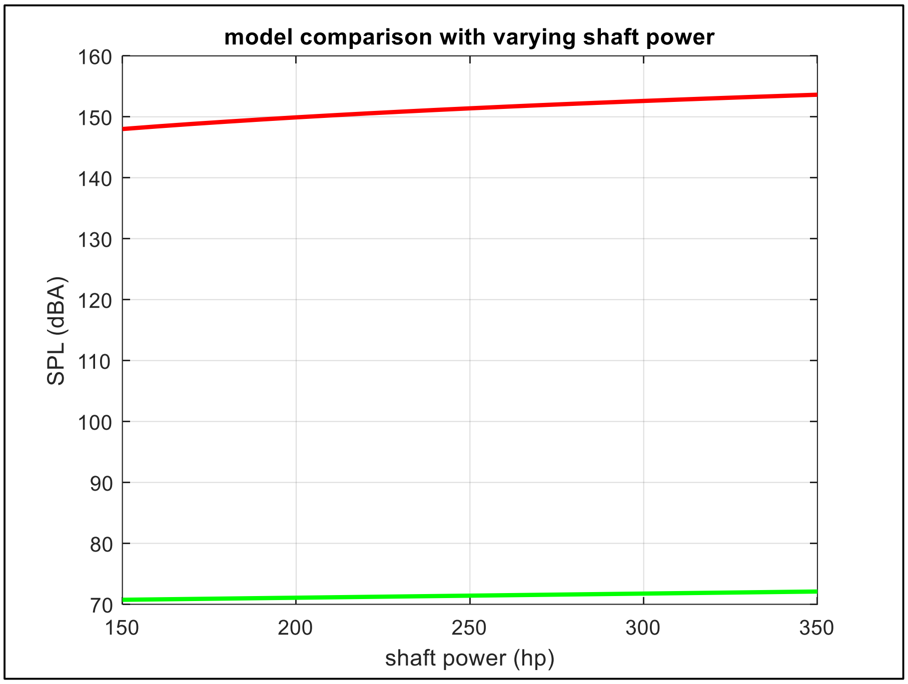


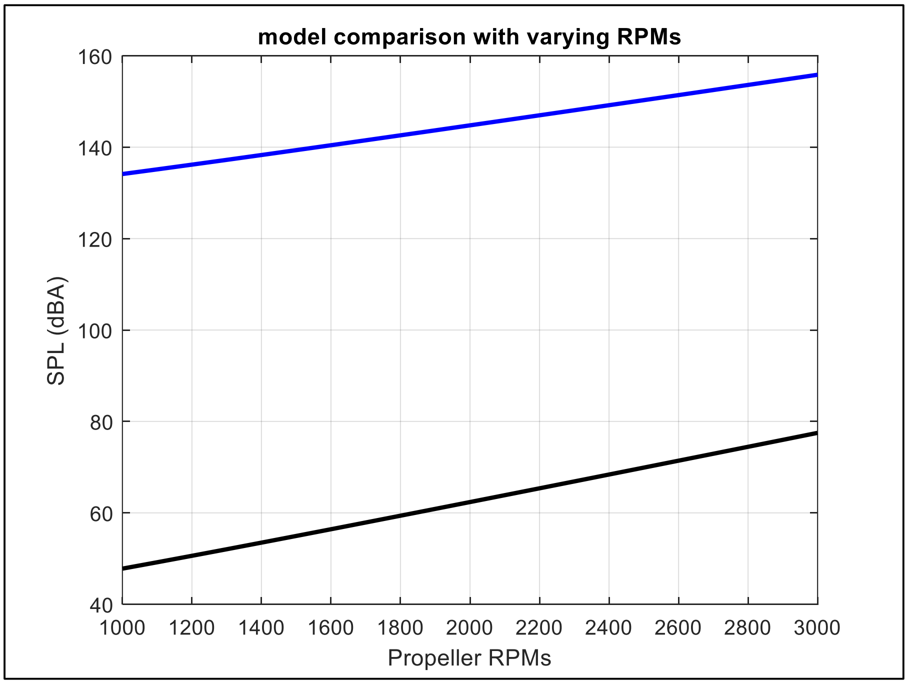


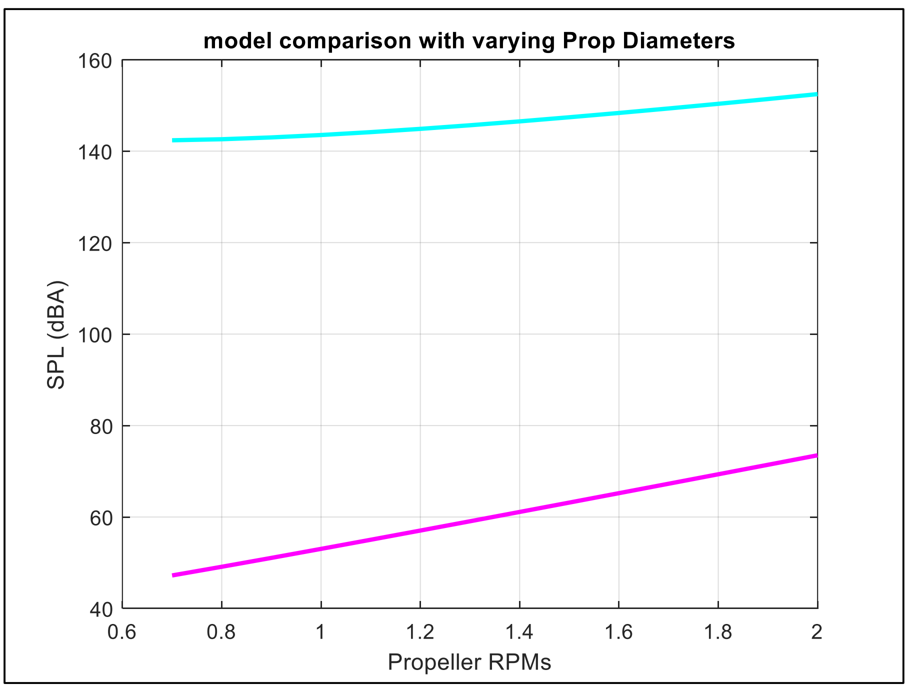
The answer to the question of whether model 7 or model 10 is better, has been answered by the paper itself. Judging from all statistical modelling parameters used to check the validity of all models mentioned in the paper, Model 10 is the best model.

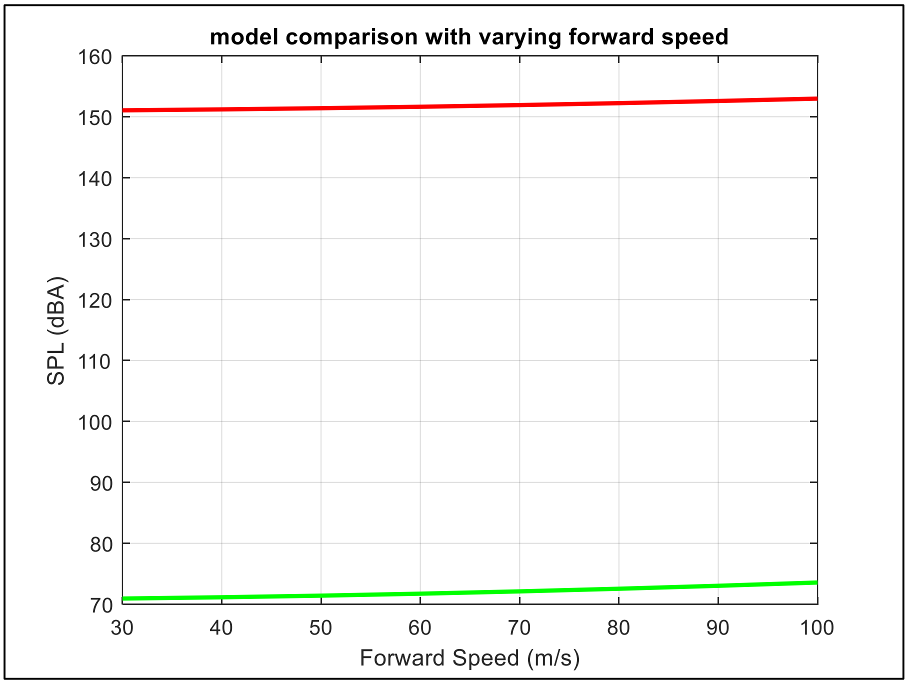
3. Comparison between Empirical Model and Linear Regression Model

The results from the above two models show high degree of mismatch. This is possibly due to difference in output units and a resolution to this potential issue is in the works. The comparison done so far is as follows:




model comparison with varying RPMs
SPL (dBA)
160
140
120
100
80
60
40
1000
1200
1400
1600
1800
2000
2200
2400
2600
2800
3000
Propeller RPMs


model comparison with varying Prop Diameters
SPL (dBA)
160
140
120
100
80
60
40
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Propeller RPMs



