Number Systems [Decimal] → All base are power of 10

→ Each Digit [0, 1, 2, .. 9]

$734 = 7*100 + 3*10 + 4*1$

$6594 = 6*10^3 + 5*10^2 + 9*10^1 + 4*10^0$

4 3 2 1

$10^3\ 10^2\ 10^1\ 10^0$

n

5 digit $= 10^4$ ✓ N digit $= 10^{N-1}$

Number Systems

1) Decimal
2) Octal → All base value will be power of 8
3) Ternary
4) Binary

$(0132)_8 = 0*8^3 + 1*8^2 + 3*8^1 + 2*8^0$

$8^3\ 8^2\ 8^1\ 8^0$

$= 64 + 24 + 2 = 90$

$(0125)_8 = 1*8^2 + 2*8^1 + 5*8^0$

$8^3\ 8^2\ 8^1\ 8^0$

$= 64 + 16 + 5 = 85$

In Octal Representation: [Each Digit 0-7]

$(10100)_8$ ✓ $(38745)_8$ X

Ternary : base 3

$$(0\ 2\ 1\ 0\ 1) = 2*3^3 + 1*3^2 + 0*3^1 + 1*3^0$$

$$3^4\ \ 3^3\ \ 3^2\ \ 3^1\ \ 3^0$$

$$= 54 + 9 + 1 = 64$$

In Ternary Each digit : [0 - 2]

Binary : 2 Each Digit : [0, 1]

$$(1\ 0\ 1\ 1\ 0) \Rightarrow 16 + 4 + 2 = 22$$

$$2^4\ \ 2^3\ \ 2^2\ \ 2^1\ \ 2^0$$

Decimal → Binary

$2\,|\,28 - 0$
$2\,|\,14 - 0$
$2\,|\,7 - 1$
$2\,|\,3 - 1$
$2\,|\,1 - 1$
0

$(1\ 1\ 1\ 0\ 0)_2 = 28$

$2\,|\,35 - 1$
$2\,|\,17 - 1$
$2\,|\,8 - 0$
$2\,|\,4 - 0$
$2\,|\,2 - 0$
$2\,|\,1 - 1$
0

Bottom up

$(1\ 0\ 0\ 0\ 1\ 1)_2 = 35$

$2\,|\,19 - 1$
$2\,|\,9 - 1$
$2\,|\,4 - 0$
$2\,|\,2 - 0$
$2\,|\,1 - 1$
0

$(10011)_2$

$2\,|\,25 - 1$
$2\,|\,12 - 0$
$2\,|\,6 - 0$
$2\,|\,3 - 1$
$2\,|\,1 - 1$
0

$(11001)_2$

Add 2 Decimal Numbers

$$
\begin{array}{ccccc}
 & 1 & 1 & 1 & \\
 & 3 & 4 & 5 & 9 \\
0 & 2 & 8 & 4 & 7 \\
\hline
0 & 6 & 3 & 0 & 6
\end{array}
$$

$$
\begin{array}{ccccc}
 & 12/10 & 12/10 & 12/10 & \\
 & 1 & 1 & 1 & \\
 & 2 & 4 & 5 & 8 \\
6/10 \quad 0 & 3 & 7 & 6 & 4 \\
\hline
0 & 6 & 2 & 2 & 2 \\
 & 6\%10 & 12\%10 & 12\%10 & 12\%10
\end{array}
$$

$c = sum/10$

$d = sum\%10$

Add 2 Binary Numbers

$c = sum/2$, $d = sum\%2$

Ex 1:

$$
\begin{array}{ccccc}
 & 1/2 & 3/2 & 2/2 & 1/2 \\
 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 \\
\hline
 & 1\%2 & 3\%2 & 2\%2 & 1\%2 \\
1 & 1 & 1 & 0 & 1
\end{array}
$$

Ex 2:

$$
\begin{array}{cccccc}
 & 2/2 & 2/2 & 3/2 & 2/2 & \\
 & 1 & 1 & 1 & 1 & \\
3/2 & 1 & 1 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 & 1 \\
\hline
 & 3\%2 & 2\%2 & 2\%2 & 3\%2 & 2\%2 \\
(1 & 1 & 0 & 0 & 1 & 0)_2
\end{array}
$$

Ex 1.

