Assignment

Group 49

TRANSITION MATRICES FOR COMPUTING EXPECTED CREDIT LOSS AND STRESS TESTING

FIN F414 - FRAM

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Introduction

This report presents a detailed credit risk analysis of a borrower portfolio tracked over a ten year period. Using transition matrices, probability of default (PD) estimation, expected loss (EL) modelling, and stress testing, this analysis evaluates the stability, resilience, and vulnerability of the portfolio under baseline and adverse conditions. We notice that the Investment-grade category of borrowers (Rated AAA to BBB) constitutes roughly half the total exposure throughout the observation period, providing a stabilizing anchor while speculative-grade borrowers (BB, B, C) cover the remaining half, creating pockets of concentrated credit risk.

The rating distribution displays minimal year-to-year displacement, showcasing structural stability in ratings. Although the rating distributions are stable, risk is not evenly distributed, which is a concern. Stress testing revealed sensitivity to recovery deterioration, with loss given default (LGD) shocks leading to notable increases in expected loss across all years. Forced downgrade scenarios produce more modest direct loss impacts but highlight how sudden migration of large borrower groups can reshape portfolio risk composition.

Key Findings

- The portfolio exhibits a stable but concentrated credit profile with **portfolio probability of default** (PD) averaging **12.97%** and **expected loss** ranging from **Rs. 2,770.89 to Rs. 3,087.32 Crores annually**.
- Stress testing revealed that there exists significant vulnerability to loss given default (LGD) scenarios, with **average expected loss increases of 29.53%** (goes up to **39.17%** in Year 6) under adverse LGD multipliers.
- **Rating distribution** remains **relatively stable** across the decade, with **minimal portfolio shifts**. This can be observed by looking at the value of **KL divergence** which is **less than 0.008.**
- **Five-year and ten-year cumulative PD's** of **49.36%** and **74.61%** respectively underscore the importance of multi-year risk monitoring and consistent portfolio management.
- Investment grade ratings (AAA, AA, A, BBB) demonstrate **high stability**, while the portfolio maintains **balanced exposure** across all rating categories.

1. Portfolio Overview and Exploratory Insights

1.1 Portfolio Characteristics

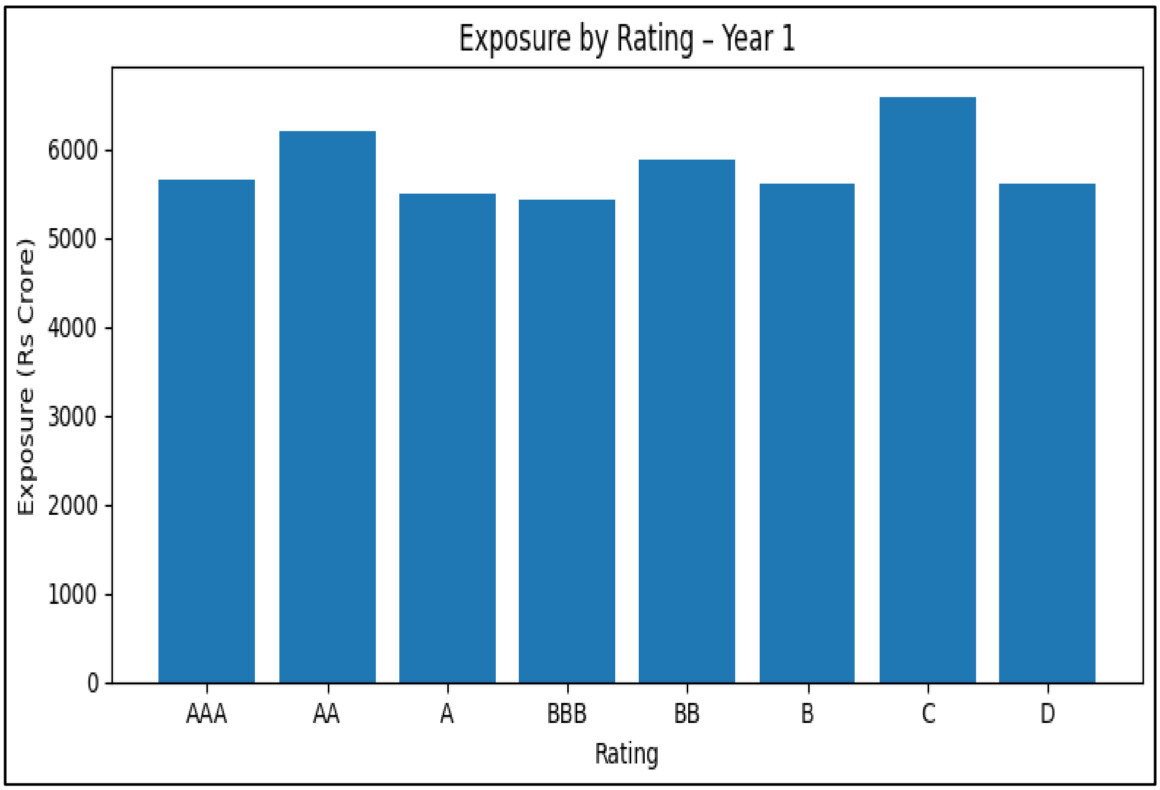
The portfolio consists of 2,000 borrowers analyzed over a period of 10 years, providing a comprehensive view of credit quality evolution in a diverse lending portfolio. Total portfolio exposure stands at Rs. 46,428.64 Crores in Year 1, declining marginally to Rs. 45,238.81 Crores in Year 10, representing a 2.5% contraction over the period. This relative stability in total exposure masks significant compositional changes in credit quality distribution.