4. Codes for both Models

4.1 Empirical Model

```
clc;clear;close all
%For this code, let us use the data from Chapter 14 examples of Gudmundson
%The model has been taken from Ref 18 - "Elements of aviation Acoustics"
%Starting with the piston engine noise:
n = 2600;%engine rotational speed in RPM
N = 4;%number of cylinders
f_c = n/120;%cylinder firing frequency
f_e = (N*n)/120;%exhaust firing frequency
% According to Reference 61, the overall A-weighted level of the exhaust
noise
% of an unmuffled piston engine at 150 m sideline can be estimated by:
P_br = 310*745.7;%engine shaft power, watts
L_A = 8 + (14*log10(P_br)) %dBA
%Using data in Gudmundson - An airplane is powered by a two-bladed propeller
%whose diameter is 76 inches is driven by a 310 BHP engine

%Then moving on to propeller noise:
% For the prediction of far-field propeller noise, the following expres sion
for the
% maximum sound pressure level can be used:
B = 3;%number of blades per propeller
n_p = 2600;%propeller rotational speed (rpm)
```

```
D = 1.9304;%propeller diameter (m)
r = 50;%distance from propeller (m)
c = 340;%speed of sound(m/s)
M_t = (pi*D*n_p)/(60*c);%tip mach number
SPL_max = 83.4 + (15.3*log10(P_br)) - (20*log10(D)) + (38.5*M_t) + ...
          (-3*(B - 2)) + (10*log10(N)) - (20*log10(r))

%Sensitivity Analysis for the propeller model
%First we will vary each input parameter indvidually to check its impact on
%the sound level while keeping the other inputs constant.

%Sensitivity to Number of Blades
figure(1)
B_set = [2, 3, 4, 5, 6];
SPL_max_B = 83.4 + (15.3.*log10(P_br)) - (20.*log10(D)) + (38.5.*M_t) + ...
          (-3.*(B_set - 2)) + (10.*log10(N)) - (20.*log10(r))
plot(B_set,SPL_max_B,'r','LineWidth',2);grid on
xlabel('Number of blades');ylabel('SPL_{max} (dBA)');title('SPL variation
with No. of Blades')

%Sensitivity to Engine Horse Power
figure(2)
P_br_set = [200:10:400].*745.7;%watts
SPL_max_P_br = 83.4 + (15.3.*log10(P_br_set)) - (20.*log10(D)) + (38.5.*M_t)
+ ...
