CME241

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Assignment-1 Solution

1 Defining the MDP

We can write the stochastic differential equation for the wealth process as

$$dW_t = (\pi_t * (\mu - r) + r) * W_t * dt + \sigma * \pi * W_t * dB_t$$

Defining the MDP:

Rewards=

$$0, t \neq T$$

$$-e^{-aW_T}/a, t = T$$

States= t, W_t -¿ $(t+1, W_{t+1})$;
with a discount factor of γ

2 HJB Equation

Since this is a continuous time process, I wish to use the continuous time formulation for the problem:

$$\max_{x_t}(E[dV^*]) = 0$$

Expanding using Ito's Lemma, we get:

$$\max_{x_t}(\frac{\partial V^*}{\partial W_t} * (x_t * (\mu - r) + r * W_t) + \frac{\partial^2 V^*}{\partial W_t^2} * (x_t)^2 * \sigma^2/2) = 0$$

Substituting $V^* = -b_t \exp\{-c_t W_t\}$,

$$\frac{\partial V^*}{\partial W_t} = b_t c_t \exp\{-c_t W_t\}$$

$$\frac{\partial^2 V^*}{\partial W_t^2} = -b_t c_t \exp\{-c_t W_t\}$$

Substitute in main equation:

$$\max_{x_t}(b_t c_t \exp\{-c_t W_t\} * (x_t * (\mu - r) + r * W_t) - b_t c_t^2 \exp\{-c_t W_t\} * (x_t)^2 * \sigma^2/2) = 0$$

At argmax, the partial derivative w.r.t $x_t = 0$. Solving,

$$x_t^* = (\mu - r)/c_t \sigma^2$$

;

Substitute in above and simplify, b_t cancels out and

$$c_t^* = -(\mu - r)^2/2rW_t\sigma^2$$

By comparison of the last recursive equation:

$$-b_T \exp\{-c_T W_T\} = \max_{x_T}(-\exp\{-aW_T\}/a + 0)$$

we get $b_T = 1/a, c_T = a$
The general recursive formula is:

$$-b_{t-1} \exp\{-c_{t-1} W_{t-1}\} = \max_{x_{t-1}}(\gamma(-b_t \exp\{-c_t W_t\}))$$

Solving the above recursion, we get:

$$b_t = \gamma^{T-t}/a$$

Hence, we have found b,c and x.