Kalshi Exchange Statistical Arbitrage

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Contents

1 Background

1.1 Introduction to Kalshi

Kalshi is a federally regulated exchange that allows traders to speculate on real-world events through the trading of "event contracts." These contracts are priced according to the collective expectations of the market of an event to occur, providing a unique and innovative platform that blends financial markets with prediction markets. Kalshi's offerings span a wide range of topics, from traditional economic indicators to unconventional events, such as weather conditions and even U.S. presidential election outcomes, making it the first legal market to enable betting on political events. Operating under the oversight of the Commodity Futures Trading Commission (CFTC), Kalshi ensures a robust regulatory framework for traders.

Event contracts are structured similarly to binary options, with a payout of 100 cents if the predicted event occurs and 0 cents otherwise. For example, if the market estimates a 40% chance that the S&P 500 Index (SPX) will close within a specific range—say, \$5250 to \$5500—on a given day, the corresponding contract is priced at 40 cents. Upon expiration, the contract settles at its payoff value based on the event outcome, allowing traders to profit from accurate predictions. This binary nature aligns closely with option contracts, where probabilities and market dynamics drive pricing. The focus of this project is on Kalshi contracts tied to the year-end closing price of the S&P 500, which define payout "buckets" for specific price ranges (e.g., \$6000–\$6199.99).

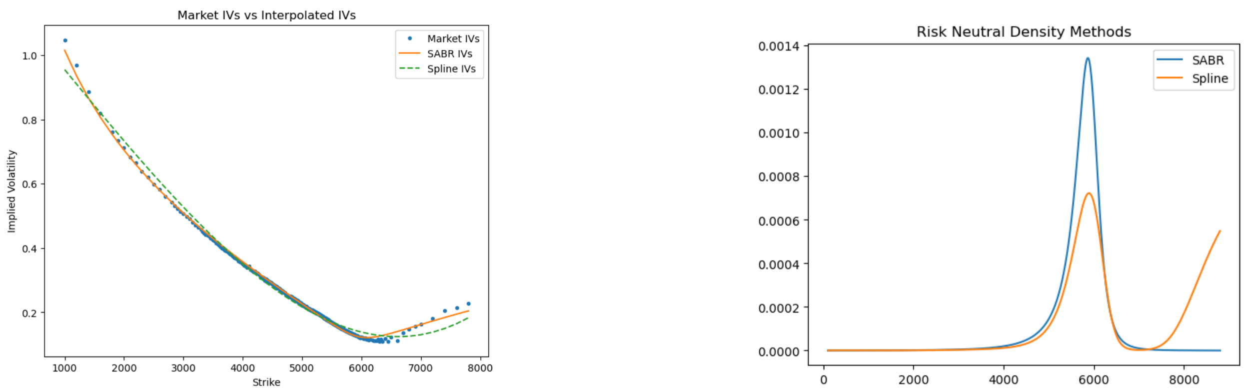
1.2 Strategy Overview and Relevance to FE670

What makes this project novel is its application of advanced algorithmic trading techniques to a nascent and less liquid market. Unlike well-established markets such as the NYSE or NASDAQ, Kalshi's lower trading volumes and wider bid-ask spreads create inefficiencies that can be exploited using statistical arbitrage. By treating the more liquid SPX options market as a benchmark, this study constructs a risk-neutral density (RND) model to identify mispricings in Kalshi's contracts. These inefficiencies are then leveraged using a systematic and algorithmic approach. Additionally, unique features offered by this exchange (such as a 3.95% APY earned on open positions) introduce new mathematical benefits to a portfolio that can be exploited through more research.

The methodologies employed in this project draw directly from concepts learned in algorithmic trading, emphasizing both theory and application. The data in the Kalshi order book is analyzed to understand bid/ask dynamics and optimize trade execution. A statistical arbitrage framework is built to compare SPX options pricing against Kalshi contracts, identifying opportunities to profit from pricing discrepancies. A robust backtesting system is designed to tune these strategies efficiently, accounting for operational factors like Kalshi's flat fee structure, extended trading hours, and low liquidity. These systems incorporate features such as risk management, order sizing, and adaptive execution based on market conditions as shown in the results.