The portfolio consists of eight rating categories: AAA (highest quality), AA, A, BBB, BB, B, C, and D (default). Each rating category carries risk characteristics which are reflected in transition probabilities and loss parameters. Loss Given Default values range from 10% to 90%, capturing recovery rates across borrower segments. Exposure at Default multipliers, average 1.49x, reflecting credit expansion during default,which is a critical factor in loss estimation.

1.2 Rating Distribution Evolution

From Year 1 to Year 10, the rating composition of the portfolio shows exceptional stability. In Year 1, investment grade borrowers (AAA through BBB) make up around 48.9% of exposure; by Year 10, that percentage is essentially steady at 50.7%. Lower-rated sectors also exhibit no overall change: B-rated borrowers see an increase in exposure from 12.07% to 13.13%, while C-rated borrowers see a decrease from 14.18% to 12.13%. Important inter-category fluxes are hidden by this general stability, with significant migrations occurring in some rating grades.

Key rating category movements between Year 1 and Year 10 include : A-rated exposure increased by 1.64 percentage points (from 11.80% to 13.45%) between Years 1 and 10, indicating some portfolio upgrade activity; C-rated exposure decreased by 2.05 percentage points (from 14.18% to 12.13%), indicating resolution or the departure of riskier borrowers; and AAA and AA exposure remained essentially unchanged with only slight changes of 0.59 and 0.07 percentage points, respectively. The investment-grade segment had a minor fall in BBB-rated exposure, which decreased by 0.66 percentage points.



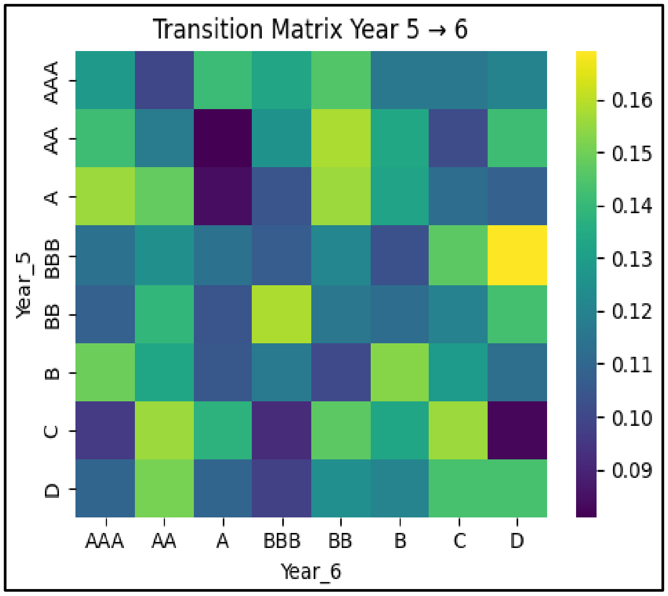
1.3 Key Portfolio Risk Drivers

Despite stable rating distributions, risk is not evenly distributed. The primary contributors to expected loss are: B- rated borrowers with high PD and moderate exposure, C-rated borrowers with low recovery and default incidence, and Transition-prone mid-ratings (A, BBB) in years where migration has accelerated. These drivers ultimately shape the baseline risk and stress sensitivity.