$c = 4/2$

$$
\begin{array}{ccccccc}
 & & 1 & 1 & 1 & 1 & \\
 & & 1 & 0 & 1 & 1 & 0 \\
 & 1 & 1 & 0 & 1 & 1 & \\
 & 1 & 0 & 1 & 0 & 1 & \\
\hline
2/2 & 2 & 2\%2 & 4\%2 & & 3\%2 & 2\%2 \\
1 & 0 & 0 & 0 & 1 & 1 & 0
\end{array}
$$

$2^3 \quad 2^2 \quad 2^1 \quad 2^0$

$\rightarrow 32 + 16 + 4 + 2 = 54$

$$
\left.
\begin{array}{r}
22 \\
11 \\
21
\end{array}
\right\} \Rightarrow 54
$$

Naming Conventions

1 → Set
0 → unset

Ex:

6	5	4	3	2	1	0
0	1	0	1	1	0	1

Is 3 bit Set : YES
Is 4 bit Set : No

[1 N] : N Numbers   [0, N-1] = N Numbers

[0 N] : N+1 Numbers   [0, N-2] = N-1 Numbers

[a b] = b-a+1

$2^0 + 2^1 = 3 \ (2^2-1)$

$2^0 + 2^1 + 2^2 = 7 \ (2^3-1)$

$2^0 + 2^1 + 2^2 + 2^3 = 15 \ (2^4-1)$

$2^0 + 2^1 + 2^2 + 2^3 .. 2^{N-1} = 2^N - 1$

GP / Geometric Progression

$2^0 + 2^1 + 2^2 + 2^3 + .. \ 2^{N-1}$

Sum of T terms in GP

$$\frac{(a)(r^t-1)}{r-1}$$

a = 1st term value
r → common ratio

$a=1, \ t=N, \ r=2$

$$\frac{(1)(2^N-1)}{2-1} = 2^N - 1$$

MSB – Most Significant Bit / Left Most Bit

LSB – least Significant Bit / Right Most Bit

8 bit binary Number : 10

				2^4	2^3	2^2	2^1	2^0
	0	0	0	0	1	0	1	0

8 bit binary Number : -10

{1 0 0 0 1 0 1 0}

1	0	0	0	1	0	1	0
-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$1 * (-2^7) + 1 * (2^3) + (1) * 2^1$

$\Rightarrow -128 + 8 + 2 = -118$

Ex2:

8 bit :

1	0	1	0	0	1	0	1
-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$-128 + 32 + 4 + 1 = -91$

8 bit :

0	0	0	1	0	1	0	1
-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$16 + 4 + 1 = 21$

4 bit Number

1	0	0	1
-2^3	2^2	2^1	2^0

$= -7$

6 bit Number

1	0	1	0	1	1
-2^5	2^4	2^3	2^2	2^1	2^0

$-32 + 8 + 2 + 1 = -21$

In N bit Number MSB $= -2^{N-1}$

→ Computer Store Negative Numbers ⇒ YES

8 bit Number [MSB value can dominate all other bits combined]

1	1	1	1	1	1	1	1
-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$2^7 - 1$

1	0	0	0	0	0	0	0
-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$= -128$

4 bit Number

	1	1	1
-2^3	2^2	2^0	2^1

Can we store +10 here = ✗ Max 7

Can we store −10 here = ✗ Min −8

# bits	Min	Max
2	$\begin{array}{cc} 1 & 0 \\ -2^1 & 2^0 \end{array} = -2$	$\begin{array}{cc} 0 & 1 \\ -2^1 & 2^0 \end{array} = 1$
3	$\begin{array}{ccc} 1 & 0 & 0 \\ -2^2 & 2^1 & 2^0 \end{array} = -4$	$\begin{array}{ccc} 0 & 1 & 1 \\ -2^2 & 2^1 & 2^0 \end{array} = 3$
4	$\begin{array}{cccc} 1 & 0 & 0 & 0 \\ -2^3 & 2^2 & 2^1 & 2^0 \end{array} = -8$	$\begin{array}{cccc} 0 & 1 & 1 & 1 \\ -2^3 & 2^2 & 2^1 & 2^0 \end{array} = 7$
5	$\begin{array}{ccccc} \_ & \_ & \_ & \_ & \_ \\ -2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array} = -16$	$\begin{array}{ccccc} 0 & 1 & 1 & 1 & 1 \\ -2^4 & 2^3 & 2^1 & 2^2 & 2^0 \end{array} = 15$