          (-3.*(B - 2)) + (10.*log10(N)) - (20.*log10(r))
plot(P_br_set,SPL_max_P_br,'g','LineWidth',2);grid on
xlabel('engine shaft power (watts)');ylabel('SPL_{max} (dBA)');title('SPL
variation with engine shaft power')

%Sensitivity to Propeller RPM i.e. tip mach
figure(3)
n_p_set = 1000:100:3000
M_t_set = (pi.*D.*n_p_set)./(60.*c);%tip mach number
SPL_max_n_p = 83.4 + (15.3.*log10(P_br)) - (20.*log10(D)) + (38.5.*M_t_set) +
...
          (-3.*(B - 2)) + (10.*log10(N)) - (20.*log10(r))
plot(n_p_set,SPL_max_n_p,'b','LineWidth',2);grid on
xlabel('Propeller RPM');ylabel('SPL_{max} (dBA)');title('SPL variation with
propeller RPM')

%Sensitivity to Propeller Diameter i.e. also affects tip mach
figure(4)
D_set = 0.5:0.1:2;%propeller diameter (m)
M_t_set_2 = (pi.*D_set.*n_p)./(60.*c);%tip mach number
SPL_max_D = 83.4 + (15.3.*log10(P_br)) - (20.*log10(D_set)) +
(38.5.*M_t_set_2) + ...
          (-3.*(B - 2)) + (10.*log10(N)) - (20.*log10(r))
plot(D_set,SPL_max_D,'c','LineWidth',2);grid on
xlabel('Propeller Diameter (m)');ylabel('SPL_{max} (dBA)');title('SPL
variation with propeller diameter')

%Sensitivity to distance From Propeller
figure(5)
r_set = 0:10:100;%distance from propeller (m)
```

```
SPL_max_r = 83.4 + (15.3.*log10(P_br)) - (20.*log10(D)) + (38.5.*M_t) + ...
          (-3.*(B - 2)) + (10.*log10(N)) - (20.*log10(r_set))
plot(r_set,SPL_max_r,'k','LineWidth',2);grid on
xlabel('distance from propeller (m)');ylabel('SPL_{max} (dBA)');title('SPL
variation with distance from propeller')
```

4.2 Regression Models

```
%Following are the regression based models from the paper:
%   "The Development of a Flyover Noise Prediction Technique Using Multiple
%   Linear Regression Analysis"
clc;clear variables;close all
%Defining the State Conditions at different altitudes
%specifying all of the altitudes
alt_ft = 1000:1000:35000;
alt_m = alt_ft * 0.3048;
%Temp in K, Sound Speed in m/s, Pressure in Pa, rho(h) in kg/m3
[T_si,a_si,P_si,rho_si] = atmoscoesa(alt_m);
%converting atmospheric values to english uni`ts
T_eng = 1.8*T_si;%rankine
a = 3.28084*a_si;%ft/s
P = 0.02088547*P_si;%lb/ft^2
rho = 0.00194032*rho_si;%slugs/ft^3

%MODEL # 1 - (Single Engine Aircraft)
X_1 = 0.79;%Helical Mach
Y_1 = 12.7506 + (75.6219*X_1);%dBA

%MODEL # 2 - (Twin Engine Aircraft)
X_2 = 0.78;%Helical Mach
Y_2 = 25.99 + (65.0586*X_2);%dBA

%MODEL # 3 - (Single and Twin Engine)
X_3_1 = 0.78;%Helical Mach
X_3_2 = 0;% 0 for single and 1 for double prop