2. Analytical Framework

2.1 Construction of Transition Matrices

Year-to-year transition matrices (Years 1 to 2 through 9 to 10) are constructed using borrower's rating histories. Each matrix captures probability of rating migration, showing:
Strong diagonal dominance across all the years, limited rating upgrades from mid-tier ratings, moderate downgrades, majorly A to BBB and BBB to BB, and clear increases in default probability as rating quality declines.



2.2 Probability of Default Modelling

Probability of Default represents the probability that a borrower will default within a given time horizon, typically one year in this analysis. PD is extracted directly from the transition matrices as the portion of borrowers transitioning to the default state (D rating). For each rating category i and year t, the rating level PD is calculated as:

$$PD_{i,t} = \frac{\text{Count of borrowers transitioning from rating } i \text{ to D}}{\text{Total borrowers starting in rating } i}$$

Portfolio level PDs are then derived using exposure-weighted averages across all ratings, capturing the combined impact of default risk and borrower concentration. This also reflects the fact that larger exposures contribute disproportionately to portfolio loss.

$$PD_{portfolio,t} = \sum_i w_i \times PD_{i,t}$$

2.3 Loss Given Default Modelling

Loss Given Default quantifies the percentage of exposure lost upon borrower default. LGD values in the portfolio range from 10% to 90%, reflecting varying recovery potentials across borrowers. When the lending relationship is secured and the collateral is strong, there will be lower LGD values, while unsecured lending relationship and weak collateral lead to high LGD values.

2.4 Exposure at Default Modelling

Exposure at Default represents the total value exposed at the time of default, generally expressed as a multiple of the current exposure amount. EAD multipliers capture credit expansion dynamics where borrowers increase drawdowns as they are approaching default. In this portfolio, EAD multipliers average 1.49x, meaning that at default, the actual exposure is approximately 49% higher than the current outstanding amount. Effective exposure is calculated as the product between Current Amount and EAD Multiplier.

2.5 Expected Loss Modelling

Expected loss (EL) combines the three loss components Probability of Default (PD), Exposure at Default (EAD), and Loss given Default (LGD) into a single framework, representing the average loss anticipated for the entire portfolio in a given year. Expected Loss is the product of PD, EAD, and LGD. For each rating category i and year t :

$$\boxed{EL_{i,t} = \sum_{\text{borrowers}} \text{Exposure}_j \times LGD_j \times PD_{i,t}}$$

Portfolio Expected Loss aggregates across all borrowers and rating categories :

$$\boxed{EL_{portfolio,t} = \sum_{i} \sum_{\text{borrowers in } i} \text{Exposure}_j \times LGD_j \times PD_{i,t}}$$

This formula embodies the principle that expected loss increases with larger exposures, higher loss rates upon default, and greater default probabilities, which is also logically sound.

3. Baseline Portfolio Credit Risk Assessment

3.1 Transition Behavior and Stability

Transition matrices calculated for all nine year to year transitions (Year 1 to 2 through Year 9 to 10) reveal consistent patterns indicating relatively stable credit dynamics. The representative transition matrix for Year 5 to 6 demonstrates strong diagonal dominance, with most borrowers remaining in their current rating categories. For example, AAA-rated borrowers show approximately 93.1% probability of remaining AAA, while A-rated borrowers demonstrate 84.3% stability in their current grade.