N bit

Min

$$\left.\begin{array}{cccccccccc} 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ -2^{N-1} & & & & & \cdot & 2^2 & 2^1 & 2^0 \end{array}\right\} = -2^{N-1}$$

Max

$$\left.\begin{array}{cccccccccc} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2^{N-1} & 2^{N-2} & 2^{N-3} & \cdot & \_\_ & \cdot & 2^2 & 2^1 & 2^0 \end{array}\right\} = 2^{N-1} - 1$$

$$2^0 + 2^1 + 2^2 + 2^3 + \dots 2^{N-2} = 2^{N-1} - 1$$

$$\left\{ \left[-2^{N-1} \; , \; 2^{N-1} - 1 \right] \right\}$$

byte : 8bit Number $\Rightarrow \{-2^7, 2^7 - 1\} \to \{-128, 127\}$
1 byte

short int : 2 bytes $\to$ 16 bits $\Rightarrow \{-2^{15}, 2^{15} - 1\} \Rightarrow \{-32768, 32767\}$

int : 4 bytes $\Rightarrow$ 32 bits $\to \{-2^{31}, 2^{31} - 1\} \approx [-2 \times 10^9, 2 \times 10^9]$

$[\underbrace{-2,147,483,648}_{\text{INT\_MIN}} \quad \underbrace{2,147,483,647}_{\text{INT\_MAX}}]$

ar[i]
$1 \leq 10^9$

long : 8 bytes $\to$ 64 bits $\to \{-2^{63}, 2^{63} - 1\} \approx [-8 \times 10^{18}, 8 \times 10^{18}]$

Approximation

$$2^{10} = 1024 \approx 1000 = 10^3$$

$$2^{10} \approx 10^3$$

Apply whole 3 on both sides $\to$

$$(2^{10})^3 \approx (10^3)^3 \Rightarrow 2^{30} \approx 10^9 \Rightarrow 2^{31} \approx 2 \times 10^9$$

Mult by 2

$$(2^{10})^6 \approx (10^3)^6 \Rightarrow 2^{60} \approx 10^{18} \Rightarrow 2^{60} * 8 \approx 10^{18} * 8$$

Mult by 8

$$2^{63} \approx 8 \times 10^{18}$$

10:50PM break

Bit wise operators : (&, |, ^, ~, <<, >>) → Tomorrow Session

Truth Table

same : 0 / diff : 1

~a : 1 → 0, 0 → 1

a	b	a&b	a|b	a^b	~a
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
1	1	1	1	0	0

	-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
byte a = 29 :	0	0	0	1	1	1	0	1
byte b = 18 :	0	0	0	1	0	0	1	0

		Decimal
print(a & b) :	0 0 0 1 0 0 0 0 :	16
print(a | b) :	0 0 0 1 1 1 1 1 :	31
print(a ^ b) :	0 0 0 0 1 1 1 1 :	15
print(~a) :	1 1 1 0 0 0 1 0	