2 Data

To properly calibrate our model and determine mispricings between Kalshi and the Options markets, we collected daily end-of-day option quotes from Refinitiv for SPXW contracts, from the start of 2022 to 2024-11-19. From Kalshi, we queried daily end-of-day bid/ask data from the Kalshi API from the start of 2022 to 2024-12-07. To better parse our data, we found the options that matched Kalshi's expirations, and gathered implied market terms such as discount factor, forward price, implied volatility, etc. utilizing Least-Squares. We remove illiquid quotes to ensure that we would be able to easily exit and enter positions.



(a) Calibration Methods

(b) RND Methods

Figure 1: Pricing Mechanisms

3 Pricing Mechanism

3.1 Theoretical Foundation

To determine a fair-value for the Kalshi prices, a risk-neutral density was derived from the market options prices. To do this, the Breeden-Litzenberger Formula was used to derive the density $f_{S_T}(K)$:

$$f_{S_T}(K) = e^{rT} \frac{\partial^2 C(K,T)}{\partial K^2}$$

where S_T is the stock price at our expiration T, r is the interest rate, and K is the strike price. With this risk-neutral density, we can derive the price P for a contract with lower and upper strikes K_1 and K_2 with:

$$P(K_1, K_2; T) = \int_{K_1}^{K_2} f_{S_T}(x,T)dx$$

3.2 Deriving the Risk-Neutral Density from Market data

Noise within the market data makes it impossible to use the call mid-prices directly. Interpolations on the call price surface as well as the implied volatility surface do not maintain certain market assumptions, such as being free from arbitrage. This causes problems when deriving the Risk Neutral Density as it leads to unexpected behavior in the second derivative, seen in Figure 1. Therefore, a explicit volatility model was required to create a smooth implied volatility surface.

The SABR volatility model was chosen to create a smooth implied volatility surface. This is a stochastic volatility model that models the a forward stock price F and the volatility of the forward, σ with a system of SDEs:

$$dF_t = \sigma_t (F)^\beta dW_t$$

$$d\sigma_t = \alpha dZ_t$$

Here, W_t and Z_t are two correlated Brownian motions with correlation ρ. Luckily, a closed-form solution exists for implied volatility, which we can use to calibrate this model on market data. Given our loss function defined as

$$\mathcal{L}(\alpha, \beta, \rho) = \sum_{i=1}^{N} \left[\sigma_{SABR}(K_i; \alpha, \beta, \rho) - \sigma_{market}(K_i)\right]^2 \ ,$$

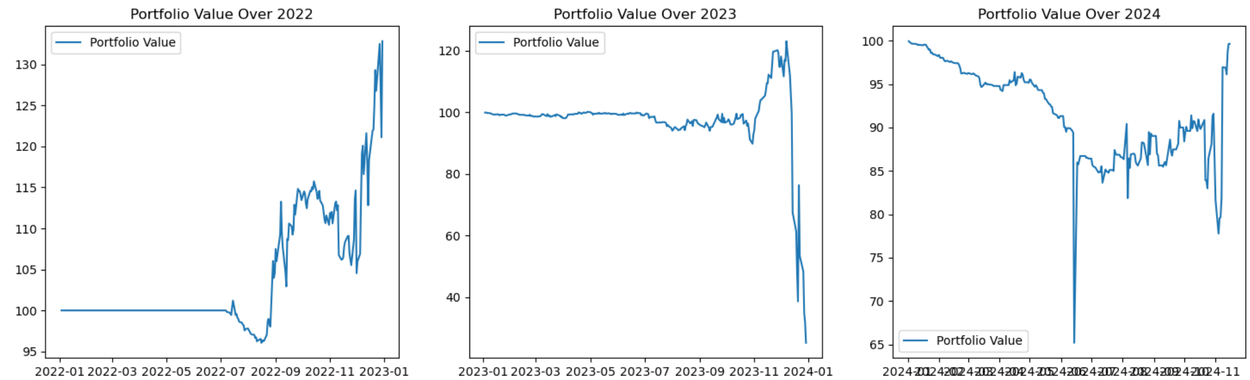


Figure 2: Portfolio Performances

we can minimize this to get the optimal parameters. Finally, we can use the Black-Scholes formula to get a dense and smooth call surface, and differentiate twice with respect to the strike to get our Risk-Neutral Density.