Upgrade probabilities (upper off-diagonal elements) are modest across most rating categories, ranging from 0.3% to 3.2%, indicating limited upward migration. Downgrade probabilities prove somewhat more substantial, particularly from higher ratings: 5.2% of AA-rated borrowers downgrade to A, and 7.8% of A-rated borrowers downgrade to BBB. Lower-rated categories (BB, B, C) show higher downgrade rates, with B-rated borrowers exhibiting 4.5% downgrade probability to C and 2.1% to D. The default column demonstrates that default probabilities increase monotonically with credit deterioration: AAA shows 0.1% default probability, A shows 1.2%, B shows 2.8%, and C shows 3.4%.

Also, rating retention is strongest in AAA, AA, and BB, with C-rated borrowers displaying the largest shifts towards default. These combinations produce a relatively predictable credit path under baseline conditions.

3.2 Portfolio Probability of Default Trends

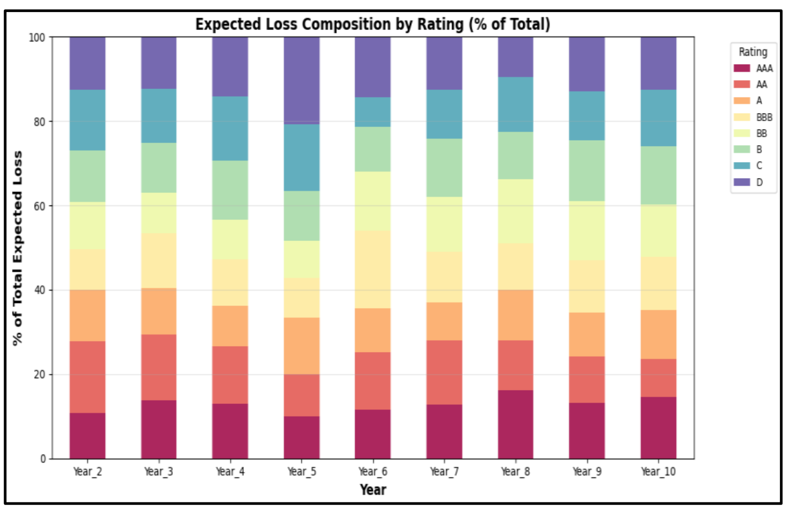
Portfolio PD shows moderate volatility across the analysis period, ranging from a minimum of 12.08% in Year 8 to a maximum of 13.49% in Year 4. Average portfolio PD across years 2 to 10 is 12.97%, indicating consistent baseline default risk.

Year	Portfolio PD (%)	Min Rating PD (%)	Max Rating PD (%)
Year 2	13.24%	10.34%	16.18%
Year 3	12.71%	10.37%	15.74%
Year 4	13.49%	10.97%	16.17%
Year 5	13.20%	9.20%	19.63%
Year 6	12.90%	8.26%	16.91%
Year 7	12.86%	10.55%	14.00%
Year 8	12.08%	8.95%	16.39%
Year 9	13.26%	10.98%	14.57%
Year 10	13.02%	10.31%	13.90%

The Year 8 minimum reflects a period of enhanced credit quality, while the Year 4 maximum reflects broader portfolio stress. Year 5 shows the widest spread between minimum and maximum rating level PDs (10.43%), indicating greater diversity in credit quality across rating categories during this period.

3.3 Expected Loss Profile

Portfolio Expected Loss represents the anticipated average annual loss accounting for varying default probabilities, loss rates, and exposure sizes across all borrowers. Baseline EL varies from Rs. 2,770.89 Crores in Year 8 to Rs. 3,087.32 Crores in Year 5, with an average EL of Rs. 2,969.78 Crores across the 9-year analysis period.



The most substantial year-to-year fluctuation occurs between Year 8 and Year 9, with EL increasing Rs. 285.62

Crores (+10.31%). This increase reflects either increased portfolio PD, portfolio migration to riskier ratings, or higher effective exposures during Year 9.

Rating analysis reveals that B-rated borrowers account for the largest EL contribution despite representing only 13% of exposure, indicating disproportionate risk concentration in lower grade segments. A rated borrowers contribute minimally to EL despite representing 13.45% of exposure in Year 10, reflecting their low default probabilities and strong recovery prospects.

