ST303/ST633 Linear Models
Assignment Sheet 2

Due: Fri 17th November, 11:59am (noon).

- Only one, randomly chosen question will be marked.
- Answer all questions. Submit questions 2-5.
- If you are familiar with RMarkdown, you may wish to use it to knit your results to a .pdf file (but this is not strictly necessary).
- If so, place your name and student number under author in the YAML header, e.g.

```
---
title: "Assignment 2"
output: pdf_document
author: John Doe 87654321
---
```

- Either way, your handwritten and/or typed work should be submitted in a single, combined .pdf along with relevant output.
- Make sure you attend the tutorial scheduled in the week ahead of the assignment submission. The tutor will work through Qu 1, answer questions and help students getting started with R (finding/reading in data, knitting an Rmd file etc).

1. An experiment was conducted to assess the relationship between storage temperature of milk and the bacterial count in the milk after a period of time. The results are given in bacteria.csv.

   (a) Fit a simple linear regression model to these data and provide appropriate graphics to assess the fit of the model. Identify the issues with the model fit.

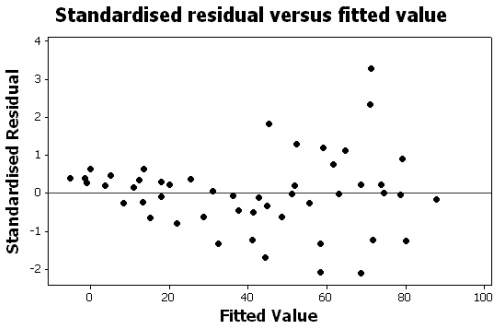
   (b) Try appropriate transformations to the response and / or predictor to find a model that is better fit to the data than the model in (a). Briefly describe each of the models you fit, discuss how well each model fits and indicate which one you deem most appropriate to model the data.

   The code you may need is below:

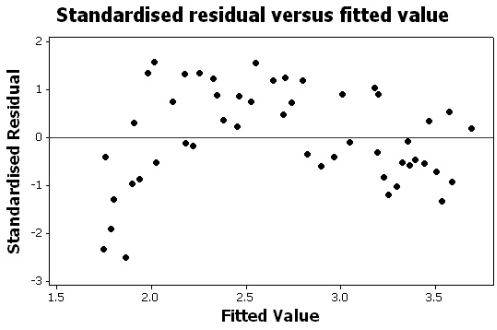
```
library(tidyverse)
bacteria <- read_csv("Bacteria.csv")
fit <- lm(count ~ temp, data = bacteria)
summary(fit)
bacteria |>
  ggplot(aes(x = temp, y = count)) +
  geom_point() +
  geom_smooth(method = "lm")
plot(fit, which = 1)
plot(fit, which = 2)
fit2 <- lm(sqrt(count)~temp, data = bacteria)
fit3 <- lm(log(count)~temp, data = bacteria)
```

2. A simple linear regression model was fitted to four different data sets and the resulting plots of standardised residual versus fitted value are shown. In each case, discuss whether the simple linear regression model is appropriate. In any case where it is not, suggest the most appropriate way to remodel the data giving reason(s) for your answer.

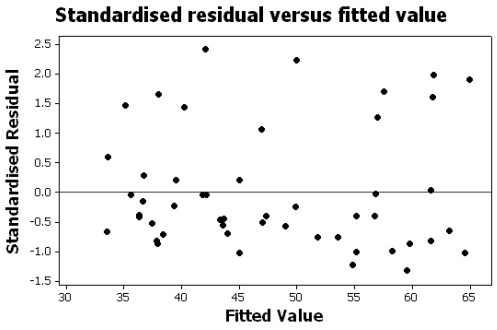
Data set 1



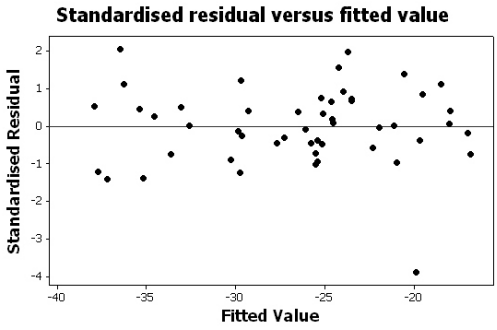
Data set 2



Data set 3



Data set 4



3. Construct some data as follows:
For x generate a column of values $1, 2, ..., 30$.
For y construct a column with values $10 + 2x + \epsilon$, where ϵ contains 30 values randomly generated from the Normal distribution with a mean of 0 and a standard deviation of 4.

   (a) Plot y versus x for the following generated data. Fit a line

   $$y = a + bx$$

   by least squares and draw it on the above plot.

   (b) Fit a line

   $$x = c + dy$$

   by least squares and draw it on the above plot.(Hint: look at ?abline to add a line to base R plot).

   (c) Are the lines in parts (a) and (b) the same? Explain.

4. The data set pollen.txt gives the proportions of pollen removed and visit duration by bumblebee queens and honeybee workers. For the bumblebee queens only (code=1):

   (a) Plot pollen removed versus time spent on flower. Fit the regression of pollen removed on time spent on flower. Plot the residuals versus the fitted values. Does the linear regression model seem appropriate? What problems are evident in the response versus predictor plot? What problems are evident in the residuals versus fitted values plot?

   (b) Do log transformations of Y and / or X help resolve the problems in (a)? (You do not need to include plots, just give a brief text explanation.)

   (c) Try fitting the regression only for those times less than 31 seconds (i.e. excluding the two longest times). Does this fit better?

   The code you may need is below:

```
library(tidyverse)
pollen <- read_table("pollen.txt")
pollen |> head()
pollen <- pollen |> filter(code==1)
pollen_c <- pollen  |>
  filter(duration < 31)
```

5. The data set UN1.txt contains

   - PPgdp: the2001 gross national product per person in US dollars,
   - Fertility: the birth rate per 1000 females in the population in the year 2000.
   - Locality: mostly UN member countries

   (a) Let's say we wish to study the conditional distribution of Fertility given PPgdp. Plot the Fertility (vertical axis) versus PPgdp (horizontal axis) for each locality on a scatterplot. Does a straight-line mean function seem to be a plausible for a summary of this graph?

   (b) Fit the regression of Fertility given PPgdp and plot the residuals versus the fitted values. Does the linear regression model seem appropriate? What problems are evident in the response versus predictor plot? What problems are evident in the residuals versus fitted values plot?

   (c) Do log transformations of Y and / or X help resolve the problems in (b)?

6. Simmulate a dataset given the code below:

```
library(tidyverse)
a <- 1.5
b <- 1
dat <- as_tibble_col(runif(100,0,2), column_name = "x") |>
  mutate(y1 = a * x + b + rnorm(100,0,0.5),
         y2 = a * exp(x) + b + rnorm(100,0,0.5),
         y3 = (a * x + b + rnorm(100,0,0.5))^2,
         y4 = a * x + b + rgamma(100,1.5))
```

   For each of the four responses (y_1, ... y_4) plot the response (vertical axis) versus x (horizontal axis) on a scatterplot. Also for each y_1, ... y_4 fit the regressions on x and make diagnostic plots (residuals vs fitted values and normal qq-plot of the residuals. For each, comment on whether linear regression model seems appropriate and what are the problems you detect from the diagnostic plots. In cases where simple linear regression model is not appropriate, suggest if there is a way to remodel the data.