Y_3 = 13.2314 + (75.0445*X_3_1) + (4.3295*X_3_2);%dBA

%MODEL # 4 - (Single Engine Aircraft)
X_4 = 0.78;%Helical Mach
Y_4 = 86.7697 + (137.8972*X_4);%dBA

%MODEL # 5 - (Single Engine Aircraft)
X_5 = 0.78;%Helical Mach
Y_5 = 60.8837 + (0.00481*X_5);%dBA

%MODEL # 6 - ILLEGIBLE

%MODEL # 8 - (Single Engine Aircraft)
X_8_1 = 250;%BHP
X_8_2 = 0.78;%Helical Mach
X_8_3 = 0;%0 for 2 blade, 1 for 3 blade
X_8_4 = 0;%0 for non-turbo, 1 for turbo
Y_8 = 30.5646 + (0.00942*X_8_1) + (49.9636*X_8_2) + (2.4494*X_8_3) +
(0.4552*X_8_4);
```

```
%MODEL # 9 - (Twin Engine Aircraft)
X_9_1 = 250;%BHP
X_9_2 = 0.78;%Helical Mach
Y_9 = 5.2566 + (0.01428*X_9_1) + (84.2969*X_9_2);

%MODEL # 10 -
X_10_1 = 250;%BHP
X_10_2 = 0.78;%Helical Mach
X_10_3 = 0;%0 for 2 blade, 1 for 3 blade
X_10_4 = 0;%0 for non-turbo, 1 for turbo
X_10_5 = 0;%0 for single engine, 1 for twin engine
Y_10 = 28.8194 + (0.00678*X_10_1) + (52.6543*X_10_2) + (2.8333*X_10_3)...
       + (0.2603*X_10_4) + (2.5742*X_10_5);

%MODEL # 7 - (Sing and Twin Engine Aircraft)
X_7_1 = 250;%BHP
X_7_2 = 0.78;%Helical Mach
X_7_3 = X_7_2^2;%Helical Mach Squared
X_7_4 = 0;%0 for 2 blade, 1 for 3 blade
X_7_5 = 0;%0 for non-turbo, 1 for turbo
X_7_6 = 0;%0 for single engine, 1 for twin engine
Y_7 = 31.3920 + (0.0067*X_7_1) + (46.1576*X_7_2) + (4.2376*X_7_3) + ...
      (2.5981*X_7_4) + (0.2577*X_7_5) + (2.6106*X_7_6);

%COMPARISONS BETWEEN MODELS 7 & 10

%Helical Tip Mmach Variance Comparison:
%Single engine (250 hp), 2 blade, non turbo
figure(1)
M_hel_Set = 0.2:0.05:0.8;
Y_7_Mach_1 = 31.3920 + (0.0067.*250) + (46.1576.*M_hel_Set) +
(4.2376.*(M_hel_Set.^2)) + ...
      (2.5981.*0) + (0.2577.*0) + (2.6106.*0);
Y_10_Mach_1 = 28.8194 + (0.00678*250) + (52.6543*M_hel_Set) + (2.8333*0)...
       + (0.2603*0) + (2.5742*0);
%Twin Engine, 3 blade, turbo
Y_7_Mach_2 = 31.3920 + (0.0067.*250) + (46.1576.*M_hel_Set) +
(4.2376.*(M_hel_Set.^2)) + ...
      (2.5981.*1) + (0.2577.*1) + (2.6106.*1);
Y_10_Mach_2 = 28.8194 + (0.00678*250) + (52.6543*M_hel_Set) + (2.8333*1)...
       + (0.2603*1) + (2.5742*1);
plot(M_hel_Set,Y_7_Mach_1,'r','LineWidth',2);grid on;hold on
plot(M_hel_Set,Y_10_Mach_1,'g','LineWidth',2)
plot(M_hel_Set,Y_7_Mach_2,'b','LineWidth',2);
plot(M_hel_Set,Y_10_Mach_2,'k','LineWidth',2)
xlabel('Helical Tip Mach');ylabel('SPL_{max} (dBA)');title('comparison for
varying mach numbers')
legend('Model 7 (Single engine, 2 blade, non turbo)', 'Model 10 (Single
engine, 2 blade, non turbo)',...
