

# Linear Algebra



**SDAIA**  
الهيئة السعودية للبيانات  
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Saudi Data & AI Authority

# Agenda

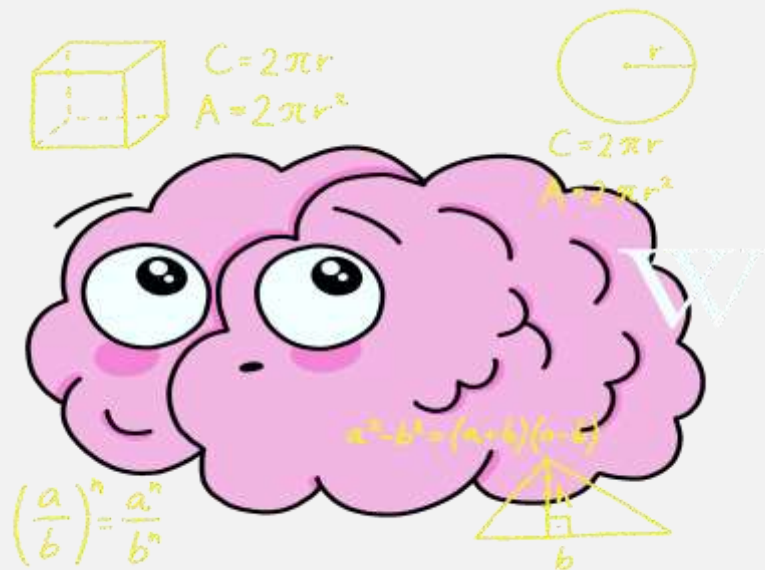
1. Introduction to Linear Algebra
2. What is the role of Linear Algebra in Machine Learning?
3. Key areas of Linear Algebra
4. Scaler, Vector, Matrix and Tensor
5. Matrix and Vector operations
6. Linear Independence of Vectors
7. Eigenvector and Eigenvalues
8. Exercise Time



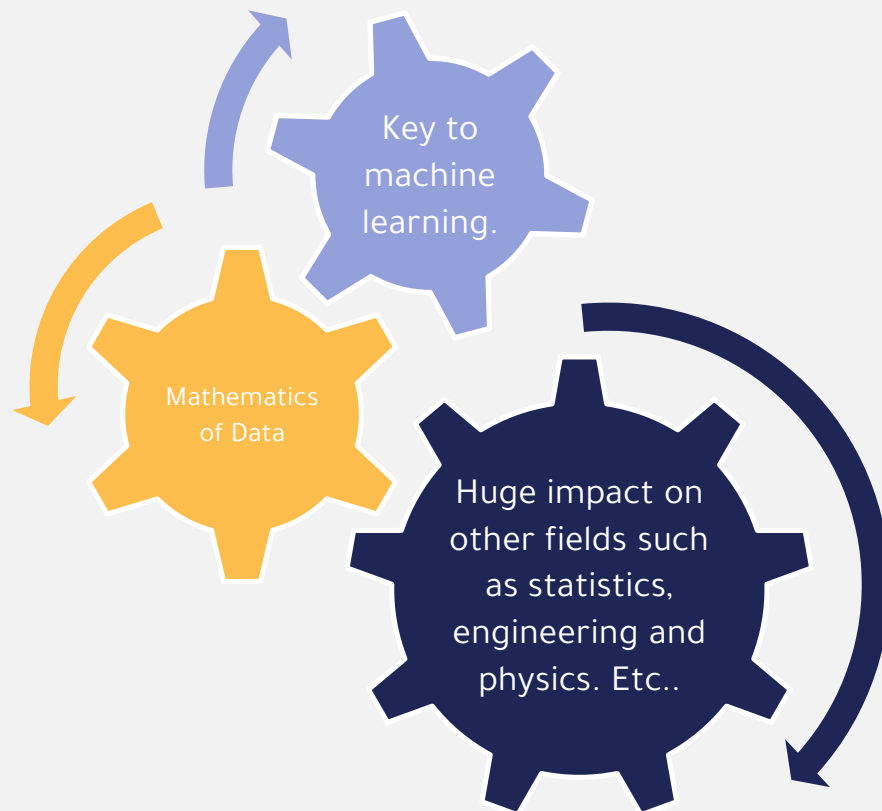
# ► Introduction to Linear Algebra

*Linear algebra is a sub-field of mathematics concerned with vectors, matrices, and operations on these data structures.*