$-128 + 64 + 32 + 2 = -128 + 98 = -30$

byte a = 13 : 0 0 0 0 1 1 0 1

byte b = 10 : 0 0 0 0 1 0 1 0

Decimal

print(a & b) : 0 0 0 0 1 0 0 0 : 8

print(a | b) : 0 0 0 0 1 1 1 1 : 15

print(a ^ b) : 0 0 0 0 0 1 1 1 : 7

Properties :

byte a = 10 $\boxed{LSB = 0}$

a = 0 1 0 1 0

1 : 0 0 0 0 1

a&1 : 0 0 0 0 0

```
if(a&1 == 0){
    LSB of a = 0
    a is even
}
```

byte a = 11 odd value 2^0

a = 0 1 0 1 1

1 = 0 0 0 0 1

a&1 : 0 0 0 0 1

```
if( a&1 == 1){
    LSB of a = 1
    a is odd
}
```

// a & 0 = ____ a & a = ____

byte $a = 10$

$$\begin{array}{rccccc} a = & 0 & 1 & 0 & 1 & 0 \\ 1 = & 0 & 0 & 0 & 0 & 1 \\ \hline a|1 = & 0 & 1 & 0 & 1 & 1 \end{array}$$

$\underbrace{01011}_{11}$

$a|1 == (a+1)$

a is even

byte $a = 11$

$$\begin{array}{rccccc} a = & 0 & 1 & 0 & 1 & 1 \\ 1 = & 0 & 0 & 0 & 0 & 1 \\ \hline & 0 & 1 & 0 & 1 & 1 \end{array}$$

$\underbrace{01011}_{11}$

$a|1 == (a)$

a is odd

byte $a = 10$

$$\begin{array}{rccccc} a = & 0 & 1 & 0 & 1 & 0 \\ 1 = & 0 & 0 & 0 & 0 & 1 \\ \hline & 0 & 1 & 0 & 1 & 1 \end{array}$$

$\underbrace{01011}_{11}$

$(a \wedge 1) == a+1$

a is even

byte $a = 11$

$$\begin{array}{rccccc} a = & 0 & 1 & 0 & 1 & 1 \\ 1 = & 0 & 0 & 0 & 0 & 1 \\ \hline & 0 & 1 & 0 & 1 & 0 \end{array}$$

$\underbrace{01010}_{10}$

$(a \wedge 1) == (a-1)$

a is odd

$\Rightarrow a \wedge a = 0$ $\quad a \wedge 0 = a$

$a \,\&\, b = b \,\&\, a$

$a \mid b = b \mid a$

$a \wedge b = b \wedge a$

Commutative property

$(a \,\&\, b \,\&\, c) \rightarrow (a \,\&\, c \,\&\, b)$

$\rightarrow (b \,\&\, c \,\&\, a)$

$(a \,\&\, (b \,\&\, c)) \rightarrow ((b \,\&\, c) \,\&\, a)$

$(a \,\&\, (c \,\&\, b)) = (a \,\&\, c \,\&\, b)$

Associative property

$a \mid b \mid c = a \mid c \mid b = b \mid c \mid a$

$a \wedge b \wedge c = a \wedge c \wedge b = b \wedge c \wedge a$

observations

$a \wedge b \wedge a \Rightarrow a \wedge a \wedge b = 0 \wedge b = b$

$a \wedge c \wedge a \wedge b \wedge b = \underbrace{a \wedge a}_{0} \wedge \underbrace{b \wedge b}_{0} \wedge c = c$

$\cancel{a} \wedge \cancel{c} \wedge \cancel{d} \wedge \cancel{c} \wedge \cancel{d} \wedge c \wedge \cancel{d} = c$

1Q) Given N array element, every element repeats twice except 1 elem find unique element

arr[7] = 4 ^ 7 ^ 6 ^ 4 ^ 8 ^ 7 ^ 6 → ans 8

arr[5] = 1 ^ 4 ^ 4 ^ 3 ^ 1 → ans 3

Sol: XOR of all Elements

// sum of all array elements

```
s = 0
i = 0; i < N; i++){
    s = s + ar[i]
}
```

If we replace + with ^ that becomes xor of all Elements

Note: Bitwise operators are faster when compared to arithmetic operators.

if(a & 1 == 0) — a is even

if(a % 2 == 0) — a is even

Doubts:

& → bitwise operator

&& → logical operator

int → 4B → 32bits → MSB = $\boxed{-2^{31}}$

arr[5] = | 1 | 4 | 4 | 3 | 1 |

ans = 0 | 1 | 1 4 | 1 | 1 3 | 8 |

8bit:

— — — — — — — —

-2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

32 bit = -2^{31}

int