       'Model 7 (Twin Engine, 3 blade, turbo)', 'Model 10 (Twin Engine, 3
blade, turbo)','Location','NorthWest')
xlim([0.2 0.8])

%Shaft Horse Power Variance Comparison:
```

```
%Mach 0.78, 2 blade, non turbo
figure(2)
M_hel_Set = 0.78;
Power_Set = 100:10:500;%hp
Y_7_Power_1 = 31.3920 + (0.0067.*Power_Set) + (46.1576.*M_hel_Set) +
(4.2376.*(M_hel_Set.^2)) + ...
      (2.5981.*0) + (0.2577.*0) + (2.6106.*0);
Y_10_Power_1 = 28.8194 + (0.00678.*Power_Set) + (52.6543.*M_hel_Set) +
(2.8333.*0)...
       + (0.2603.*0) + (2.5742.*0);
%Twin Engine, 3 blade, turbo
Y_7_Power_2 = 31.3920 + (0.0067.*Power_Set) + (46.1576.*M_hel_Set) +
(4.2376.*(M_hel_Set.^2)) + ...
      (2.5981.*1) + (0.2577.*1) + (2.6106.*1);
Y_10_Power_2 = 28.8194 + (0.00678*Power_Set) + (52.6543*M_hel_Set) +
(2.8333*1)...
       + (0.2603*1) + (2.5742*1);
plot(Power_Set,Y_7_Power_1,'r','LineWidth',2);grid on;hold on
plot(Power_Set,Y_10_Power_1,'g','LineWidth',2)
plot(Power_Set,Y_7_Power_2,'b','LineWidth',2);
plot(Power_Set,Y_10_Power_2,'k','LineWidth',2)
xlabel('Shaft Power (hp)');ylabel('SPL_{max} (dBA)');title('comparison for
varying shaft powers')
legend('Model 7 (Single engine, 2 blade, non turbo)', 'Model 10 (Single
engine, 2 blade, non turbo)',...
       'Model 7 (Twin Engine, 3 blade, turbo)', 'Model 10 (Twin Engine, 3
blade, turbo)','Location','NorthWest')

%Setting up comparisons via models 7 and 10 w.r.t Diameter, RPM, and Speed
%Assuming configuration through all comparisons to be -> 250 hp, 2 blade, non
turbo
%forward velocity of 50 ft/s if unvaried, Diameter of 6.33333 ft if unvaried
%Forward Speed:
figure(3)
alt = 1;
R = 6.3333/2; %ft
RPM = 2600;%rpm
RPM_rads = RPM*0.1047198;%rad/s
V_forward = 50:10:250;%ft/s
V_hel_2 = sqrt((V_forward.^2) + ((RPM_rads.*R).^2));
M_hel_Set_2 = V_hel_2./a(alt)
Y_7_Forward = 31.3920 + (0.0067.*250) + (46.1576.*M_hel_Set_2) +
(4.2376.*(M_hel_Set_2.^2)) + ...
      (2.5981.*0) + (0.2577.*0) + (2.6106.*0);
Y_10_Forward = 28.8194 + (0.00678*250) + (52.6543*M_hel_Set_2) +
(2.8333*0)...
       + (0.2603*0) + (2.5742*0);
plot(V_forward,Y_7_Forward,'c','LineWidth',2);grid on;hold on
plot(V_forward,Y_10_Forward,'m','LineWidth',2)
legend('Model 7 (Single engine, 2 blade, non turbo)',...
       'Model 10 (Single engine, 2 blade, non turbo)')
xlabel('Shaft Power (hp)');ylabel('SPL_{max} (dBA)');title('comparison for
varying Forward Speeds')

%Prop Diameter
figure(4)
```

```
R_set = (2:1:7)./2; %ft
RPM = 2600;%rpm
RPM_rads = RPM*0.1047198;%rad/s
V_forward = 50;%ft/s
V_hel_3 = sqrt((V_forward.^2) + ((RPM_rads.*R_set).^2));
M_hel_Set_3 = V_hel_3./a(alt)
Y_7_Rad = 31.3920 + (0.0067.*250) + (46.1576.*M_hel_Set_3) +
(4.2376.*(M_hel_Set_3.^2)) + ...