It provides a framework for representing and solving systems of linear equations and studying geometric concepts such as lines, planes, and their higher-dimensional counterparts.



# Introduction to Linear Algebra



## ► What is the role of Linear Algebra in Machine Learning?

**Machines or computers only understand numbers.**

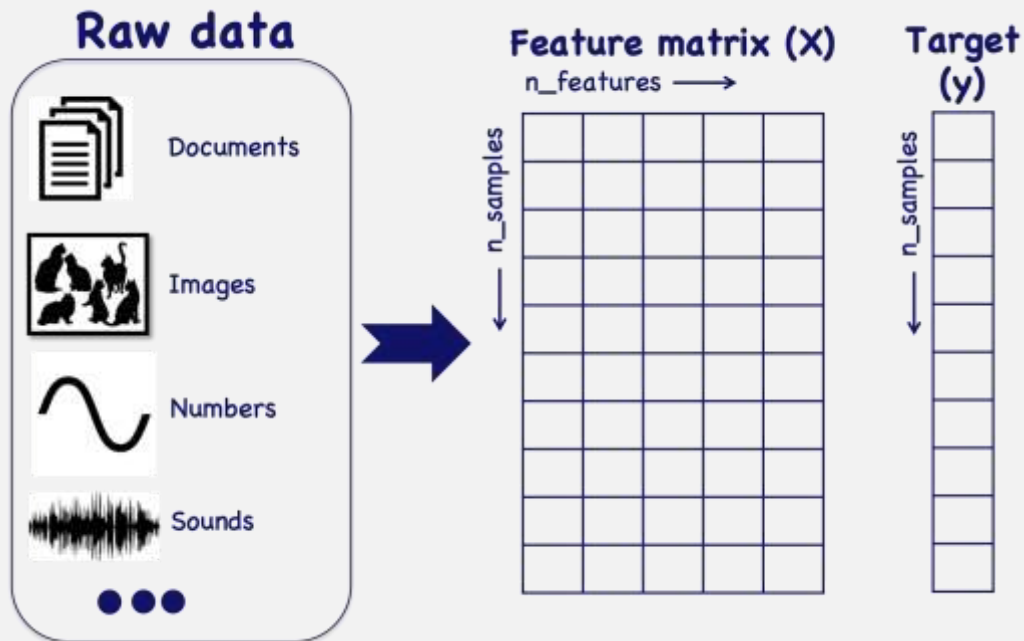
- ❖ Unlike traditional programming, where instructions are predefined, machine learning operates by extracting **patterns** from data.



Machine Learning

# What is the role of Linear Algebra in Machine Learning?

Numerical representation of data facilitates machine understanding and problem-solving.



# ▶ What is the role of Linear Algebra in Machine Learning?

## *REASONS FOR LEARNING LINEAR ALGEBRA BEFORE MACHINE LEARNING*

To gain a better understanding of how the different machine learning algorithms really work under the hood.

*Better Machine  
Learning Algorithms*

*Enhanced graphic  
processing*

*Improving Statistics*



## ► What is the role of Linear Algebra in Machine Learning?

*REASONS FOR LEARNING LINEAR ALGEBRA  
BEFORE MACHINE LEARNING*

Mastery of relevant linear algebra concepts is essential for becoming proficient in Data Science and Machine Learning field.



