Kalman filter

Monday, March 19, 2018 9:46 AM

State:

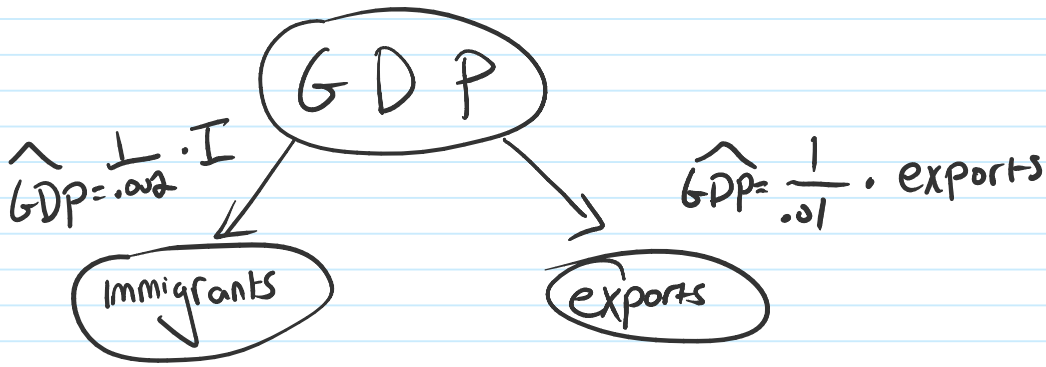
A country's gdb could be measured by their current GDP, and their "velocity" or increase in GDP per year.

Transition Model:

The transition model is simple: the next expected GDP is the current GDP + the velocity. We assume the velocity stays the same.

Sensor Model:

The sensors for our model are **Exports** and **Immigrants**.
The higher the GDP, the more exports and immigrants we expect.
We assume that there will be .002 immigrants per GDP unit (millions of USD) and .01 export per GDP unit.



No Sensor Information

$$x_0 = \begin{bmatrix} 1000 \\ 10 \end{bmatrix} \text{ (given)}$$

$$\hat{x}_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1000 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 1000 + 10 \\ 10 \end{bmatrix}$$

$$\hat{x}_1 = \begin{bmatrix} 1010 \\ 10 \end{bmatrix}$$

Sensor inputs

$$Z = \begin{bmatrix} 3 \\ 12 \end{bmatrix} \begin{matrix} \text{Immigrants} \\ \text{Exports} \end{matrix}$$

$$\hat{Z} = \begin{bmatrix} .002 & 0 \\ .01 & 0 \end{bmatrix} \begin{bmatrix} 1010 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 2.02 \\ 10.1 \end{bmatrix}$$

residual

$$R = Z - \hat{Z}$$

$$= \begin{bmatrix} 0.98 \\ 1.90 \end{bmatrix}$$

X Covariance

$$X\text{-}Cov_0 = \begin{bmatrix} 1 & 0 \\ .1 & .1 \end{bmatrix} \quad \text{Given}$$

$$X\text{-}Cov_1 = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$X\_Cov_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ .1 & .1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Sensor Covariance

$$Sensor\_Cov_1 = \begin{bmatrix} .002, & 0 \\ .01, & 0 \end{bmatrix} \cdot X\_Cov_1 \cdot \begin{bmatrix} .002 & .01 \\ 0 & 0 \end{bmatrix}$$

$$K\_gain = X\_Cov_1 \cdot \begin{bmatrix} .002 & .01 \\ 0 & 0 \end{bmatrix} Sensor\_cov^{-1}$$

$$= \begin{bmatrix} -.0336 & 0.1872 \\ -.0056 & 0.0312 \end{bmatrix}$$

$$\hat{x}_1 = \hat{x}_1 + K\_gain \cdot R$$

$$= \begin{bmatrix} 1010.323 \\ 10.05 \end{bmatrix}$$

So, our prediction would be updated higher by our sensor input.