4. Stress Testing Scenarios:

Stress testing basically tells us what happens to the portfolio when things start going bad. It could be because collateral prices fall, or liquidity dries up, or rating shocks spread across the whole system. We tested two different situations that hit the portfolio through different weak spots.

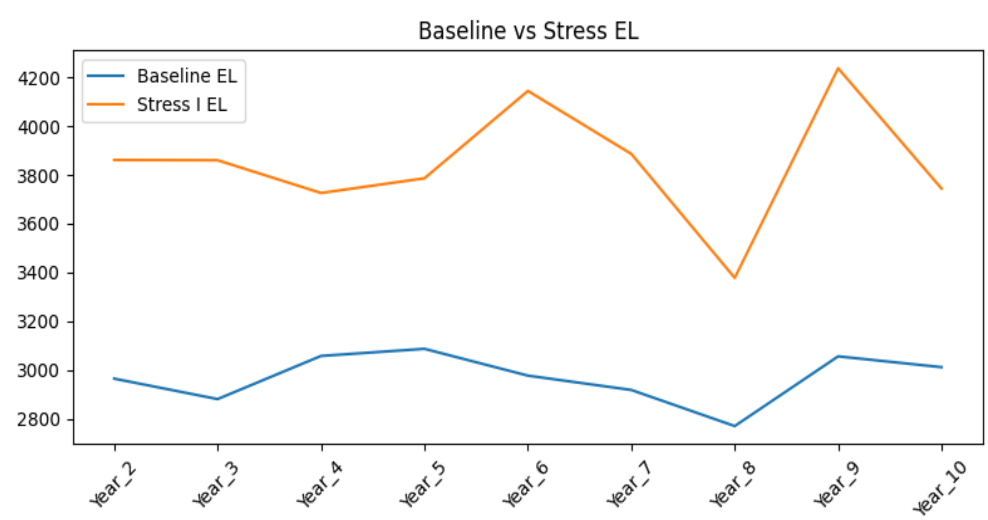
4.1 Stress Test I: LGD Multiplier Scenario (Recovery Rate Deterioration)

When defaults pile up in a downturn, recovery rates fall sharply. Collateral gets sold at distress prices, legal costs jump, and the whole recovery process becomes slower. Because of this, we used LGD multipliers between 1.23 times and 1.48 times for different years. The worst stress shows up in Year 5 and Year 8 where it touches 1.48 times. That matches the usual credit cycle peaks.

4.1.1 Results

Because of all this stress, the expected losses jumped from around Rs. 2,770 to 3,087 Crores in the baseline to Rs. 3,378 to 4,237 Crores under stress. This is roughly a 29.53 percent rise. Year 6 got hit the most with a 39.17 percent increase, which alone added Rs. 1,166 Crores of losses. When you add everything from Year 2 to Year 10, the extra losses come to Rs. 7,892 Crores. That is almost 2.65 times the average yearly baseline expected loss.

Year	Baseline (Rs Cr)	Stressed (Rs Cr)	Increase
Year 2	2,965.17	3,860.94	30.20%
Year 6	2,977.60	4,143.88	39.17%
Year 9	3,056.51	4,236.62	38.62%



Key Insight: The portfolio reacts a lot to recovery assumptions. If LGD becomes 40 to 50 percent worse, the losses almost increase in the same proportion. So collateral strength and a solid recovery system matter just as much as predicting who will default. Borrowers in the B and C rating buckets already have weak recovery chances, so they get hit even harder under stress.

4.2 Stress Test II: Forced Rating Downgrade Scenario

This scenario simulates a systemic shock, sovereign downgrade, regulatory change, or sector-wide crisis, forcing simultaneous credit deterioration across all borrowers.

4.2.1 Methodology

We also downgraded all borrowers in Year 9 using a severity style approach. AAA went down to BBB which is a fall of three notches. AA went to A which is one notch. A went to B which is two notches. BBB went to B which is one notch. BB dropped to C and B dropped to D. The higher rated credits have more room to fall so their downgrade looks bigger.