**What exactly that I need to learn to be a successful Data Scientist?**





# Key areas of Linear Algebra

## A. NOTATIONS

*Notation in linear algebra enables you to read algorithm descriptions in papers, books, and websites to understand the algorithm's working.*

$$\vec{x}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$





# Key areas of Linear Algebra

## *B. OPERATIONS*

Vectors and matrices can make concepts clearer, and it can also help in the description, coding, and even thinking capability.

In linear algebra, it is required to learn the basic operations such as **addition, multiplication**, etc.





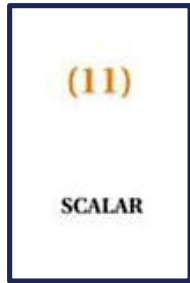
# Key areas of Linear Algebra

## *C. MATRIX FACTORIZATION*

- Matrix factorization is commonly utilized in solving systems of linear equations, image and signal denoising, and reducing the dimensionality of data for analysis and visualization purposes.

# ► Scaler, Vector, Matrix and Tensor

## SCALAR



5 3 7

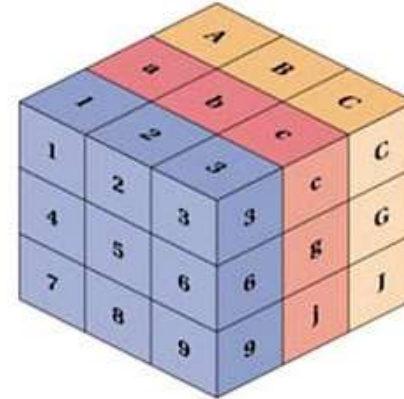
Row Vector  
(shape 1x3)

5  
1.5  
2

Column Vector  
(shape 3x1)

4 19 8  
16 3 5

MATRIX



TENSOR

- A single point or a single data is called a scalar. It cannot be divided further.
- In the figure above 11 is a scalar, which is a single data. Scalars are zero-dimensional.

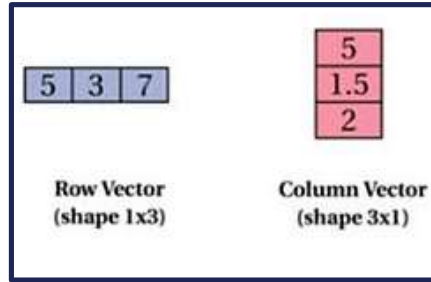


# ► Scaler, Vector, Matrix and Tensor

## VECTOR

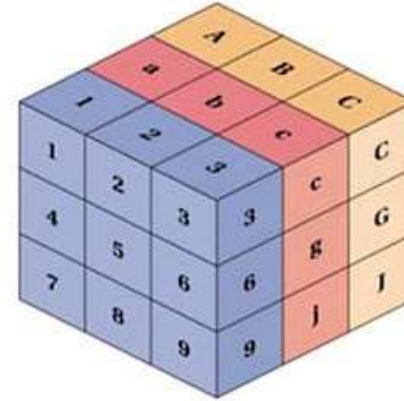
(11)

SCALAR



4	19	8
16	3	5

MATRIX



TENSOR

- If we have multiple scalars, then it is called a vector.
- Vectors are of 2 types. Row vector which is horizontally filled and Column Vector which is vertically filled. Vectors are one-dimensional.



# ► Scaler, Vector, Matrix and Tensor

## *MATRIX*

(11)

SCALAR

5	3	7
---	---	---

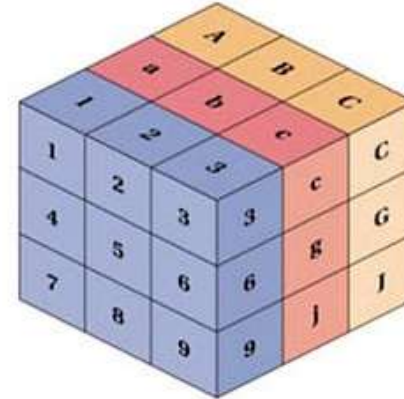
Row Vector  
(shape 1x3)

5
1.5
2

Column Vector  
(shape 3x1)

4	19	8
16	3	5

MATRIX



TENSOR

- A matrix is a group of vectors. It has a vector, filled horizontally and a vector filled vertically.
- Matrices(plural of matrix) are two-dimensional. A table is a common example of matrix.