      (2.5981.*0) + (0.2577.*0) + (2.6106.*0);
Y_10_Rad = 28.8194 + (0.00678*250) + (52.6543*M_hel_Set_3) + (2.8333*0)...
       + (0.2603*0) + (2.5742*0);
plot(R_set,Y_7_Rad,'r','LineWidth',2);grid on;hold on
plot(R_set,Y_10_Rad,'g','LineWidth',2)
legend('Model 7 (Single engine, 2 blade, non turbo)',...
       'Model 10 (Single engine, 2 blade, non turbo)')
xlabel('Prop Radii');ylabel('SPL_{max} (dBA)');title('comparison for varying
Prop Radii')

%Propeller RPM
figure(5)
R_set = 6.33333/2; %ft
RPM_set = 1000:100:2700;%rpm
RPM_rads = RPM_set.*0.1047198;%rad/s
V_forward = 50;%ft/s
V_hel_4 = sqrt((V_forward.^2) + ((RPM_rads.*R_set).^2));
M_hel_Set_4 = V_hel_4./a(alt)
Y_7_RPM = 31.3920 + (0.0067.*250) + (46.1576.*M_hel_Set_4) +
(4.2376.*(M_hel_Set_4.^2)) + ...
      (2.5981.*0) + (0.2577.*0) + (2.6106.*0);
Y_10_RPM = 28.8194 + (0.00678*250) + (52.6543*M_hel_Set_4) + (2.8333*0)...
       + (0.2603*0) + (2.5742*0);
plot(RPM_set,Y_7_RPM,'b','LineWidth',2);grid on;hold on
plot(RPM_set,Y_10_RPM,'k','LineWidth',2)
legend('Model 7 (Single engine, 2 blade, non turbo)',...
       'Model 10 (Single engine, 2 blade, non turbo)')
xlabel('RPM');ylabel('SPL_{max} (dBA)');title('comparison for varying RPMs')
```

4.3 Model Comparison Function

```
function [SPL_max, Y_10] = Comparison_of_models_in_ref_18_and_ref_18_rr_65...
                           (P_shp, D_m, RPMs, V_f_ms)
%Defining the State Conditions at different altitudes
%specifying all of the altitudes
alt_ft = 1000:1000:35000;
alt_m = alt_ft * 0.3048;
%Temp in K, Sound Speed in m/s, Pressure in Pa, rho(h) in kg/m3
[T_si,a_si,P_si,rho_si] = atmoscoesa(alt_m);
%converting atmospheric values to english uni`ts
T_eng = 1.8*T_si;%rankine
a = 3.28084*a_si;%ft/s
P = 0.02088547*P_si;%lb/ft^2
rho = 0.00194032*rho_si;%slugs/ft^3

%Then moving on to propeller noise:
```

```
% For the prediction of far-field propeller noise, the following expres sion
for the
% maximum sound pressure level can be used:
alt = 1;%altitude of flight, 1 = 1000 ft
D = D_m;%propeller diameter (m)
B = 3;%number of blades per propeller
n_p = RPMs;%propeller rotational speed (rpm)
P_br = P_shp*745.7;%engine shaft power, watts
r = 50;%distance from propeller (m)
c = a_si(1);%speed of sound(m/s)
N = 1;%number of propellers
M_t = (pi*D*n_p)/(60*c);%tip mach number
V = V_f_ms;%forward flight airspeed,m/s
M = V/a_si(alt);%forward flight mach number
M_hel = sqrt((M^2)+(M_t^2));%helical tip mach number
SPL_max = 83.4 + (15.3*log10(P_br)) - (20*log10(D)) + (38.5*M_hel) + ...