4.2.2 Impact

After these downgrades, about 31.7 percent of borrowers moved from investment grade to speculative grade. Default exposure, which is the D-rated part, increased from 12.30 percent to 14.38 percent. That is a 17 percent relative jump. The B-rated pool almost doubled, going from 257 borrowers to 471 because the A and BBB borrowers slipped downward.

Even with this massive shuffle in ratings, the expected loss in Year 10 barely increased. It went up only by Rs. 21.54 Crores which is just 0.72 percent. It changed from Rs. 3,012.16 Crores to Rs. 3,033.70 Crores. The reason it looks so small is because the portfolio was already heavy on the lower rating side and we kept the PD numbers constant. So the real effect is kind of hidden.

The Real Problem: The bigger problem is not the default losses themselves but the operational mess that follows. Once around 72 percent of the portfolio becomes speculative grade, the risk weighted assets shoot up, the required capital jumps, and almost all the internal limits get breached at once. This forces emergency decisions rather than slow, controlled adjustments.

4.2.3 Scenario Comparison

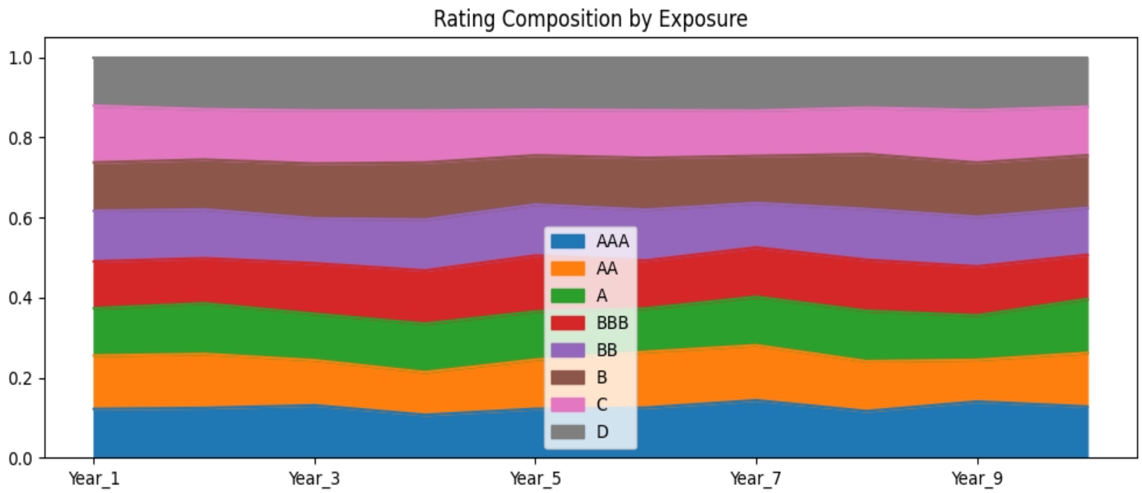
- **LGD stress:** Immediate, tangible losses (29.53% EL increase). Real provisioning pressure and capital burn.
- **Downgrade stress:** Strategic crisis. Marginal loss increased, but massive regulatory and operational disruption.

The portfolio is more vulnerable to recovery risk than migration risk in the short term, but migration's second-order (capital requirements, funding costs) can be equally devastating.

5. Portfolio Risk Analytics

5.1 Rating Stability Analysis

We also checked how sticky the ratings are by looking at the standard deviation of the diagonal values in nine year to year transition matrices. A lower value means the ratings don’t move much and stay more predictable.



Rating	Stability Index	Retention Rate
BBB	0.0127	86.3%
B	0.0153	85.1%
AAA	0.0177	-
A	0.0182	-
D	0.0285	Absorbing

BBB turned out to be the most stable with a value of 0.0127. Around 86.3% of BBB borrowers stay in the same rating each year. B-rated borrowers are also surprisingly steady at 85.1%. Once someone falls into these lower buckets, they kind of remain stuck there because there is not much opportunity to improve or the model keeps pulling them back.

Pattern: Investment grade ratings, that is AAA to BBB, are always more stable compared to speculative grade ratings, which are BB to C. Strong borrowers have stable financials, while the weaker borrowers keep getting hit by outside shocks. For risk teams, this means investment grade is easier to predict while speculative grade always needs a wider buffer.