# ► Scaler, Vector, Matrix and Tensor

## TENSOR

(11)

SCALAR

5	3	7
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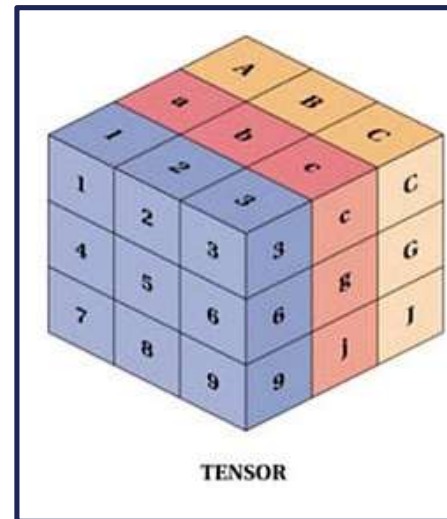
Row Vector  
(shape 1x3)

5
1.5
2

Column Vector  
(shape 3x1)

4	19	8
16	3	5

MATRIX



- A tensor is a group of matrices which is three-dimensional.
- A common example is Rubik's cube which is 3D.



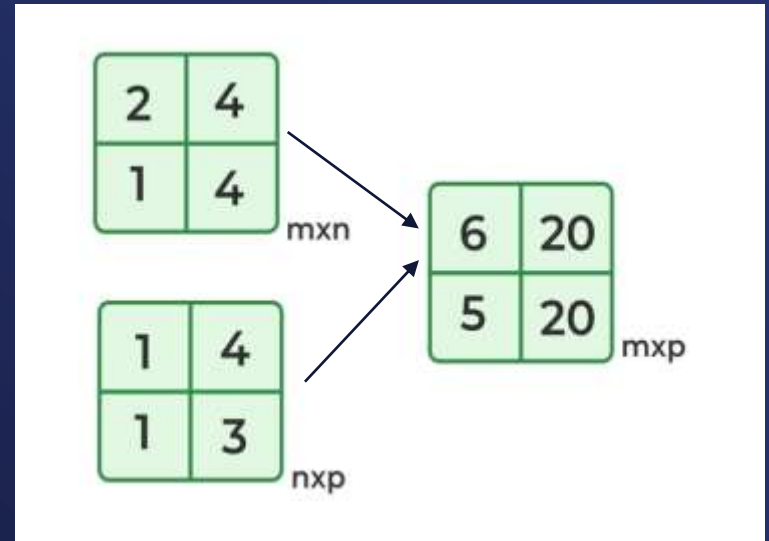
# Matrix and Vector operations

## WHAT ARE MATRIX OPERATIONS?

**Matrix operations are the operations that are used to combine various matrices to form a single matrix.**

The following are some of the important matrix operations.

- Addition - matrix operations
- Subtraction - matrix operations
- Multiplication - matrix operations
- Transpose operation of a matrix
- Inverse operation of a matrix





# ▶ Matrix and Vector operations

## *MATRIX-SCALAR OPERATIONS*

If you multiply a scalar to a matrix, you do so with every element of the matrix.

The image below illustrates this perfectly for multiplication:

$$\begin{array}{ccc} 3 & \cdot & \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} \\ \uparrow & & \uparrow \\ \text{scalar} & & \text{matrix} \end{array}$$

# ▶ Matrix and Vector operations

## *MATRIX-VECTOR MULTIPLICATION*

Multiplying a matrix by a vector can be thought of as multiplying each row of the matrix by the column of the vector.