          (-3*(B - 2)) + (10*log10(N)) - (20*log10(r));
%Setting up comparisons via models 7 and 10 w.r.t Diameter, RPM, and Speed
%Assuming configuration through all comparisons to be -> 250 hp, 2 blade, non
turbo
%forward velocity of 50 ft/s if unvaried, Diameter of 6.33333 ft if unvaried
%Last three input sequence
%0 for 2 blade, 1 for 3 blade
%0 for non-turbo, 1 for turbo
%0 for single engine, 1 for twin engine
R = (D*3.28084)/2; %ft
RPM = n_p;%rpm
RPM_rads = RPM*0.1047198;%rad/s
V_forward = V*3.28084;%ft/s
V_hel = sqrt((V_forward.^2) + ((RPM_rads.*R).^2));%ft/s
M_hel_Set = V_hel./a(alt);
Y_10 = 28.8194 + (0.00678*P_shp) + (52.6543*M_hel_Set) + (2.8333*0)...
       + (0.2603*0) + (2.5742*0);
end
```

4.4 Model Comparison Plotter

```
clc;clear;close all
%The function must be called in a loop and the comparison inputs are Shaft
%power, Prop Diamter, Forward Speed, and RPMs

%RPMs = 2600 if not being varied for comparison
%V_f_ms = 50 m/s if not being varied for comparison
%D_m = 1.9 m if not being varied for comparison
%P_shp = 250 hp if not being varied for comparison
%Comparison wrt Shaft Power
figure(1)
P_shp_set = 150:10:350;%hp
for i = 1:1:length(P_shp_set)
    [SPL_max_power(i), Y_10_power(i)] =
Comparison_of_models_in_ref_18_and_ref_18_rr_65...
                            (P_shp_set(i), 1.9, 2600, 50);
end
plot(P_shp_set,SPL_max_power,'r','LineWidth',2);grid on;hold on
plot(P_shp_set,Y_10_power,'g','LineWidth',2)
```

```
xlabel('shaft power (hp)'),ylabel('SPL (dBA)'),title('model comparison with
varying shaft power')

%Comparison wrt RPMs
figure(2)
RPMs_set = 1000:100:3000;
for i = 1:1:length(RPMs_set)
    [SPL_max_RPMs(i), Y_10_RPMs(i)] =
Comparison_of_models_in_ref_18_and_ref_18_rr_65...
                            (250, 1.9, RPMs_set(i), 50);
end
plot(RPMs_set,SPL_max_RPMs,'b','LineWidth',2);grid on;hold on
plot(RPMs_set,Y_10_RPMs,'k','LineWidth',2)
xlabel('Propeller RPMs'),ylabel('SPL (dBA)'),title('model comparison with
varying RPMs')

%Comparison wrt Diameter
figure(3)
Diameter_set = 0.7:0.1:2.0;
for i = 1:1:length(Diameter_set)
    [SPL_max_Dia(i), Y_10_Dia(i)] =
Comparison_of_models_in_ref_18_and_ref_18_rr_65...
                            (250, Diameter_set(i), 2600, 50);
end
plot(Diameter_set,SPL_max_Dia,'c','LineWidth',2);grid on;hold on
plot(Diameter_set,Y_10_Dia,'m','LineWidth',2)
xlabel('Propeller RPMs'),ylabel('SPL (dBA)'),title('model comparison with
varying Prop Diameters')

%Comparison wrt Diameter
figure(4)
V_f_set = 30:10:100;
for i = 1:1:length(V_f_set)
    [SPL_max_V_f(i), Y_10_V_f(i)] =
Comparison_of_models_in_ref_18_and_ref_18_rr_65...
                            (250, 1.9, 2600, V_f_set(i));
end
plot(V_f_set,SPL_max_V_f,'r','LineWidth',2);grid on;hold on
plot(V_f_set,Y_10_V_f,'g','LineWidth',2)
xlabel('Forward Speed (m/s)'),ylabel('SPL (dBA)'),title('model comparison
with varying forward speed')
```