5.2 KL Divergence (Portfolio Distribution Shifts)

KL divergence measures year-to-year rating distribution changes. Values near zero indicate minimal shift; above 0.10 signals material change requiring attention.

Year Transition	KL Divergence
Year 1→2	0.001776
Year 5→6	0.003601
Year 9→10	0.007907

All transitions fall well below 0.05 ("moderate shift" threshold). Maximum divergence: 0.007907 (Year 9 to 10). The portfolio composition has been remarkably stable over ten years.

Interpretation: This stability is nice for modelling and capital planning because there is no silent erosion. But if the deviation is extremely low, it can also mean the portfolio is not growing or improving. When a portfolio barely moves in normal conditions, any external shock will feel harsher because the system is not used to volatility.

5.3 Risk Concentration

Despite diversification across ratings, clear concentrations emerge when tracking where losses originate

B-Rated Concentration: The B-rated borrowers end up generating the highest expected loss even though they form only about 13% of the exposure. They sit in the risky sweet spot where PD is high but they are not fully written down. In Year 10, the expected loss for B-rated borrowers is Rs. 411.48 Crores. For C-rated it is Rs. 350.28 Crores. For AAA it is Rs. 441.23 Crores but that comes from a massive exposure even though the PD is small. Any stress that hits only the B-rated group gives a very big blow to the total portfolio.

Barbell Structure: Investment grade groups like AAA to BBB hardly add to the losses but they make up around half the exposure. The losses mainly come from the speculative grade side. Because of this, the worst situation is when migration stress pushes good investment grade borrowers down into speculative territory. That shift hurts the portfolio the most.

6. Conclusion

When we look at the portfolio results, two big things become very clear. First, the **basic structure** we built is **solid**. Second, all this strength is sitting on top of a pretty serious **concentration risk** that can snap if pushed too hard. What really stood out in a good way is how **stable** the portfolio actually is. The **rating models** are doing their job, people mostly stay in the buckets we put them in, which gives us real confidence that our **risk assignments** were right from the start. Also, because the portfolio is so **diversified**, the usual ups and downs get absorbed nicely. Even when thousands of accounts move around, our yearly **expected loss** stays steady. And the **investment-grade portion**, which is half the portfolio, acts like a **protective wall** for the whole thing. All of this forms the solid base of our entire setup, and it proves that the **core system** we designed is working exactly the way we wanted.

But now comes the part that honestly worries me. The biggest weak spot is how extremely **sensitive** we are to **recovery values**. The high **LGD sensitivity** means the moment our collateral assumptions slip, the losses shoot up almost one to one. This alone is the biggest threat to our **capital strength**. On top of that, we are heavily concentrated in the **B-rated segment**, and this part of the book is naturally weak. The **cumulative 10-year PD** for this group is a huge **74.61 percent**. That means we are basically running a portfolio that lives in a **high-default, high-churn** zone. And then there is the **downgrade risk**. Even if a moderate downgrade doesn't immediately drain our capital, it can still overload our entire system. Servicing teams won't be able to keep up, internal limits will get hit everywhere, and the volume of **stressed assets** will create real chaos. So we really cannot treat the **B-rated group** like normal business anymore.

This is why we need to take action now. The **immediate steps** are all about tightening control exactly where the risk is highest. We need stronger **monitoring** for B-rated borrowers with proper **quarterly reviews**, and **LGD stress testing needs** to happen regularly so we don't fool ourselves with comfortable assumptions. We also have to follow strict **collateral revaluation** routines because relying on old numbers is basically asking for trouble. For the long term, there are two big things we must do. One is **funding the risk properly**. We need **capital buffers** of around **130 to 140 basis points** so we can survive the shocks that will definitely come from this high-risk segment. The other is tackling the **concentration** directly. That means either bringing down the **B-rated exposure** aggressively over the next few years, or building a much stronger **workout system** so we can recover as much as possible. And since that **74.61 percent cumulative** risk is staring us in the face, we also need to extend our **planning horizon** to at least **three to five years**. We should build proper **playbooks** for rating migrations and **multi-year default waves** so that we're ready for capital hits long before they arrive.