The output will be a vector that has the same number of rows as the matrix.

$$\begin{array}{ccc} \begin{bmatrix} 1 & 0 & 3 \\ 3 & 1 & 1 \end{bmatrix} & \times & \begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 13 \\ 24 \end{bmatrix} \\ \uparrow & & \uparrow \\ \text{Matrix} & & \text{Vector} \end{array}$$

# Matrix and Vector operations

## *MATRIX-MATRIX ADDITION AND SUBTRACTION*

**The matrices must have the same dimensions and the result is a matrix that has also the same dimensions.**

You add or subtract each value of the first matrix with its corresponding value in the second matrix.

$$\begin{bmatrix} 8 & 5 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 9 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 17 & 10 \\ 3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 5 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 9 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$$

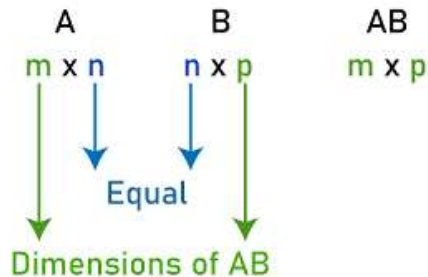
# Matrix and Vector operations

## *MATRIX-MATRIX MULTIPLICATION*

**You can only multiply matrices together if the number of the first matrix's columns matches the number of the second matrix's rows.**

The result will be a matrix with the same number of rows as the first matrix and the same number of columns as the second matrix.

$$\begin{pmatrix} 2 & 3 & 10 \\ 8 & 0 & 9 \end{pmatrix} \begin{pmatrix} -6 & -4 \\ 5 & 1 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 3 & * \\ * & * \end{pmatrix}$$



# ▶ Matrix and Vector operations

## *TRANPOSE OPERATION OF A MATRIX*

The transpose of matrix  $A$  is an operator that flips a matrix over its diagonal. In other words, it switches the row and column indices of a matrix. This operation produces another matrix of order  $n \times m$  denoted by  $A^T$

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2 \times 3} \quad A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}_{3 \times 2}$$

# ▶ Matrix and Vector operations

## *IDENTITY MATRIX*

**The number 1 is an identity because everything you multiply with 1 is equal to itself.** Every matrix that is multiplied by an identity matrix is equal to itself.

$$1 \times 1 \quad [1]$$

$$2 \times 2 \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$3 \times 3 \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

etc.

# ▶ Matrix and Vector operations

## *INVERSE OPERATION OF A MATRIX*

For any matrix **A** its inverse is found only when **A** is a square matrix and its determinant is equal to 1, such that:

$$A \times A^{-1} = A^{-1} \times A = I, \text{ where } I \text{ is the identity matrix.}$$

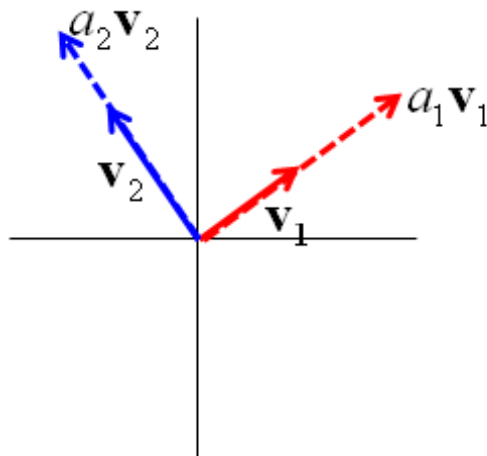
For a 2x2 matrix the inverse is:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$\uparrow$   $A^{-1}$                        $\uparrow$  Determinant

## ► Linear Independence of Vectors

A set of vectors is said to be linearly independent if no vector in the set can be expressed as a linear combination of the other vectors in the same set.





## ► Linear Independence of Vectors

Let  $S$  be a linear space. Some vectors  $x_1, x_2, x_3, \dots \in S$  are said to be linearly independent if and only if they are not linearly dependent.