4 Strategy Considerations

5 Results

Given that we are using contracts that expire at the end of each year, we test our strategy for each year separately, incorporating the payoff of our held positions at the end of the year. Each year provides a unique market environment:

- **2022** is characterized by extremely low liquidity and infrequent, as the exchange only started offering this contract mid-way through the year. The lack of attention to the platform during this time theoretically should provide many arbitrage opportunities.
- **2023** is also relatively illiquid, but has quotes for most of the year across many contracts. This acts as our baseline.
- **2024** has the most volume, but more notably, SPX rose outside of the bounds of these 'bucket' contracts, meaning reduced liquidity as we are only trading OTM contracts.

5.1 Performance

We show the performance over each of the three years as seen in Figure 2.

The performance is summarized as such:

	Annualized Return	Annualized Volatility	Sharpe Ratio	Payoff
2022	31.68%	26.40%	1.086	\$209
2023	-80.0%	119.9%	-0.691	\$270
2024	12.2%	50.6%	0.184	

5.2 Strategy Characteristics

We see some incredibly interesting results. First, in 2022 we see an exceptional return, most likely motivated by extremely low liquidity, followed by a large end-of-year payoff from our accumulated positions. Observing the strategy closely, we accumulate many short positions that are underpriced, which end up paying off very well at the end of the year. 2023 is a strange year. For the first few months we seem to not be making many trades, followed by a large spike and drop. The drop being so close to the end of the contracts indicates that there is something wrong in our pricing strategy that is adverse to large moves, as it is not delta-neutral. However, the payoff in 2023 is even larger than 2022, which completely makes up for the portfolio value losses themselves. Finally, 2024 seems to have a large drawdown and subsequent rise. However, without the end-of-year payout we cannot be sure of the fate of our portfolio.

Overall, this strategy exhibits extremely large swings and volatility, and would benefit from more robust risk-management, such as keeping itself delta-neutral. However, this proof-of-concept strategy seems to suggest there is value that can be arbitraged out of Kalshi.

6 Conclusion

This study provides a detailed exploration of statistical arbitrage opportunities between Kalshi event contracts and SPX options, leveraging the unique characteristics of both markets. By constructing a risk-neutral density (RND) from the highly liquid SPX options market, we identified pricing discrepancies that could potentially yield profitable arbitrage opportunities. Our findings underscore the promise of this approach, but also highlight several critical challenges and avenues for improvement.

A primary insight from the study is the impact of liquidity constraints on this emerging market. The low trading volume and wide bid-ask spreads make execution timing crucial, often limiting the scalability of the strategy. Additionally, the single-lot trading constraint, while useful for ensuring order fulfillment, hampers the ability to capitalize on large-scale inefficiencies. Future iterations of the strategy could benefit from incorporating Kalshi's bid/ask size data to tailor order sizes more dynamically.

The flat transaction fee and Kalshi's extended trading hours beyond 4 PM add further complexity. Although these factors introduce operational challenges, they also open unique opportunities for sophisticated strategies, such as executing trades when traditional SPX options markets are closed. Furthermore, Kalshi's 3.9% APY on open contracts and cash presents an underexplored dimension that could enhance returns, particularly in environments where arbitrage spreads are narrow.

Our analysis of the RND construction reveals that while the model effectively identifies inefficiencies, the optimization of free parameters, such as implied volatility and discount factors, remains a key area for refinement. A more robust RND could better capture the underlying market dynamics, increasing the accuracy of predicted mispricings. Testing on a larger data set, including historical data from 2022 onward, would provide deeper insight into the resilience of the strategy under varying market conditions.

Expanding beyond S&P 500 contracts, Kalshi's broader suite of products, including contracts for Bitcoin, Crude Oil, and other assets, represents a fertile ground for further research. These markets often exhibit different volatility patterns and liquidity profiles, potentially allowing for diverse arbitrage strategies. Furthermore, exploring shorter-term contracts, such as daily or weekly expirations, could mitigate some of the liquidity challenges observed with annual contracts.

Future enhancements should also consider the use of Kalshi limit orders, which eliminate transaction fees and allow more precise execution. Such an approach could reduce slippage and improve overall profitability, especially in volatile market conditions. Integrating alternate payoff structures, such as binary contracts with call-like features, could also expand the strategy's applicability.

In conclusion, this study demonstrates the viability of statistical arbitrage between the Kalshi and SPX options markets while addressing the challenges inherent in an emerging exchange. By refining the RND model, accounting for operational nuances, and expanding the scope to other contract types, this strategy has the potential to deliver significant returns. As Kalshi continues to mature, the increasing liquidity and product diversity will further enhance the feasibility of such arbitrage approaches, making this an exciting area for continued exploration and development.