It follows from this definition that, in the case of linear independence,

$$\alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n = 0$$

Implies

$$\alpha_1 = \dots = \alpha_n = 0$$

In other words, when the vectors are linearly independent, their only linear combination that gives the zero vector as a result has all coefficients equal to zero.



## ► Linear Independence of Vectors

Example Let  $x_1$  and  $x_2$  be  $2 \times 1$  column vectors defined as follows:

$$x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Consider a linear combination of these two vectors with coefficients  $a_1$  and  $a_2$



# ► Linear Independence of Vectors

Example Let  $x_1$  and  $x_2$  be  $2 \times 1$  column vectors defined as follows:

Consider a linear combination of these two vectors with coefficients  $a_1$  and  $a_2$

$$\begin{aligned} & a_1 x_1 + a_2 x_2 \\ &= a_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} a_1 \cdot 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ a_2 \cdot 1 \end{bmatrix} \\ &= \begin{bmatrix} a_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ a_2 \end{bmatrix} \\ &= \begin{bmatrix} a_1 + 0 \\ 0 + a_2 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \end{aligned}$$

Then, we have that  $a_1 x_1 + a_2 x_2 = 0$

if and only if

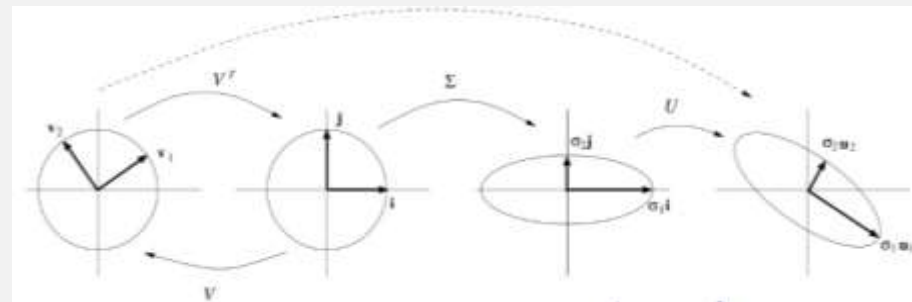
$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



# ► Eigenvalue and Eigenvector

**Eigenvalues are associated with eigenvectors**  
**in Linear algebra.**

The word 'Eigen' is of German Origin which means 'characteristic'. These are the characteristic value that indicates the factor by which eigenvectors are stretched in their direction.





# Eigenvalue and Eigenvector

## **Eigenvalues and Eigenvectors and their use in Machine Learning and AI**

- In machine learning, eigenvalues and eigenvectors are used to represent, perform operations on data, and to train machine learning models.
- They are also used to develop algorithms for tasks such as image recognition, natural language processing, and robotics.



# ► Eigenvalue and Eigenvector

## What is Eigenvectors Formula?

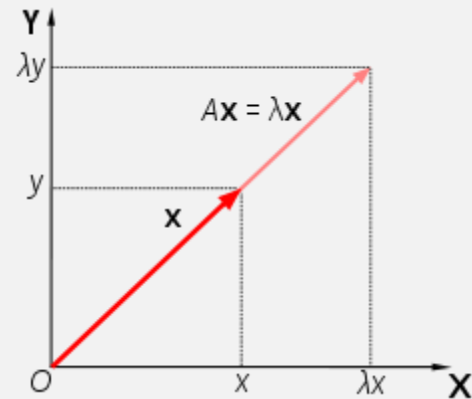
The eigenvector of any matrix is calculated using the formula,

$$A\mathbf{v} = \lambda\mathbf{v}$$

where,

$\lambda$  is the eigenvalue

$\mathbf{v}$  is the eigenvector



# Eigenvalue and Eigenvector

## Examples of EigenValue and EigenVector Applications:

**Principal Component Analysis (PCA)** is a widely used technique to reduce the number of features while retaining as much information as possible.

**Singular Value Decomposition (SVD)** is a technique for image compression commonly used in applications like image storage and transmission.



# Thank You



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